

United Aircraft Research Laboratories



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Study of Trajectories and Upper
Stage Propulsion Requirements for
Exploration of the Solar System

Contract NAS2-2928
Final Report

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Study of Trajectories and Upper Stage Propulsion
Requirements for Exploration of the Solar System

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SUMMARY

The contents of this report are the results of NASA Contract NAS2-2928 which was issued to determine the trajectories and upper-stage propulsion requirements for exploration of the solar system. Many types of high-thrust and low-thrust propulsion systems are analyzed for mission applications, ranging from chemical and advanced nuclear rockets to electric propulsion and radioisotope power. The objective bodies include all the planets, one moon each of Jupiter and Saturn, the asteroid Ceres, the comet Schwassmann-Wachmann II, and the Sun.

The results of the study are reported in two parts; one for high-thrust upper-stage propulsion systems and one for low-thrust missions. The primary parameter used for determining the relative merits of the various propulsion systems is the initial mass required in an Earth parking orbit. This parameter is also used to determine the applicability of existing and hypothetical launch vehicles.

A complete set of scaling laws is derived which defines the inert weights of the propellant tanks and associated structure, engine weights, and the required mass of the manned spacecraft. For high-thrust propulsion, the resulting transportation systems are integrated with the trajectory data from which the energy requirements are derived.

The missions include unmanned flyby and orbital probes, manned orbital stopovers, and manned landing missions. Major input variables include payload, engine specific impulse, engine specific weight, stay time at the planet, and excursion module weight.

All missions start from a 278-km Earth parking orbit. Each major impulse is performed with a separate, disposable stage. Midcourse correction contingencies are included, based on a 430-sec specific impulse chemical system.

No consideration was given to launch window constraints in this study.

All planetary captures are performed by propulsive braking, except for the Earth return which is assumed to be by atmospheric braking. All landing maneuvers are initiated from circular planetary parking orbits. The excursion modules descend and ascend with chemical propulsion and are left in planetary orbit prior to departure. The total stay time varies from zero to 60 days except for Mercury, in which case the stay time is governed by the mission optimization procedure.

For low-thrust systems, variable-thrust, constant-power trajectories are utilized with hyperbolic excess speeds at each terminal. An average thruster efficiency of 0.80 was used for all computations, and was considered to affect the conversion of powerplant output into jet power.

Low-thrust missions include orbital probes to Mercury, Venus, Mars, and Jupiter, and manned missions to Mars. These missions are also analyzed with a mixture of high-thrust and low-thrust propulsion, with the latter being applied heliocentrically. Mixed engines are also used in the high-thrust analysis, where combinations of upper stages are employed.

The data are presented in various forms such that trade-off parameters can be established. Most of the results are based on nominal values of the scaling laws. However, sufficient perturbations of the scaling constants are performed such that combined or individual effects can be determined. Because of the large amount of data and the many variables involved, the reader is cautioned to firmly establish the bases of comparison.

CONCLUSIONS

As with all large parametric studies, there may be definite conclusions of broad scope and other conclusions which may be inferred and substantiated only after a detailed analysis of the data. Since the scope of this study includes mission analyses of the entire solar system with many types of high- and low-thrust propulsion systems, the conclusions will be general, with detailed results to be extracted by the reader.

It should be pointed out that, with the exception of the solid-core nuclear rocket, the performance of all advanced nuclear rocket systems is extremely uncertain at this time. Therefore, the differences between the characteristics of the advanced nuclear rocket systems represent, at this stage, differences in the effect of the different values of specific impulse, thrust-to-weight ratios, and minimum engine size assigned to these engines. For instance, both the light weight advanced nuclear

rocket and one of the gaseous-core nuclear rockets are assumed to have nominal values of specific impulse of 2000 sec. The minimum weight of this light weight advanced nuclear rocket is assumed to be 907 kg, while that of the gaseous-core nuclear rocket is assumed to be 13,600 kg. Therefore, any conclusions about the superiority of one system over the other should be interpreted as a conclusion about the desirability of lower minimum engine weights.

It should also be pointed out that the solid-core nuclear rocket is in an advanced stage of development, while the more advanced nuclear rocket systems are in various research stages. Although the advanced engines provide considerably better performance in many instances than a solid-core nuclear rocket, the only nuclear propulsion which will be available for a number of years will be the solid-core nuclear rocket, and demonstration of the actual performance attainable from the advanced systems will require a number of years of vigorous effort in the research and development area.

Propulsion System Comparisons

An important consideration in the comparison of propulsion systems for all missions is the effect of minimum permissible engine weight; a large value of minimum engine weight can negate the initial-mass advantage normally associated with high specific impulse, especially for small payloads and/or moderate mission velocity requirements.

Unmanned Probe Missions

In order to minimize initial mass in Earth orbit, flyby probes with payloads less than 500 to 800 kg should use chemical Earth-escape stages. Payloads in the 1000 kg to 10,000 kg class would benefit from a light weight advanced nuclear stage. The 3500-sec specific impulse gaseous-core nuclear rocket is desirable should the payload requirements lie between 10,000 and 200,000 kg. Above this payload, the pulsed-nuclear rocket becomes attractive for minimizing the mass requirements.

For orbiter-probe missions, chemical propulsion should be used for Venus and Mars trips with small payloads and minimal-energy trajectories. Missions to the outer planets generally favor the choice of advanced nuclear systems.

The use of a Jupiter swingby for flyby probe missions to the outer planets can reduce trip times by 50% for given values of initial mass. However, these swingbys are of no value for orbital stopovers for the time period and trip times used in this study.

The Atlas-Centaur booster with a chemical upper stage is capable of sending payloads of 500 kg to the inner planets and 100 kg to Jupiter and Saturn.

One-AU orbiters inclined 60 deg to the ecliptic can be performed for less mass than a 45-deg orbiter if three plane-change maneuvers are employed.

The probes of certain flyby missions can be retrieved or brought to close proximity of the Earth by properly selecting the flyby distance at the target body.

Manned Missions

Mass requirements for direct manned missions to the outer planets are quite insensitive to stay times unless short trip times are used. These missions favor the use of advanced nuclear upper stages for minimizing initial mass requirements.

The Venus swingby maneuver shows no mass advantage for 1980 manned Mars missions using propulsive braking at Mars, but provides a reduction in Earth entry speed.

Most manned landing missions can be performed if advanced nuclear upper stages are combined with a post-Saturn launch vehicle. Excursion modules that comprise as much as 50% of the useful payload can be employed.

Manned orbiters to Mercury, Venus, and Mars can be performed by Earth-orbital rendezvous of two Saturn V launch vehicles with advanced nuclear upper stages. Orbiters to the outer planets require a post-Saturn booster. All landing missions require a post-Saturn booster and advanced nuclear upper stages.

Trade-Off Parameters

Variations in propellant tank structural weights have little effect on system mass. Insulation effects are moderately large for missions to the outer planets. The meteoroid protection model can have a drastic effect on total system mass, especially if single-thickness protective skin is employed. The use of a Whipple bumper will considerably reduce the sensitivity of the total mass to the protection models for missions in which the protection requirements are normally large.

Engine thrust-to-weight ratios have a minor effect on the mass requirements.

Combining extreme values of the scaling laws can, in some instances, change the mass requirements by an order of magnitude.

Hybrid Systems

For minimal-energy orbiters with small payloads, the planetary capture stage should be chemical, regardless of the type of Earth-escape stage.

Manned missions would benefit the most from the use of advanced nuclear stages for planetary capture and departure, regardless of type at Earth-escape propulsion.

The data presented in this report for the lightweight advanced nuclear rocket with a specific impulse of 1400 sec is representative of expected liquid-core nuclear performance.

Combined high- and low-thrust systems can be more desirable than all high-thrust systems for many high-energy missions. Low-thrust propulsion can be mixed with chemical and solid-core nuclear systems for probe missions to neighboring planets, but requires combination with advanced nuclear systems for outer-planet missions.

High- and low-thrust systems can be combined in an optimum manner, thus greatly reducing the effect of low-thrust powerplant specific weight on system mass for unmanned orbiters. This conclusion is not necessarily valid for manned missions.

All missions involving low-thrust propulsion benefit from extended trip times.

Although the mass computations for the high- and low-thrust hybrid missions are based on variable-thrust, nuclear-electric systems, it is felt that the same hybrid missions using constant thrust will yield mass results which are comparable. This conclusion stems from the comparison of variable- and constant-thrust heliocentric transfer trajectories which indicate similar values of J , and also from the assumption that the division of the total hyperbolic excess speed between the high-thrust and low-thrust systems will be comparable.

RECOMMENDATIONS FOR FUTURE STUDY

The minimum engine weights used in the high-thrust studies play an important role in defining system mass, especially for the smaller payloads. Emphasis should be placed on the development of realistic engine weight vs engine thrust curves for each type of engine, rather than using arbitrary values of engine thrust-to-weight ratio. The resulting comparisons of powerplant performance would then be more accurate.

Limits should be determined and imposed on the Earth entry speed, above which storable propellants should be used for retro-braking.

Human factors should be determined and a limit should be set on trip time. For example it may be desirable to eliminate analyses of missions whose duration is the better part of an astronaut's useful life span. This philosophy also applies to probe missions, where the gain of knowledge may surpass the accumulation of data, thereby making probes obsolete.

Penalties in initial mass requirements should be determined for nonoptimum flyby probes which return to the vicinity of Earth or may even be recovered. These missions could also use powered-flyby techniques.

Investigation should be made of the use of a single advanced nuclear engine for all major impulses at a given mission as opposed to an optimum-sized engine for each stage.

The mission analyses for low-thrust systems should be expended to include constant-thrust trajectories coupled with the hybrid systems, and the results compared with the variable-thrust data.

Unless specific weights can be reduced an order of magnitude below currently expected values (10 to 20 kg/kw), the development effort should be concentrated on other aspects of powerplant performance and operation.

The combined effects of power decay and specific weight on initial mass requirements should be determined.

INTRODUCTION

With man's ever-increasing desire for knowledge of the universe in which he lives, exploration of this environment appears imminent. Since this enormous task will require billions of dollars for research, development, and hardware, and will continue for many years, it is necessary to consider each individual mission as part of a long-range, systematically planned space program. This space program, as presently envisioned, will include both manned and unmanned missions. These missions will have extremely wide ranges of energy requirements, communication distances, travel times, and useful payloads, thereby implying that the propulsion requirements for upper-stage engines will also vary widely. Many of the earlier missions will employ available Earth-launch vehicles and associated launch facilities, thus limiting the range of weights which can be placed in Earth orbit. Consequently, the requirements imposed upon upper-stage propulsion systems could have a critical role in determining the extent to which the solar system can be explored. Simultaneously, the

payload capability of future boosters may become evident, and/or the requirements for Earth orbital rendezvous may be defined.

As an aid to the planning of future space activities, this study was performed to determine the upper-stage propulsion requirements for exploration of the solar system. The objective bodies include all planets, an asteroid, the sun, and a comet. The missions consist of unmanned flybys and orbiters, manned orbiters, and manned landings. The propulsion systems consist of chemical, solid-core, gaseous-core, pulsed nuclear, and light-weight advanced nuclear high-thrust systems; and low-thrust systems using nuclear-electric, radioisotope, and solar power.

The energy requirements for high-thrust systems are derived from conventional Keplerian Laws governing heliocentric trajectories. The utilization of gravitational fields to shape vehicle trajectories is analyzed to seek advantages in propulsion requirements, Earth-entry velocities, and other mission parameters.

Different types of propulsion systems are combined for the upper stages to determine whether certain combinations are more efficient for some missions than others. Particular emphasis is directed toward hybrid high- and low-thrust systems.

It was originally intended that the low-thrust mission analysis would be performed with constant-thrust, optimum-coast trajectories. The solution of constant-thrust trajectory optimization problems, as approached by the Newton-Raphson algorithm, encountered unforeseen difficulties. Consequently, an available variable-thrust program was employed for the study. Near the termination of this study, the constant-thrust trajectory computer program had been successfully developed, but not in time for mass computations. In comparing the two programs, it was found that the trajectories, the coast periods, and the corresponding J values are very similar. Thus it is expected that the mass results would be comparable.

SCALING LAWS AND SUBSYSTEM WEIGHTS

The propulsion requirements derived from this study are dependent upon the various necessary assumptions and approximations made at the initiation of the study. When expressing propulsion efficiency and propulsion requirements in terms of payload fraction, it is necessary to derive expressions for the inert weights of the various vehicle components and to define the engine performance such that the results appear "reasonable". Since the study involves the 1980-2000 time period and a projection of at least 15 years in the state of the art, it is meaningless to dictate

strict values for the various parameters; consequently, certain scaling laws have been derived which represent the variations in the pertinent parameters of the study. Along with the scaling laws, a range of values is assumed for each variable. This range of values reflects uncertainties in the environments to be encountered and assumptions as to technological advancement in the various disciplines. Although these ranges tend to represent possible high and low values about some nominal, their real value is in establishing trade-off parameters from which one may interpolate the results at will. The scaling laws are described in the following subsections.

Engine Weights

One measure of engine performance is the engine weight. For this study, some engine weights are determined from given values of engine T/W. Other engine weights are expressed as functions of thrust, number of engines, and a bulk weight.

Chemical Systems

For chemical propulsion systems, the engine weights are derived empirically from the data tabulated in Fig. 1. The curve is defined by the equation

$$W_e = \tau T + Z = 0.0125T + 45 \quad (1)$$

where W_e = engine weight, kg

τ = d (engine weight)/d (thrust)

T = engine thrust, kg

Z = constant

Solid-Core Nuclear Systems

The engine weights for the solid-core nuclear system involve a contingency for clustering, since a maximum single-engine thrust level of 160,000 kg is imposed. Since clustering is required for the more ambitious missions, the engine weight variation must be expressed as a function of the thrust level and the number of engines. This relationship is shown in Fig. 2 and compared with the data from which it was derived (Ref. 1). Although the reference data terminate with a seven-engine cluster, the following empirical expression for total engine weight was generated for any number of engines required to achieve a given thrust level.

$$W_e = (4535 + 0.095 T/n)n^{1.12}$$

Upon the suggestion of NASA, the engine weight equation was altered to

$$W_e = (4535 + 0.110 T/n)n^{1.12} \quad (2)$$

where W_e = total engine weight, kg

T = thrust required, kg

n = number of engines

Light Weight Advanced and Gaseous-Core Nuclear Systems

The engine weights for the light weight advanced and gaseous-core rockets are generally derived from assumed values of T/W_e which are varied as shown in the following table.

<u>Type</u>	<u>I_{sp}</u>	<u>T/W_e</u>
Gaseous-Core	1400 to 2500	5 to 20
Light Weight Advanced	1400 to 2500	5 to 20
Advanced Gaseous-Core	3500	1 to 3

It should be noted that a lower limit is placed on the weight of these advanced nuclear engines in order to account for the probable impossibility of fabricating these engines below some limiting size, as indicated on page 18.

Pulsed Nuclear Rocket

The engine weights for the pulsed nuclear systems are also derived from assumed values of engine T/W_e . The values of engine T/W_e vary from 1 to 5, while the specific impulse range is from 1200 to 1800 sec with a thrust level of 180,000 kg. Normally, the thrust level for the 1800-sec engine would be about 360,000 kg. However, it was assumed that advanced technology would increase the effectiveness of the energy source such that the I_{sp} could be increased from 1200 to 1800 sec by decreasing the equivalent "mass flow".

Propellant Tank Weights

For the propulsion systems that require propellant tanks, it is necessary to express the total tankage weights in terms of the independent variables. The total tank weight is defined as the total of all component weights minus the engine. These components include tank structure and accessories, a boil-off contingency, insulation, and meteoroid protection.

Tank Structure and Accessories

The contribution of the tank structure and accessories to the total tankage weight is derived empirically from existing hardware data and from theoretical calculations. For this study, it is assumed that the tanks are launched to orbit in a "full" configuration. Therefore, the propellant tanks which are to be used for upper stages must be subjected to the same dynamic conditions as the launch booster. It is then feasible to use launch booster data as a guide to determining the tank structural weights. Figure 3 presents a compilation of booster data presented as a function of propellant weight and specific gravity. From these data an excellent correlation can be derived by the following expression.

$$W_s = \frac{AW_p^{0.9}}{\sigma^{0.533}} \quad (3)$$

where W_s = total tank weight, including pressure shell, thrust structure, baffles, feed lines, reserves, residuals, and other tank accessories excluding the engine

W_p = propellant weight

σ = propellant average specific gravity

A = constant

In English units, a representative value for "A" is 0.21, which corresponds to a value of about 0.194 in metric units. Since these data represent the present technology and since this report is concerned with the 1980-2000 time period, the data of Fig. 3 are used as an upper limit.

A nominal value for "A" and a check on the basic form of Eq. (3) is derived from data supplied by NASA. Figure 4 compares the NASA data with the basic equation for liquid hydrogen. With a value for A of 0.10 and a constant of 500 kilograms included, the correlation is phenomenal.

Data for liquid oxygen and liquid hydrogen were compared with the same equation derived from the liquid hydrogen correlation. This comparison is presented in Fig. 5. For propellant weights above 100,000 kg, the correlation is quite good. Below this value, the two approximations diverge until the tank weight is different by a factor of two. It was necessary to resolve this difference and establish the best estimate.

In analyzing the NASA data, it is noted that the hydrogen tanks weigh over twice as much as the O_2/H_2 tanks for large propellant weights. This result is expected, since, for a given propellant weight, the H_2 tank will have a surface

area 2.76 times that of the O_2/H_2 tank, based on average specific gravity. Also at these large weights, the assorted fixed weights are a relatively small fraction of the total tank weight. For smaller tanks (less propellant) the accessory weights become more influential in the comparison, such that the ratio of H_2 tank weight to O_2/H_2 tank weight is assumed to approach unity. Figure 6 is a plot of this ratio versus propellant weight. The NASA data are quite scattered and have tank weight ratios of less than unity up to propellant weights of 10,000 kg.

The continuous line was derived from the equation

$$W_s = \frac{0.10 W_p^{0.9}}{0.533} + 500 \quad (4)$$

by substituting the appropriate average specific gravity value. For the larger values of propellant weight, the ratio is 2.25. This indicates that the final ratio is less than the ratio of the respective surface areas. This can be expected since all tank structure weights are not proportional to surface area. Let the total tank weight be expressed as

$$W_T = W_s + W_A$$

where W_s = skin weight

W_A = accessory and structure weight

then

$$\frac{W_{TH_2}}{W_{TO_2/H_2}} = \frac{W_{sH_2} + W_{AH_2}}{W_{sO_2/H_2} + W_{AO_2/H_2}} = 2.25$$

Assuming $W_{AH_2} = W_{AO_2/H_2}$ and letting $W_{sH_2} = 2.76 W_{sO_2/H_2}$,

$$\frac{2.76 W_{sO_2/H_2} + W_A}{W_{sO_2/H_2} + W_A} = 2.25$$

Solving for W_A , one finds that for hydrogen tanks, the skin weight is about 87% of the total tank weight. Likewise, for O_2/H_2 , the skin weight is about 71% of the total. These fractions are considered reasonable and have been adopted for the scaling law analysis.

The general expression for propellant tank structural and accessory weight is then

$$W_T = \frac{AW_p^{0.9}}{\sigma^{0.533}} + 500 \quad (5)$$

Boil-Off and Insulation Weights

The weight contingencies for propellant boil-off and insulation are interdependent and can be considered as one contribution to the total inert weight of the system. The boil-off weight is expressed as

$$W_{BO} = \frac{KA t \Delta T}{\delta L} \quad (6)$$

and the insulation weight is expressed as

$$W_i = \rho \delta A \quad (7)$$

where

K = insulation thermal conductivity, kcal/hr-m-K

A = surface area, m^2

t = exposure time, hours

ΔT = temperature differential, K

δ = insulation thickness, m

L = heat of vaporization, kcal/kg

ρ = insulation density, kg/m^3

W_{BO} = weight of boil-off, kg

W_i = insulation weight, kg

The total contingency for boil-off and insulation is then

$$W_{BO} + W_i = \frac{KA t \Delta T}{\delta L} + \rho \delta A \quad (8)$$

The optimum insulation thickness which will minimize the total weight can be obtained by differentiating the weight, Eq. (7), with respect to δ and setting the result equal to zero. Hence,

$$\delta_{opt} = \left(\frac{Kt \Delta T}{L\rho} \right)^{1/2} \quad (9)$$

The minimum weight is then

$$(W_{Bo} + W_i)_{min} = 2A \left[\frac{\rho Kt \Delta T}{L} \right]^{1/2} \quad (10)$$

For an assumed tank geometry, the surface area can be expressed in terms of the propellant weight and specific gravity. All propellant tanks are assumed to be cylindrical with hemispherical ends and a length-to-diameter ratio of 3.0. The surface area is then

$$A = 0.0575(W_p/\sigma)^{2/3} \quad (11)$$

where

A = surface area, m^2

W_p = propellant weight, kg

σ = propellant specific gravity

The properties of the insulation material are assumed as follows

$$K = 4.44 \times 10^{-5} \text{ kcal/hr-m-K}$$

$$\rho = 80 \text{ kg/m}^3$$

The thermal conductivity of the insulation was assumed to be that of a superinsulation such as Linde SI-4. With these insulation properties, and substituting Eq. (10) into Eq. (9), the expression for the combined boil-off and insulation weights becomes

$$W_{Bo} + W_i = 0.0336 \left(\frac{W_p}{\sigma} \right)^{2/3} \left(\frac{t \Delta T}{L} \right)^{1/2} \quad (12)$$

where t is now in days.

or

$$W_{Bo} + W_i = B \left(\frac{W_p}{\sigma} \right)^{\frac{2}{3}} \left(\frac{t \Delta T}{L} \right)^{\frac{1}{2}}$$

The exposure time and temperature differential are functions of the mission definition. These parameters are developed in Appendix A of this report.

Meteoroid Protection Weights

The flux model used for this study is the NASA-MSD adoption of Whipple's 1963a distribution of near-Earth cometary particles. The cometary flux is expressed as

$$\log Q_c = -1.34 \log m - 14.48 + 2.68 \log (0.44/\rho) \quad (13)$$

where

$$\rho = 0.5 \text{ gm/cc}$$

which yields

$$\log Q_c = -1.34 \log m - 14.32 \quad (14)$$

where the units of Q_c are no./m²-sec (number of encounters per unit area and time)

Converting the units to no./m²-day, the flux equation becomes

$$Q_c = 10^{-9.384} m^{-1.34} \quad (15)$$

Poisson's approximation to the binominal distribution is given by:

$$Q = \frac{1 - P_0}{At} \quad (16)$$

where Q = flux, no./m²-day

A = area, m²

t = time, days

Combining Eqs. (14) and (15) yields

$$m = \left[10^{-9.384} \frac{At}{1-P_0} \right]^{1/1.34} \quad (17)$$

The Nysmith-Summers penetration theory for penetration depth is non-dimensionally expressed as

$$\frac{\delta}{d} = 2.28 \left(\frac{\rho_p}{\rho_t} \right)^{2/3} \left(\frac{V}{c} \right)^{3/2} \quad (18)$$

where

d = diameter

$$d = \left(\frac{6m}{\pi\rho} \right)^{1/3}$$

therefore:

$$\delta = 2.83 \left(\frac{\rho_p m V^2}{\rho_t^2 c^2} \right)^{1/3} \quad (19)$$

A factor of 1.5 is included as the difference in penetrating a semi-infinite slab compared with a thin-wall target thickness. This general analysis does not consider sandwich construction, such as the Whipple bumper.

Using a particle density, ρ_p , of 0.5 gm/cc and an average particle velocity of 20 km/sec, the shielding thickness becomes

$$\delta = \frac{0.1147}{(\rho_t c)^{2/3}} \left(\frac{At}{1-P_0} \right)^{1/4} \quad (20)$$

where

δ = shielding thickness, cm

ρ_t = shielding material density, gm/cc

c = sonic velocity in shielding material, km/sec

A = exposed area, m²

t = exposure time, days

P_0 = probability of no penetration

The total shielding weight can then be written as

$$W_M = 10\rho_t A \delta \quad (21)$$

Combining Eqs. (10), (19), and (20)

$$W_M = 0.0575 \left(\frac{W_p}{\sigma} \right)^{5/6} t^{1/4} \quad (22)$$

where W_M = total shielding weight, km

W_p = propellant weight, km

σ = propellant specific gravity

t = exposure time, days

Equation (21) assumes that the shielding material is aluminum with a density of 2.77 gm/cc and a sonic velocity of 5.1 km/sec.

Finally the required meteoroid protection weight is expressed generally as

$$W_M = C \left(\frac{W_p}{\sigma} \right)^{5/6} t^{1/4} \quad (23)$$

Total System Inert Weight

The expression for the inert weight of the entire propulsion system is formulated by combining the various components. The weight of the engines can be derived from Eq. (2) in general terms as

$$W_E = \left[Z + \tau \left(\frac{T}{n} \right) \right] n^{1.12} \quad (24)$$

Combining Eqs. (5), (12), (23), and (24), the inert weight equation becomes

$$W_I = \frac{AW_p^{0.9}}{\sigma^{0.533}} + 500 + B\left(\frac{W_p}{\sigma}\right)^{2/3} \left(\frac{t \Delta T}{L}\right)^{1/2} + C\left(\frac{W_p}{\sigma}\right)^{5/6} t^{1/4} + \left[Z + \tau \left(\frac{T}{n}\right)\right] n^{1.12} \quad (25)$$

The thrust, T, in the last term of Eq. (25) can be expressed as

$$T = g_i (W_{pL} + W_p + W_I) \quad (26)$$

where

g_i = initial acceleration, g's

W_{pL} = payload, kg

W_p = propellant weight, kg

W_I = inert weight, kg

Substituting (26) into (25) yields

$$W_I = \frac{\frac{AW_p^{0.9}}{\sigma^{0.533}} + B\left(\frac{W_p}{\sigma}\right)^{2/3} \left(\frac{t \Delta T}{L}\right)^{1/2} + C\left(\frac{W_p}{\sigma}\right)^{5/6} t^{1/4}}{1 - \tau g_i n^{0.12}}$$

$$\frac{Zn^{1.12} + \tau g_i (W_{pL} + W_p) n^{0.12} + 500}{1 - \tau g_i n^{0.12}} \quad (27)$$

Parameter Limitations and Variations

The scaling laws describe how each contributor to the total inert weight of the propulsion system varies with the independent variables. It is further necessary to establish upper and lower limits on each component to enable one to properly analyze the entire system by determining the trade-off partial derivatives of each major variable. These limits represent possible deviations from a nominal value which is an average of a proposed state of the art for the 1980 time period. The lower limits are based on projected advances in the state of the art and more favorable environmental conditions, while the upper values tend toward the present state of the art and account for degradations in assumed performance and environmental conditions.

Engine Parameters

As stated in Eq. (24), the engine weights can be expressed as

$$W_e = \left[Z + \tau \left(\frac{T}{n} \right) \right] n^{1.12}$$

where Z and T are the variables of interest. The following table presents the assumed limits for the pertinent parameters, including the range of specific impulse used in the study.

		τ	Z, kg	I_{sp} , sec
Chemical	High	0.0150	90.6	470
	Nominal	0.0125	45.3	430
	Low	0.0100	0	400
Solid Nuclear	High	0.115	5450	900
	Nominal	0.110	4536	800
	Low	0.105	3630	700
Pulsed Nuclear	High*	1.0	0	3600
	Nominal	0.33	0	1800
	Low	0.20	0	1200
Gaseous & Light Weight Advanced Nuclear	High	0.20	0	2500
	Nominal	0.10	0	2000
	Low	0.05	0	1400
Adv. Gaseous Nuclear	---	1.0-0.33	0	3500

* Not used in study

For the low-thrust nuclear-electric systems, the powerplant weights are derived from assumed values of specific weight which range from 3 to 18 kilograms per kilowatt (electric).

Limits have been imposed on maximum engine thrust and minimum engine weight as shown in the following table.

<u>Engine</u>	<u>Maximum Thrust, kg</u>	<u>Minimum Weight, kg</u>
Chemical	4,535,000	45
Solid Nuclear	160,000	4,356
Light Weight Adv. Nuclear	908,000	907
Gaseous Nuclear	908,000	13,600

The light weight advanced nuclear rocket connotation in these tables. could apply to a highly advanced, liquid-core nuclear rocket with very high

(2000 sec) specific impulse or a gaseous-core nuclear rocket of extremely low minimum weight. Thus, one hypothetical engine can represent projected states of the art in two different disciplines. Actually, the 1400-sec specific impulse could be considered a nominal value for the liquid-core nuclear rocket.

Tankage Constant

As shown in Fig. 4, the constant "A" in Eq. (5) has a nominal value of 0.10. The upper limit of "A" was derived from Fig. 3 and has a metric equivalent of about 0.20. A lower value of 0.05 was arbitrarily chosen to represent a very advanced state of the art.

Boil-Off and Insulation Constant

From Eq. (12), the nominal value of the constant "B" is 0.0336 or approximately 0.034. The high and low values were chosen as twice and half this value, respectively. The high value of 0.068 corresponds to the use of superinsulations which may have conductivities which are four times the assumed value due to installation losses and heat leaks. The lower limit of 0.017 represents an advanced state of the art.

Meteoroid Protection Constant

The constant "C" from Eqs. (22) and (23) is assumed to be 0.06 for the nominal case. To allow for various definitions of the particle velocity, density, concentration, and probability of penetration, upper and lower values of "C" were chosen as 0.12 and 0.03, respectively.

It is expected that an increase in particle size and concentration will exist in the asteroid belt, which has been placed at 2.0 to 4.0 AU from the sun. At the suggestion of NASA, all values of "C" were increased 66% for those missions which pass through the asteroidal region.

The inert weight constants are summarized in the following table.

	A	B	<u>C</u>	
			<u>Asteroid</u>	<u>Nonasteroid</u>
High	0.200	0.068	0.200	0.120
Nominal	0.100	0.034	0.100	0.060
Low	0.050	0.017	0.050	0.030

Payload Definition

The payloads used for this study correspond to manned and unmanned systems. For manned systems, an entire spacecraft weight is postulated as a function of several mission variables. The first variable considered is the crew size. For missions which land a manned excursion module on the objective body, the total crew number is eight. Missions involving no excursion module have a five-man crew.

The spacecraft definition varies with the type of propulsion employed. For high-thrust systems, the spacecraft consists of the following major subsystems.

Command Module

The command module is the control center of the spacecraft, part of which is the Earth-entry vehicle. Although the Earth-entry vehicle design can be optimized for each entry speed to achieve minimum weight, it is assumed that the entry module weighs 9000 kg and is capable of direct entry at speeds up to 20 km/sec. A vehicle L/D of 0.60 will provide an entry corridor of 16 km. Theoretically, the entry vehicle weight should decrease with reduced entry speed. However, an entry vehicle entering at parabolic speed would require a total weight of about 7000 kg. It is shown later that the 2000-kg difference in entry vehicle weight is a small fraction of the total spacecraft weight, thereby making it practical to assume a constant Earth-entry vehicle weight without changing the conclusions derived from the mission studies. The total command module, including the Earth-entry vehicle, weighs 13,600 kg.

Service Module

The service module serves as the crew's living quarters and recreational area, and also houses the primary life support equipment. Its total weight excluding the life support equipment, is 4550 kg.

Solar Shelter

A shelter is provided for protection against solar flare protons. Although this proton flux varies with the distance from the sun, the integrated effect over the varying trip durations enables one to fix a given shelter weight without introducing prohibitive error. Therefore, the solar shelter weight is fixed at 11,350 kg.

Power Supply

The power supply furnishes primary electric power for the entire spacecraft. While the power source may be from a solar boiler or a nuclear source, depending on the mission, the weight is fixed at 1360 kg.

Basic Structure

It is assumed that the spacecraft is a rotating vehicle capable of providing an artificial gravity of 0.4 g for the crew. The basic structure has a 25-meter spin radius and a central hub which serves as the solar shelter. The total basic structure weighs 9080 kg. A miscellaneous weight of 2270 kg is added as an extra contingency.

Radiation Shielding

For high-thrust nuclear systems, radiation shielding is required. This shielding weight is set at 4535 kg and is based on a radiation dose of 5 rem/yr. The nuclear-electric systems include the radiation shielding weights in the powerplant specific weight parameter.

Life Support Systems

The weight of the life support system is a function of the total trip duration and the number of crewmen, and can be expressed as

$$W_{Ls} = [1.48(n-4) + 7.1] (T-200) + 500 (n-4) + 2370 \quad (28)$$

where

W_{Ls} = life support system weight, kg

n = number of crewmen

T = total trip duration, days

For crew sizes of eight and five men, the life support weights become, respectively,

$$W_{Ls} = 13.02 (T-200) + 4370 \text{ kg}$$

$$W_{Ls} = 8.58 (T-200) + 2870 \text{ kg}$$

Total Spacecraft Weight

Combining all of the components, the total spacecraft weight can be derived as a function of trip duration, crew size, and type of propulsion employed. For nuclear-electric systems the radiation shielding and power supply contributions are excluded. The following table summarizes the spacecraft weights.

<u>Propulsion</u>	<u>Crew Size</u>	<u>Spacecraft Weight, kg</u>
High Thrust	8 men	51,115 + 13.02 (T-200)
	5 men	49,615 + 8.58 (T-200)
Low Thrust	8 men	45,220 + 13.02 (T-200)
	5 men	43,720 + 8.58 (T-200)

For the unmanned missions, the payload is assumed and varied over a wide range, the range being a function of the type of propulsion employed. Missions using chemical propulsion have payloads ranging from 45 kg to 45,350 kg, while the nuclear propulsion systems have a range varying from 454 kg to 453,500 kg. In cases where the payload capability can be increased to be compatible with booster capabilities, the maximum payload has been increased above the proposed limits.

STUDY RESULTS

Trajectory Analyses for High-Thrust Systems

It is desirable and, in many cases, necessary to analyze the trajectory characteristics of the various types of missions independently of the transportation system. This type of analysis defines the total energy requirements in terms of characteristic velocities at each boundary of the mission profile and enables one to greatly reduce the areas of interest for the ensuing mass computations derived from the integration of the transportation system with the characteristic velocities.

Minimum Energy Approximations

For flyby missions, the only characteristic velocity is the Earth departure speed. Since the flybys for this study are unmanned, the transfer duration is of secondary importance and contributes little toward defining optimum (minimum mass) missions. It is therefore necessary to locate trajectories with minimum Earth departure speeds.

The unmanned orbital stopover missions can be defined similarly to the flybys, except for the energy requirements at the objective body for establishing an orbit about that body. Again, it is necessary to minimize the total energy requirements of the mission.

A simple model can be formulated by superimposing the orbital traces of the objective body and those of the Earth on a plot of heliocentric longitude vs Julian date. It can be assumed that all orbits are coplanar. The plots are shown in Figs. 7 through 15 for Mercury, Venus, Mars, Ceres, Jupiter,

Saturn, Uranus, Neptune, and Pluto. On each plot is superimposed a Hohmann transfer (180 deg), shown as a dashed line. The intersections of the dashed line and the Earth orbital traces define Earth-departure dates for Hohmann transfers. For many objective bodies, there are numerous intersections which represent areas of interest. However, some points of intersection can be eliminated. Note that the heliocentric longitudes of the planet's perihelion and aphelion are superimposed on trace plots. Since the orbits of the objective bodies are eccentric, certain launch dates are more desirable than others. For transfer trajectories to exterior bodies, less energy is required to arrive at the perihelion of the planet's orbit than at any other point. Since the transfers traverse 180 deg of heliocentric arc, the heliocentric longitude of the Earth at launch should coincide with the heliocentric longitude of the planet's aphelion. For transfers to interior bodies, the reverse is true; that is, the Earth launch heliocentric longitude should coincide with the planet's perihelion. Due to the ephemerides of the planets, the actual launch dates will not necessarily correspond to the planet's aphelion or perihelion. Thus, the closest point is chosen as a guide to defining the optimum missions.

In Fig. 9, a desirable Earth launch date occurs at Julian date 2444185. From the trajectory computer program, the minimum hyperbolic excess speed occurs at a Julian date of 2444180. Small errors in optimum Earth departure date and/or transfer times are due to the inclination of the target's orbit to the ecliptic.

Because of the relatively high 7-deg inclination of Mercury's orbit, the trace model is useful only for flyby missions. For orbital stopover missions, the Earth departure speeds and Mercury arrival speeds do not minimize simultaneously. It is therefore necessary to find non-Hohmann transfers which will minimize the total characteristic velocity. Figures 16 through 20 present velocity contours for various transfer times. The minimum-velocity flyby departs Earth at approximately Julian date 2444575 and is a 100-day transfer. However, the corresponding Mercury arrival velocity is very large. An Earth-departure date of 2444350 results in an increased departure speed, but a much lower Mercury arrival speed occurs. This type of trade-off is performed for each objective body, with the final optimum trips being selected from the results of the computer analysis. With the exception of Mercury, the Earth departure dates are similar for both flyby and orbital stopover missions.

Round-Trip Trajectories

The trajectory analyses for the round-trip missions require an integration of outbound and inbound transfers to define the characteristic velocities. It is expedient to fix a planetary arrival date and determine the outbound and inbound transfers which yield the minimum total characteristic velocity. This procedure is repeated for a range of planetary arrival dates until an over-all minimum total characteristic velocity is determined. This same procedure is used to define the optimum missions which have a given stay time at the planet.

The Earth return transfer begins at the end of the stay time instead of on the arrival date, which is used for zero-day stopovers.

Tables I, II, and III summarize typical minimum-energy missions for flybys, orbital stopovers, and round-trip missions, respectively. For the outermost planets, the effects of stay time on the total velocity are very small. For Ceres and the inner planets the effects of stay time are quite pronounced.

The analysis of Mercury missions in the 1980 to 1985 time period indicates that, generally, three launch opportunities arise each Earth year; this is evident from Fig. 7. The velocity requirement variation for a given year is indicated in figures 16 through 20. Of the three opportunities, one has a low Earth departure speed available for a flyby mission, while the other two have higher velocity requirements of varying degrees. However, for orbital probes, there is generally only one logical opportunity each year since Mercury arrival speeds must be considered. The launch dates for flybys and orbiters differ considerably. The analysis of Mercury round trips is unique in that small stay times are all but impossible from an energy requirement standpoint. When low-energy transfers are made to Mercury, there is an absence of desirable return legs until a stay time of about 80 days has elapsed. Figure 21 presents velocity contour charts, showing the Earth departure and Mercury departure speeds for a range of Julian dates. The minimum velocities occur at about 2444675 and 2444755 Julian dates at Mercury, indicating that an 80-day stopover is near optimum. Likewise, a zero-day stopover would correspond to characteristic velocities of well over 0.70 EMOS. It is therefore necessary to establish the optimum stay time and then reduce the total trip time by varying the stay time together with the outbound and inbound transfer times.

It is desirable to determine the velocity requirements as a function of trip time for each objective body. The transfer durations to the outer planets are so large with minimum-energy (Hohmann) trajectories, that the feasibility of such missions becomes questionable, especially for manned missions. When analyzing round-trip missions for reduced total duration, it is found that the favorable trip times occur at specific intervals, i.e., the variation of total velocity with trip time is not a smooth function. The minimum-velocity trips occur in increments defined as:

$$\Delta t = \frac{2\pi}{\mu_E - \mu_P} \quad (29)$$

where Δt = incremental total trip time, days

μ_E = mean daily motion of Earth, rad/day

μ_P = mean daily motion of planet, rad/day

As an example, a Hohmann transfer to Uranus requires about 6000 days, making a minimum-energy round trip of about 12,000 days. From Table III, the total duration is 12,030 days. Using Eq. (29) the next desirable total trip time is 369 days less, or about 11,660 days. Representing the trip time as

$$t = 12,030 - 369 n \quad (30)$$

where n is an integer, other trip times can be approximated. Assuming $n = 12$, the trip time becomes 7600 days. Again the Uranus data in Table III indicate a desirable trip duration of 7620 days. This approach is not meant to absolutely define the series of best trip durations, but acts as a very good approximation and greatly reduces the search for the desirable trips. Figure 22 shows the actual variation of total velocity with trip time for Uranus, where the dashed line merely indicates the trend of the minima, with one occurring about every 370 days. A most important fact is that the lowest Earth entry speeds occur at the minima. Between the minima, the Earth entry speeds increase to hyperbolic excess speeds of over 1.0 EMOS.

For planets in which the Δt in Eq. (29) is larger than the Hohmann transfer time, the approximation must be viewed somewhat differently. Using Mars as an example, Hohmann transfers require about 250 days. However, the Δt is 778 days, indicating that the variation of trip time with total velocity will be a smooth continuous curve and that minimum-energy trips will occur every 2.13 years.

For interior planets, Eq. (29) is not applicable, since negative values of Δt result. If the absolute value of the denominator were imposed, positive values of Δt would occur. However, the values would be greater than the Hohmann transfer times. For Venus, Δt would equal 588 days as compared with a Hohmann transfer of about 150 days. Also for Mercury, Δt would equal 117 days as compared with about 100 days for the Hohmann transfer.

Retrieval of Unmanned Planetary Flyby Probes

The flyby missions analyzed in the study were optimized by selecting the transfer trajectories which would minimize the initial mass requirements for each selected trip duration. Since, for many of the objective bodies, the communication distances from the planetary encounter to the Earth would dictate extremely large on-board power requirements, an analysis was performed to determine how each planetary gravitational field could perturb the flyby trajectory such that the resultant trajectory would come in close proximity to the Earth for communication advantages. This distance was arbitrarily chosen as one AU. No attempt was made to alter the Earth-planet trajectory to seek more advantageous flyby conditions. Instead, the previously optimized flyby missions were analyzed. The flyby distance was varied between 1.1

planetary radii and infinity. The arrival heliocentric velocity and flight path angle were transformed into planetocentric coordinates such that gravitational bend angles could be determined as a function of the passage distance and hyperbolic excess speed. This bend angle, k , can be expressed as

$$\csc \frac{k}{2} = 1.0 + \frac{m_o}{m_p} r_p v_\infty^2 \quad (31)$$

where k = flyby bend angle, deg

m_o = mass of sun

m_p = mass of target body

r_p = planetary passage distance, AU's

v_∞ = hyperbolic excess speed, EMOS

A graphical reference is shown in Fig. 23. An analytical procedure may be employed to accomplish the planetocentric vector analysis, but a graphical solution is less tedious and allows a greater understanding of the basic phenomena. In Figs. 24 to 31, subscripts are given the following identification:

1	Asymptotic arrival conditions
2	Asymptotic departure conditions
p	Target body
L.S.	Light side pass
D.S.	Dark side pass
E	Escape (heliocentric) conditions

The heliocentric speeds of the approaching probe and the target body are known, as are their respective flight path angles. The hyperbolic excess velocity is given by:

$$\bar{v}_\infty = \bar{v}_{H_1} - \bar{v}_p \quad (32)$$

A circle of radius v_∞ may now be drawn. This circle defines all possible asymptotic departures and each point represents planetary passage at a different planetary passage distance. However, all points must be discarded at which r_p is less than 1.0. After determining the heliocentric escape speed, V_e , at the heliocentric distance of the given target body, a circle of radius V_e is drawn.

If the two circles intersect, all points of asymptotic departure at which the heliocentric departure speed is greater than escape must be excluded. These points represent departure (heliocentric) velocities which produce heliocentric hyperbolic trajectories and are of no value in terms of probe retrieval (Fig. 32). As in Fig. 26 the counterclockwise intersection, as rotated from the arrival hyperbolic excess vector, is at 6.4 planetary radii; therefore, all such passes of Saturn for the given arrival conditions must be between 6.4 Saturn radii and ∞ . If the hyperbolic excess vector is rotated clockwise, a planetary passage distance of 1.1 Saturn radii is reached before heliocentric escape speed occurs - for these arrival conditions the corresponding passes may occur for any Saturn passage distance from 1.1 radii to ∞ .

If there is no intersection of the departure loci and the escape circle, two possibilities arise:

1. If all departure loci are external to the escape circle, then no elliptic return is possible; all return trajectories are hyperbolic (Fig. 24).
2. If all departure loci are interior to the escape circle, then all return trajectories are elliptic.

In Fig. 29, for planetary passage distances greater than 1.1 Neptune radii, all return trajectories are retrograde.

Once a spectrum of minimum mass trips is determined, without return considerations, over a specific range of Earth departure dates, diagrams as in Fig. 29 can be constructed. The procedure followed should be:

1. Eliminate all locus points which are not applicable
2. Find the minimum passage distances
3. Determine the respective heliocentric departure conditions
4. Establish the return trajectory including ephemerides of the Earth and the returning probe
5. If the closest communication distance is favorable, examine the next case; if not, return to (2) with a slightly larger passage distance, and repeat (3) and (4)

The missions depicted in Figs. 24 to 31 were examined for planetary passage distances of 1.1 planetary radii. No favorable Earth return was found. Various other passage distances were chosen within a limit of practical mission desirability, and no favorable return could be established. Either the probe will not return within a desirable communication distance or the trip time taken by the returning probe is considered excessive.

On each of the above figures, the planetary passage distance for a heliocentric departure flight path angle of 0 deg, $r_p \gamma=0$, is depicted. The corresponding heliocentric peripoint distance, R_{π} , is also displayed. The heliocentric, 0-deg departure is either the maximum or minimum total-energy return trajectory. This trajectory exhibits an extremum of peripoint distance as a function of departure heliocentric flight path angle under the restriction of a circular heliocentric target orbit. The system of equations developed to examine the peripoint distance is:

$$R_{\pi} = a(1-e) \quad (33)$$

$$a = K / \left\{ \frac{2K}{r} - V^2 \right\} \quad (34)$$

$$e = \left\{ 1 - \frac{r^2 V^2 \cos^2 \gamma}{Ka} \right\}^{\frac{1}{2}} \quad (35)$$

$$v_{\infty}^2 = V^2 + V_p^2 - 2V_p V \cos (\gamma + \gamma_p) \quad (36)$$

where K = gravitational constant

γ = departure flight path angle

r = heliocentric distance to target body

V_p = heliocentric target body speed

V_{∞} = planetocentric hyperbolic excess speed

V = heliocentric speed of departing probe

a = return trajectory semi-major axis

e = return trajectory eccentricity

R_{π} = heliocentric peripoint distance of return trajectory

The first three equations of the system are the standard two-body conic section relationships for elliptic trajectories. The last equation is a form of the law of cosines for an oblique triangle. If one assumes that the target bodies are travelling in heliocentric circular orbits, then it can be shown that $\gamma = 0$ yields not only an extremum of the system, but a minimum (Fig. 33) subject to the R_{π} departure solution described below. Since V is quadratic in $\cos \gamma$, two solutions exist: R_{π_1} , which represents the greater departure

speeds, and R_{π_2} , which represents the smaller departure speeds (Fig. 34). At $\gamma = 0$, the maximum R_{π_1} , for all choices of flight path angle, implies that r is actually the periapsis of a heliocentric ellipse and that R_{π_2} , the minimum for all choices of γ , is the apoapsis of a heliocentric ellipse. In Figs. 25 to 31 the apoapsis extremum of heliocentric peri-distance is displayed. The tangential flight path angle, γ_{limit} , represents the crossover from the R_{π_1} type to the R_{π_2} type solution. Figure 35 depicts the semi-major axis and eccentricity of the return elliptic trajectories. As v_{∞} is increased, the intersection situation of escape circle and departure loci arises. This implies that the maximum region of R_{π_1} is eliminated from $\gamma = 0$ symmetrically outward and that now two equal and symmetric maxima exist with respect to the $\gamma = 0$ axis.

With the generation of a similar curve for planetary passage distance as a function of flight path angle, the tradeoff in passage distance and peripoint distance may be observed. Again, if no satisfactory return is found, one must either resort to special techniques, e.g., powered flybys, or to an increase in MEO to change the arrival conditions so that, possibly, a more favorable return may be found.

Once a desired passage distance is found which produces a reasonable peripoint distance, the orbital elements of the returning (elliptic) probe trajectory may be used in a computer program to generate return communication distances. A computer program was written for direct elliptic return in the ecliptic plane. Return in the ecliptic appears to be reasonably valid, since all arrival inclinations were small. The program generates ephemerides for both the Earth and the returning probe and calculates the communication distance and elapsed return time of the probe from the target body as shown in Figs. 36 to 50. Return time is limited to 1000 days for inner planetary flybys and 2000 days for outer planetary flybys. All traces depicted have planetary passage distances of 1.1 planetary radii except for that of Fig. 50 ($r_p = 11.0$).

Upon examination of these traces, it may be concluded that

1. Communication with the returning probes may occur approximately 400 days after arrival at the target body.
2. A communication window will last for approximately 100 days.
3. Solar interference to communications will usually occur near maximum separations (indicated points on traces), i.e., the sun is not between the probe and the Earth at favorable communication times.

It is suggested that a future study be made of tradeoff parameters of MEO, target body passage distance, peripoint return distance, return trip time, and communication time. The complete solution of the retrieval of interplanetary

flybys cannot be given here, for it is a study in itself. The problem is a sophisticated one which requires subtlety and exactness. It is hoped that this description gives an indication of what must be done.

Mass Computations for High-Thrust Systems

The combinations of missions and objective bodies analyzed in this study are presented in Table IV. For given combinations of objective bodies, trajectories, and propulsion systems, the initial mass required in Earth orbit is computed by a computational program which is described in Appendix B of this report. In general, this program incorporates the scaling laws and the resultant inert weight equation, Eq. (27), with the velocity requirements derived from the trajectory analysis.

The velocity requirements are expressed in terms of hyperbolic excess speed and are normalized with respect to the Earth's mean orbital speed (EMOS). Using the physical and orbital properties of the objective bodies involved, the mass program defines a new set of hyperbolic speeds which accounts for gravity losses due to the body's gravitational field. These revised velocities are used in conjunction with the propulsion characteristics in the rocket equation to determine the ratio of initial mass to burn-out mass.

The program determines the initial acceleration which minimizes the sum of the inert and propellant weights for each major propulsion maneuver. The results of the computations are presented in several ways to enable one to determine the effect of each parameter used in the analysis on the initial mass requirements.

For the unmanned missions, the mass requirements were computed as a function of payload and specific impulse for each objective body and propulsion system. All data are derived from nominal values of the engine and inert weight parameters. The payload capability of various Earth launch boosters is superimposed on these data to provide a ready reference for determining the deliverable payload to the objective body of interest.

The missions correspond to the longest trip durations chosen for each objective body as referenced in Tables I and II and the effects of trip time are determined for each objective body and propulsion system. The inert weight parameters are also varied to establish tradeoff parameters with initial mass.

Unmanned Flyby Missions

The mass requirements for ballistic flybys are shown in Figs. 51 through 110 for the objective bodies outlined in Table I and the longest trip times. These mass requirements were computed for a range of payloads and specific

impulses for each propulsion system. The payload ranges from 454 kg to 453,500 kg for all propulsion systems except chemical, which has a payload range of 45 kg to 45,350 kg.

The mass requirement variation with specific impulse varies with payload and energy (velocity) requirement. For Venus, where the velocity requirements are about the lowest of any objective body, an increase in specific impulse from 400 to 470 sec, for chemical propulsion, decreases the initial mass requirements about 20% throughout the entire range of payloads. A flyby to Mercury has a corresponding reduction of 24%.

A more ambitious flyby to Jupiter indicates that, with the same improvement in specific impulse, the mass savings are about 39% for the smaller payloads and 29% for the larger payloads. Finally, a Pluto flyby indicates a 46% savings in mass for large payloads and a 62% savings for small payloads.

Comparable results exist for other propulsion systems. As an example, consider a Pluto flyby using a light weight advanced nuclear rocket. Increasing the specific impulse from 1400 to 2500 sec reduces the mass requirements about 40% at the large payloads and 65% for the smaller payloads. These percentages correspond to a 79% increase in specific impulse. Essentially the same percentages were obtained for Pluto with a 17% increase in the chemical propulsion specific impulse.

Payload fractions can be readily obtained by dividing the payload at any point by the corresponding mass requirement.

It is desirable to compare the performance of the various types of propulsion systems for a given mission. This comparison is achieved by superimposing the nominal specific impulse data for each propulsion system. These composite plots are shown in Figs. 111 through 120. Chemical propulsion appears desirable for flyby missions with small payloads. For example, an Atlas-Centaur is capable of sending payloads of 500 kg to Mercury, Venus, Mars, Ceres, and Schwassmann-Wachmann II. With the same launch vehicle, a payload of 100 kg could be delivered to Jupiter and Saturn, but a larger booster is required for an all-chemical Pluto flyby.

There are two curves for the gaseous-core nuclear engine. The curve with the higher mass requirements has a specific impulse of 2000 sec, while the other has a specific impulse of 3500 sec.

Of particular interest is the comparison of the light weight advanced nuclear, the gaseous-nuclear, and the pulsed nuclear propulsion systems. The light weight advanced nuclear propulsion system appears to be a good candidate for flyby missions to all objective bodies when considering payloads between 1000 kg and 10,000 kg. A critical factor in this observation is minimum engine weight. As an example, for a selected payload of 4535 kg, the Neptune flyby

composite (Fig. 117) suggests the use of the light weight advanced nuclear engine. The mass requirement is 11,600 kg. The engine weight for this mission is 2060 kg with an engine T/W of 10. However, if a minimum engine weight of 5000 kg were imposed, the mass requirement would increase to about 20,000 kg.

The adjacent gaseous-core nuclear curve has a specific impulse of 3500 sec and a minimum engine weight of 13,600 kg with an engine T/W of 2. For a payload of 4353 kg, the mass requirement is 26,727 kg. In comparing this gaseous nuclear curve with the light weight nuclear curve, it becomes apparent that the 2000-sec specific impulse system is more desirable for this Neptune flyby payload than the gaseous nuclear system with a specific impulse of 3500 sec. However, if one considers a payload of 45,000 kg, the 3500-sec specific impulse gaseous nuclear system is more advantageous, since the engine weight becomes a smaller fraction of the total inert weight, thus permitting an advantage for the higher specific impulse.

Generally, because of the lower minimum engine weight limit, and the higher engine T/W ratio, a light weight advanced nuclear rocket would provide lower mass requirements than a gaseous-core nuclear system of higher specific impulse over a range of payloads between 1000 kg and 10,000 kg.

It may be interesting to compare a 1400-sec specific impulse, which represents a nominal value for a liquid nuclear engine, to the 3500-sec gaseous nuclear engine. At a payload of 4500 kg, the two systems require initial masses which are approaching equality, again indicating the penalty associated with the high minimum allowable weight assumed for the gas-core nuclear rocket.

For the pulsed nuclear systems, the total mass of structure is somewhat different from other advanced nuclear systems, since no "propellant" tank inert weights are involved. The engine weight derived from given values of engine T/W includes all "inerts" associated with the energy source. Therefore, the total mass is defined as

$$MEO = W_{pL} + W_E + W_P$$

as compared to the more conventional relationship expressed as

$$MEO = W_{pL} + W_E + W_P + W_I$$

where MEO = mass in Earth orbit

W_{pL} = payload

W_e = engine weight

W_p = propellant weight

W_I = tankage inert weight

The pulsed nuclear system with an engine T/W of 3 and a nominal specific impulse of 1800 sec is comparable with the light weight advanced rocket and gaseous-core systems only at very large payloads, such as 200,000 kg and above. Here, the very large inert weights associated with the continuous-thrust systems greatly penalize the effectiveness of the high specific impulse.

It should be emphasized that the data shown in the composite plots are for nominal values of specific impulse and engine T/W ratio. Also the "nominal" values may actually be "high" or "low" values, depending on one's postulated state of the art. Comparisons of propulsion systems with other values of specific impulse can be made by utilizing the composite plots and the individual plots (Figs. 51 through 110).

Composite plots of mass requirement versus trip time are presented in Figs. 121 through 130 for all propulsion systems and a payload of 4535 kg. As the energy requirements increase with reduced trip time, the chemical and solid nuclear systems indicate rapid increases in the mass requirements. Although the light weight advanced nuclear system is desirable for most of the longer trips to the planets, the 3500-sec gaseous-nuclear system becomes desirable for the shorter trip times. An example is shown in Fig. 126 for a Uranus flyby. The crossover point between the light weight and gaseous nuclear systems occurs at a trip time of about 800 days. Again it should be emphasized that the minimum-engine-weight limit imposed on the engines will alter the conclusions somewhat. However, as described earlier, the light weight nuclear system appears desirable for the longer trip durations to the respective planets, even if one were to impose a large (5000 kg) minimum engine weight limit and consider payloads in the vicinity of 4500 kg.

Jupiter Swingbys

Flyby missions to Saturn, Neptune, and Uranus were computed using Jupiter swingbys to perturb the transfer trajectories. A range of payloads was used for the various types of propulsion systems.

The Jupiter swingby velocity requirements, launch dates, and passage distances are shown in Table V. Figures 131 through 145 present the mass requirements as a function of trip time and payload. The circled points represent actual trips, with the connecting lines merely indicating the trends.

A direct comparison of swingby-flybys and ballistic flybys is made in Figs. 146 through 157 for a payload of 4535 kg and for various propulsion systems. Generally, large savings in trip time can be realized for a given initial mass; conversely, considerable mass savings may occur for a given trip duration. For chemical propulsion, the mass requirements are reduced about 20% to 40% for Jupiter swingbys to Saturn and about 40% to 50% for swingbys to Uranus and Neptune. The mass savings with solid-core nuclear propulsion are about 30% to 40%. The 2000-sec specific impulse light weight advanced nuclear and gaseous nuclear propulsion systems indicate about a 30% reduction. Finally, the mass savings with a 3500-sec gaseous nuclear system are very small; generally less than 10%.

Solar Flybys

Flybys which pass within 0.05 AU of the sun were computed for various payloads and Earth launch dates. The launch dates were chosen to coincide with Earth's perihelion and aphelion positions. The results of the mass computations are shown in Figs. 158 and 159. The effect of Earth launch date on initial mass is small, while the difference in trip time is only three days.

Unmanned Orbital Probes

The mass requirements for the orbital probes are shown in Figs. 160 through 202 for the objective bodies outlined in Table II. The same range of payloads as for the flyby missions are also used here. Again, the mass requirement variation with specific impulse is a function of payload and target planet. Examining chemical propulsion missions to Venus, an increase in specific impulse from 400 to 470 sec decreases the mass requirement by about 30% for the entire payload range. A flyby to Mercury shows a mass reduction of 67% for small payloads and 50% for large payloads.

An orbital stopover with chemical propulsion to an outer planet, such as Neptune, is very difficult from an energy standpoint. In Fig. 192, it is shown that an increase in specific impulse from 400 to 470 sec reduces the initial mass requirements by about 80%. From Fig. 193, the same reduction in mass for a light weight advanced nuclear system would require a specific impulse increase from 1400 to 2500 sec for a 10,000 kg payload.

As with flybys, it is convenient to combine the results of the individual engine data in composite plots as shown in Figs. 203 through 211. For Venus and Mars stopovers, the chemical system is desirable for payloads below about 500 kg. For Mercury and the outer planets, where the energy requirements are high, the chemical system mass requirements are very large compared with other propulsion systems.

The light weight advanced nuclear system with a specific impulse of 2000 sec appears desirable for payloads of 1000 kg to 10,000 kg, as with the flyby

probes. The same conclusions can be formulated when comparing the light weight, gaseous, and pulsed nuclear systems. In Fig. 205, a Mars orbiter with a payload of 4535 kg and using light weight advanced nuclear engines requires an initial mass of 10,630 kg. The engine weight for the Mars capture stage and the Earth departure stage is 907 kg. If a minimum engine weight of 3000 kg is imposed, the mass requirement becomes about 17,000 kg, which is still considerably less than either of the gaseous-core nuclear systems. A 5000-kg min engine weight limit would increase the light weight nuclear mass requirement to 25,000 kg, which is still less than the other nuclear systems and about comparable with the chemical system.

For low-energy missions, such as Mars and Venus, and with the nominal values of specific impulse chosen, the light weight advanced nuclear system provides minimum mass requirements for payloads in the vicinity of 4500 kg with the minimum engine weight limitation of 907 kg used for this study. Increasing the minimum weight to 5000 kg does not change the conclusion. However, the 5000-kg limitation would make the chemical system more desirable for payloads below 2000 kg.

The pulsed nuclear system is advantageous only for very large payloads such as 200,000 to 300,000 kg.

A difficult mission to perform is to establish a synchronous orbit about the sun. Both the Earth departure speed and the velocity required to inject into the synchronous orbit are very high. Consequently, this mission can only be performed with advanced nuclear propulsion as shown in Fig. 210. The mass requirements for delivering various payloads to a rendezvous with the comet Schwassman-Wachmann II are shown in Fig. 211.

Since the previously mentioned figures represent the optimum orbital stopovers for each respective planet, it is desirable to investigate the effects of trip time on the initial mass requirements. Figures 212 through 219 present the initial mass requirements for a payload of 4535 kg.

For all planets except Venus and Mars, the mass requirements for chemical and solid-core nuclear systems are very high and are not competitive with the other propulsion systems. For Mercury, Venus, and Mars orbital stopovers, the light weight advanced nuclear system appears desirable. Trips to the outer planets generally favor the 3500-sec specific impulse gaseous-core nuclear system, especially for greatly reduced trip durations.

Jupiter Swingbys

As with the flyby probes, the possibility of using Jupiter swingbys for orbital stopover missions to the outer planets was investigated. The trajectory data and velocity requirements are presented in Table VI. The corresponding mass requirements are shown in Figs. 220 through 233 for

various payloads and trip times. These data are superimposed on the direct stopover data for comparable trip times and a payload of 4535 kg and presented in Figs. 234 through 242. Generally, these composite plots indicate that there is no advantage in utilizing Jupiter swingbys for orbital stopover missions to the outer planets. Although the Earth-departure velocities are lower, as in the swingby-flybys, the planetary arrival velocities are higher for comparable total trip durations. The resulting mass requirements are equal to or higher than the mass requirements for the direct stopover trips. In comparing Tables VI and II, it can be seen that the Earth launch dates for the swingbys are quite different from those for the ballistic missions, since it is necessary to have the swingby planet and target planet in the proper relative heliocentric positions.

One-AU Inclined Orbits

The scope of this study includes an analysis of establishing inclined circular orbits about the sun with a radius of one AU. These orbits are inclined at angles of 15, 30, 45, and 60 deg to the ecliptic. The energy requirements are based on an optimum triple plane-change model, such that the required inclined circular orbit is established by three impulses and three plane changes (Ref. 2). The following table summarizes the pertinent parameters of each maneuver for a payload of 45,360 kg.

Inclination deg	First Impulse		Second Impulse		Third Impulse	
	ΔV - kps	ΔI - deg	ΔV - kps	ΔI - deg	ΔV - kps	ΔI - deg
15	3.24-3.43	2.0	6.95	11.0	0.388	2.0
30	3.28-3.54	3.7	13.50	22.6	1.19	3.7
45	3.70-3.97	4.7	14.30	35.6	3.27	4.7
60	7.40-7.91	1.7	3.90	56.6	10.40	1.7

The range in ΔV for the first impulse is due to the varying gravity losses which occur for different types of propulsion systems.

Figures 243 through 247 present the mass requirements as a function of payload for various types of propulsion. The first noticeable trend is that more mass is required for the 45-deg inclination than for the 60-deg inclination.

The total characteristic velocities are about the same for inclinations of 45 and 60 deg as shown in the above table. However, the ΔV distribution among the three impulses is more favorable for the 60-deg orbit. A theoretical optimum distribution of three ΔV 's would occur if each ΔV were equal, or approximately 7 kps. Consequently, since the ΔV distribution for the 60-deg orbit is more nearly optimum, the staging effects result in lower mass requirements.

The mass computer program indicates that 15-deg orbits can be achieved with chemical and solid-core nuclear propulsion, but not 30-deg or higher inclinations. Also, the mass requirements are so large that an Earth launch booster larger than a Saturn V would be required without recourse to orbital assembly.

With upper stages of gaseous nuclear propulsion, payloads of 20,000 kg to 40,000 kg can be put into a 15-deg inclined orbit using a Saturn V Earth launch booster. For higher inclinations a light weight advanced nuclear rocket or a gaseous-core nuclear rocket with a 3500-sec specific impulse would be required. The hypothetical post-Saturn booster would make it possible for the light weight and gaseous nuclear systems to deliver payloads of 60,000 kg to 300,000 kg to a 45-deg orbit and payloads up to 500,000 kg to a 15-deg orbit.

A short analysis was performed, considering a single impulse to achieve a required inclined orbit. This analysis assumes that the parking orbit at Earth is inclined at the proper angle to the ecliptic such that only the Earth departure impulse is required. With this assumption, the velocity requirements are generally less than for the three-impulse maneuver.

A single impulse maneuver from a parking orbit which is in the ecliptic would have velocity requirements considerably larger than those for the optimum three-impulse technique. The following table summarizes these total velocity requirements.

Type of Maneuver	Total Velocity - kps			
	Orbital Inclination			
	15 deg	30 deg	45 deg	60 deg
Optimum 3-Impulse	10.6	17.0	21.3	22.0
Single Impulse from Inclined Parking Orbit	5.7	11.3	17.6	24.0
Single Impulse from Parking Orbit in Ecliptic	14.6	18.5	22.8	28.8

Mixed-Propulsion Orbital Stopovers

Previous data have applied to transportation systems which use the same type of propulsion systems for each major impulse. For orbital stopovers, the Earth escape and planetary capture maneuvers are performed by the same type of engine. Since the payload of each stage varies considerably for the various objective bodies, it is desirable to investigate the mixing of propulsion types such that the most desirable characteristics of each engine may be exploited. Orbital stopovers to Venus, Jupiter, and Neptune were analyzed with propulsion consisting of chemical, solid nuclear, light weight advanced nuclear, and gaseous nuclear systems. The corresponding mass requirements

are presented in Figs. 248 through 259. The dashed line on each curve represents the use of the same capture-stage propulsion system as is used for Earth escape.

For a minimal-energy mission, such as to Venus, Figs. 249, 250, and 251 indicate that, for payloads up to 500 kg, a chemical capture stage will provide minimal mass requirements for each type of Earth-escape stage, with the light weight nuclear Earth escape providing the minimum mass combination. For larger payloads, up to 60,000 kg, the light weight nuclear capture stage appears the most desirable for each type of Earth escape stage. Above 60,000 kg, the gaseous nuclear capture stages become more efficient and produce the lower over-all mass requirements.

Orbital stopovers to an outer planet, such as Neptune (Figs. 256 to 259), have different characteristics with mixed propulsion systems. The chemical capture stage is not as efficient as the high-specific-impulse nuclear systems throughout the entire payload range. Light weight advanced nuclear propulsion is desirable as a capture stage up to payloads of about 20,000 kg, above which the gaseous-core nuclear system is more favorable.

The chemical Earth-escape mission can benefit by using a higher-specific-impulse capture stage. This conclusion also applies to the solid-core nuclear Earth escape propulsion. For a payload of 10,000 kg, the over-all minimum mass occurs with a 3500-sec specific impulse gaseous-core nuclear Earth escape system and a light weight advanced nuclear engine for the Neptune capture stage.

Manned Orbital Stopovers

The analysis of manned interplanetary missions includes orbital stopovers. For these missions, no excursion module is utilized. The vehicle establishes an orbit about the objective body for a designated stay time and returns to Earth. The payloads for these missions are derived from the spacecraft scaling laws and vary with the mission trip time because of the definition of the life support system weight as defined in Eq. (28). Five men are employed for the orbital stopover mission. The stay time in planetary orbit is varied from zero to 60 days, with the exception of Mercury missions as described under the section on trajectory analyses.

For each type of propulsion system, the mass requirements are computed as a function of specific impulse, trip time, and planetary stay time. Figures 260 through 299 present the variation of initial mass with specific impulse for all objective bodies and propulsion systems. Also indicated on these curves are maximum and minimum values of mass, corresponding to variations in the inert weight scaling laws and the engine T/W ratio. The "maximum" values were derived by combining all the highest inert weight parameters with the lowest engine T/W ratio to obtain a hypothetical upper limit. Likewise, the "minimum" values were obtained by combining the lower inert

weight parameters with the highest engine T/W ratio. The range of mass requirements between the maximum and minimum points should encompass the most optimistic and pessimistic viewpoints, with the nominal representing a good approximation.

The effects of specific impulse on initial mass requirements are a function of the objective body and the type of propulsion system. For chemical propulsion, an increase in specific impulse from 400 to 470 sec reduces the mass requirements by 38% for a low-energy trip, such as to Venus, and by as much as 68% for a mission to Pluto. The Mercury mission indicates a mass saving of over 70%.

Increasing the specific impulse of a solid-core nuclear system from 700 to 900 sec provides a mass saving of 41% for the easier Venus mission and 75% for the difficult Mercury mission. The Pluto mission (Fig. 296) with a 700-sec engine did not run on the mass computational program, thus indicating that the combination of lower specific impulse, relatively high engine weight, and high velocity requirements result in a mission which cannot be performed unless the stages are larger than required for the payload.

For all objective bodies except Mercury, the variation of mass with trip time is computed for nominal values of specific impulse and for stay times ranging from zero to 60 days. It is more convenient to present the Mercury data (Figs. 300 to 305), as a function of outbound and return transfer durations, since the stay times cannot be conveniently held as independent variables and still define optimum missions. The stay times for each mission are tabulated on each figure and are defined by symbols.

Figures 306 through 345 present the mass requirement variation with trip time and stay time. For the outer planets, including Jupiter, the effects of stay time on mass are quite small for the long-duration trips, becoming more pronounced at the shorter trip durations until a factor of two in total mass occurs between a zero-day and a 60-day stopover for a 1010-day Jupiter manned orbital stopover (Fig. 318).

Mars, Venus, and Mercury missions are quite sensitive to stay time, especially with chemical and solid-core nuclear propulsion. Even the high-specific-impulse nuclear systems show appreciable increases in mass with an increase in stay time. An example is shown in Fig. 314 for a light weight advanced nuclear system. The 250-day Mars mission utilizing a 60-day stopover requires three million kilograms of initial mass compared to about one-half million kilograms for a zero-day stopover.

Although there are many parameters involved in defining the system mass, a general comparison of the relative merits of the various types of propulsion systems can be made from composite plots as shown in Figs. 346 through 352. Here, the nominal values of specific impulse are retained. As would be

expected the high-specific-impulse nuclear systems are very efficient with the high payloads involved with the manned missions. For the longer trip times to Venus and Mars, either a light weight nuclear rocket or the 3500-sec gaseous nuclear rocket is desirable. For shorter trip times and missions to Mercury and the outer planets, the 3500-sec gaseous nuclear engine produces the lowest system mass.

Venus Swingbys

Recent studies throughout the industry have indicated that a Venus swingby on either the outbound or inbound leg of a Mars stopover mission may provide an over-all reduction in the mass requirements. Many of these studies have considered atmospheric braking at Mars as a capture maneuver. Since this braking scheme has the same effect on the mass requirements as would a very high specific impulse, the capture ΔV does not affect the system mass appreciably. However, if propulsive braking is used for Mars capture, the capture ΔV is influential. From Ref. 3 an optimum Venus swingby mission has an Earth escape hyperbolic excess speed of 0.167 EMOS, a Mars capture speed of 0.258 EMOS, a Mars departure speed of 0.100 EMOS, and a trip time of 540 days with a 10-day stopover.

A comparable standard stopover would have speeds of 0.1124, 0.1209, and 0.262 EMOS for Earth escape, Mars capture, and Mars escape, respectively. Mass requirements for these missions are shown in Figs. 312 through 316 for the various propulsion systems. The Venus swingby requires 60 to 70 days more duration and has mass requirements slightly higher than the zero to 20-day standard stopovers. This result does not necessarily indicate that swingbys cannot be found which will have lower mass requirements. However, the flexibility in choosing swingby missions is quite limited. It is necessary to do a complete analysis of Venus swingbys with propulsive braking at the planet to define the optimum missions. Obviously, the optimum missions assuming atmospheric braking do not suffice when propulsive braking is substituted.

Mixed-Engine Missions

As with the unmanned orbital probe analysis, it is desirable to investigate the effects on system mass of mixing the types of propulsion for the manned orbital stopovers. For this analysis, one engine type is used for planetary capture and departure and another for Earth escape. Figures 353 through 376 present the mass requirements as a function of trip time and stay time for Mars and Jupiter missions.

Composite plots are presented in Figs. 377 through 384 for Mars 20-day stopovers and Jupiter 60-day stopovers. The light weight advanced and gaseous nuclear systems are desirable as planetary stages for all Earth-escape propulsion systems, especially for chemical and solid nuclear propulsion. The

lowest mass requirement for the Mars mission occurs with an all-light weight nuclear system as shown in Fig. 379. However, this observation reflects a fine-line difference between an arbitrary use of light weight advanced nuclear rockets and 3500-sec gaseous-core nuclear rockets for the various stages. This small difference is evident when comparing Figs. 379 and 380.

Similar observations for the Jupiter missions reveal that missions with chemical or solid-core nuclear Earth escape stages benefit the most from gaseous-core nuclear upper stages ($I_{sp} = 3500$ sec).

Manned Landing Missions

An analysis of manned landing missions to Mercury, Mars, Ceres, and the largest moons (Table III) of Jupiter and Saturn was performed, using various sizes of excursion modules. The excursion module size is defined as a fraction of the total spacecraft weight, which includes the excursion module. The spacecraft weight is defined as for the manned orbital missions except the crew size is increased to eight men. The excursion module plus the spacecraft comprise the total useful payload. The excursion module is then expressed as a fraction of this payload. Figures 385 through 409 present the mass requirements as a function of specific impulse and excursion module weight fraction. The mass requirements for chemical and solid-core nuclear systems are beyond the single launch capability of a post-Saturn vehicle with an orbital payload capability of 680,400 kg to 907,200 kg. For Mars, excursion module weight fractions of 0.50 to 0.60 are achievable with the light weight advanced or gaseous-core nuclear systems with 2000-sec specific impulse. Generally, manned landing missions can be performed with either light weight advanced nuclear or gaseous nuclear systems for excursion module weight fractions of about 0.50, and a single post-Saturn Earth launch booster.

As with the manned orbital stopovers, the effects of trip time on total mass are presented in Figs. 410 through 439 as a function of stay time and excursion module weight fraction. A direct comparison of engine performance is presented in Figs. 440, 441, 442, and 443 for Mercury, Mars, Jupiter, and Saturn for an excursion module weight ratio of 0.20. The 3500-sec, gaseous nuclear propulsion system provides minimum-mass landing missions to Mars, Jupiter, and Saturn. This conclusion is based only on the comparison of engine performance using "nominal" values of specific impulse.

Limits on System Mass

The bulk of the data presented has been derived from nominal values of the inert weight constants and nominal values of engine T/W ratio. If one combines all high values of the inert weight constants simultaneously, an upper limit can be inferred. Likewise, a combination of minimum values defines an optimistic viewpoint toward the mass requirements.

Flyby Probe Missions

The effects of combining the maximum and minimum values of the inert weight constants on total mass are shown in Figs. 444 through 452 for Mars, Jupiter, and Neptune flyby missions in combination with the various propulsion systems and for various payloads. For Mars flybys, the mass requirements vary about 60% for small payloads (454 kg), to about 25% for a payload of 45,360 kg. Comparable variations between the maximum and minimum values occur for solid nuclear propulsion. For light weight advanced nuclear systems, the variation is only about 15% for all payloads.

The mass variations for the more ambitious flybys to Jupiter and Neptune are quite sizable, especially for chemical and solid-core nuclear systems. This variation is as much as an order of magnitude.

Orbital Probe Missions

As with the flybys, the effects of the maximum and minimum combinations of the inert weight scaling constants and engine T/W ratios on the mass requirements are analyzed. The data are shown in Figs. 453 through 460 for Mars, Jupiter, and Neptune.

Since two major impulses are involved for orbital stopover, the spread between the maximum and minimum limits is increased substantially due to the compound effects of the two impulses. With chemical propulsion, the mass varies as much as 200% for small payloads, to 60% for large payloads, as compared with equivalent flyby percentages of 60% and 25%, respectively. The mass variation for liquid nuclear propulsion is about 30%.

Figure 456 shows a minimum curve only. Previously, the nominal data were too high for the mass computational program to produce values of mass required. The combination of nominal scaling constants yielded a situation in which the inert weight fractions were too high for the propulsive energy.

Manned Landing Missions

The manned missions, which use three major propulsive maneuvers, show an even greater sensitivity to the inert weight constants than either the flybys or orbital probes, considering the relatively large payloads which tend to reduce this sensitivity. Figures 461 through 480 present the manned landing data as a function of excursion module weight fraction. One extreme is shown in Fig. 477, where the variation between the limits is an order of magnitude.

Increasing specific impulse tends to reduce the sensitivity of the inert weight scaling constants. Conversely, increasing energy requirements tends to increase the sensitivity.

Parameter Variations

Although upper and lower limits on system mass can be defined by varying the scaling constants simultaneously, it is desirable to separate the effects of each variable and determine its sensitivity. To illustrate these variations, orbital probe missions to Mars, Jupiter, and Neptune were analyzed using a 4536-kg payload. Figures 481 through 483 present the effects on total mass by varying the propellant tank structural constant, A , defined in Eq. (27) and keeping all other variables nominal. For the light weight nuclear engine, in Fig. 481, there is no variation in mass between " A " values of 0.05 to about 0.10. This phenomenon is quite proper. In the mass computational program, if the meteoroid-protection skin thickness is greater than the structural skin-thickness requirement, the former is used in the mass computations. For the Neptune stopover mission (Fig. 483) no variation in mass occurs for the entire range of " A ".

The variation of system mass with the insulation and boil-off constant is presented in Figs. 484, 485, and 486. For the relatively low-energy Mars mission, doubling the insulation weight increases the total mass by only 10% to 15%. However, for Jupiter, the same change in insulation will increase the mass by 70%.

For Neptune, doubling the insulation weight from the nominal value increases the total mass by 75%.

A critical parameter governing the system mass is the weight (skin thickness) requirement for meteoroid protection, especially for missions of long duration and/or that pass through the asteroid belt. These requirements are shown in Figs. 487 through 489. For the Mars mission, a 50% change in the meteoroid protection constant will change the mass requirements 5% to 10%. The Neptune mission (Fig. 489) indicates that a 50% change in " C " from the nominal will change the system mass by a factor of 2.5 for the light weight advanced nuclear engine and a factor of 5.0 for the chemical engine.

Incorporating the Whipple-bumper fabrication technique for the Mars mission will have little effect on the total initial mass requirements or the sensitivity of this mass to the protection requirements. However, for a mission, such as to Neptune, which has a long duration and passes through the asteroid belt, the Whipple bumper would have a considerable effect on the system mass, as illustrated in Fig. 469. The Whipple bumper data points assume the following definitions for the protection weights, as suggested by NASA.

$$\Delta W_m = 0 \text{ if } 0 \leq W_m \leq W_s$$

$$\Delta W_m = W_m - W_s \text{ if } W_s < W_m \leq 4/3 W_s$$

$$\Delta V_m = \frac{1}{3} W_s \text{ if } \frac{4}{3} W_s < W_m \leq 5\frac{1}{3} W_s$$

$$\Delta W_m = \frac{1}{4} W_m - W_s \text{ if } W_m > 5\frac{1}{3} W_s$$

where W_m = meteoroid protection weight using single skin thickness

W_s = tank weight (skin thickness)

ΔW_m = incremental weight to be added to W_s

The meteoroid protection requirement alone can dictate whether a mission can be performed with a given launch booster. From Fig. 489 it can be seen that a 2000-sec specific impulse, light weight advanced nuclear propulsion system coupled with a Saturn V booster could deliver 4563 kg to an orbit about Neptune if the nominal value of "C" is used. However, if "C" is increased only 10% the mission cannot be performed.

The effects of engine T/W ratio on system mass were analyzed for orbital missions to Mars, Jupiter, and Neptune, using a 4563 kg payload. For chemical systems, increasing the engine T/W ratio from 66 to 100 decreases the mass requirement by only 3.5% for the Neptune mission. The Mars mission indicates 0.5% decrease in mass.

An advanced light weight nuclear system with an I_{sp} of 2000 sec shows essentially no effect on mass for the Mars mission when the engine T/W is increased from 5 to 20. The Jupiter mission indicates a 30% decrease in mass, while the Neptune mission shows a mass reduction of about 11%.

Earth Entry Velocities

One ground rule of this study was that all Earth-return missions will use atmospheric braking for decelerating the entry vehicle upon Earth arrival, and that missions would be chosen such that the entry speed would not exceed about 20 km/sec, which corresponds to a hyperbolic excess speed of about 0.56 EMOS.

The Earth entry speeds are tabulated in Table III in terms of hyperbolic excess speed. For Mercury missions, the entry speed decreases as the return leg time and stay time decrease. The optimum 1980 Mars mission indicates high entry speeds which encourage the use of Venus swingbys to reduce this speed, even if a mass advantage is not apparent.

Venus missions have relatively low Earth entry speeds which increase slightly as the stay time is increased. The Ceres missions show high entry speeds for the longest trip duration and a greatly reduced entry speed when the trip time is reduced about four months.

The Earth entry speeds returning from missions to the outer planets generally vary between 15 km/sec and 18 km/sec, except for greatly reduced trip times where the speeds can increase to 22 km/sec.

For some planets, it is possible to find return trajectories which have lower Earth arrival speeds. However, the planetary departure speeds increase simultaneously.

As an example, consider the first Ceres mission tabulated in Table III where the Earth arrival speed is 0.4677 EMOS. If the return transfer time is reduced 30 days, the Earth entry speed becomes 0.3619 EMOS. However, the Ceres departure speed increases to 0.2644 EMOS from 0.2502 EMOS. For this particular departure date of 2444790, a minimum Earth arrival speed of 0.3375 EMOS is possible with a return transfer duration of 370 days. The corresponding Ceres departure speed is 0.2766 EMOS.

Generally, for the outer planets, the Earth arrival speeds minimize simultaneously with the corresponding planet departure speeds. Thus, the Earth entry speeds are essentially minimized for the specified return transfer time.

Trajectory Analysis for Low-Thrust Systems

Variable Thrust-Constant Power Trajectories

For power-limited propulsion systems, the final mass of a low-thrust interplanetary vehicle is given by

$$\frac{1}{M_f} = \frac{1}{M_0} + \int_0^T \frac{a^2}{2P(X,t)} dt$$

where M_f and M_0 are the final and initial masses of the vehicle, respectively, $a(t)$ is the thrust acceleration of the vehicle over the powered flight time, T , and $P(X,t)$ is the kinetic energy in the exhaust jet relative to the vehicle; X represents the vector of state and control variables. For the particular case of a thruster that is characterized by completely variable specific impulse, I_{sp} , Ref. 4 has shown that by minimizing the integral

$$J = \int_0^T \frac{a^2}{2P(X,t)} dt$$

payload mass is maximized and powerplant mass is minimized. It is noted that for the case of completely variable I_{sp} , considered in this analysis, the

thrusting and steering schedule, $a(t)$, is completely independent of the powerplant characteristics during the minimization of J , but the selection of a powerplant and its associated propulsion parameters is contingent upon the condition that the chosen power source must have the capability of delivering the required time-varying thrust magnitude, $|M(t) \cdot a(t)|$, where $M(t)$ is the vehicle mass. Except for this coupling, the analysis leading to optimization of the powerplant characteristics and the analysis leading to optimal thrusting (in the sense that J is minimized) may be carried out independently.

Optimal low-thrust trajectories employing variable and unconstrained I_{sp} are not consistent with present propulsion system technology, and therefore steering and thrusting schedules obtained using this analysis cannot be used in actual missions. Yet, such solutions do have practical value in that they yield upper bounds on attainable payload masses for the missions considered and are therefore valuable in the evaluation of the performance of vehicles powered by either constant-thrust or constant- I_{sp} , variable-thrust engines. References 5, 6, 7, and 8 have demonstrated that important comparisons can be made between the optimal variable-thrust and optimal constant-thrust trajectories and propulsion parameters, where the engines are operating at constant power. The present analysis makes comparisons of the trajectories and the associated propulsion parameters for vehicles which have basically different propulsive power sources but operate at completely variable thrust magnitude. Again, the conclusions which might be drawn as a result of such comparisons (as to which power mode is most advantageous for a particular mission) could not be implemented by actual thrusters, but it is possible that such analysis may assist the systems engineer to more easily match powerplant requirements to over-all mission requirements where the flights are made by means of the more feasible constant- I_{sp} , variable-low-thrust propulsion system.

This analysis was performed under the following assumptions:

1. The vehicle is capable of delivering completely variable I_{sp} . The thruster efficiency, $\eta(I_{sp})$, is taken as constant throughout the powered flight. The validity of this assumption is subject to substantiation after the optimal thrusting schedule has been obtained. If the power in the exhaust jet is specified above a certain limit, then the assumption is valid with relatively small error (Ref. 5). The functional form of $P(X,t)$ is chosen to represent constant power, power obtained through radioisotope decay, or power obtained through the conversion of the sun's energy to electrical power by the use of solar cells.
2. The departure and destination planets associated with the trajectories are nongravitating points in space; the only gravitational force acting upon the vehicle is that induced by the sun's mass.
3. The planetocentric and heliocentric trajectories are computed separately and matched such that the asymptotic velocity of the

vehicle in the planetocentric frame is added vectorially to the heliocentric velocity of the planet to give the boundary value of velocity for the interplanetary trajectory.

The governing differential equations for optimal trajectories are derived in Appendix D; also, the transversality conditions which must be satisfied at the boundaries for the different trajectory modes are set forth there. The associated two-point boundary value problem is solved numerically by use of the finite-difference Newton-Raphson algorithm as set forth in Appendix C. The explicit form of the matrices required by this algorithm is set forth in Appendix D.

Power Modes

The constant-power mode for low-thrust interplanetary transfer has been treated extensively in the literature (Refs. 4 through 8). In particular, this problem has been recently investigated through the use of the Newton-Raphson algorithm in Refs. 9, 10, and 11, and the optimal trajectories and mass fractions generated for several missions are set forth there. Therefore, the constant-power, variable-low-thrust transfers are used as the nominal optimal transfers in this analysis, and the trajectories and system parameters of the other power modes are compared with them.

The use of radioisotope fuels for auxiliary power supply has been treated in the literature, e.g., Refs. 12 and 13, and has been implemented in satellites; the use of radioisotope fuel for the propulsive power supply in interplanetary low-thrust vehicles has been advocated recently in Ref. 14. The dominant problem in constructing such a propulsion system is the lack of availability, in the necessary amounts, of radioisotopes that satisfy these basic criteria:

- a. The isotopic source should be of minimum size and weight; that is to say, the specific power, thermal kilowatts per kilogram, should be large.
- b. The half-life should be such that power-flattening requirements are a minimum.
- c. Radioisotope cost should be low.
- d. Necessary shielding should be low.
- e. The isotope must be chemically compatible with its containment material.
- f. The isotopic form must be compatible with mission safety requirements.

The feasibility of low-thrust missions performed with radioisotope power propulsion can be demonstrated by a brief example. Consider the 300-day Mars

flyby or impactor mission depicted in Fig. 490. The value of J is 1.225×10^{-3} kw/kg (or J is $1.225 \text{ m}^2/\text{sec}^3$, since $1 \text{ m}^2/\text{sec}^3 \equiv 10^{-3} \text{ kw/kg}$); the radioisotope is Po^{210} with a half-life of 0.38 years (Ref. 13). The rocket equation for power-limited systems given above may be rewritten as:

$$\begin{aligned} \frac{M_0}{M_T} &= 1 + \frac{M_0}{M_W} \alpha \int_0^T \frac{a^2}{2e^{-\gamma t}} dt \\ &= 1 + \frac{M_0}{P_0} J \end{aligned}$$

where $P_0 = M_W/\alpha$ is the electrical power delivered to the thruster at time $t = 0$, M_W is the powerplant mass (i.e., the mass of the power supply and power conversion machinery) and α is the powerplant specific mass, kilograms per kilowatt (Ref. 4). The thruster efficiency, $\eta(I_{sp})$, is assumed equal to 100% for this example. Then

$$P_0 = M_0 J \left(\frac{1}{M_0/M_T - 1} \right).$$

If it is assumed that M_0 is 10^3 kg and that M_T is 9.5×10^2 kg, then P_0 becomes 23.3 kw. With the additional assumption that the efficiency of conversion of thermal energy to electrical energy is 30%, the thermal power generated by the power source must be equivalent to 77.7 kw. The radioisotope Po^{210} is available in these amounts and has modest shielding requirements with a large specific power rating; further, its half-life of about 100 days meets minimum requirements. However, it is very costly - about \$26,500 a gram or a \$15,400,000 fuel bill for this mission. If the cost of this fuel can be reduced, as is envisioned for the future, it is seen that such a primary propulsion system is both feasible and desirable for a number of applications.

No attempt is made to choose a "best" radioisotope fuel, but a comparison of the trajectories associated with the use of various isotopes is made for a particular Mars rendezvous as illustrated in Fig. 491. The following sources or types of power are assumed:

<u>Radioisotope</u>	<u>Half-Life, yr</u>	<u>Decay Constant, γ</u>
(Constant Power)	∞	0.0
Pu ²³⁸	89.0	0.124×10^{-2}
Tl ²⁰⁴	4.0	0.276×10^{-1}
Ce ¹⁴⁴	0.78	0.141
Tm ¹⁷⁰	0.35	0.315

Since the thrusters using power sources having half-lives which are large compared with the trip time operate at very nearly constant power throughout the mission, the resultant trajectories (and fuel consumptions) are very close to those of the constant-power case. For this reason, only the envelope of the trajectories is given. As the power source half-life is decreased, the trajectories "drift" outward from that of the constant-power mode. This implies that as the half-life decreases further, the mission may be physically impossible to perform with such power sources. That this is so can be appreciated by considering the vehicle at its encounter with the destination planet, where the thrusting requirements for matching the planet's velocity could not be met if, in the case of radioisotopes with very short half-lives, the power available becomes vanishingly small.

Finally, there is a feature of the radioisotope power mode that considerably simplifies mission analyses, as was illustrated above in a cursory fashion. The exhaust power obtainable from a particular radioisotope at any given time is directly proportional to its mass at time $t = 0$. Hence, the mission thrusting requirements embodied in J lead directly to power source mass and shielding requirements. The result is a quick and reliable feasibility survey for missions utilizing this power mode.

The use of the sun's energy as a source of propulsion power for interplanetary vehicles has been discussed and investigated in the literature (Refs. 15, 16, 17, and 18). The present analysis has postulated that the power in the exhaust jet is proportional to $1/R^n$ where R is the heliocentric radius of the vehicle. The literature has shown that high temperatures greatly decrease the efficiency of solar cells (e.g., Ref. 15). Therefore, solar power expressed in this fashion gives generality to the solution that allows for the degradation of the power delivered to the exhaust jet during close passage by the sun. Figure 492 illustrates the form of the exhaust jet power in the solar-power mode for various values of n , normalized such that the power is unity at Earth departure. The heavier profile represents the case where $n = 2$ for $R \geq 1.0$ AU and $n = 1$ for $R < 1.0$ AU; it is seen that n can change values at $R = 1.0$ AU only, while still maintaining continuity in the power profile. In actuality the power derived from the solar cell would drop off markedly at about 0.65 AU, and the power profile shown would not be valid. However, it is felt that by inclining the cells to the sun's rays, the effect of temperature upon energy conversion efficiency

would be lessened and the power level could be maintained at some constant level in the close environs of the sun. No attempt is made in this analysis to choose a "best" value for n in the region $R < 1.0$ AU; the value $n = 2.0$ is used for the power mode comparisons that follow.

Power Mode Comparison

The comparison of the three power modes is separated into two parts; trips to interior planets and trips to exterior planets. Specifically, only trips to Venus and Mars are presented here, and the conclusions based upon these results are subject to substantiation by calculations for other interior and exterior planets.

As indicated above and set forth in detail in Refs. 4 and 6, for the variable low-thrust missions considered here, extremization of the payload fraction and the powerplant fraction is implicit in the operation of minimizing the integral J . For any J obtained in this analysis, whether or not it be the global minimum for the trajectory mode, power mode, and destination planet under consideration, the payload fraction and powerplant fraction are given by

$$\mu_{p1} \equiv \frac{M_{p1}}{M_0} = (1-\delta)^2$$

$$\mu_w \equiv \frac{M_w}{M_0} = \delta(1-\delta)$$

where M_{p1} is the mass of actual payload vehicle structure mass and $\delta^2 \equiv \alpha J$. These ratios are presented here and considered in the evaluation and comparison of the power modes. Further, these results are qualitative only, since the power in the exhaust jet at Earth departure, P_0 , has been set equal to unity in all numerical calculations. This does not affect the formal minimization of J , but merely implies that the rating of the power source is arbitrary. The analysis is done without the addition of excess velocities at the boundaries although the algorithm can handle this case (see Appendix D).

Mars Trips

Figures 493, 494, and 495 illustrate a comparison of the power modes for a Mars rendezvous trip of 300 days. The differences among the three trajectories are negligible as might be expected, since this particular set of departure and trip times is that of an optimum impulsive transfer listed elsewhere in this report, and therefore the J 's associated with these trajectories are in the neighborhood of the global minimum value of J . Inspection of the mass fractions indicates that constant power is best for this mission in the sense that, for acceptable values of α ($\alpha < 20$), the powerplant and payload fractions are minimum and maximum, respectively.

As indicated above, at the encounter of the destination planet, thrust magnitude requirements are increasing; yet, for the radioisotope power (the fuel is Po^{210} which has a half-life of 0.38 years) and the solar-power vehicles, the exhaust power is decreasing, thereby indicating that these power modes do not suit this trajectory mode as well as the constant-power mode.

Figures 490, 496, and 497 illustrate the comparison of the power modes for a Mars flyby or impactor mission for the same trip time and departure date. Again the differences among the three trajectory profiles are negligible. The transversality conditions for this trajectory mode cause the thrust magnitude to vanish at the final time (see Appendix D). This indicates that the decreasing power modes would be best suited to the flyby mode, but this example does not bear this out. J_c , the J for the constant power mode is the smallest and, again, leads to the best powerplant and payload fraction for acceptable values of α .

Further insight into these comparisons is obtained if trips that do not yield global minimum values for the associated J 's are investigated. Figures 498, 499, and 500 illustrate such a comparison for a Mars rendezvous. The radioisotope fuel used here has a half-life of 9.5 years; the power gained from its decay is essentially constant over the duration of the trip, approximately 0.45 years. Hence, the trajectories and corresponding J 's of the constant-power and radioisotope-power modes very nearly coincide. It is interesting to note that the solar-power trajectory for this mission moves inward toward the sun from the constant-power trajectory during the early portion of the trip and outward during the final portion. If such a maneuver is to be performed to obtain an increase in power, a larger increment of energy is gained by "dipping" toward the sun at the smaller distances from the sun precisely as is depicted in this example.

Figures 501, 502, and 503 illustrate a comparison of the power modes for a Mars flyby for the same trip time and departure date as the preceding example. Of significance here is the fact that the radioisotope power mode (with Ce^{144} as fuel with a half-life of 0.78 years) yields the minimum value of J , $J_r = 2.15 \times 10^{-3}$ kw/kg; correspondingly, the best values for the powerplant and payload fractions are produced by this power mode. Additional insight into this fact is gained from Fig. 504. This figure gives comparisons of the exhaust power to the thrust acceleration over the trip duration for each power mode. Clearly, the area under the thrust acceleration curve in the radioisotope power mode is the smallest of the three modes, a strong indication that this mode would yield the minimum value of J .

Venus Trips

Figures 505, 506, and 507 illustrate a comparison of the power modes for a Venus rendezvous trip of 170 days. Here, as in a Mars example, the

departure date and trip are those for an optimum impulsive transfer, and the differences among the three trajectories are negligible. Notable in this example is the fact that the solar power mode is the best in the sense that J_R is the smallest of the three modes. Whereas in the Mars trajectories the solar power was decreasing upon encounter with the destination planet, here the solar power is increasing as the thrust magnitude requirements are increasing. The question as to whether the use of a less advantageous form of the dependence of power upon the radius, say $\text{power} \propto 1/r^{1.5}$, would also lead to the conclusion that the solar power is the best mode for this mission, can only be answered by further investigation.

Figures 508, 509, and 510 illustrate a comparison of the power modes for a Venus flyby for the same trip time and departure date as the above rendezvous trajectory. Again the differences among the three trajectories are negligible. The values of J for the constant-power and solar-power modes very nearly coincide; these two modes appear as strange bedfellows based upon the results obtained from the exterior-planet analysis. The mass fractions for all three modes compare favorably with each other; however, this indicates that other factors, such as ease of powerplant construction, powerplant reliability, or powerplant cost, might be the overriding criteria in the final choice of a "best" power mode.

Figures 511, 512, and 513 illustrate comparisons of the power modes for a Venus rendezvous mission with a trip time and departure date that do not yield global minimum values of J . The radioisotope fuel has a half-life of 9.5 years and, as above, there is little difference between the constant power and radioisotope power trajectories and J 's. Again, the solar power trajectory passes closest to the sun along that portion of the trip where the distance from the sun of the nominal trajectory (constant exhaust power) is the smallest.

Figures 514, 515, and 516 illustrate comparisons of the power modes for a Venus flyby for the trip time and departure date of the preceding example. The solar power modes again yield the best mass fractions and execute the dipping toward the sun during the portion of the trajectory of closest passage.

Summary

The comparisons presented above indicate the following general statements with regard to variable low-thrust interplanetary flight.

- a. Rendezvous flights to the exterior planets are performed most advantageously by the use of the constant-power mode. Rendezvous flights to the interior planets are performed most advantageously by the use of the solar-power mode.

- b. Flyby or impactor flights to the exterior planets are performed most advantageously by the use of the radioisotope-power mode. Flyby or impactor flights to the interior planets are performed most advantageously by the use of the solar-power mode.

Since only relatively few missions have been investigated here, it is quite evident that a much more extensive investigation into power mode comparisons must be made before these general statements can be put forth as valid and well substantiated conclusions.

In particular, the model assumed for representation of the power obtained from solar cells in the region less than one astronomical unit from the sun, i.e., $\text{power} \propto 1/R^2$, is universally recognized as inaccurate. The determination of the precise model for solar power in this region must be the first step in the further study of this mode. Secondly, as indicated above, flights to all planets must be included in the investigation; a power mode found to be most advantageous for a Mars flyby cannot be arbitrarily assumed to be the most advantageous for a Pluto flyby. Finally, it is remembered that the power generated at time $t = 0$ by all power supplies was set equal to unity for these comparisons. This has the desired effect of putting the different power modes on an equal basis for the trajectory analysis, but it falsely implies that the cost per kilowatt delivered is the same for each power mode. This cost factor must also enter into conclusions regarding the best choice for a power mode.

Solar Probe Trajectory

An additional variation of the finite-difference, Newton-Raphson algorithm has been prepared. Appropriate modifications of the algorithm have been made to satisfy the transversality conditions required by a probe to a specified distance from the sun (see Appendix D). The examples illustrated here are flown in the constant-power mode only.

Figure 517 illustrates an exterior probe to 1.7 AU; the algorithm converges rapidly to this solution (in four iterations) from an optimal Mars rendezvous trajectory used as the initial guess. Figure 518 illustrates an interior probe to 0.35 AU. This solution was not as highly convergent as the exterior probe. It is noted in Appendix D that in the coefficient matrix, B_j , there are terms with R^7 as their denominator. At $R = 0.5$ AU, for example, this denominator is equal to 0.0078; it is readily seen how numerical difficulties could be encountered in interior probes. The convergence problems are overcome by "tracking" the solution from a more easily obtained solution by successively reducing the probe radius by a specified decrement after each solution has converged. The operation is illustrated in Fig. 518 where the initial guess was a Venus flyby solution and successive decrements of 0.10 AU were applied to the probe radius. Convergence of the algorithm was slow.

The trajectory calculations depicted in the figure took eight minutes of machine time on an IBM 7094. Figure 519 is an extrapolated curve based upon the values of J obtained from the above interior probe results. It is noted that as the probe passes closer to the sun the value of J increases, as is expected.

Constant-Exhaust Power, Constant-Thrust-With-Coast Trajectories

The analyses and computations presented thus far have yielded optimal variable- I_{sp} , low-thrust interplanetary trajectories. The more practical and useful case of constant thrust, constant exhaust power trajectories is investigated in the following paragraphs. The detailed analysis is presented in Appendix D; presented below are the salient points of that analysis and an examination of the results of the numerical computations of some typical Mars rendezvous missions.

As has been shown in Ref. 4, the solutions for (1) optimal thrusting and steering schedules and (2) optimal propulsion parameters cannot be uncoupled for the general case of power-limited, constant I_{sp} interplanetary flights. Clearly, when the exhaust power, P , and the thrust magnitude, $|T|$, are specified for a particular mission, i.e.,

$$P = \frac{1}{2} \dot{m} c^2 \equiv \beta_1$$

$$|T| = \dot{m} c \equiv \beta_2$$

the jet exhaust velocity, c , and the (outward) mass flow rate, $\dot{m}$, are completely defined throughout the mission. Further, $P = \eta M_w / \alpha \equiv \beta_1$; that is to say, for a given powerplant specific mass, the powerplant mass is also specified. Implicit in this statement is the assumption that the power available at the powerplant is converted with constant efficiency, $\eta(I_{sp})$, into kinetic energy at the thruster nozzle. In addition, the choice of β_1 and β_2 directly affects the value of the integral $\int_0^T a^2 dt$. In short, specification of β_1 and β_2 for a mission admits a solution for optimal control during the flight, but this solution will not, in general, lead to a maximization of payload mass, the ultimate goal.

However, an approximate method has been presented in the literature (Ref. 19) that allows the two operations of (1) optimizing the control and (2) optimizing the propulsion parameters to be uncoupled for the purpose of numerical computation of trajectories that are optimal in the sense that payload mass is maximized. These trajectories will, in general, include an optimum coast period. The basic assumptions of this method are:

- a. The minimum value of $\int_0^T a^2 dt$ is invariant with respect to the powerplant fraction, μ_w , the ratio of powerplant mass to initial total mass.

- b. The average thrust acceleration, $\bar{a}$, over a trajectory with minimum $\int_0^T a^2 dt$ also is invariant with respect to μ_w .

The validity of these assumptions is borne out by actual trajectory solutions and it will be seen that these approximations lead to very good comparison with exact results.

Analysis

In Appendix D it is shown that the governing differential equations are

$$\begin{aligned}
 \ddot{x} - \frac{\dot{m}c}{1-\dot{m}t} \frac{\alpha_p}{p} \frac{u}{R^3} + \frac{x}{R^3} &= 0 \\
 \ddot{y} - \frac{\dot{m}c}{1-\dot{m}t} \frac{\alpha_p}{p} \frac{v}{R^3} + \frac{y}{R^3} &= 0 \\
 \ddot{z} - \frac{\dot{m}c}{1-\dot{m}t} \frac{\alpha_p}{p} \frac{w}{R^3} + \frac{z}{R^3} &= 0 \\
 \ddot{u} - \frac{u(2x^2 - y^2 - z^2) + 3x(vy + wz)}{R^5} &= 0 \\
 \ddot{v} - \frac{v(2y^2 - x^2 - z^2) + 3y(wz + ux)}{R^5} &= 0 \\
 \ddot{w} - \frac{w(2z^2 - x^2 - y^2) + 3z(ux + vy)}{R^5} &= 0
 \end{aligned} \tag{37}$$

over the interval $0 \leq t \leq T$ with

$$\alpha_p = \begin{cases} 1, & 0 \leq t \leq t_1 \text{ and } t_2 \leq t \leq T \\ 0, & t_1 \leq t \leq t_2 \end{cases} \tag{38}$$

where t_1 , t_2 are the thruster switch-off and switch-on times, respectively, $p = (u^2 + v^2 + w^2)^{1/2}$ and $R^2 = x^2 + y^2 + z^2$. In addition, the following equations define the scaling of the Lagrange multipliers and the determination of the switching times

$$[p(0)]^2 = 1 \tag{39}$$

$$[p(t_1)]^2 = [p(t_2)]^2 \tag{40}$$

It is noted that for a rendezvous mission, when Eqs. (37) and the appropriate boundary conditions are expressed as central difference equations at each of n mesh points, Eqs. (37), (39), (40) and the boundary conditions represent $6n + 2$ equations in the $6n + 2$ unknowns $x_k, y_k, z_k, u_j, v_j, w_j, t_1, t_2$; $k = 0, 2, 3 \dots n-1, n+1, j = 1, 2 \dots n$.

In addition, the following two equations define the approximate values of μ_w and c , the exhaust velocity, such that the payload mass is maximized (Ref. 23):

$$\mu_w \left[1 - \left(\frac{2\mu_1}{1+\mu_1} \right) \left(\frac{\eta' c}{\eta} \right) \right] - \mu_1 (1 - \mu_1) = 0 \quad (41)$$

$$\bar{a} \alpha c - 2\mu_w \eta - \frac{\alpha J}{2} = 0 \quad (42)$$

where

$$\frac{1}{\mu_1} = 1 + \frac{\alpha J}{2\eta\mu_w},$$

$$\eta = \frac{1}{1 + (d/c)^2},$$

$$\frac{d\eta}{dc} \equiv \eta' \text{ and}$$

μ_1 is the ratio of total final mass to total initial mass.

Thus, the solution to the coupled problem may be computed in parts through the use of two computer routines. Assuming the initial values for $\dot{m}c$ and c and specifying the parameters α and d , Eqs. (37), (39), (40), and expressions representing the boundary conditions are solved by the modified Newton-Raphson algorithm yielding values for $J = \int_0^T a^2 dt$ and $\bar{a}$. These two values are submitted to the second algorithm, which is a search routine for finding the c and μ_w that simultaneously satisfy Eqs. (41) and (42). This solution is then utilized to up-date the values of $\dot{m}c$ and c :

$$\dot{m} = \frac{2\mu_w * \eta}{\alpha(c*)^2}, \quad \dot{m}c = \frac{2\mu_w * \eta}{\alpha c*}$$

where here $\eta = 1/1 + (d/c^*)^2$ and c^* and μ_w^* are the up-dated values of c and μ_w . These values, in turn, are resubmitted to the Newton-Raphson routine, which then computes up-dated values of J and $\bar{a}$. This process is repeated until the changes in μ_w and c between successive iterations satisfy an appropriate convergence criterion. This process has proved to be very strongly convergent for the cases considered; so much so, in fact, that the values of J and $\bar{a}$ may be up-dated after each iteration that is internal with respect to the modified Newton-Raphson algorithm (i.e., not requiring that J and $\bar{a}$ converge to definite values for each up-dated guess of $\dot{m}c$ and c), thus considerably shortening computation time. That this up-dating procedure can be efficiently carried out after each internal iteration leads to the conviction that this problem can be solved in an even more economical fashion by incorporating the propulsion parameter equations [(41) and (42)] into the Newton-Raphson algorithm. In this regard, see Appendix F.

A brief explanation of the method of attaining a first guess for the state, control, and propulsion variables required in order to initiate the computations is necessary here. As is well documented in the literature (e.g., Ref. 8), the value of J_v for an optimal constant-power, variable- I_{sp} trajectory is approximately 10% to 15% less than the J_c : $J_c = 1.15 J_v$. The initial guess for $\bar{a}$ is the geometric mean of the thrust acceleration for the variable-thrust solution,

$$\bar{a} = \left(\frac{J_v}{T} \right)^{\frac{1}{2}}$$

The initial guesses on the switching times t_1 and t_2 are then obtained in this manner: Letting T_p be the powered time for the constant thrust case, $T_p = T - (t_2 - t_1)$, and determining T_{min} to be the time at which the magnitude of the thrust acceleration of the variable-thrust case reaches a minimum value, one can write

$$T_p = \frac{J_c}{\bar{a}^2},$$

utilizing the expression for the geometric mean thrust acceleration for the constant-thrust case. Centering the coast phase, $t_2 - t_1$, about $t = T_{min}$, one has

$$t_1 = T_{min} - \frac{T - T_p}{2}$$

$$t_2 = T_{min} + \frac{T - T_p}{2}$$

The initial guesses of the state and control time histories are taken from the corresponding variable-thrust solution. Although these initial approximations seem rather gross (the possibility of better starting solutions certainly cannot be discounted), the numerical results presented in the next paragraphs required only 10 sec or less of computing time for the cases studied; the program is presently operational only for Earth-Mars and Mars-Earth rendezvous missions.

Computations and Results

Figure 520 is a projection upon the ecliptic plane of an Earth-to-Mars rendezvous trajectory using the constant-thrust, constant-power mode. In this case the values of I_{sp} and initial thrust acceleration, a_0 , are held fixed, hence the resultant payload fraction is not a maximum. The values of I_{sp} and a_0 are given in the figure and yield these values for the mass fractions: μ_{p1} is 0.8460 and μ_w is 0.232 where α and η are 1.0. Although the figure does not illustrate this fact, it was found that the constant-thrust, constant-power trajectory profile differed to only a very small degree from the variable-thrust profile. The value of J_v for the variable-thrust case was $5.78 \text{ m}^2/\text{sec}^3$. This yields mass fractions, μ_{p1} and μ_w , equal to 0.9010 and 0.0508, respectively. It is noted that the variable-thrust case, while requiring the larger powerplant, yields a 6.1% increase in payload over the constant-thrust case. Figure 521 is a graphical representation of the optimal thrusting and steering schedule for this case. The coasting phase is represented on the thrust direction curve by a dotted line; during this period the thruster must be reoriented for engine restart after 166 days of flight.

Finally, a comparison is made in Fig. 522 of the vehicle parameters generated by Melbourne and Sauer in Ref. 19 and those generated by the modified Newton-Raphson algorithm coupled with the external routine to maximize payload mass. The results of the former are obtained through a rigorous analysis utilizing the calculus of variations throughout and are considered to be exact numerical solutions to the problems considered. The solutions generated by the Newton-Raphson algorithm are the results of the approximate analysis presented in Ref. 19 and repeated above.

One factor has had an effect on the numerical solutions (although to what extent the values have been altered cannot be determined). The dates of Earth departure are not set forth explicitly for the trips cited in Ref. 23. Hence, these dates were approximated as well as possible through interpolation of curves depicting J versus Earth launch date which are presented by Melbourne and Sauer elsewhere (Ref. 8). It is felt that this fact has contributed to a small degree to the differences noted in the figure. The modified Newton-Raphson algorithm yields an exact value of the integral $\int_0^T a^2 dt$ for the mass flow rate and specific impulse to which the routine converges. Therefore, the best comparison between these

two methods is obtained at those trip times for which the respective values of J are most nearly equal. As an example, where T is 180 days and α is 1 kg/kw, the values of this integral differ by just 2 parts in 8080 and the corresponding mass fractions illustrate very good comparison. In fact, the two methods illustrate good to excellent comparison over all the missions that were examined.

The analysis and results presented above and in Appendix D bring together two strong methods in the realm of optimal trajectory and propulsion parameter investigations. The Newton-Raphson algorithm, on the one hand, modified to yield constant-thrust-with-coast trajectories, optimal in the sense that $\int_0^T a^2 dt$ is minimized, contributes to the solution the optimal control schedule and optimal positioning (in time) and duration of the coast phase plus its inherent rapid computing time. On the other hand, the approximate method of maximizing the payload fraction for various ranges of propulsion parameters (a la Melbourne and Sauer) contributes its excellent comparison to the results of the complete calculus of variations analysis and, also, imparts the uncoupling (for the purposes of numerical computation) of the optimal control portion and the optimal propulsion parameter portion of optimal constant-exhaust power, constant-thrust-with-coast trajectory investigations.

Mass Computations for Low-Thrust Systems

Hybrid-Thrust Optimization Procedures

The general approach to the study consisted first of determining the combination of hyperbolic excess speeds and low-thrust trajectory requirement which yields over-all maximum payload ratio for the ideal hybrid-thrust vehicle, and second, using the above information to compute the vehicle mass required on Earth parking orbit (with velocity losses). It was found expedient to utilize the above approach in order to eliminate the velocity loss aspect from the high-thrust mass ratio equations, thereby simplifying the optimizing procedure considerably.

Insofar as the actual mass calculations are concerned, the hyperbolic excess speeds and the low-thrust trajectory requirement found by the above means are used with the assumption that the vehicle gross mass computed therefrom is minimum with respect to the given payload. No further iterations are made. It should also be pointed out that the results of a given optimization are valid only for a fixed pair of departure and arrival dates and therefore do not include the influence of such dates on the hybrid-thrust system. The procedure must be repeated for a change in either the departure or arrival dates or both.

For given departure and arrival dates and the corresponding hyperbolic excess speeds, the trajectory program computes the minimum value of the

quantity $J = \int_0^T a^2 dt$, where a is the thrust acceleration over the powered time, T . This is the usual quantity associated with the variable low-thrust trajectory and is somewhat analogous to incremental velocity for high-thrust systems.

The following discussion briefly outlines the formulation and analysis of the high-low thrust system optimization. The analysis of the variable-thrust trajectory optimization under hybrid-thrust operation is discussed in the "Study Results" section under the subsection entitled "Trajectory Analysis for Low-Thrust Systems". The associated computer program is described in Appendix D.

High + Low + High-Thrust Operation

The flight profile usually associated with a planet-bound transfer (manned or unmanned orbiters) involves a high-thrust (chemical or nuclear) Earth-departure system, a low-(variable)-thrust heliocentric transfer, and a high-thrust capture into a parking orbit about the planet. The over-all payload-to-gross weight ratio for this mode is

$$\mu_L = \left[\frac{1 - \mu_1 \beta_1}{(1 - \beta_1) \mu_1} \right] (1 - \gamma_m \Gamma)^2 \left[\frac{1 - \mu_2 \beta_2}{(1 - \beta_2) \mu_2} \right]$$

where

$$\mu_{1,2} = \exp \left[\sqrt{v_{A,B}^2 \left(\frac{V_{\infty A,B}}{C_{1,2}} \right)^2 + \left(\frac{V_{e1,2}}{C_{1,2}} \right)^2} - \left(\frac{V}{C} \right)_{1,2} \right]$$

$$\gamma_m^2 \equiv \frac{\alpha_w J_m}{2}, \quad \Gamma^2 \equiv \frac{J}{J_m}$$

The term $(1 - \gamma_m \Gamma)^2$ represents the low-thrust system payload-to-gross weight ratio which has been maximized with respect to the powerplant mass. The other two terms are the payload ratios of the departure and capture stages. The expression μ represents the ideal mass ratio for the high-thrust system at departure from (subscript 1) or arrival onto (subscript 2) a circular parking orbit.

The quantities v_A , v_B , and Γ , respectively, represent the hyperbolic excess speeds on the initial and final boundaries and the intervening low-thrust J as a result of these speeds. They are normalized with respect to the all-high-thrust hyperbolic excess speeds ($V_{\infty A}$, $V_{\infty B}$) and the all-low-thrust J ($\equiv J_m$). The powerplant specific weight is α_w , and the high-thrust rocket exhaust velocity is C . The high-thrust step inert weight fraction, β , is defined as the ratio of the step inert weight to the weight of the

propellant plus inerts. The escape velocity V_e is evaluated at the parking orbit radius where the circular velocity is V .

From work performed under Contract NAS8-11309 (MSFC), the dependence of Γ^2 on v_A and v_B may be easily computed for a given set of departure and arrival dates. Hence for given values of γ_m , V_∞ , V_e , C , and V , the problem is to maximize

$$\mu_L = \mu_L (v_A, v_B, \Gamma)$$

subject to

$$\Gamma = \Gamma (v_A, v_B)$$

High + Low-Thrust + Atmospheric Entry

Employing an atmospheric entry system is assumed to be a mode suitable for Earth return legs wherein the crew and scientific materials are recovered, but the return spacecraft is not. In this case the payload ratio is given by

$$\mu_L = \frac{\left[\frac{1-\mu_1\beta_1}{(1-\beta_1)\mu_1} \right] (1-\gamma_m\Gamma)^2}{1 + \left(\frac{m_e'}{m_s} \right) \rho_E(v_B)}$$

where $\rho_E(v_B)$ represents the growth of a reference ablative entry system, m_e' , with the (normalized) entry hyperbolic speed v_B . In the present analysis $\rho_E(v_B)$ may be represented by a linear or exponential growth. The quantity m_s is strictly the return spacecraft exclusive of the entry system and the low-thrust propulsion system. As before, the dependence of Γ on (v_A, v_B) may be computed for a given set of departure and arrival dates.

High-Thrust + Constant-Thrust + High-Thrust

In this flight mode, the intervening low-thrust system operates under constant rather than variable thrust. The high-thrust systems on the boundaries function as before. The over-all payload ratio is thus

$$\mu_L = \left[\frac{1-\mu_1\beta_1}{(1-\beta_1)\mu_1} \right] \mu_{PL} \left[\frac{1-\mu_2\beta_2}{(1-\beta_2)\mu_2} \right]$$

where the notation used before applies and, in addition,

$$\mu_{PL} = \frac{1}{1 + \frac{(\gamma_m \Gamma)^2}{\mu_w \eta}} - \mu_w$$

which is the constant-thrust payload ratio to be maximized by determining the appropriate value of the powerplant fraction μ_w (Ref. 19). Note that, in contrast to the payload ratio $(1 - \gamma_m \Gamma)^2$ for the variable-thrust system, the maximum ratio must be found for each set (v_A, v_B) and, hence, for the resulting Γ . The optimum powerplant fraction which maximizes μ_{PL} is given by

$$\mu_{w0} = \frac{\mu_b (1 - \mu_b)}{1 - \frac{2\mu_b}{1 + \mu_b} \left(\frac{\eta'}{\eta} \right)}$$

where

$$\frac{1}{\mu_b} = 1 + \frac{(\gamma_m \Gamma)^2}{\mu_w \eta}$$

$$\bar{a} \alpha_w = 2\mu_w \eta + (\gamma_m \Gamma)^2$$

$$\gamma_m^2 = \frac{\alpha_w J_m}{2}$$

The thruster efficiency as a function of thruster exhaust velocity C could assume the form

$$\eta = \frac{1}{1 + \left(\frac{d}{C}\right)^2} \quad d = 20 \text{ km/sec (hypothetical, Ref. 19)}$$

or

$$\eta = \frac{1}{1 + \left(\frac{5000}{I_{sp} + 5000}\right)^2} - 0.03 \quad \text{electron bombardment}$$

or

$$\eta = \frac{1}{1 + \left(\frac{2000}{I_{sp} + 2000}\right)^2} - 0.06 \quad \text{heavy molecule}$$

The derivation of the equation for optimum μ_w is based on the assumptions that the average thrust acceleration $\bar{a}$ for a trajectory with given Γ is invariant with μ_w and that the minimum value of Γ is also invariant with μ_w . Thus for different values of Γ (i.e., v_A and v_B), the average thrust acceleration must be determined, and this value is fixed during the solution of the foregoing series of equations.

Numerical Optimization Procedure

The major problem of the entire analysis is in deriving a functional form of $\Gamma(v_A, v_B)$. The trajectory optimization program has not been modified to include the system optimization as part of the over-all optimization primarily because of the various modes of operation desired and the numerous propulsion parameters required as input. It was felt expedient for present purposes to keep the hybrid-thrust system optimization separate and use a parametric approach to the computation of optimum v_A and v_B for a given set of dates. In this way, at fixed dates, the resulting Γ surface may be used many times for flight profile variations and different propulsion parameters rather than computing the optimum trajectories as would be necessary if it were part of an over-all optimization procedure.

At first it appeared feasible to fit some functional form representing a geometric surface in three dimensions to the high-low thrust mix data. In a particular case for an Earth-to-Mars trajectory this fit yielded surprisingly accurate results if the surface was assumed to be approximated by an elliptic paraboloid of the form

$$\Gamma = \frac{[-d_2(v_A - 1) + d_1(v_B - 1)]^2}{a^2} + \frac{[d_1(v_A - 1) + d_2(v_B - 1)]^2}{b^2}$$

with vertex at $v_A = 1$, $v_B = 1$, and where a , b , d_1 , d_2 are constants determined in the process of fitting the surface. An example of the Γ surface is shown in Fig. 523.

As the analysis proceeded, it was noted that considerable time may be saved if, instead of attempting to fit an analytic expression to the different Γ surfaces, a table of Γ for various (v_A, v_B) was used. In this way, the table may be generated by properly sequencing the series of computations in the trajectory optimization program for the different pairs of hyperbolic excess speeds (v_A, v_B) . Furthermore, it is not reasonable to expect, without further theoretical study, that the Γ surface can always be approximated by an elliptic paraboloid or any other simple geometric surface. An example of such a table is given in Table VII for an Earth-to-Venus variable-thrust transfer. Note that the entries in the table are Γ , the form required by the optimization equations. The different J 's found under the various pairs of hyperbolic excess speeds are normalized to form $\Gamma^2 = J/J_m$.

For interpolating within the table, a nonlinear fit is applied to the four adjacent points. The accuracy of the interpolation naturally depends on the number of rows (columns) used for v in the range 0 to 1.0. From simple sensitivity studies performed using various table sizes, it was concluded that a 5 x 5 or 6 x 6 table is sufficient for the current mission analysis purposes. Some results of the sensitivity studies are shown in Table VIII. Note that the quantity of final interest, μ_L , is slightly affected even if the corresponding hyperbolic excess speeds differ significantly among the various Γ sources, i.e., the function, or the 9 x 9 or 5 x 5 table. The differences noted are referenced to the values obtained using the function, i.e., elliptic paraboloid.

Once the dependence of Γ on (v_A, v_B) is determined, the corresponding payload ratio equations are optimized using a numerical procedure termed "direct search" (Ref. 20). This procedure involves straightforward search strategies which do not necessarily require the usual classical techniques. In brief, a starting solution, i.e., first guesses for (v_A, v_B) , is introduced into the appropriate equations and the solution evaluated. Small perturbations (or explorations) are made in one independent variable while keeping the others fixed and a "direction" is determined which increases the value of the payoff. After each variable has acquired a direction, a "move" is then made which involves changing all variables by the determined amounts. After a move is made, the value of the payoff function is evaluated and compared to previous results to insure that the directions used are successful. This procedure is repeated until a failure results, at which time exploratory moves are again instituted until a new set of directions is determined. If no improvements are made, the step size (or exploration) is decreased, and the procedure is started with the latest estimates for (v_A, v_B) . A solution is obtained when the step size becomes less than some input tolerance. A more detailed discussion of the systematic search technique, and the associated program logic, are presented in Appendix G.

This simple numerical procedure has been quite successful in attacking the problems so far analyzed. Figure 524 typifies the numerical results obtained from a sample optimization. Note that the structural factor of a given high-thrust step is assumed constant during the optimization. Since the over-all approach is to obtain an estimate of the hyperbolic excess speeds to use in the actual mass computation, it is necessary to estimate what the structural factors would be after such computations have been performed. Fortunately the maximum payload ratio is not significantly affected (about 10%) by drastic changes in the structural factors. Thus a reasonable guess for these factors based on high-thrust steps previously computed should provide results close to the optimum. This approximate approach to the structural factors has been found to be expedient rather than attempting to correlate step inert weight growth with payload or propellant weight in order to include it in the optimization procedure itself.

Mass Analysis

The mass computations utilize the basic high-thrust program which includes velocity losses and variations in step hardware design. These aspects are discussed more fully in the high-thrust system studies. For the current effort, the effect of the high-thrust propulsion system was accounted for by varying the specific impulse and the coefficients in the inert weight equation. Three sets of values for these parameters were considered: nominal, optimistic, and pessimistic. In practically all of the cases, nominal values were used because of the number of low-thrust parameters that were to be considered.

It should be noted that the type of powerplant assumed in these studies is an onboard nuclear space power system providing 100% of its initial power rating throughout the trip time. The design sophistication of the power system is represented in part by the specific weight, i.e., the amount of powerplant mass required to produce a unit of power.

The presentation of study results is categorized by planet. The first to be discussed is the manned mission to Mars, with the unmanned missions (i.e., orbital probes) presented in a planetary sequence: Mercury, Venus, Mars and Jupiter. As stated previously, the complexity of optimizing hybrid-thrust round-trip missions precluded the possibility of analyzing manned missions to other planets. Mars was selected for the round-trip mission primarily because of the interest in manned planetary trips compatible with the time period and technology utilized and, further, because of the amount of trajectory information then extant.

Manned Mars Mission

The mass requirements for the 1980 mission as a function of powerplant specific weight and trip duration are shown in Figs. 525 and 526, which are essentially cross plots of each other. The basic mission objective assumed was delivery into a Martian parking orbit (926 km altitude) of a 45 metric ton (100,000 lb) excursion module which lands, stays for 30 days, and returns to the parking orbit with 454 kg of scientific materials. Optimum nuclear-plus-electric-plus-nuclear operation is employed for the outbound leg, while for the return leg optimum nuclear-plus-electric-plus-atmospheric entry is used. Entry at Earth is accomplished by an advanced Apollo type ablator vehicle accommodating eight astronauts.

As expected, the mass requirements are reduced considerably and continue to go down monotonically as longer trip times are utilized. This is caused mainly by the significant reduction in low-thrust requirements as the trip time is increased. The influence of the powerplant specific weight is reduced considerably at the extended trip times, although the effect on mass does not become less than about 10% for a 5 kg/kw change in specific weight. Note also that for a given extended trip time (e.g., 630 days, Fig. 525), decreases in specific weight are not significant until a specific weight

of about 10 kg/kw is attained. This would indicate that if a powerplant to be used for an extended duration mission possesses a high specific weight initially, the development effort required to decrease the specific weight is not worth the return in mass savings unless a very low (about 5 kg/kw) specific weight can be achieved.

The required powerplant rating is quite dependent on the specific weight, as shown in Fig. 527. This rating is determined from the mass computation, which uses the optimum powerplant-to-gross-weight fraction for maximum payload, and reflects the power which must be produced by a powerplant under the assumed specific weight. It is not implied that a nuclear space power system can be built to yield the necessary power at the indicated specific weight. Figure 527 merely indicates that for the parameterized specific weight, the corresponding power rating is that required to perform the mission.

It is of interest to note that as the specific weight increases, the required power rating decreases, or, equivalently, the powerplant mass decreases. This effect is due primarily to the hybrid-thrust optimization which depends on specific weight as a parameter. Thus as higher specific weights are used, more of the planetary transfer requirements are shifted to the high-thrust systems which results in a lower trajectory requirement on the low-thrust propulsion system. The power requirements are consequently reduced.

At the extended trip times (530 and 630 days), the low-thrust trajectory requirements are quite low initially, and hence changes in the hyperbolic excess speeds at the various specific weights and different trip times do not materially affect the power requirements. This result is graphically shown in Fig. 527.

The major high-thrust propulsion system for the Mars round-trip mission is the Earth departure nuclear step. This system constitutes in most cases almost 25% of the total vehicle mass. The effect of the powerplant specific weight on the propellant mass (hydrogen) of the nuclear departure step is depicted in Fig. 528. The propellant requirements level off rather sharply at the longer trip times, pointing out one attractive characteristic of the hybrid-thrust system which may be useful. That is, for a trip time greater than 500 days, essentially the same size Earth departure nuclear propulsion system may be used for the manned mission regardless of specific weight and of mission duration.

In the results discussed so far, the output of the powerplant was assumed constant at its initial rating regardless of the operating time. Hence the vehicle mass requirements are somewhat optimistic. If the powerplant is assumed to be a Rankine cycle nuclear system, then some of the results obtained in Contract NAS8-11309 may be used to determine the increase in vehicle mass due to power degradation. Using a statistical analysis based on assumed

component failure rates, various probable decreasing power profiles as a function of time were derived for different probability levels. The probability level is interpreted as the probability that at a given time the fraction of power being delivered will be at least indicated by the profile.

The power decay with time is integrated into the trajectory computation, the results of which enter into the optimization of the hybrid-thrust system. The final results of the mass computation using the foregoing information are summarized in Fig. 529 for the 530-day mission and two probability levels, 0.99 and 0.999. The number of component spares provided gives an indication of the maintenance capability and, furthermore, influences the power decay curve.

As can be seen in Fig. 529, for a given probability level the addition of the capability of maintaining certain components in the power system has a significant effect on the vehicle mass requirements. The choice of probability level depends on other considerations concerning over-all probability of crew survival and mission success. It is also interesting to note that for the maintained system at the lower probability level, the sensitivity of the vehicle mass to powerplant specific weight is reduced. The relationship between maintenance capability and specific weight is needed, however, to properly interpret the results of Fig. 529.

Mercury Orbital Probes

The major problem encountered in analyzing the Mercury missions was the computation of the trajectory requirements for the longer trip times. The longer durations were desirable because of the prohibitive J's encountered in trips of less than 300 days. Because of these difficulties it was decided to concentrate the trajectory computation effort on the 300-day trip time and determine the corresponding minimum-J trajectory.

Even in this restricted case, converging to the proper stationary solution became a problem. This aspect is illustrated in Fig. 530 wherein the objective was to determine the lowest J for the 300-day trip time by changing the arrival date at Mercury. A converged solution for the arrival date of 4380 was utilized to compute the trajectory corresponding to an arrival only 20 days later. As can be seen the converged solution did not produce the expected J.

The results of the trajectory computation led to the incomplete data shown in Fig. 531. It finally became necessary to use the lowest-J trajectory encountered which, nevertheless, appeared to be close to the minimum for the 300-day transfer. Because convergence is sensitive in the region of the minimum-J trajectory, it was expected that the addition of hyperbolic excess speeds would further aggravate the trajectory computation. This in fact became true; therefore it was decided to compute the mass requirements using

zero hyperbolic excess speeds. That is, the high-thrust systems at departure and arrival accommodate only escape conditions with respect to the planet. The hybrid-thrust mix used is thus nonoptimum.

The results of the mass computations are summarized in Figs. 532 and 533 for the chemical propulsion system and in Figs. 534 and 535 for the nuclear system. In all cases the data presented are payload ratio, mass on Earth orbit, and powerplant rating as a function of either powerplant specific weight or the payload delivered.

In all cases, the mass requirements are within the single-launch capability of Saturn V. The powerplant output required for the low-thrust systems remains relatively unaffected by the wide range of values in specific weight. For comparison purposes, optimistic and pessimistic values of specific impulse, specific weight, and high-thrust step inert weight requirements were used. Even in the pessimistic cases, the mass requirements are still less than 100 metric tons.

As expected, the payload has a significant effect on the vehicle requirements as shown in Figs. 533 and 535. For the nominal assumptions indicated therein, the Saturn V can deliver about 10 metric tons of payload using either the nuclear or chemical propulsion systems. Improving the surface launch booster payload capability to the lower values associated with the post-Saturn systems allows a factor of 10 increase in payload delivered to Mercury.

The growth in powerplant rating with payload approximately parallels that of the vehicle mass, assuming a fixed specific weight of 12 kg/kw. Note that regardless of the type of high-thrust propulsion system used the powerplant rating reaches one megawatt at a payload of 10 metric tons. It is not implied, however, that the required powerplant rating can be achieved by a system having a specific weight of 12 kg/kw. Actually the matching requirements between necessary power rating and design technology must be imposed on the results such as given in Figs. 533 and 535 in order to determine the payload capability of a given type powerplant.

Venus Orbital Probes

The trajectory data utilized in this study were taken from trajectory plots given in Figs. 536 and 537, wherein rendezvous (i.e., parabolic) conditions at the boundaries were assumed. Note that the minimum J for the 300-day trip is about $1 \text{ m}^2/\text{sec}^3$ whereas for the 150-day trip it is about $7 \text{ m}^2/\text{sec}^3$. The trajectory information presented in Figs. 536 and 537 is useful for combining the outbound and inbound legs to find the total J requirements for a given round-trip time. Figure 538 depicts the results for a sample round-trip mission of 450 days.

The Venus probes were analyzed according to two categories: nonoptimum hybrid-thrust mix for 300-day transfer time and optimum hybrid-thrust mix for 150-day transfer. This will permit an illustration of the capability of the optimum hybrid-thrust system to reduce trip time considerably yet require essentially the same total vehicle mass.

The results of the nonoptimum hybrid system are depicted in Figs. 539, 540, 541, and 542 which contain the same information as described previously for the Mercury missions. The trip time is 300 days assuming nominal values for the propulsion parameters, although optimistic and pessimistic values were employed for selected points.

The specific weight has virtually no effect on both the vehicle mass and the payload ratio, assuming a fixed payload. The mass and power rating change considerably, however, as payload is varied for a given specific weight. The payload that can be delivered to a Venus parking orbit by the Saturn V booster is about 15 to 20 metric tons under the assumptions used here. Utilization of a post-Saturn system increases the payload by an order of magnitude.

Compared with the chemical system, the nuclear system yields significant savings in mass. For example, if the payload is 10 metric tons, the chemical system mass requirement is about 740 tons (Fig. 540) compared with the nuclear system's 530 tons (Fig. 543).

The 150-day mission selected for analysis using the optimum hybrid-thrust system was that displaying minimum J, Fig. 536. The resulting mass requirements are illustrated in Fig. 543 for both chemical and nuclear propulsion systems and a range of payloads and specific weights. As in the nonoptimum case, the specific weight has but slight effect on the vehicle mass, but the influence of payload is considerable. Further, the nuclear hybrid system requires significantly less mass compared to the chemical-hybrid system.

Note that in the chemical and nuclear-hybrid vehicle (Figs. 517 and 519) the mass requirements are essentially the same as, in fact somewhat higher than, those given in Fig. 543. For example, at a payload of 45,359 kg the 300-day nonoptimum nuclear hybrid-thrust vehicle mass is about 200 metric tons (Fig. 542), whereas Fig. 543 indicates that the optimum mixed-thrust system can deliver the same payload for somewhat less than 200 tons. Thus the trip time has been reduced by half with no penalty in mass provided optimum mixing of high- and low-thrust systems is employed.

For the same 150-day transfer time, the optimum mixed-thrust vehicle requires less mass than the corresponding all high-thrust system. This result is shown in Fig. 544, wherein both chemical and nuclear systems are compared against the hybrid vehicle for various payloads. The mass savings are about 10% and, while not being overly impressive, nevertheless indicate

the usefulness of hybrid-thrust systems for even this low-energy mission. The one possible drawback is the operational problems associated with a vehicle employing mixed propulsion system types compared to one using all high-thrust propulsion. These operating complexities must be traded off against the mass reduction.

The Earth departure high-thrust propulsion system is affected only slightly by the powerplant specific weight as indicated by Fig. 545. As expected, however, the payload exerts a strong influence. The fact that the step weight is insensitive to specific weight indicates that a fixed-size Earth departure propulsion system may be used to deliver the same payload even as powerplant specific weight may improve to lower values. Viewed differently, it appears from the results so far that reductions in powerplant specific weight yield negligible returns in decreasing either mass required on Earth parking orbit or departure step propulsion mass. This implies that development effort for space powerplants should be concentrated on some other aspect, e.g., reliability.

The powerplant rating required to complete the mission increases considerably below a specific weight of about 15 kg/kw (Fig. 546). Conversely, as specific weight goes up, the powerplant rating is reduced primarily because of the optimized hybrid system. That is, as specific weight is increased, more of the mission requirement is placed on the high-thrust system which results in decreased low-thrust trajectory requirement. The net result is that the powerplant mass is reduced somewhat, thereby leading to lower power requirements. As mentioned previously, the design capability of a certain type of onboard powerplant considered for the mission must be matched against the rating curve to identify the operating points of the candidate powerplant.

Mars Orbital Probes

Figure 547 compares the optimum hybrid-thrust system with an all-chemical high-thrust system for a particular trip of 280 days and a given powerplant specific weight of 10 kg/kw. For this set of assumptions at least, the chemical system always yields lower payload ratios regardless of the departure and arrival step inert weight fractions.

Further study is necessary to include the effects of departure and arrival dates for a fixed trip time using both the hybrid- and nonhybrid-thrust systems. A more inclusive comparison thus can be made if each system is optimized on dates for the given trip time.

Using the trip described in the previous figure the combined nuclear high-thrust and electric system was compared against the all-high-thrust nuclear propulsion system. Under the same set of assumptions, Fig. 548 shows that the advantage of the optimum hybrid-thrust system is now not so definite. Thus if the arrival step inert weight fraction under hybrid operation is about

0.40 to 0.45, the corresponding payload ratios are essentially the same as the high-thrust system if the inert weight fractions in the latter system are all about 0.30 to 0.35. Hence, determination of the favored mode of operation for a particular trip requires actual computation of the vehicle system mass under both propulsion modes. This is because the variation of inert weight fraction with incremental velocity and accelerated mass has not been accounted for in the payload ratio computations.

An attempt to isolate the general effects of powerplant specific weight is shown in Fig. 549. For the particular trip and propulsion profile indicated, the maximum payload-to-gross-weight ratios were computed using various powerplant specific weights. The plot is based on the assumption that the nuclear propulsion system step inert weight fractions remain the same even though powerplant weight (hence the fraction of all-high-thrust hyperbolic speeds) changes. Two sets of assumed values for inert weight fractions (in parenthesis) are shown for high-thrust steps leaving Earth and arriving at Mars. The payload fractions are optimum only for the specific trip fixed by the Julian dates.

As expected, the sophistication of design in the propulsion steps for minimizing inert weight affects the payload ratios significantly. All three steps are represented in the accompanying figure; the two high-thrust systems by the inert fractions and the low-thrust system (to a major extent) by the powerplant specific weight. The relationship between inert fraction, incremental velocity and mass to be accelerated precludes actually accounting for the inert fractions in a general curve as illustrated here.

It is of interest to note that if the inert weight fractions do not change considerably from an assumed set of values, then decreases in specific weight do not yield significant increases in payload fraction until the specific is reduced to below 10 kg/kw. Hence reducing the specific weight from 25 to 15 kg/kw is apparently not worth the return in payload capability if such a reduction is not "easily" attained. Note further that it is advantageous to utilize low inert weight fractions. Thus a change in the departure and arrival fractions from (0.20, 0.40) to (0.10, 0.30) results in about a 20% increase in payload fraction. The departure step fraction of 0.10, however, is beyond current technology for nuclear systems and is included here merely to indicate the importance of lower inert fractions.

A 160-day trip was selected to evaluate the capability of optimum hybrid-thrust systems against both nonoptimum hybrid systems and all-high-thrust systems. The results are presented in Fig. 550. It can be seen that the optimum hybrid system mass requirements are below the single-launch orbital payload capability of the Saturn V, regardless of the powerplant specific weight. The nonoptimum system (parabolic) exceeds the Saturn V payload at a specific weight of about 15 kg/kw.

An important result is that by use of the optimum high-low thrust system, the mass requirements do not change drastically with powerplant specific weight compared with the nonoptimum hybrid vehicle. For instance, at 3 kg/kw both the optimum and nonoptimum hybrid systems require about the same mass; however, at approximately 18 kg/kw the nonoptimum mass requirements increases by about a factor of 4 while that for the optimum system increases only 30%.

For the 4536-kg payload, the specific weight, which is generally considered a major indication of powerplant development effort, may be very high but still useful for orbital probe missions. In the chemical case, the delivery of a 4536-kg payload to a 926-km orbit about Mars is within the capability of a single Saturn V, whereas the use of the "parabolic" mode requires developing a power system with specific weight less than 15 kg/kw. To utilize the full capability of the Saturn V, the payload could be approximately doubled if the specific weight were less than 20 kg/kw.

Essentially the same comments as above may be made for the nuclear high-thrust system. In this instance the mass requirements are lower than the chemical case but not significantly so at these payloads.

If the transfer time were to be doubled, the low-thrust trajectory requirement becomes so low that the parabolic mode of operation yields less mass than the 160-day transfer using the optimum high-low mix. For a transfer of 300 days, a specific weight of 12 kg/kw, and payload of 4536 kg, the vehicle mass requirement is about 20 metric tons. However, the duration of such a trip is a stringent requirement on the reliability of the payload and related subsystems.

The comparison of combined-thrust operation is further illustrated in Tables IX and X, which present a mass breakdown of the chemical and nuclear systems respectively. In each table is shown the redistribution of mass among the three propulsion stages due to the optimization of the trajectory. Table IX indicates that while the capture step increases in mass, the electric system decreases considerably, which in turn reduces the requirement on the departure system even though the velocity increment it must provide increases over the parabolic case. Table X also compares the mass requirement for the 300-day transfer.

From the work of the high-thrust mission studies, it was found that the solid-core nuclear (all high-thrust vehicle) mass requirement is about 86.5 metric tons, which is considerably higher than even the hybrid-thrust chemical system (Fig. 550). Of the high-thrust vehicles applied to the present mission, the liquid-core nuclear propulsion system yielded the lowest mass requirement, 14.9 metric tons. Although this mass is about half that of the solid-core nuclear hybrid-thrust vehicle, the point to consider is the availability of either type of system in terms of technological development.

The Earth departure propulsion system mass changes somewhat with specific weight, as indicated in Figs. 551 and 552 for the chemical and nuclear systems, respectively. As can be seen, optimally combining the high- and low-thrust systems results in less dependence of the step mass on specific weight. While it is important in terms of Earth departure step mass to employ low specific weights, it is even more so to use the optimum hybrid-thrust mode.

Further evidence of this, in regard to the powerplant, is illustrated in Fig. 553, wherein the power rating requirements for the optimum and nonoptimum hybrid systems are compared. At a given specific weight, the stated powerplant rating is that required of the powerplant in order to complete the mission (assuming no power loss). The minimum power requirement for the parabolic case is about 670 kw at a specific weight of approximately 9 kg/kw, while about 250 kw is required in the optimum hybrid case at the same specific weight. Interpreted differently, the powerplant mass required in the parabolic case is 6 metric tons, while that required in the optimum case is just above 2 metric tons, a ratio of slightly more than 2.5 in mass and power. Actually, for a proposed powerplant, the operating points may be found by superimposing the powerplant's design operating curve over the requirements curve.

Jupiter Orbital Probes

Three different trip times were selected for the hybrid-thrust study, each being one which yields minimum mass for the nonmixed-thrust operating mode. The trips were of 1000, 600, and 400 days duration, all of which depart from Earth in 1982. The two-dimensional trajectory profiles for the low-thrust rendezvous are depicted in Fig. 554. The communications path at the time of planetary arrival is about 5 AU for the two cases shown and is not obstructed by the sun.

An indication of the tradeoff between powerplant specific weight and trip time in terms of payload ratio is illustrated in Fig. 555 for two values of assumed thruster efficiency. Note that the payload ratio applies only to the heliocentric electric propulsion stage, since Earth-to-Jupiter rendezvous trajectories were used. That is, the electric propulsion system starts under parabolic conditions at Earth and shuts down upon arrival at parabolic conditions with respect to Jupiter. The results shown in Fig. 555 indicate that, for the electric stage at least, the powerplant specific weight has a significant influence on the admissible trip times for reasonable payload ratios. If 15 kg/kw is assumed to be a representative specific weight for the 1980 time period, then the minimum trip time that may be performed is about 500 days. It is interesting to note that changing the trip time from 1000 to 600 days requires a decrease in specific weight of about 50% for a fixed payload ratio. Decreasing the trip time from 600 to 400 days requires a reduction in specific weight of about 70%. The salient point is that the 50% reduction occurs at the high specific weight values (about 30 kg/kw) while the 70% reduction must occur at low values (about 10 kg/kw), a level at which such reductions may not be feasible.

The foregoing trips were analyzed for the optimum distribution of hyperbolic excess speeds and J to maximize the payload ratio of the over-all hybrid-thrust vehicle system. The high energy requirements indicated by the impulsive transfer hyperbolic speeds and low-thrust rendezvous requirements implied that not only would a hybrid system be advantageous to minimizing mass, but that an advanced high-thrust system beyond the solid-core nuclear rocket should be utilized. This indeed was the case as indicated by the inability of the optimizing procedure to converge to a solution because of the high impulsive velocity increments and the step inert weight fractions. Consequently, a light weight advanced nuclear rocket, delivering 2000 sec specific impulse, was combined with the electric system. Sample results of this analysis are presented in Fig. 556 for a 400-day mission and a specific weight of 5 kg/kw.

Figure 556 indicates a result typical of many of the hybrid-thrust planetary missions. The fraction of the departure impulsive transfer hyperbolic excess speed seems to range from 0.50 to 0.80, while for the arrival case it ranges from 0.30 to 0.60. Although the actual optimum values of the fractional hyperbolic speeds depend on the specific weight and the step inert fractions, use of estimates within the above range of values usually yield payload fractions close to the optimum.

The results of the mass computation for the different trip times and payloads are presented in Fig. 557. In contradistinction to the information of Fig. 555, the specific weight has little effect on the over-all system, unless specific weights less than 5 kg/kw are contemplated. This is especially true as the mission duration is increased. As expected, the mass of the vehicle on Earth parking orbit increases in approximately the same proportion as the payload. In the cases shown, a ten-fold increase in payload results in a ten-to-one increase in vehicle mass.

As trip time increases, the reduction in mass becomes less significant principally because after a certain length of trip time the low-thrust trajectory requirement decreases to such a low value that the corresponding reduction in high-thrust system levels off near the parabolic case. Thus, as indicated in Fig. 556, increasing the trip duration from 400 to 600 days reduces the initial vehicle mass by 50%, while going from a 600- to a 1000-day mission reduces the mass about 20%.

Compared with the nonmixed high-thrust system, the hybrid vehicle reduces the mass requirements considerably as shown in Fig. 558. The left side of the figure compares the 400- and 600-day trips for a payload of 4536 kg, while the right side illustrates the effect of payload for the 1000-day trip. The mass reductions are quite significant for all cases, ranging from about 45% to a high of about 90%. Based on a Saturn V Earth orbital payload capability of 100 metric tons, trips as low as one year may be executed for a payload of 4536 kg. Conversely, if trip time

were to be fixed at 400 days, approximately 6800 kg could be delivered into the Jupiter parking orbit using the full capability of the Saturn V.

The payload and trip time aspects are further explored in Fig. 559, wherein the payload is fixed at 4536 kg. This hybrid-thrust curve illustrates the previously known fact that the longer trip times yield a monotonically decreasing mass requirement whereas the nonmixed system produces a distinct minimum at a trip duration of about 700 days. Mission durations as short as one year may be accomplished by a single Saturn V launch compared with at least two launches for the nonmixed-thrust system at a trip time of 600 days. The striking difference in capability between the two types of vehicle systems is shown by the hybrid-thrust mass curve at a payload of 45,359 kg. Thus for about the same vehicle mass the hybrid-thrust system can deliver ten times the payload. Note further that the higher-payload capability system even requires less vehicle mass if trips greater than 850 days are employed.

The demands on the power system output are demonstrated in Fig. 560 for a range of payloads, specific weights, and trip times. For a given trip time, the required powerplant rating varies approximately in proportion to the payload. The effect of specific weight on mass displays the same tendency regardless of payload or trip time. The application of a given type of power system (assuming 100% constant power output) to the Jupiter missions may be found by overlaying on Fig. 560 the candidate powerplant's power output specific weight performance curve. As an example, the nuclear Rankine cycle space powerplant studied in Contract NAS8-11309 can be used for the 400-day mission at a payload of 45,359 kg at a power level of just below 5000 kw, or it can be applied to the 600-day mission and same payload at a power rating of about 1000 kw.

The effect of powerplant specific weight on the Earth departure propulsion system is shown in Figs. 561, 562, and 563 for the 400-, 600-, and 1000-day mission, respectively. Referring to Fig. 557, it can be seen that approximately 40% of the vehicle mass on Earth parking orbit is made up of the Earth departure propulsion system. This system is practically unaffected by the powerplant specific weight (Figs. 561, 562, and 563) which when combined with the arrival propulsion system results in a vehicle system that is insensitive to variations in specific weight (Fig. 557). As in most of the cases discussed previously, the vehicle mass as well as the Earth departure step mass becomes sensitive to specific weight for values below 10 kg/kw.

GENERALIZATION OF RESULTS

The results of this study are presented in various forms such that one may use the contents to establish trade-off parameters for comparing combinations of specific parameters of interest. The bulk of the data are based on "nominal" values of the many parameters defining the total system mass requirements.

The trajectory analyses are conveniently divided between high-thrust and low-thrust systems. The optimum launch dates for unmanned missions using high-thrust systems were obtained for the launch periods of interest by superimposing the orbital traces of Earth, the target body, and 180-deg transfers, using a coplanar model. Once the minimum-energy points are located, determining the effects of trip time on the velocity requirements is routine. For round-trip missions, it is necessary to integrate the out-bound and inbound velocity requirements to determine the minimum combination.

For the outer planets, the variation of mass, or energy requirement, is not a smooth continuous function of trip time but consists of a series of local minima, where the minima occur at intervals defined by the mean motion of the objective body.

The usefulness of the flyby probe missions need not cease with the execution of the flyby maneuver. For some flybys, the probe returns to a close proximity of the Earth. The low-energy flybys to each planet can generally return to within 1 AU or so of the Earth in about 400 days if the passage distance is close to the planet. Many of the high-energy flybys will not return to the vicinity of the Earth since the flyby maneuver increases the heliocentric speed to hyperbolic.

The mass computations for the unmanned flyby and orbital probes indicate that chemical propulsion is desirable for payloads up to 1000 kg for the neighboring planets and up to 500 kg for the outer planets. For larger payloads, up to about 10,000 kg, a light weight advanced nuclear engine provides minimum mass. A gaseous nuclear system of the same specific impulse shows higher mass requirements due to the high value of minimum engine weight. For very large payloads the desirability of the gaseous and light weight advanced nuclear systems are about equal, except for the more ambitious missions to the outer planets. When large payloads are used in conjunction with high-energy requirements for flyby missions, the pulsed nuclear and 3500-sec specific impulse gaseous nuclear systems provide minimum mass requirements which are essentially equal. For orbital stopovers, under the same conditions, the pulsed nuclear propulsion is more desirable.

The Atlas-Centaur booster with a chemical upper stage can perform flyby probe missions to Mercury, Venus, and Mars with a useful payload of 600 kg

to 1100 kg. Similar missions to Jupiter and Saturn can be performed with payloads of 300 kg and 100 kg, respectively.

The Saturn IB booster is capable of performing a wide range of flyby probe missions and delivers about ten times the useful payload as the Atlas-Centaur, besides providing flybys to Uranus, Neptune, and Pluto with payloads of 100 kg to 200 kg.

Orbital probe missions with Atlas-Centaur and chemical upper stages can be made to Venus and Mars only, with payloads of about 200 kg. Higher payloads or other objective bodies require at least a Saturn IB booster; a Saturn V is required for orbital probe missions past Mars. The Saturn V booster and upper-stage propulsion having performance at least equal to that attributed herein to the light weight advanced nuclear engines are required for orbital probe missions to the outer planets.

A hypothetical post-Saturn booster will perform most unmanned missions with most upper-stage propulsion systems, delivering 100,000 kg or more payload.

Unmanned flyby missions to the planets beyond Jupiter can benefit from a Jupiter swingby, especially if chemical or solid-core nuclear engines are used. Moderate mass savings can be made for given trip times. Conversely, for a given mass requirement, the Jupiter swingby can reduce the trip time substantially. The Jupiter swingby does not reduce the mass requirements for orbital probes to the outer planets.

The three-impulse analysis of establishing 1-AU inclined orbits indicated that it generally requires less mass for a 60-deg orbit than a 45-deg orbit, due to the staging effects from the distribution of the incremental velocities involved. However a single impulse from an inclined Earth parking orbit requires less energy up to inclinations of about 55 deg.

Some benefits arise from mixing the propulsion systems such that different types are used for Earth escape and planetary capture. If chemical Earth escape is used, a light weight advanced nuclear capture stage should be used for a minimum mass combination.

Generally, for large payloads, a light weight advanced Earth-escape stage with a 3500-sec gaseous nuclear capture stage is desirable. For smaller payloads, the staging sequence should be reversed. The all-chemical system should be used for very small payloads of a few hundred kilograms.

As expected, the large payloads and velocity requirements associated with manned missions tend to favor the high specific impulse nuclear systems, even with their heavy engines. Generally, the 3500-sec gaseous-core nuclear rocket provides the minimum mass missions. Varying the stay time at the

planet from zero to sixty days has little effect on the mass requirements for missions to the planets past Jupiter. Likewise, the longer Jupiter missions are fairly insensitive to stay time. Stay time has a decisive effect on the mass requirements for missions to the neighboring planets, with an extreme case showing an order of magnitude increase in mass between a zero and sixty-day stopover mission to Mars using chemical propulsion.

The Saturn V booster is not large enough for the manned missions. A post-Saturn booster with a payload capability of 900,000 kg and upper stages of advanced nuclear propulsion could perform selected manned orbital stopover missions to each planet analyzed. The extra mass of an excursion module used for surface exploration greatly taxes the post-Saturn capability, especially for large landers. For landing missions to Mercury, Mars, and the largest moons of Jupiter and Saturn, the excursion module cannot be over about 60% of the useful payload, even with the advanced nuclear rockets. An appreciable reduction in trip time, say 60%, results in a limiting excursion module weight of about 20%.

For each type of engine, changes in specific impulse have a varying effect on the mass requirements. Low-energy missions, such as a Mars flyby, may show only a 1% change in mass for each percent change in specific impulse. A more ambitious Neptune orbiter may indicate a corresponding sensitivity of over 20 to 1.

The definition of the system inert weights greatly influences the resultant mass requirements. If the extremes of the upper and lower limits to the various parameters are combined, the mass requirements may vary an order of magnitude for chemical systems, down to a few percent for the advanced nuclear rockets. An analysis of the individual effects of the inert weight parameters indicate that tankage structural weights have a relatively small effect on total system mass. The insulation assumptions are primarily influential for the outer planets only. The assumed meteoroid protection model has a profound effect on the system mass and is the primary independent inert weight parameter.

The low-thrust trajectory and mass analyses have been based primarily on variable-thrust, constant-power transfers with coast. Radioisotope energy and solar energy are also considered for power sources. Rendezvous flights to the exterior planets are performed most advantageously by the use of the constant power mode. Rendezvous flights to the interior planets benefit from using solar power. Flyby or impactor missions to the exterior planets should use the radioisotope power source, while similar missions to the inner planets benefit from solar power.

Hybrid-thrust systems appear very attractive and generally produce lower mass requirements than all high-thrust propulsion for equal trip times. Unmanned probes to Mercury, Venus, Mars, and Jupiter and a manned mission to Mars are analyzed and compared with the high-thrust characteristics.

The effects of extending the trip time on required mass for the manned Mars missions are favorable. The low-thrust requirements are reduced significantly. Also, if the powerplant specific weight is high, reducing it will not substantially reduce system mass.

In all of the unmanned orbital probe missions, the use of optimum hybrid-thrust systems mitigates the effect of powerplant specific weight. Significant changes in mass do not occur until the specific weight is less than 5 kg/kw and in some cases until it is about 1 kg/kw. Consequently, the development of lower specific-weight power systems should be weighed against the return available from applying development elsewhere.

For manned missions, the reverse situation appears to prevail to some extent. The specific weight has a significant effect on total vehicle mass; however, this effect is reduced as mission duration is increased.

The required rating of the powerplant, as determined by the hybrid-thrust trajectory profile and mass requirements, does not change significantly with specific weight. For optimum hybrid-thrust systems, the power requirement tends to decrease and level off with specific weight. For a given mission and payload, the powerplant rating generally decreases with increasing specific weight.

For a fixed payload and mission, the hybrid-thrust departure step is essentially unaffected by changes in the powerplant specific weight, and may still be used to perform the mission regardless of fluctuations in specific weight. If the specific weight decreases below 10 kg/kw, the accompanying payload may be increased accordingly.

The large number of parameters involved in this study make it necessary to carefully weigh the data and trends presented herein.

Determining the relative merits of the various types of propulsion systems analyzed in this report requires a detailed understanding of the many assumptions that comprise each data point. The reader is cautioned to fully understand the content before forming conclusions.

REFERENCES

1. TRW Report No. 8423-6007-RU000.
2. Heller, H.: Optimum Transfers Between Noncoplanar Circular Orbits. Planetary and Space Science, Vol. 13, No. 2, February 1965.
3. Deerwester, J.: Initial Mass Savings Associated with the Venus Swingby Mode of Mars Round Trips. AIAA Second Aerospace Sciences Meeting, No. 65-89, January 1965.
4. Irving, J. H.: Low-Thrust Flight: Variable Exhaust Velocity in Gravitational Fields. Chapter 10, Space Technology, Edited by H. Seifert, John Wiley & Sons, Inc., 1959.
5. Melbourne, W. G.: Interplanetary Trajectories and Payload Capabilities of Advanced Propulsion Vehicles. Technical Report No. 32-68, Jet Propulsion Laboratory, Pasadena, 1961.
6. Melbourne, W. G. and C. G. Sauer, Jr.: Optimum Thrust Programs for Power-Limited Propulsion Systems. Technical Report No. 32-118, Jet Propulsion Laboratory, Pasadena, 1961. Also Astronautica Acta, Vol. VIII, 1962, FASC4.
7. Melbourne, W. G., D. E. Richardson, and C. G. Sauer, Jr.: Interplanetary Trajectory Optimization with Power-Limited Vehicles. Technical Report No. 32-173, Jet Propulsion Laboratory, Pasadena, 1961.
8. Melbourne, W. G. and C. G. Sauer, Jr.: Optimum Interplanetary Rendezvous with Power-Limited Vehicles. Technical Report No. 32-226, Rev. 1, Jet Propulsion Laboratory, Pasadena, 1962. Also AIAA Journal, Vol. 1, No. 1, January 1963.
9. Van Dine, C. P.: An Application of Newton's Method to the Finite Difference Solution of Nonlinear Boundary Value Systems. United Aircraft Research Laboratories Report UAR-D37, March 1965.
10. Van Dine, C. P., W. R. Fimple, and T. N. Edelbaum: Application of a Finite-Difference Newton-Raphson Algorithm to a Problem of Low-Thrust Trajectory Optimization. Presented at the AIAA-ION Astrodynamics Specialist Conference, Monterey, California, September 1965.
11. Ragsac, R. V.: Study of Low-Acceleration Space Transportation Systems. United Aircraft Research Laboratories Report D-910262-3.

REFERENCES (contd.)

12. Streb, A. J.: Radioisotope Power Systems for Manned Space Stations. Presented at the Third Biennial Aerospace Power Systems Conference, Philadelphia, Pa., September 1964. AIAA Preprint No. 64-711.
13. Rohrmann, C. A. and E. D. Sayre: Radioisotope Space Power-Prospects and Limitations. Presented at First AIAA Annual Meeting, Washington, D.C., July 1964. AIAA Preprint No. 64-453.
14. Romero, J. B.: Potentialities of Radioisotope Propulsion for Space Probes. J. Spacecraft, Vol. 3, No. 4, April 1966.
15. Loferski, J. J.: The Photovoltaic Effect and Solar Energy Conversion. ARS Preprint 1288-60, September 1960.
16. Research on Solar-Energy Conversion Employing Cadmium Sulfide. Technical Documentary Report No. ASD-TDR-62-69, Flight Accessories Laboratory, Wright-Patterson Air Force Base, Ohio, June 1962.
17. Wenzinger, C. J. and H. Carleton: The Solar Power Supply System for the Orbiting Astronomical Observatories. ARS Preprint No. 63-217, June 1963.
18. Lamorte, M. F.: Solar Cells Designed for Operation in the Inner Regions of the Solar System. ARS Preprint No. 2568-62, September 1962.
19. Melbourne, W. G. and C. G. Sauer, Jr.: Payload Optimization for Power-Limited Vehicles. JPL Space Programs Summary No. 37-17, Vol. IV, October 1962.
20. Hook, R. and T. A. Jeeves: Direct Search Solution of Numerical and Statistical Problems. Journal of the Association of Computing Machinery, Vol. 8, No. 2, April 1961, pp. 212-229.
21. Dempster, W. R., R. L. Evans, and J. R. Olivier: Lunar Storage of Liquid Propellants. NASA TN D117, 1962.
22. Smolak, G. R., R. H. Knoll, and L. E. Wallner: Analyses of Thermal-Protection System for Space Vehicle Cryogenic Propellant Tanks. NASA TRR-130, 1962.
23. Liquid Propellant Losses During Space Flight. Second Quarterly Progress Report. Report No. 03270-00-02, Arthur D. Little, Inc., May 1961.
24. Liquid Propellant Losses During Space Flight. First Quarterly Progress Report. Report No. 03270-00-02, Arthur D. Little, Inc., January 1961.

REFERENCES (contd.)

25. Radiation Heat Transfer Analysis for Space Vehicles. Technical Report No. ASD-TR-61-119, Part II, September 1962.

TABLE I

VELOCITY REQUIREMENTS FOR FLYBY OR IMPACTER MISSIONS

Objective Body	Earth Departure Date - Julian	Earth Departure Velocity - EMOS	Trip Time days
Mercury	2444565	0.2170	110
	2444575	0.2208	100
	2444585	0.2469	90
	2444595	0.3390	70
	2444605	0.4257	60
Venus	2445480	0.084	160
	2445500	0.120	100
	2445520	0.220	60
	2445540	0.363	40
Mars	2444180	0.1004	290
	2444190	0.1161	230
	2444200	0.1264	200
	2444220	0.1860	140
	2444230	0.2820	100
Jupiter	2444970	0.2931	950
	2444980	0.3058	800
	2444980	0.3608	500
	2444990	0.5761	300
	2445010	0.9359	200
Saturn	2446840	0.3551	1780
	2446830	0.3563	1620
	2446830	0.3602	1460
	2446840	0.3935	1100
	2446840	0.4463	860
Uranus	2447210	0.3867	4200
	2447200	0.3946	3400
	2447200	0.4129	2600
	2447210	0.4711	1800
	2447230	0.7341	1000

TABLE I (Contd.)

Neptune	2450520	0.4023	6000
	2450520	0.4106	5000
	2450520	0.4285	4000
	2450520	0.4751	3000
	2450540	0.7806	1500
Pluto	2451580	0.4256	6000
	2451580	0.4468	5000
	2451580	0.4743	4000
	2451580	0.5208	3000
	2451580	0.6560	2000
Ceres	2444400	0.2166	430
	2444420	0.2517	350
	2444420	0.3070	250
	2444430	0.4892	150
S-WII	2444280	0.2268	720
	2444220	0.2525	500
	2444260	0.2692	440
	2444260	0.2738	380
	2444260	0.3005	320
	2444260	0.3903	260

TABLE II

VELOCITY REQUIREMENTS FOR ORBITAL STOPOVER MISSIONS

Objective Body	Earth Departure Date - Julian	Earth Departure Velocity - EMOS	Planet Arrival Velocity - EMOS	Trip Time Days
Mercury	2444365	0.3231	0.2819	90
	2444380	0.3911	0.3519	70
	2444390	0.4924	0.3845	60
Venus	2445470	0.088	0.120	170
	2445500	0.099	0.100	130
	2445520	0.144	0.127	100
	2445540	0.260	0.251	60
Mars	2444180	0.1006	0.0899	300
	2444200	0.1141	0.1064	260
	2444200	0.1181	0.1363	230
	2444210	0.1378	0.1723	200
	2444230	0.1978	0.1966	170
Jupiter	2444970	0.2947	0.1925	1000
	2444970	0.2981	0.2274	800
	2444970	0.3265	0.3517	600
	2445010	0.5189	0.5874	400
Saturn	2446840	0.3551	0.2033	1820
	2446840	0.3741	0.3120	1300
	2446840	0.3935	0.3960	1100
	2446840	0.4335	0.5229	900
	2446850	0.5194	0.7179	700
Uranus	2447220	0.3913	0.1536	5400
	2447210	0.3877	0.1924	4000
	2447200	0.4001	0.2750	3000
	2447210	0.4326	0.4122	2200
	2447220	0.5507	0.7178	1400
Neptune	2450520	0.4023	0.2131	6000
	2450520	0.4285	0.3574	4000
	2450520	0.4751	0.5106	3000
	2450540	0.6112	0.8138	2000
	2450540	0.7806	1.1183	1500
S-W II	2444220	0.2525	0.2857	500
	2444260	0.2712	0.3750	400
	2444280	0.3652	0.5737	300
Sun	2444214	0.4793	0.7354	75

TABLE III

VELOCITY REQUIREMENTS FOR ROUND-TRIP MISSIONS

Stay Time Days	Earth Depart. Date Julian	Earth Depart. Velocity EMOS	Arrival Velocity EMOS	Depart. Velocity EMOS	Outbound Leg Days	Inbound Leg Days	Earth Arrival Velocity EMOS
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MERCURY ROUND TRIPS

80	2444365	0.3231	0.2819	0.3643	90	200	0.5187
80	2444375	0.3506	0.2871	0.3643	80	200	0.5187
85	2444380	0.3911	0.3519	0.3643	70	200	0.5187
85	2444390	0.4924	0.3845	0.3643	60	200	0.5187
70	2444365	0.3231	0.2819	0.4096	90	170	0.3992
70	2444375	0.3506	0.2871	0.4096	80	170	0.3992
75	2444380	0.3911	0.3519	0.4096	70	170	0.3992
75	2444390	0.4924	0.3845	0.4096	60	170	0.3992
60	2444365	0.3231	0.2819	0.4877	90	150	0.3138
60	2444375	0.3506	0.2871	0.4877	80	150	0.3138
65	2444380	0.3911	0.3519	0.4877	70	150	0.3138
65	2444390	0.4924	0.3845	0.4877	60	150	0.3138
50	2444365	0.3231	0.2819	0.6274	90	110	0.2547
50	2444375	0.3506	0.2871	0.6274	80	110	0.2547
55	2444380	0.3911	0.3519	0.6274	70	110	0.2547
55	2444390	0.4924	0.3845	0.6274	60	110	0.2547

MARS ROUND TRIPS

0	2444190	0.1124	0.1209	0.2542	250	220	0.5844
	2444240	0.2384	0.1761	0.2779	170	200	0.5574
	2444280	0.5330	0.3190	0.5300	100	150	0.6360
20	244190	0.1124	0.1209	0.2693	250	210	0.5908
	2444250	0.2876	0.1801	0.3064	160	190	0.5822
	2444290	0.5580	0.3670	0.5986	90	140	0.7029
40	2444190	0.1124	0.1209	0.2877	250	200	0.5982
	2444260	0.3445	0.1883	0.3397	150	180	0.6086
	2444290	0.6763	0.4220	0.6669	80	130	0.7703
60	2444190	0.1142	0.1344	0.2959	240	170	0.6393
	2444270	0.4091	0.2031	0.3791	140	170	0.6376

TABLE III (Contd.)

Stay Time Days	Earth Depart. Date Julian	Earth Depart. Velocity EMOS	Arrival Velocity EMOS	Depart. Velocity EMOS	Outbound Leg Days	Inbound Leg Days	Earth Arrival Velocity EMOS
<u>VENUS ROUND TRIPS</u>							
0	2445490	0.090	0.102	0.176	150	300	0.280
	2445510	0.125	0.156	0.163	100	250	0.254
	2445520	0.161	0.222	0.354	80	170	0.267
20	2445490	0.090	0.102	0.181	150	320	0.291
	2445510	0.125	0.156	0.259	100	230	0.288
	2445520	0.180	0.306	0.403	70	160	0.308
40	2445490	0.090	0.102	0.180	150	330	0.317
	2445510	0.135	0.214	0.325	90	220	0.315
	2445530	0.224	0.325	0.531	60	150	0.372
60	2445490	0.090	0.102	0.182	150	350	0.298
	2445510	0.153	0.291	0.394	80	210	0.349
	2445530	0.274	0.453	0.630	50	140	0.459
<u>CERES ROUND TRIPS</u>							
0	2444420	0.2405	0.1998	0.2502	370	410	0.4677
	2444420	0.2570	0.2160	0.2421	340	320	0.2826
20	2444420	0.2405	0.1998	0.2605	370	400	0.5093
	2444420	0.2621	0.2247	0.2351	330	310	0.2860
40	2444420	0.2405	0.1998	0.2712	370	390	0.5524
	2444420	0.2672	0.2351	0.2619	320	300	0.2897
60	2444420	0.2518	0.2089	0.2712	350	390	0.5524
	2444420	0.2727	0.2420	0.2748	310	290	0.2937
<u>URANUS ROUND TRIPS</u>							
0	2447230	0.3988	0.1501	0.1513	6190	5840	0.4007
	2447220	0.3929	0.1592	0.1669	5000	4840	0.4005
	2447210	0.3894	0.2038	0.2110	3800	3820	0.4793
	2447200	0.4055	0.3013	0.3416	2800	2640	0.4116
	2447220	0.5082	0.6116	0.6358	1600	1600	0.5205
60	2447230	0.3988	0.1501	0.1516	6190	5800	0.4012
	2447220	0.3929	0.1592	0.1685	5000	4800	0.4211
	2447210	0.3894	0.2038	0.2157	3800	3760	0.4761
	2447200	0.4055	0.3013	0.3552	2800	2570	0.4128
	2447220	0.5082	0.6116	0.6680	1600	1530	0.5186

TABLE III (Contd.)

Stay	Earth Depart. Time Date Days Julian	Earth Depart. Velocity EMOS	Arrival Velocity EMOS	Departure Velocity EMOS	Outbound Leg Days	Inbound Leg Days	Earth Arrival Velocity EMOS
<u>GANYMEDE ROUND TRIPS (JUPITER III)</u>							
0	2444970	0.2931	0.1954(.185)*	0.1972(.186)*	950	850	0.3384
	2444970	0.3075	0.2754(.218)	0.2390(.207)	700	700	0.3641
	2444980	0.3608	0.4581(.325)	0.4161(.297)	500	510	0.3590
60	2444970	0.2931	0.1954(.185)	0.2033(.188)	950	820	0.3496
	2444970	0.3075	0.2754(.218)	0.2596(.211)	700	700	0.4149
	2444980	0.3608	0.4517(.320)	0.5023(.355)	500	450	0.3906
<u>TITAN ROUND TRIPS (SATURN VI)</u>							
0	2446840	0.3551	0.2033(.145)*	0.1790(.133)*	1820	2180	0.4218
	2446840	0.3590	0.2350(.164)	0.2189(.155)	1620	1620	0.3756
	2446840	0.3771	0.3263(.230)	0.3228(.225)	1260	1260	0.4168
	2446840	0.4335	0.5229(.397)	0.5611(.430)	900	860	0.5005
60	2446840	0.3551	0.2033(.145)	0.1782(.133)	1820	2120	0.3877
	2446840	0.3590	0.2350(.164)	0.2290(.165)	1620	1560	0.3787
	2446840	0.3771	0.3263(.230)	0.3462(.248)	1260	1200	0.4250
	2446840	0.4335	0.5229(.397)	0.6180(.480)	900	800	0.5339
<u>NEPTUNE ROUND TRIPS</u>							
0	2450520	0.4023	0.1783	0.1807	7000	6840	0.4378
	2450520	0.4023	0.2131	0.2104	6000	6040	0.4611
	2450520	0.4106	0.2684	0.2533	5000	5200	0.4678
	2450520	0.4285	0.3574	0.3562	4000	4000	0.4799
60	2450520	0.4023	0.1783	0.1822	7000	6800	0.4133
	2450520	0.4023	0.2131	0.2126	6000	5960	0.4016
	2450520	0.4106	0.2684	0.2577	5000	5120	0.4099
	2450520	0.4285	0.3574	0.3640	4000	3920	0.4275
<u>PLUTO ROUND TRIP</u>							
0	2451560	0.4094	0.1922	0.2384	6800	7060	0.3978
	2451580	0.4430	0.2641	0.3366	5160	5000	0.4427
	2451580	0.5118	0.4853	0.5748	3140	2980	0.5200
	2451580	0.6210	0.7501	0.8178	2160	2120	0.6294
60	2451560	0.4094	0.1922	0.2410	6800	7000	0.3982
	2451580	0.4430	0.2641	0.3418	5160	4940	0.4425
	2451580	0.5118	0.4853	0.5888	3140	2920	0.5230
	2451580	0.6210	0.7501	0.8446	2160	2060	0.6416

* Based on moon system (see Appendix E)

TABLE IV

MISSION SUMMARY

Objective Body	Earth Departure Date	Unmanned Flyby	Unmanned Orbiter	Manned Orbiter	Manned Landing
Sun	Unspecified	0.05 AU	1.0 AU	No	No
Mercury	1980-1985	Yes	0.176 AU*	h=92.6 km	h=92.6 km
Venus	1983	Yes	h=92.6 km**	h=926 km	No
Mars	1980	Yes	h=1852 km	h=926 km	h=926 km
Ceres	1980	Yes	h=926 km	No	h=50 km
Jupiter	1981-1982	Yes	No	About Ganymede	On Ganymede
			$r_p = 2.0$ ***	h=92.6 km	h=92.6 km
Saturn	1986-1988	Yes	$r_p = 2.39$	About Titan	On Titan
				h=556 km	h=556 km
Uranus	1987-1988	Yes	$r_p = 2.0$	h=413,200 km	No
Neptune	1996-1998	Yes	$r_p = 2.0$	h=329,000 km	No
Pluto	2000	Yes	No	Yes	No
S-W II	1980-1981	Yes	Rendezvous instead of orbiter	No	No

* Synchronous orbit

** Orbital altitude

*** Orbital radius in planetary radii

TABLE V

JUPITER SWINGBYS FOR SATURN FLYBYS

EARTH DEPARTURE			JUPITER SWINGBY			SATURN ARRIVAL
Julian Date	V_{∞} (EMOS)	Total Trip Time (Days)	Julian Date	V_{∞} (EMOS)	r_p (Jupiter Radii)	Julian Date
244-3400	0.3252	1792.	244-4200	0.2091	17.590	244-5192
244-3400	0.3328	1515.	244-4100	0.2476	11.319	244-4915
244-3400	0.3453	1289.	244-4000	0.3120	6.6438	244-4689
244-3400	0.3690	1080.	244-3900	0.4137	3.6745	244-4480
244-3800	0.3787	1011.	244-4300	0.4319	12.529	244-4811
244-3850	0.3557	1223.	244-4400	0.3400	19.831	244-5073

JUPITER SWINGBYS FOR URANUS FLYBYS

EARTH DEPARTURE			JUPITER SWINGBY			URANUS ARRIVAL
Julian Date	V_{∞} (EMOS)	Total Trip Time (Days)	Julian Date	V_{∞} (EMOS)	r_p (Jupiter Radii)	Julian Date
244-3800	0.3357	3641.	244-4600	0.2157	1.0000	244-7441
244-3800	0.3437	2944.	244-4500	0.2580	5.0892	244-6744
244-3800	0.3557	2490.	244-4400	0.3262	2.5791	244-6290
244-3800	0.3787	2091.	244-4300	0.4319	1.2935	244-5891
244-4200	0.3701	2544.	244-4800	0.3354	1.2311	244-6744
244-4200	0.3910	1938.	244-4700	0.4435	5.2657	244-6138
244-4550	0.4620	1613.	244-5000	0.5762	9.1159	244-6233
244-4600	0.4068	2348.	244-5100	0.4452	23.318	244-6948

JUPITER SWINGBYS FOR NEPTUNE FLYBYS

EARTH DEPARTURE			JUPITER SWINGBY			NEPTUNE ARRIVAL
Julian Date	V_{∞} (EMOS)	Total Trip Time (Days)	Julian Date	V_{∞} (EMOS)	r_p (Jupiter Radii)	Julian Date
244-3800	0.3357	5547.	244-4600	0.2157	3.9926	244-9347
244-3800	0.3437	4695.	244-4500	0.2580	1.9592	244-8495
244-4200	0.3582	5438.	244-4900	0.2650	1.5042	244-9638
244-4200	0.3701	3751.	244-4800	0.3354	5.8011	244-7951
244-4200	0.3910	2979.	244-4700	0.4435	2.4670	244-7179
244-4600	0.4068	3258.	244-5100	0.4452	1.0538	244-7858
244-4600	0.4518	2314.	244-5000	0.6148	3.8832	244-6914
244-5000	0.4625	2892.	244-5400	0.6036	1.8781	244-7892
244-5000	0.5784	1790	244-5300	0.8852	5.3753	244-6790

TABLE VI

JUPITER SWINGBYS FOR SATURN STOPOVERS

EARTH DEPARTURE			JUPITER SWINGBY			SATURN ARRIVAL	
Julian Date	V_{∞} (EMOS)	Total Trip Time (Days)	Julian Date	V_{∞} (EMOS)	r_p (Jupiter Radii)	Julian Date	V_{∞} (EMOS)
244-3050	0.5438	1901.	244-4000	0.2044	1.1955	244-4951	0.2997
244-3400	0.3252	1792.	244-4200	0.2091	17.590	244-5192	0.2479
244-3400	0.3328	1515.	244-4100	0.2476	11.319	244-4915	0.3378
244-3400	0.3453	1289.	244-4000	0.3120	6.6438	244-4689	0.4394
244-3400	0.3690	1080.	244-3900	0.4137	3.6745	244-4480	0.5684
244-3800	0.3787	1011.	244-4300	0.4319	12.529	244-4811	0.5054
244-3850	0.3557	1223.	244-4400	0.3400	19.831	244-5073	0.3355

JUPITER SWINGBYS FOR URANUS STOPOVERS

EARTH DEPARTURE			JUPITER SWINGBY			URANUS ARRIVAL	
Julian Date	V_{∞} (EMOS)	Total Trip Time (Days)	Julian Date	V_{∞} (EMOS)	r_p (Jupiter Radii)	Julian Date	V_{∞} (EMOS)
244-3800	0.3357	3641.	244-4600	0.2157	1.0000	244-7441	0.2692
244-3800	0.3437	2944.	244-4500	0.2580	5.0892	244-6744	0.3812
244-3800	0.3557	2490.	244-4400	0.3262	2.5791	244-6290	0.4878
244-3800	0.3787	2091.	244-4300	0.4319	1.2935	244-5891	0.6172
244-4200	0.3701	2544.	244-4800	0.3354	1.2311	244-6744	0.4118
244-4200	0.3910	1938.	244-4700	0.4435	5.2657	244-6138	0.6059
244-4250	0.7213	3700.	244-4900	0.2825	1.8988	244-7950	0.2219
244-4550	0.4620	2530.	244-5100	0.4268	24.567	244-7080	0.3701
244-4600	0.4068	2348.	244-5100	0.4452	23.318	244-6948	0.4021

JUPITER SWINGBYS FOR NEPTUNE STOPOVERS

EARTH DEPARTURE			JUPITER SWINGBY			NEPTUNE ARRIVAL	
Julian Date	V_{∞} (EMOS)	Total Trip Time (Days)	Julian Date	V_{∞} (EMOS)	r_p (Jupiter Radii)	Julian Date	V_{∞} (EMOS)
244-3700	0.7157	5929.	244-4700	0.2195	1.5430	244-9629	0.2776
244-3800	0.3357	5547.	244-4600	0.2157	3.9926	244-9347	0.2978
244-3800	0.3437	4695.	244-4500	0.2580	1.9592	244-8495	0.3808
244-4200	0.3582	5438.	244-4900	0.2650	1.5042	244-9638	0.2812
244-4200	0.3701	3751.	244-4800	0.3354	5.8011	244-7951	0.4797
244-4200	0.3910	2979.	244-4700	0.4435	2.4670	244-7179	0.6467
244-4600	0.4068	3258.	244-5100	0.4452	1.0538	244-7858	0.5260

TABLE VII

DEPENDENCE OF Γ ON v_A AND v_B
 Lv $\oplus$ 5490 Ar $\ominus$ 5640
 $V_\infty = 0.090$ EMOS $V_\infty = 0.102$ EMOS

$v_A \backslash v_B$	0	0.2	0.4	0.6	0.8	1.0
0	1	0.78412	0.6300	0.5240	0.4541	0.4136
0.2	0.7865	0.5734	0.4255	0.3290	0.2704	0.2413
0.4	0.62332	0.41330	0.27126	0.18373	0.13608	0.11176
0.6	0.50897	0.30292	0.16665	0.08804	0.0505	0.03676
0.8	0.43994	0.24047	0.11081	0.04060	0.02163	0.01540
1.0	0.39963	0.20241	0.07872	0.02560	0.00986	0

$$J_n = 5.781 \text{ m}^2/\text{sec}^3$$

SENSITIVITY OF PAYLOAD RATIOS TO "Γ SURFACE"

Nuclear Propulsion, $I_{sp} = 800 \text{ sec}$ $\alpha_4 = 10 \text{ kg/kw}$

I Function			9 x 9 Table								5 x 5 Table							
β_1	β_2	ν_A	ν_B	μ_L	β_1/β_2	ν_A	ν_B	μ_L	$\% \text{ Diff}$	ν_A	ν_B	μ_L	$\% \text{ Diff}$	ν_A	ν_B	μ_L	$\% \text{ Diff}$	
.15	.15	.7156	.4212	.3200	.15/.15	.7538	.4000	.3192	5.3	5.0	.8000	.4000	11.8	5.0	.8000	.4000	.28	
	.20	.7238	.3975	.3116	.20	.7538	.4000	.3110	4.1	.63	.8000	.4000	10.5	.63	.8000	.4000	.22	
	.25	.7325	.3725	.3027	.25	.7538	.4000	.3018	2.9	7.4	.8000	.4000	9.2	7.4	.8000	.4000	.36	
	.30	.7412	.3475	.2930	.30	.7612	.3669	.2915	5.4	5.6	.8000	.4000	7.9	15.1	.8000	.4000	.68	
.20	.15	.6819	.4250	.3050	.20/.15	.6788	.4000	.3041	.45	5.8	.7944	.4000	16.5	5.9	.7944	.4000	.59	
	.20	.6894	.4012	.2969	.20	.6788	.4000	.2963	1.5	.30	.7944	.4000	15.2	.30	.7944	.4000	.51	
	.25	.6981	.3762	.2883	.25	.6788	.4000	.2874	2.8	6.3	.7944	.4000	13.8	6.3	.7944	.4000	.59	
	.30	.7069	.3506	.2790	.30	.6950	.3788	.2775	1.7	8.0	.7944	.4000	12.4	14.09	.7944	.4000	.86	
.25	.15	.6438	.4294	.2882	.25/.15	.6062	.4000	.2876	5.8	6.8	.7088	.4000	10.1	6.8	.7088	.4000	.87	
	.20	.6519	.4050	.2806	.20	.6062	.4000	.2802	7.0	1.2	.7088	.4000	8.7	1.2	.7088	.4000	.78	
	.25	.6600	.3800	.2723	.25	.6062	.4000	.2718	8.2	5.3	.7088	.4000	7.4	5.3	.7088	.4000	.81	
	.30	.6681	.3544	.2635	.30	.6131	.3900	.2623	8.2	10.0	.7088	.4000	6.1	12.9	.7088	.4000	1.1	
.30	.15	.6025	.4344	.2695	.30/.15	.6000	.4000	.2691	.41	7.9	.6256	.4000	3.8	7.9	.6256	.4000	1.0	
	.20	.6094	.4094	.2623	.20	.6000	.4000	.2622	1.5	2.3	.6256	.4000	2.7	2.3	.6256	.4000	.91	
	.25	.6175	.3844	.2545	.25	.6000	.4000	.2544	2.8	4.1	.6256	.4000	1.3	4.1	.6256	.4000	.90	
	.30	.6250	.3581	.2462	.30	.6000	.3919	.2455	4.0	9.4	.6256	.4000	1.0	11.7	.6256	.4000	1.2	

TABLE IX

COMPARISON OF CHEMICAL AND LOW-THRUST OPERATION

Mass Breakdown for Mars Unmanned Orbital Stopover

160 Day Transfer

	Optimum Mix <u>kg</u>	Parabolic* <u>kg</u>
Payload	4536	4536
Orbit Capture Step ¹	5443	280
Propellant	4233	1932
Inert	1210	648
Heliocentric (Electric) Step	9658	63219
Propellant	5638	47962
Powerplant ²	3691	14011
Inert	329	1246
Earth Departure Step ¹	35327	97129
Propellant	32458	90126
Inert	2869	7003
Mass on Earth Parking Orbit	54964 (121000 lb)	167464 (368500 lb)

1. Chemical, $I_{sp} = 430$ sec

2. Specific weight = 18 kg/kw

* High thrust operation at parabolic conditions only

TABLE X

COMPARISON OF NUCLEAR AND LOW-THRUST OPERATION

Mass Breakdown for Mars Unmanned Orbital Stopover

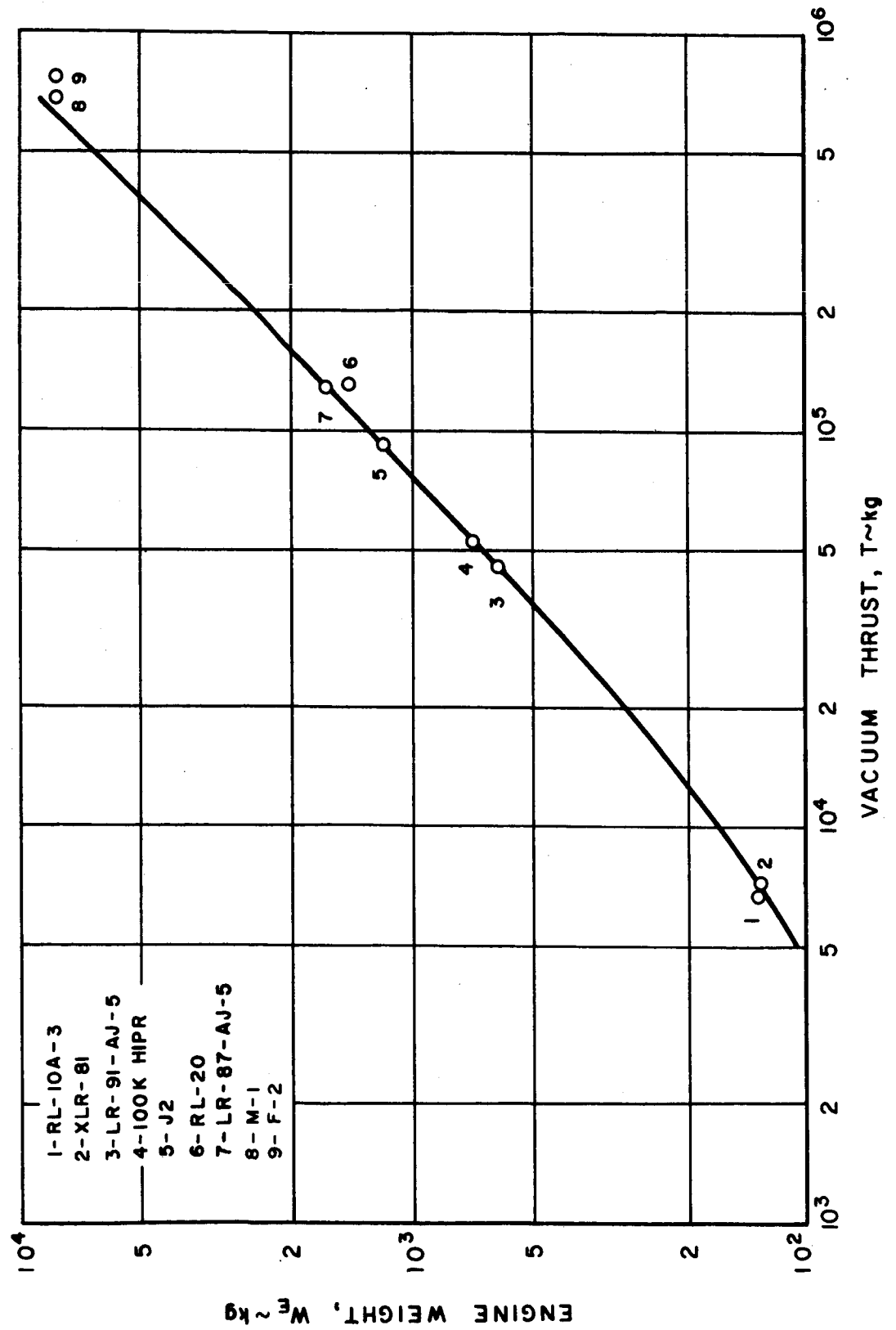
	160 Day Transfer		300 Day Transfer	
	Optimum Mix kg	Parabolic* kg		Parabolic* kg
Payload	4536	4536		4536
Orbit Capture Step ¹				
Propellant	2679	1141	1141	2508
Inert	2823	1367	1367	1141
				1367
Heliocentric (Electric) Step	7303	30706		3515
Propellant	4148	21443		1935
Powerplant ²	2784	8173		1394
Inert	371	1090		186
Earth Departure Step ¹	18505	30854		9290
Propellant	13989	23864		6905
Inert	4516	6990		2385
Mass on Earth Parking Orbit	35846 (78900 lb)	68604 (151000 lb)		19849 (43700 lb)

1. Nuclear, $I_{sp} = 800$ sec

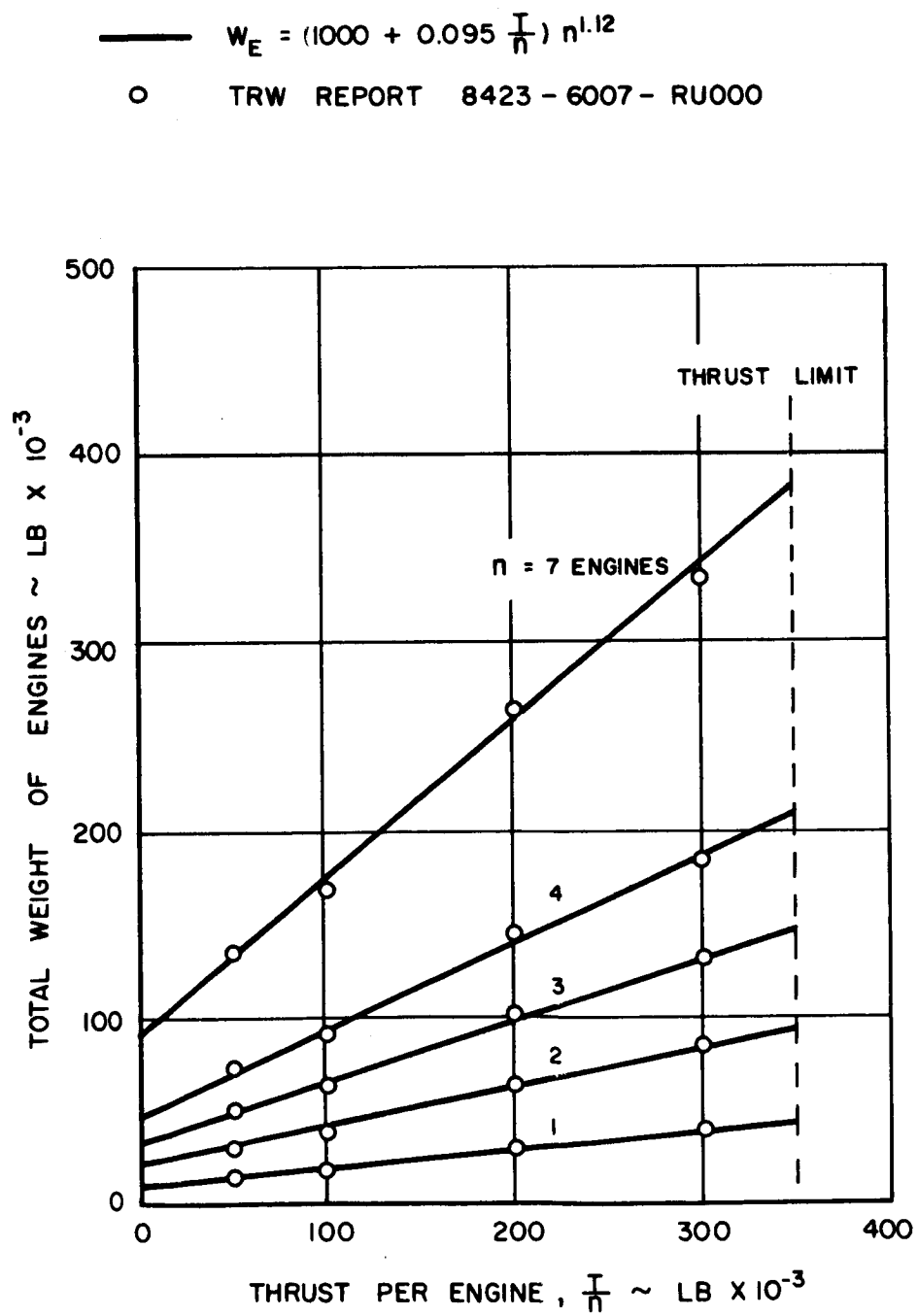
2. Specific weight = 12 kg/kw

* High thrust operation at parabolic conditions only

LIQUID-ROCKET ENGINE WEIGHT SUMMARY

ALL AREA RATIO ≥ 40 

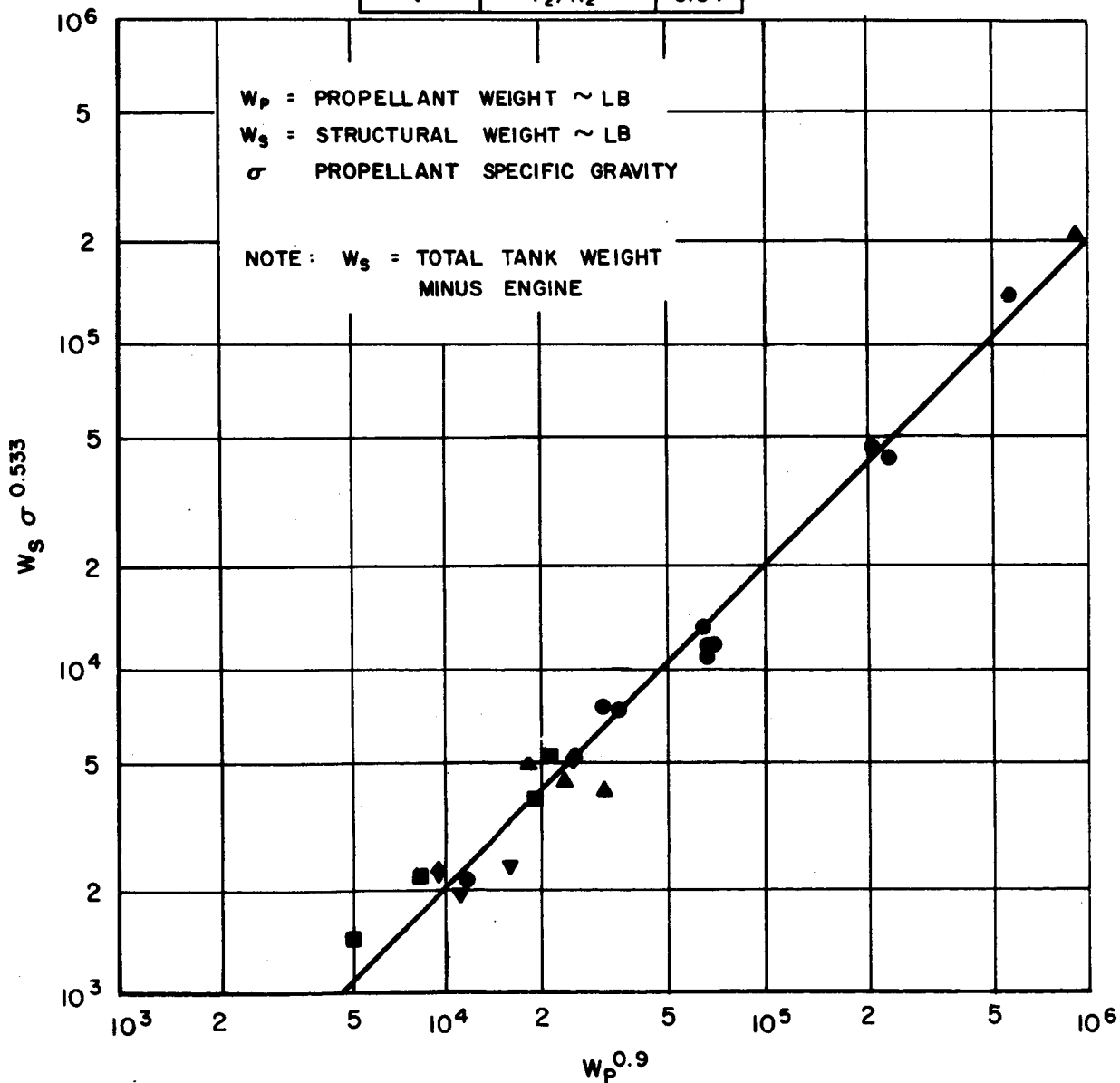
SOLID-CORE NUCLEAR ENGINE WEIGHTS



TANK WEIGHT CORRELATION FOR UPPER STAGE BOOSTERS

$$W_S = \frac{AW_P^{0.9}}{\sigma^{0.533}}$$

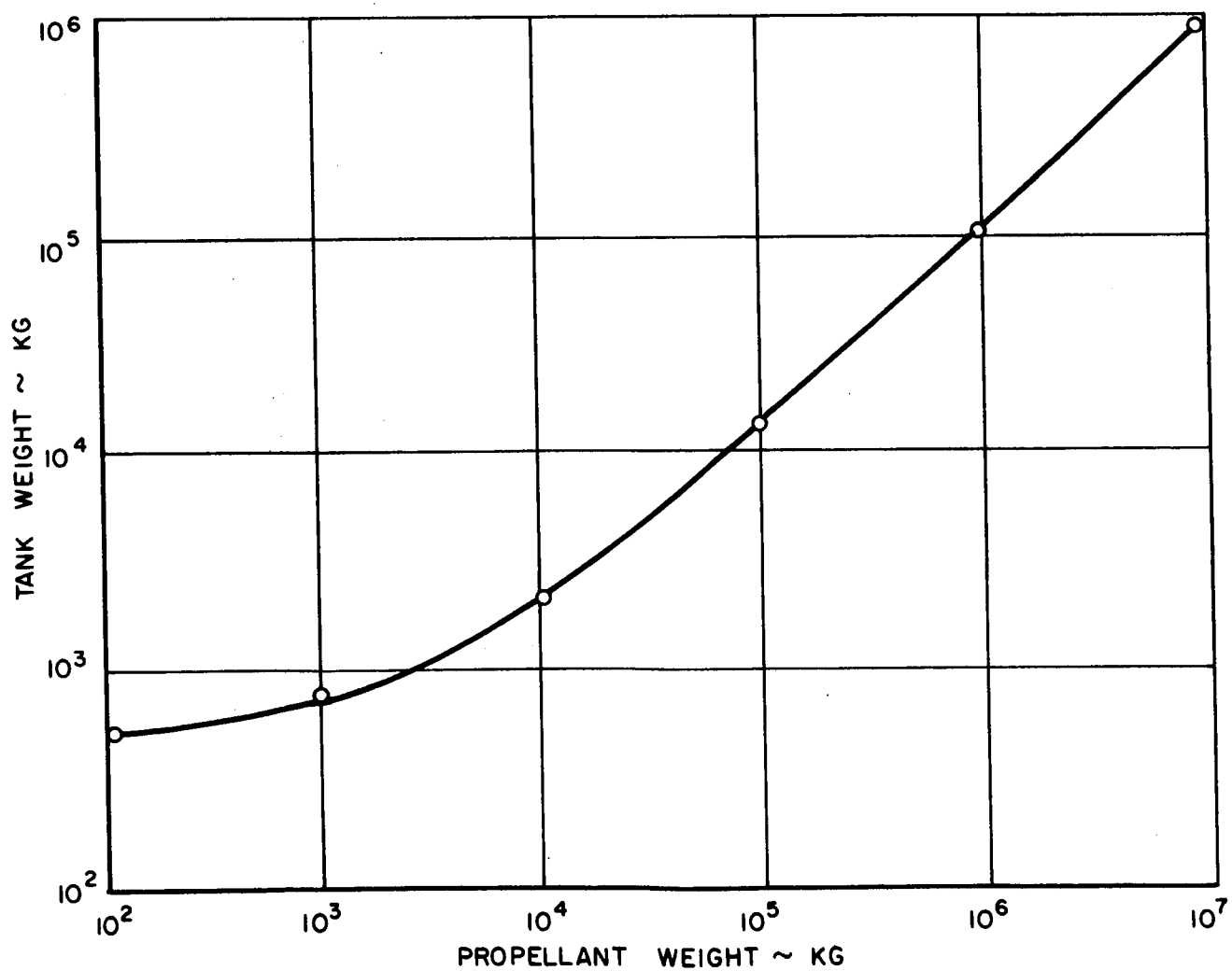
SYMBOL	PROPELLANT	$\sigma =$
●	O ₂ /H ₂	0.32
■	UDMH/IRFNA	1.20
▲	LOX/RP	0.61
◆	N ₂ O ₄ /AERO/AL	1.33
▼	F ₂ /H ₂	0.57



TANK WEIGHT CORRELATION

HYDROGEN

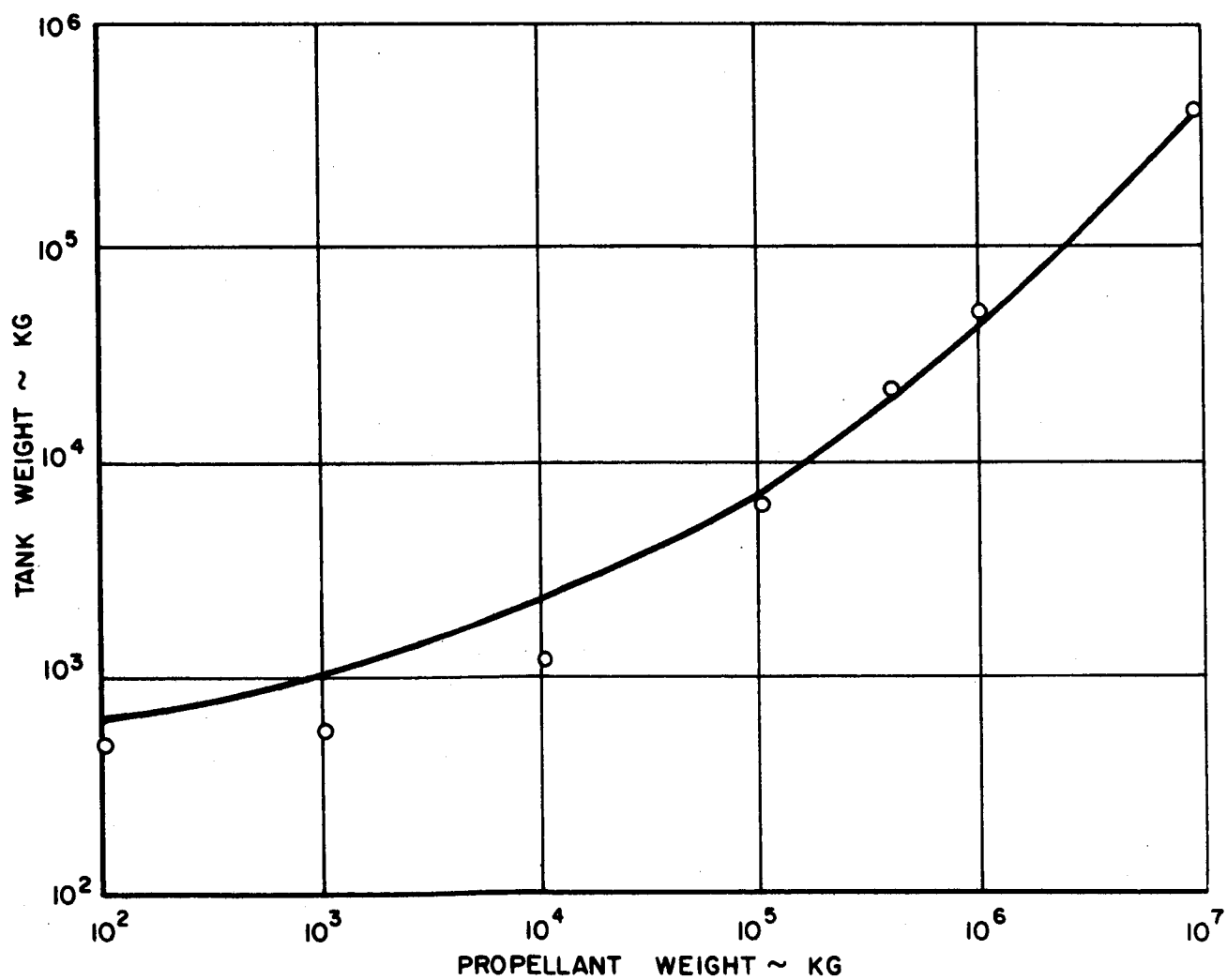
— NASA
○ UARL $\left(W_T = \frac{0.1 W_P^{0.9}}{\sigma^{0.533}} + 500 \right)$



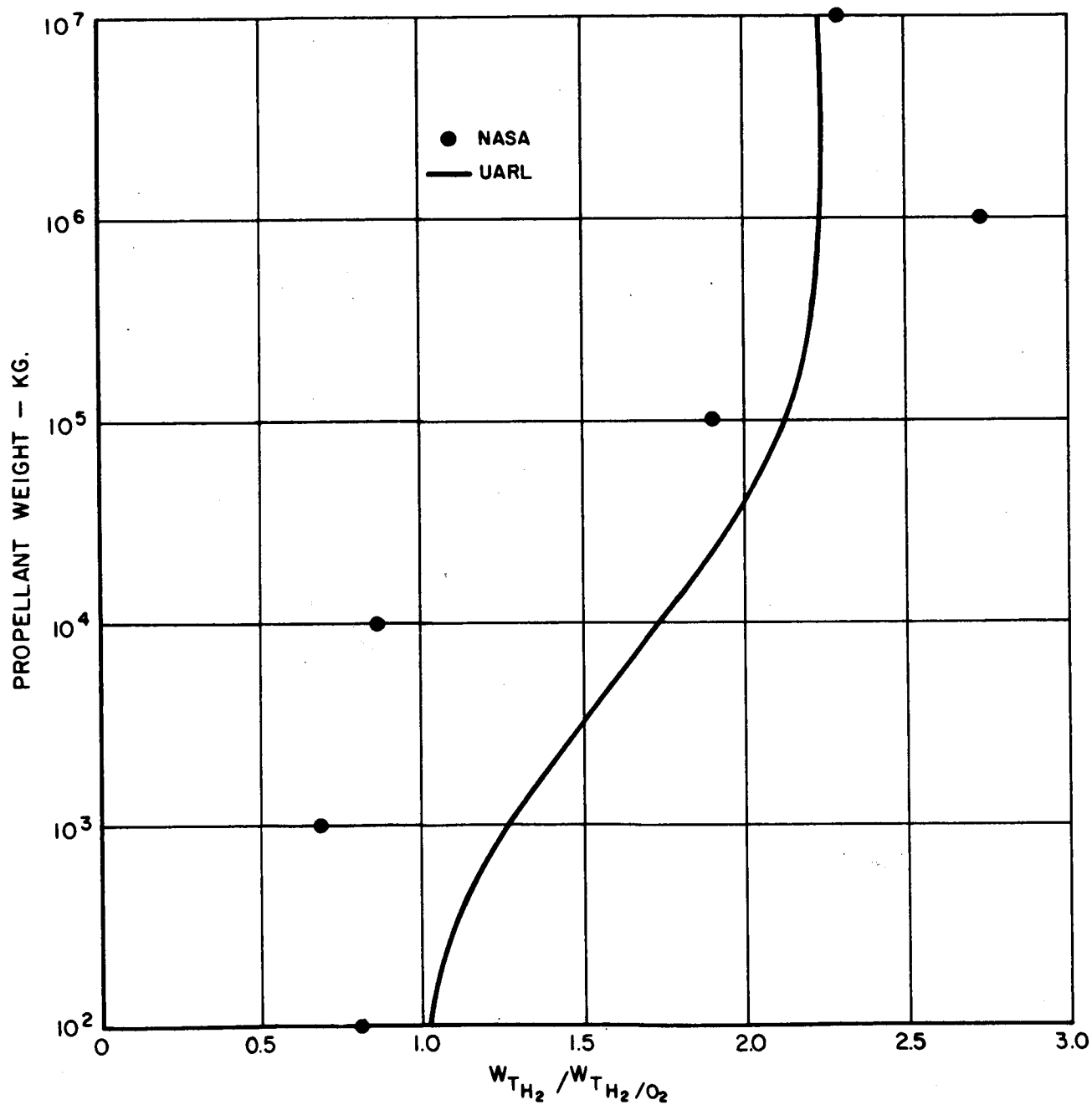
TANK WEIGHT CORRELATION

OXYGEN/HYDROGEN

— NASA
○ UARL $(W_T = \frac{0.1 W_P^{0.9}}{\sigma^{0.533}} + 500)$



TANK WEIGHT COMPARISON FOR HYDROGEN AND OXYGEN/HYDROGEN TANKS



180 DEGREE TRANSFERS TO MERCURY

(COPLANAR)

○ EARTH DEPARTURE DATES

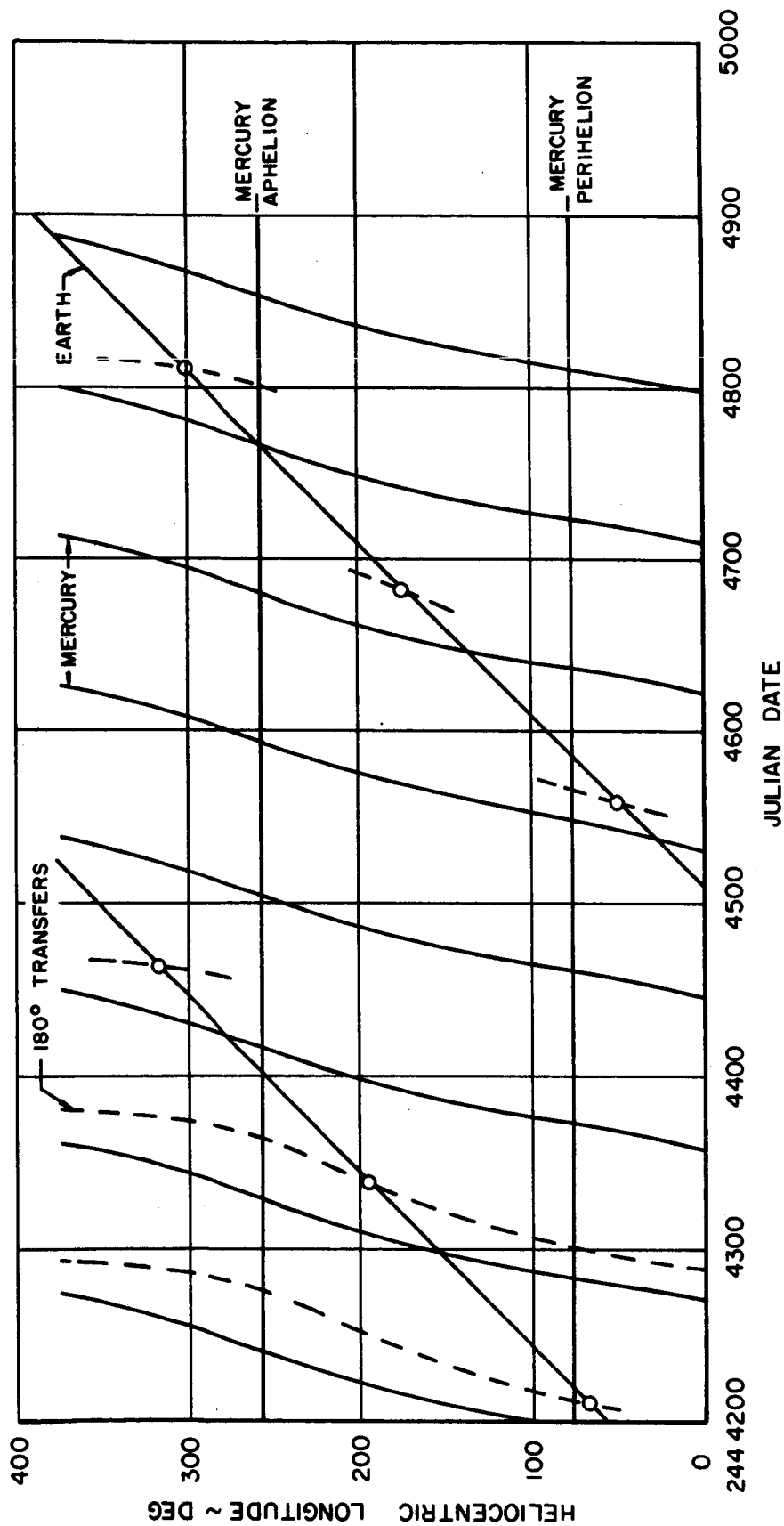
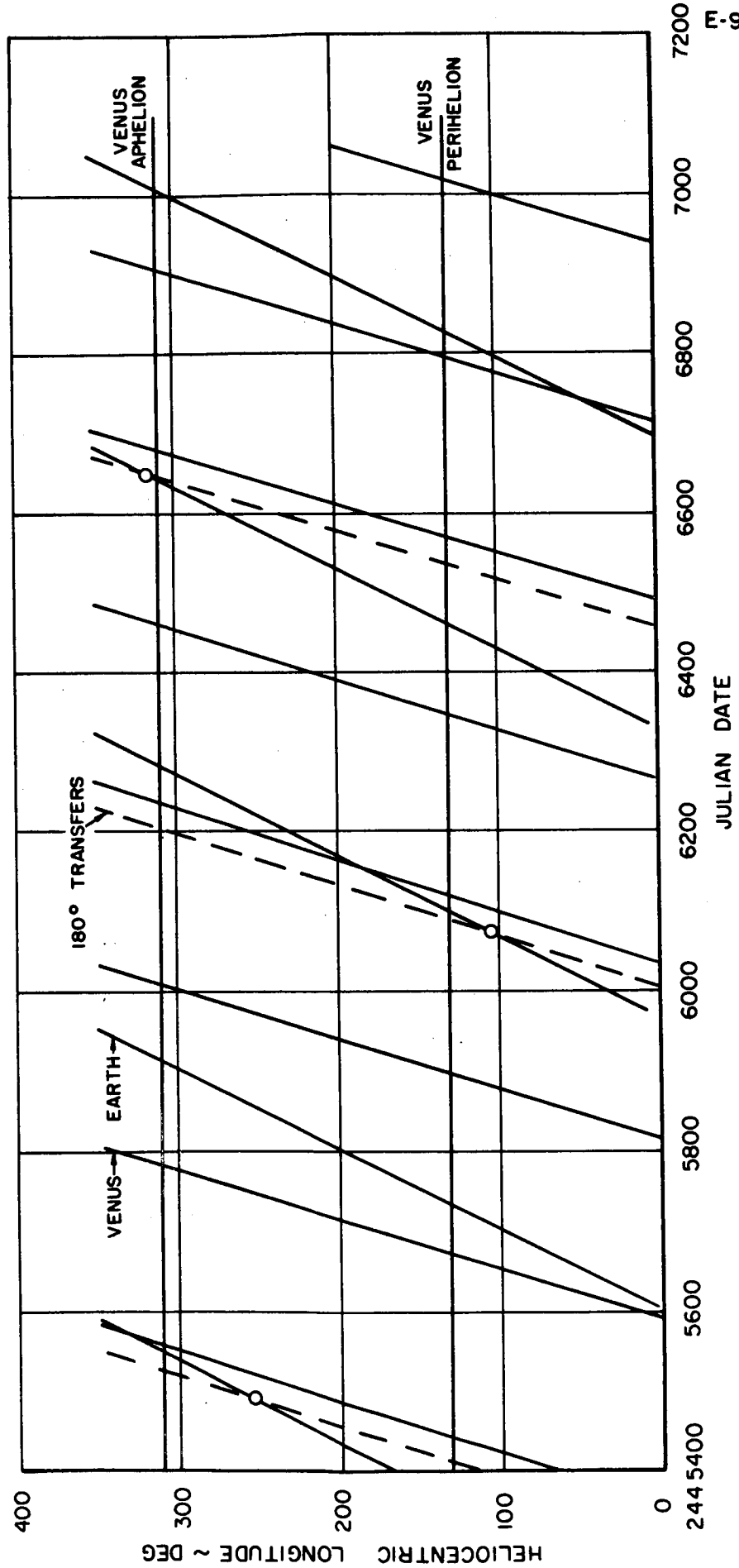


FIG. 8

180 DEGREE TRANSFERS TO VENUS

(COPLANAR)

O EARTH DEPARTURE DATES



180 DEGREE TRANSFERS TO MARS

(COPLANAR)

O EARTH DEPARTURE DATES

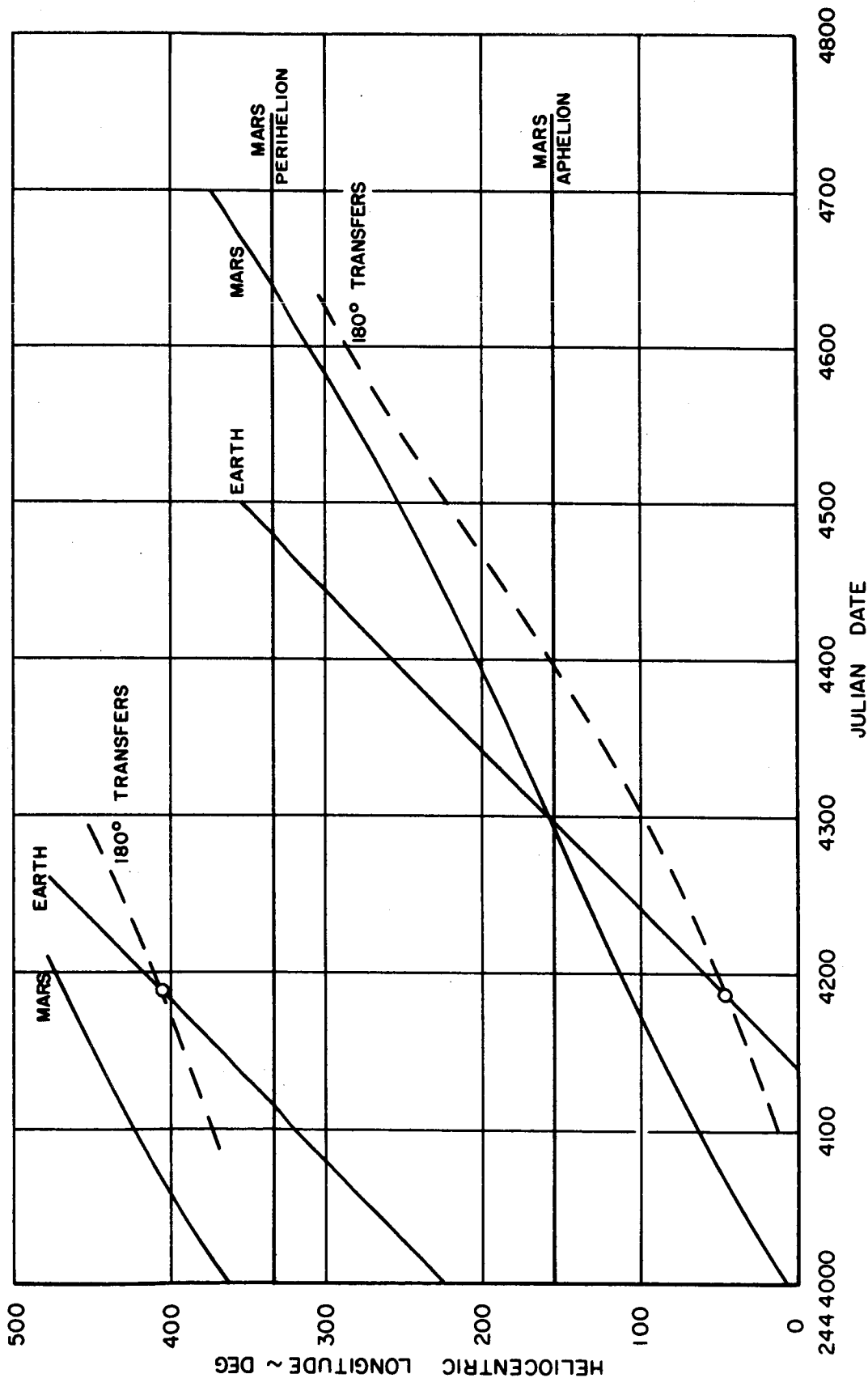
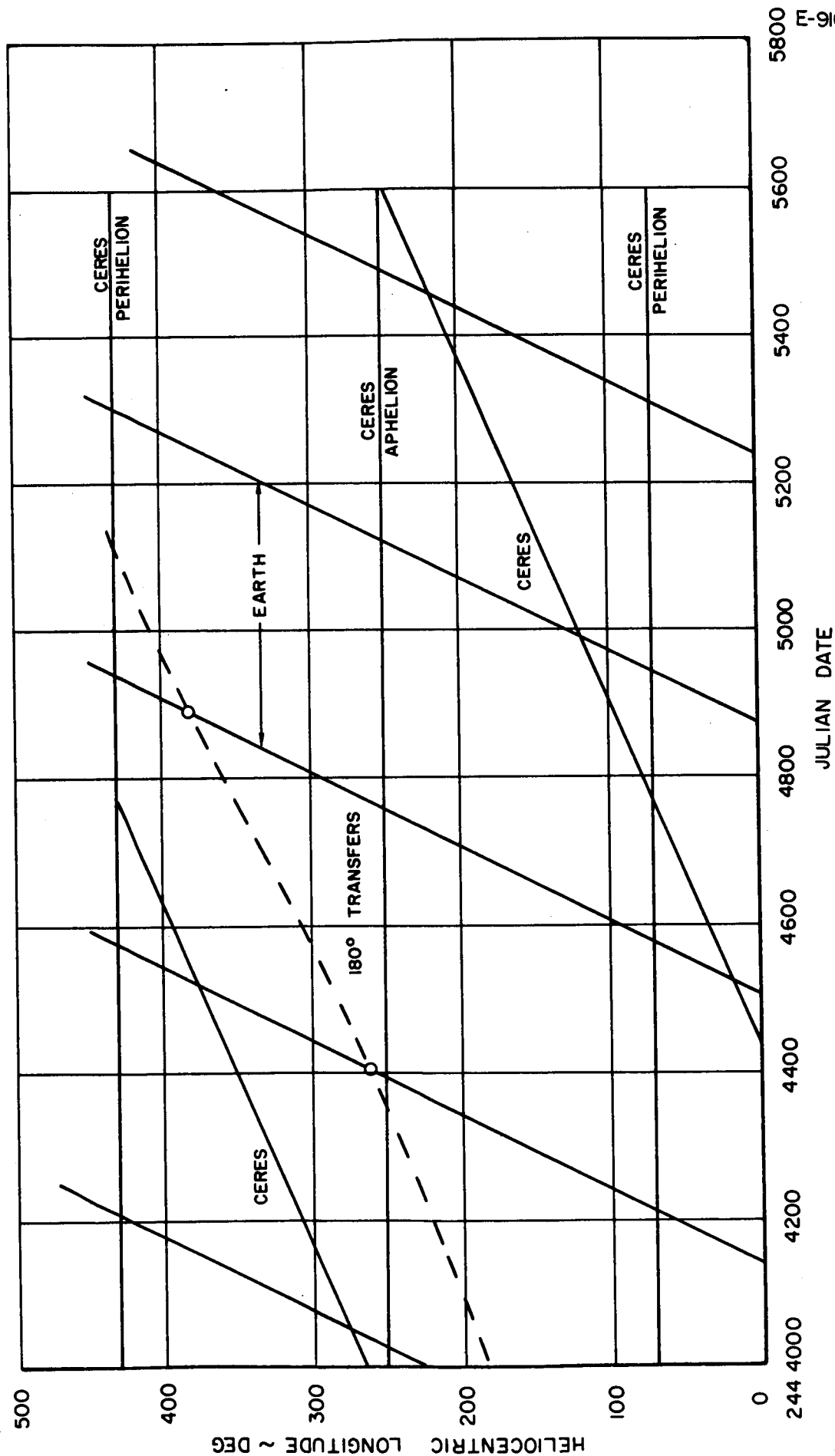


FIG. 10

180 DEGREE TRANSFERS TO ASTEROID CERES

(COPLANAR)

○ EARTH DEPARTURE DATES



180 DEGREE TRANSFERS TO JUPITER

(COPLANAR)

○ EARTH DEPARTURE DATES

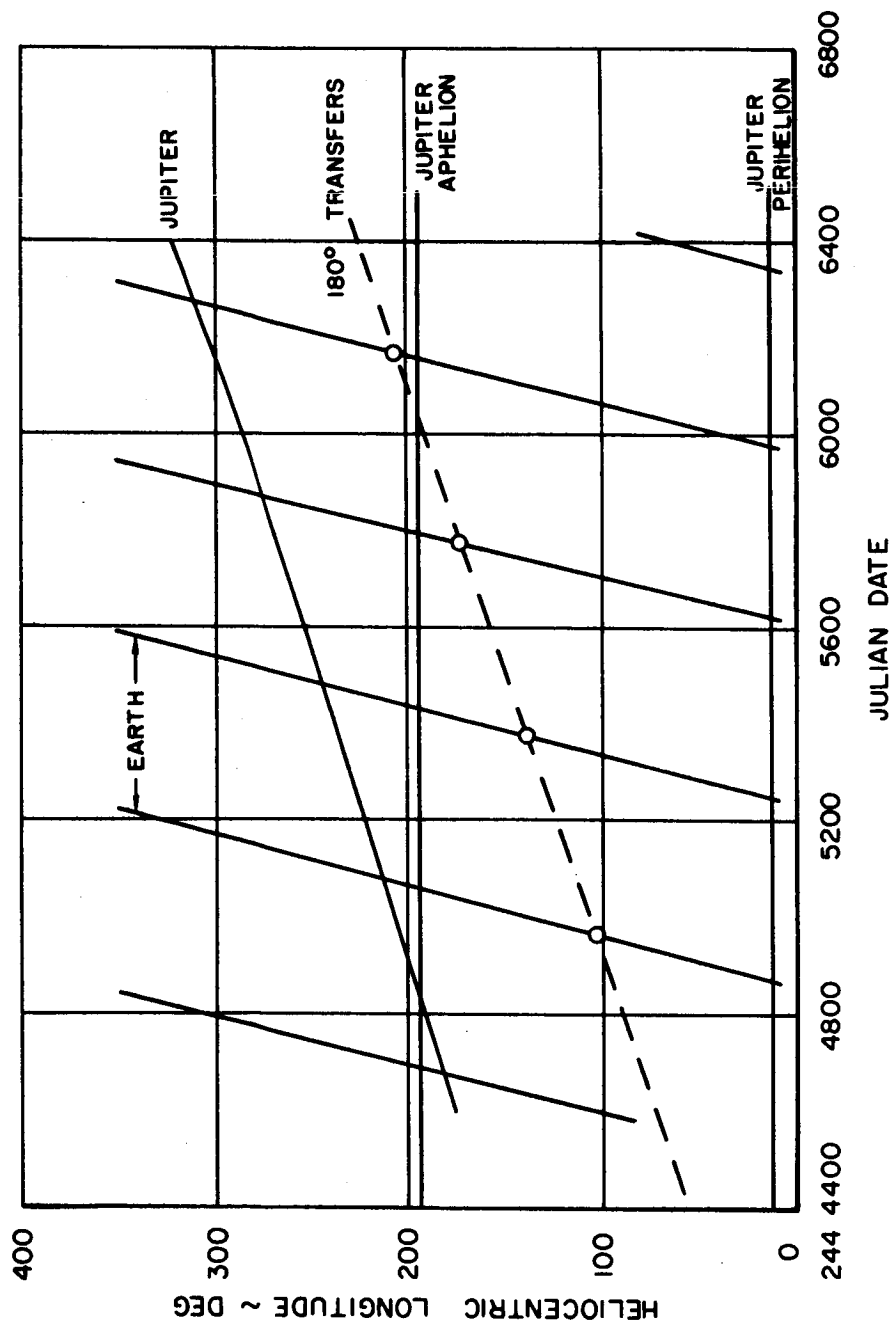


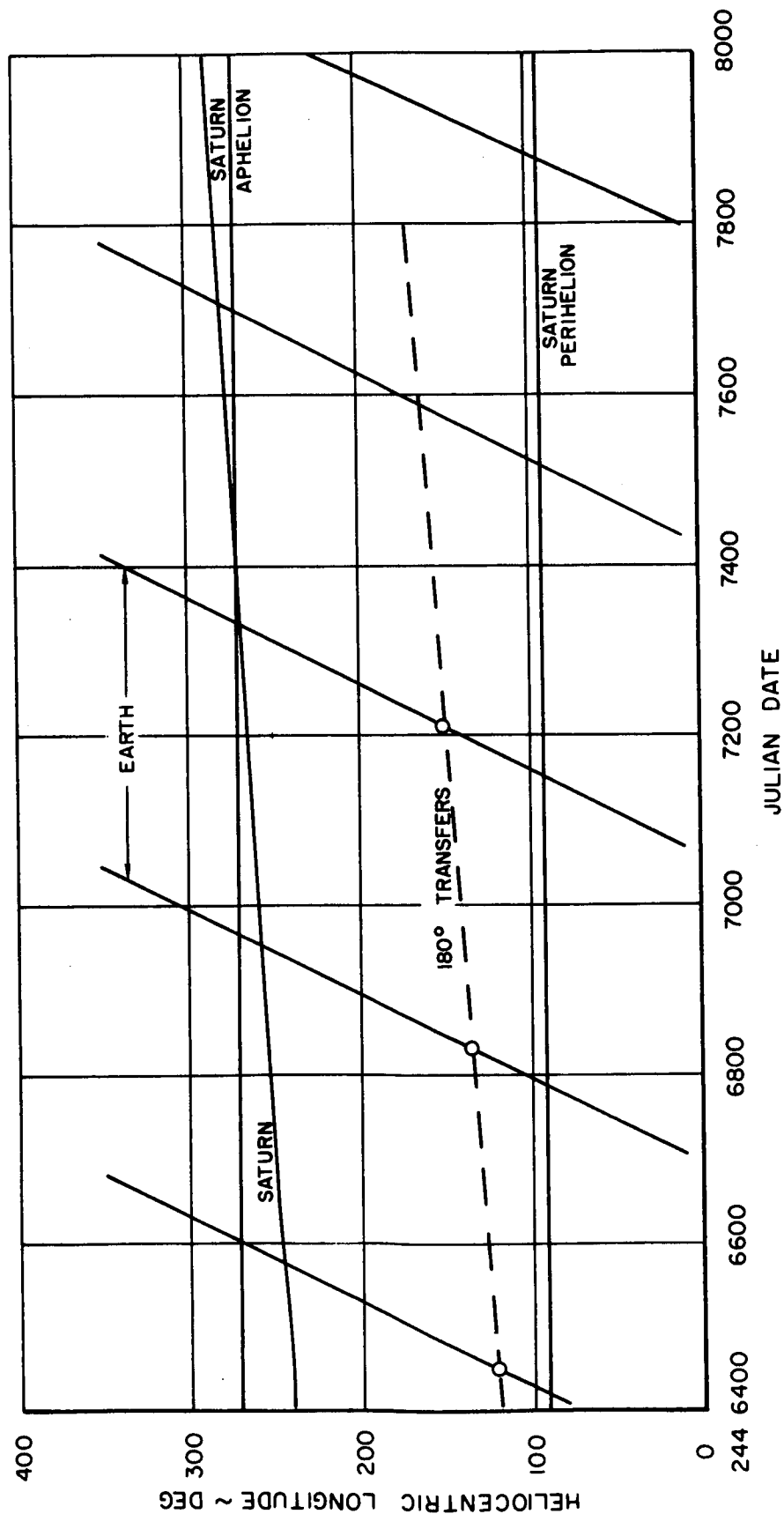
FIG.12

E-910352-9

180 DEGREE TRANSFERS TO SATURN

(COPLANAR)

O EARTH DEPARTURE DATES



180 DEGREE TRANSFERS TO URANUS

(COPLANAR)

○ EARTH DEPARTURE DATES

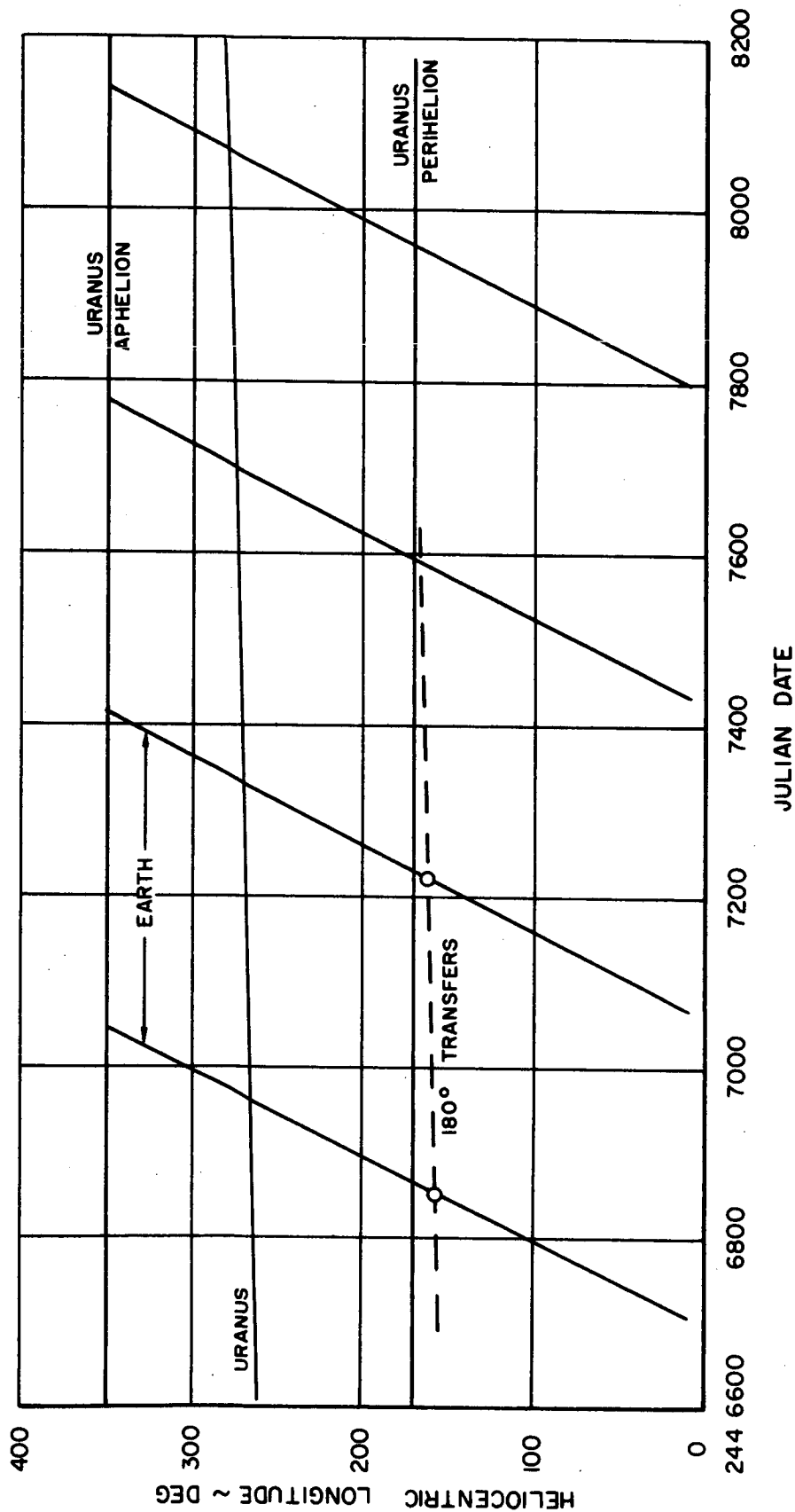


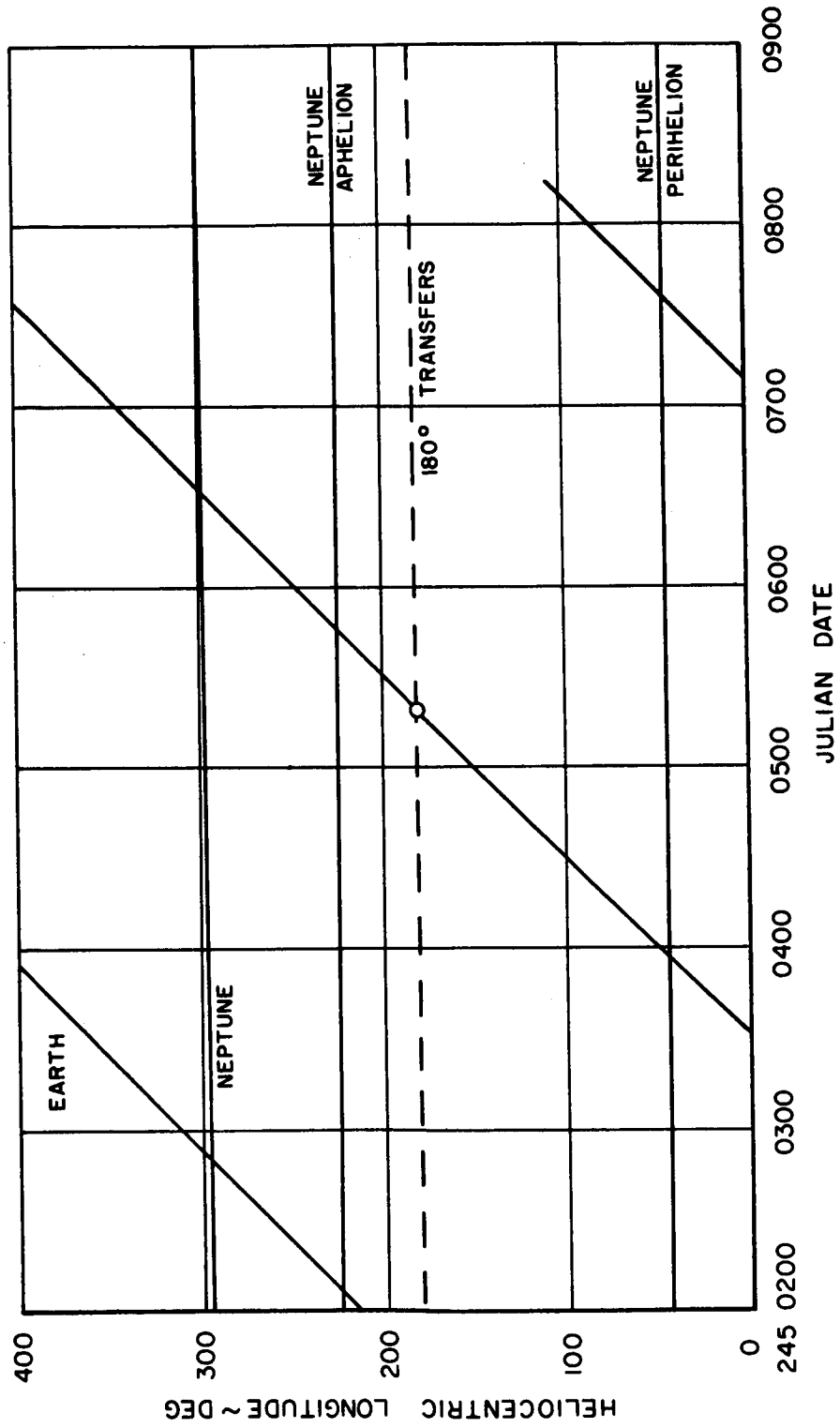
FIG. 14

E-910352-9

180 DEGREE TRANSFERS TO NEPTUNE

(COPLANAR)

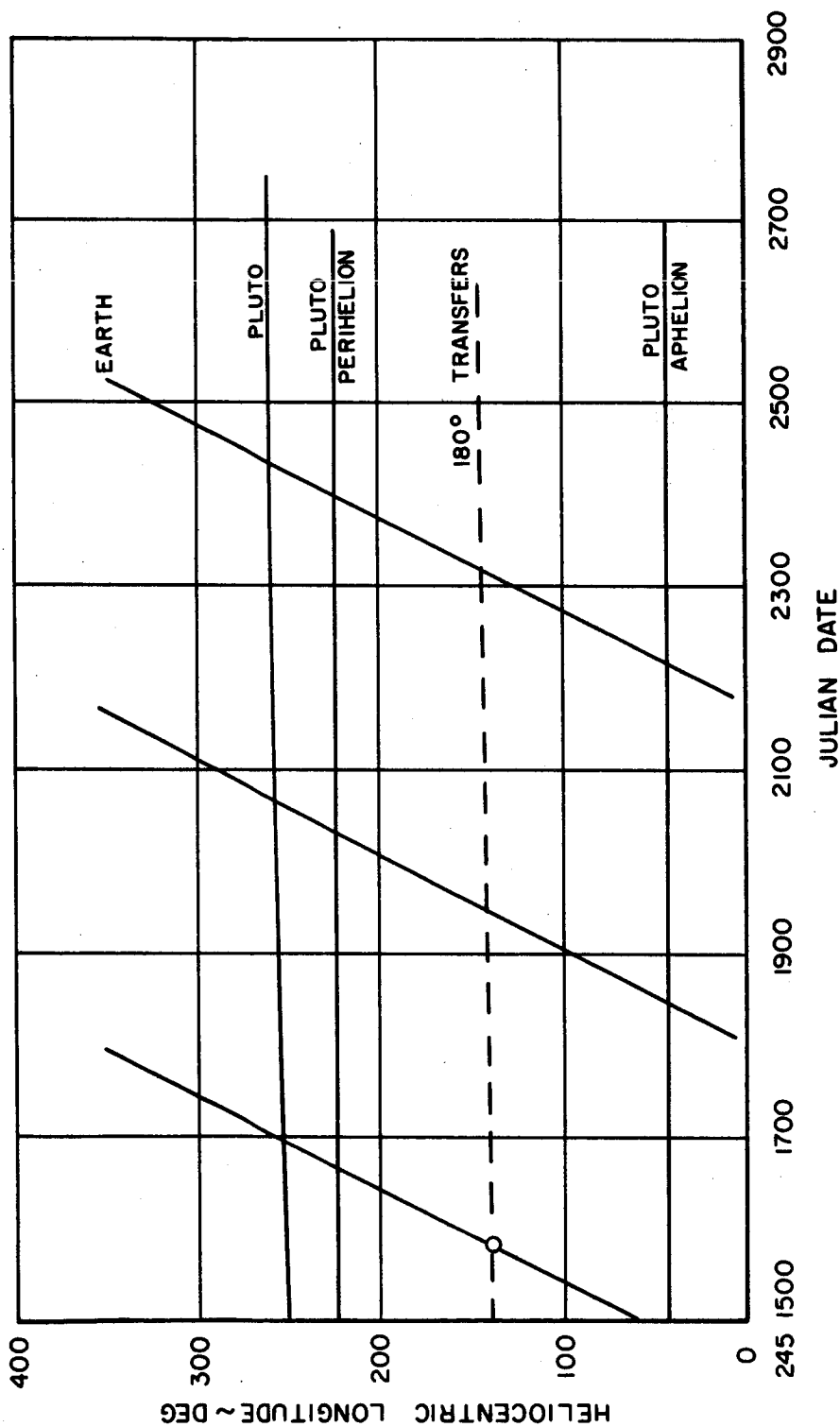
○ EARTH DEPARTURE DATES



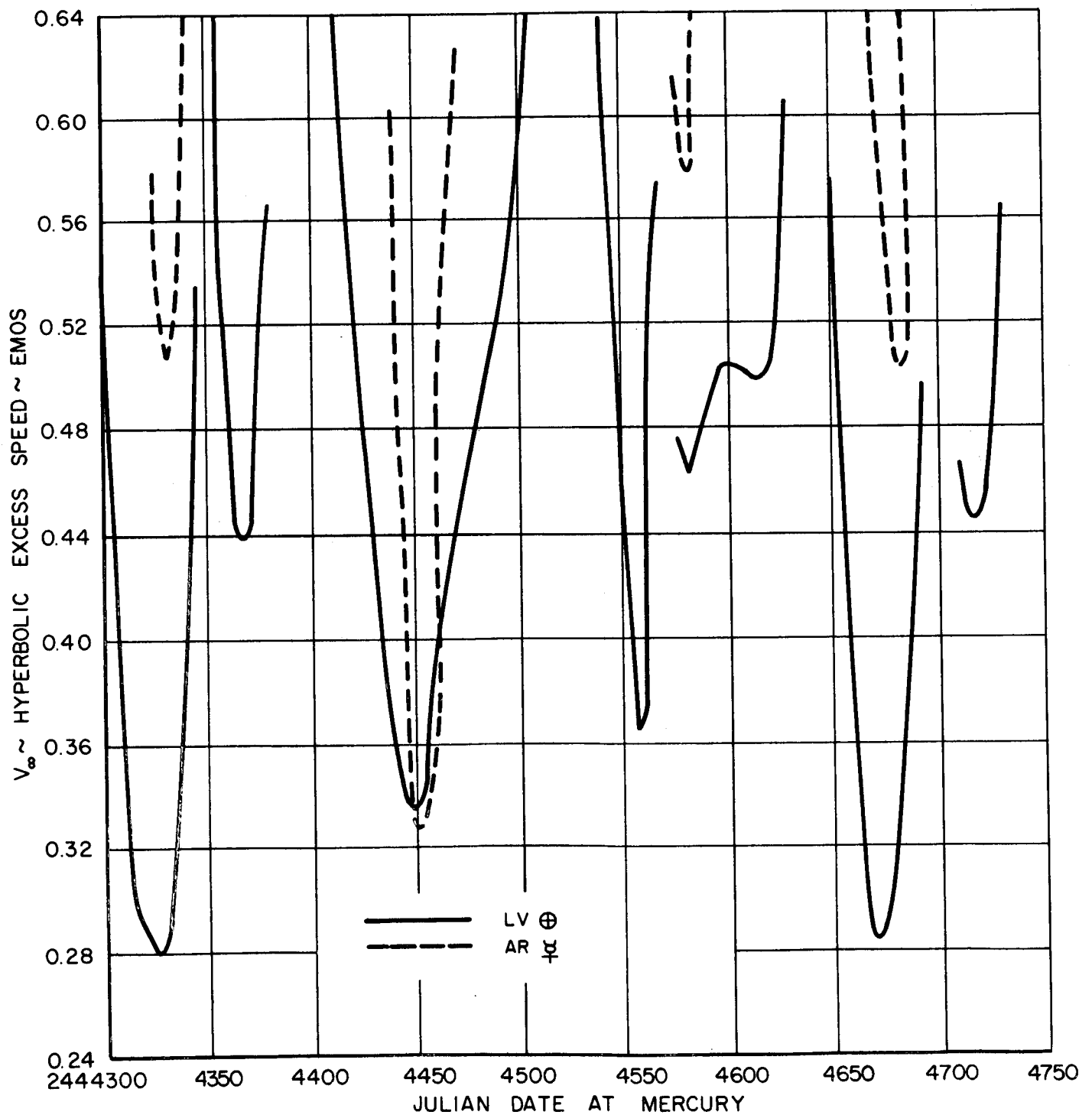
180 DEGREE TRANSFERS TO PLUTO

(COPLANAR)

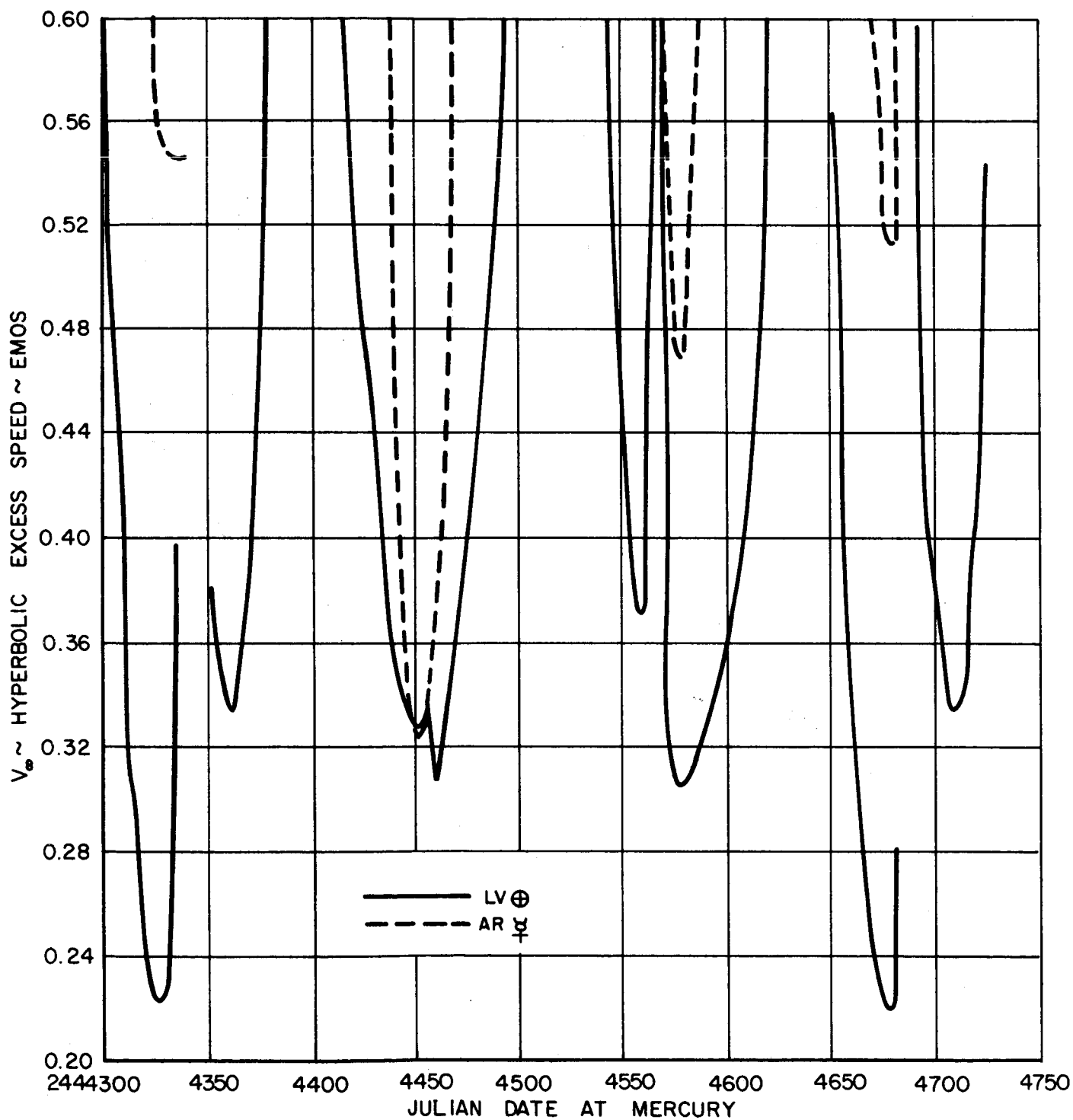
○ EARTH DEPARTURE DATES



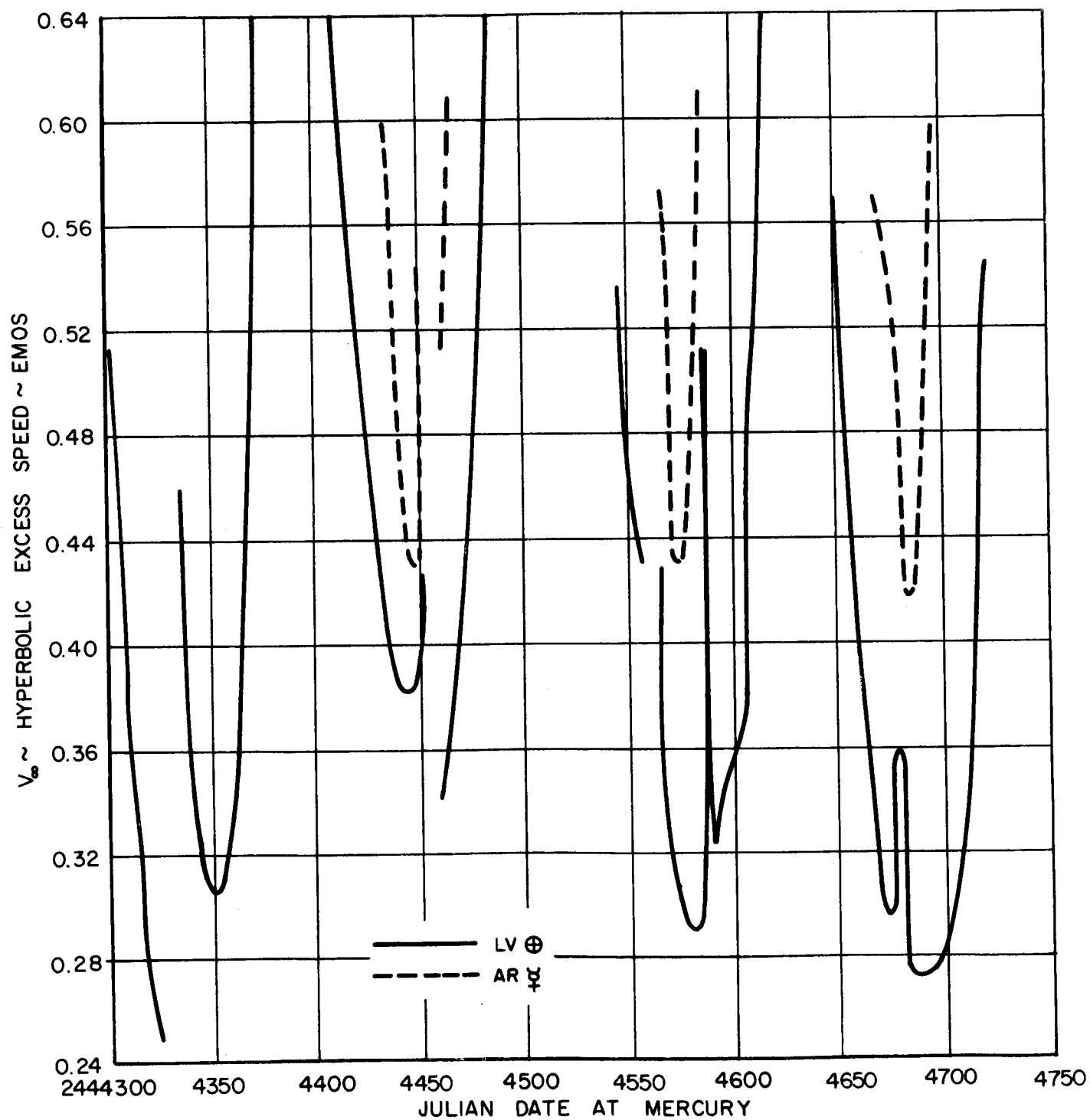
EARTH-MERCURY 80-DAY TRANSFERS



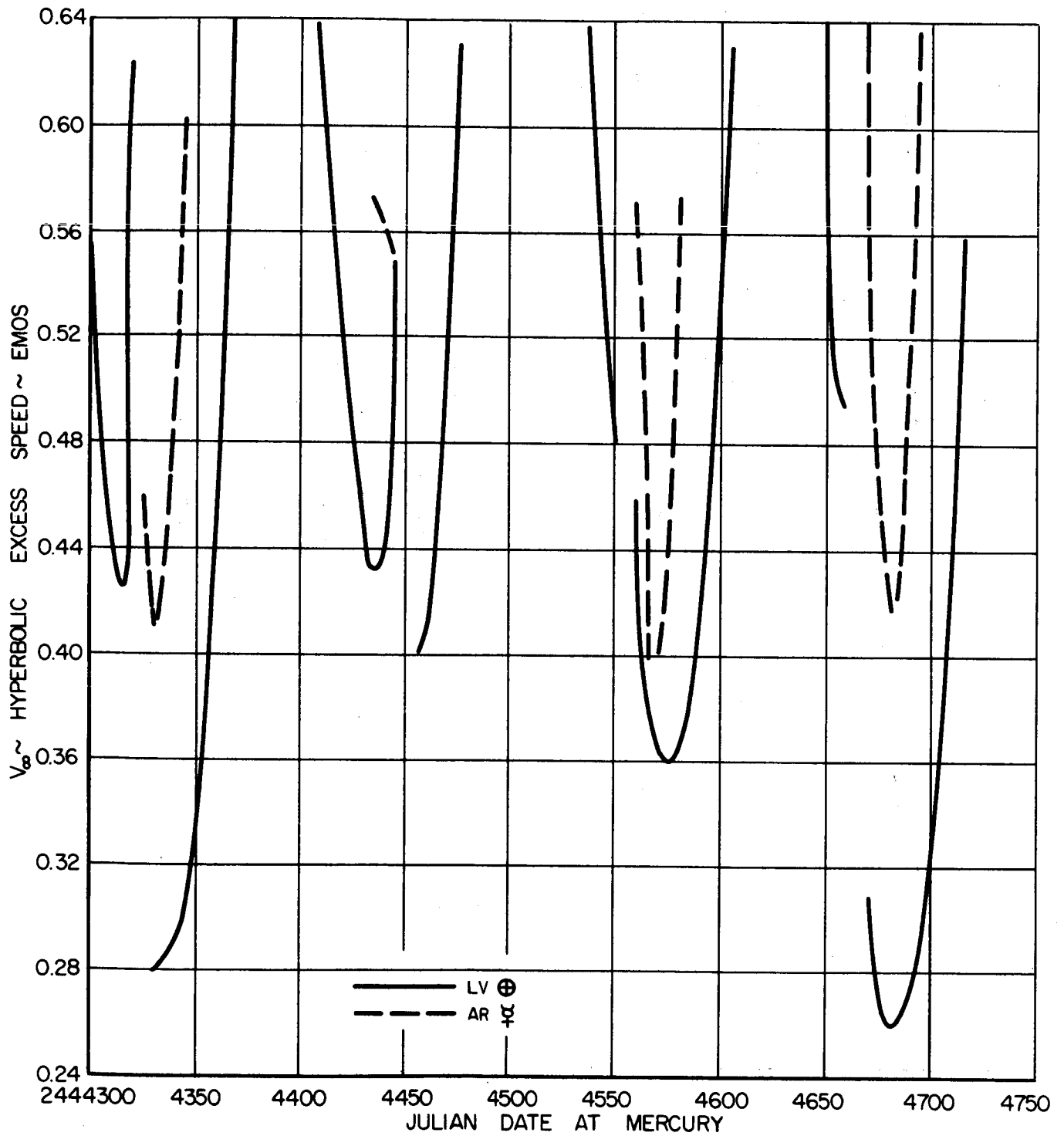
EARTH-MERCURY 100-DAY TRANSFERS



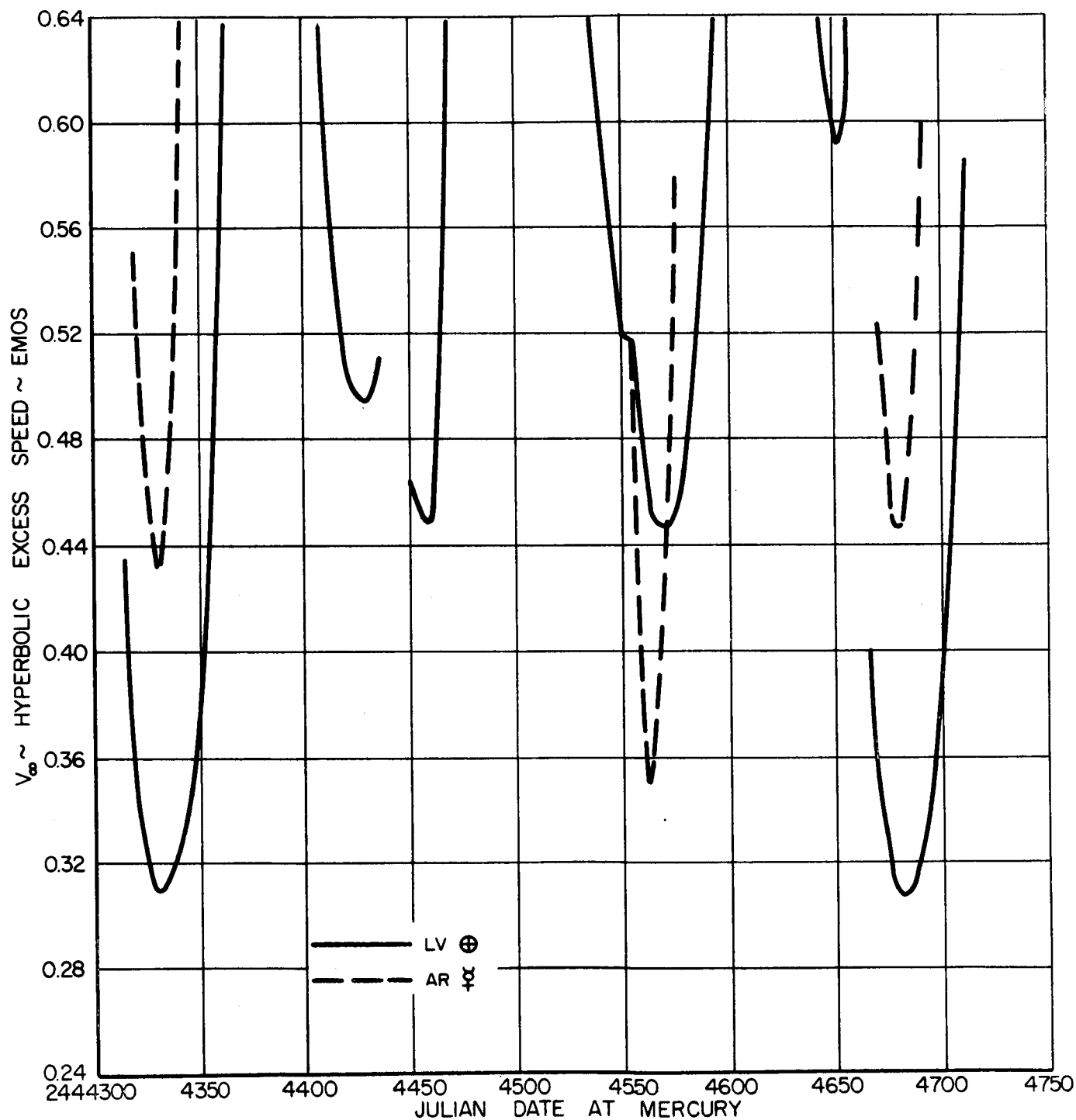
EARTH-MERCURY 120-DAY TRANSFERS



EARTH-MERCURY 140-DAY TRANSFERS



EARTH-MERCURY 160-DAY TRANSFERS



MERCURY VELOCITY CONTOURS

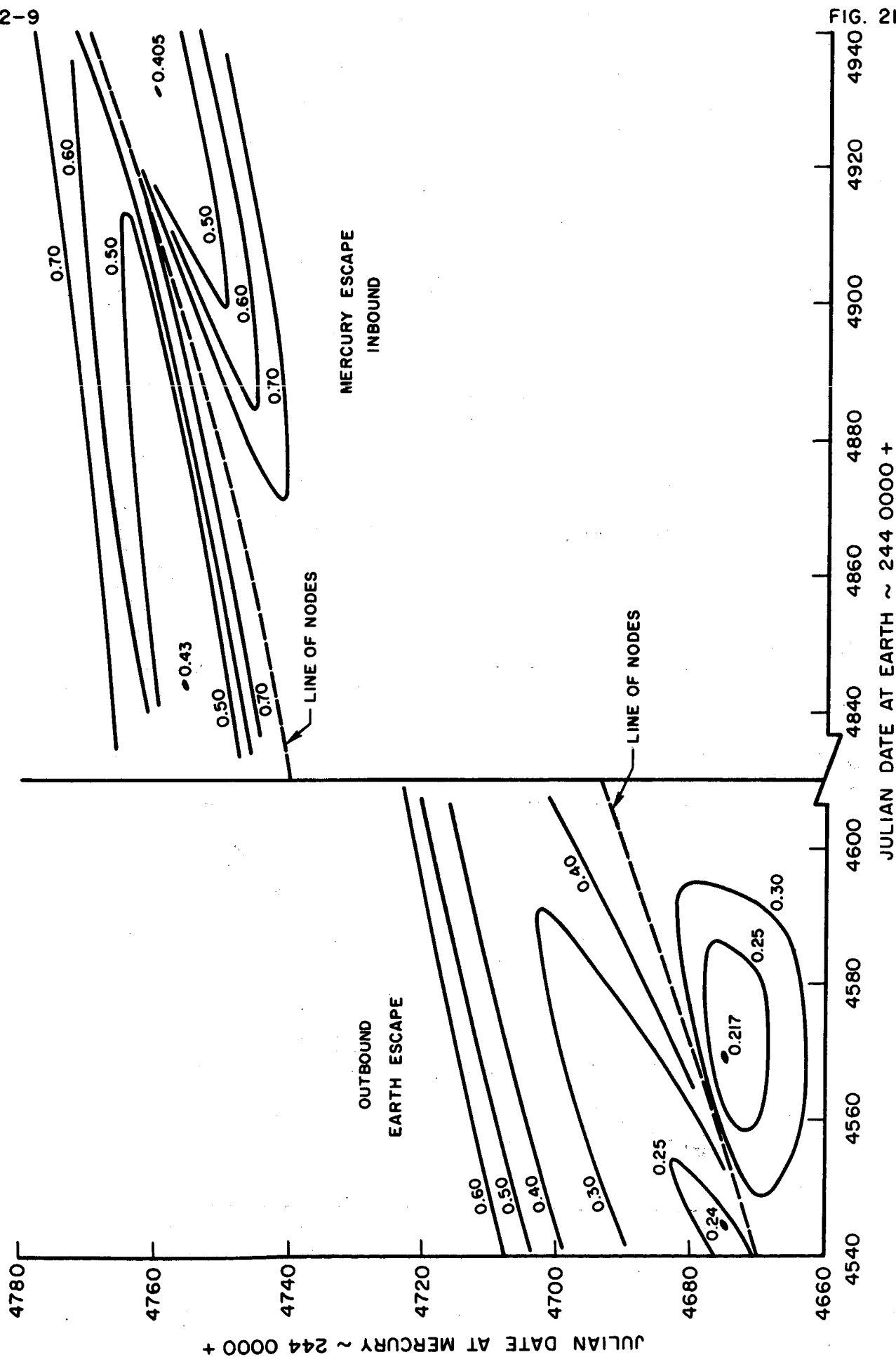
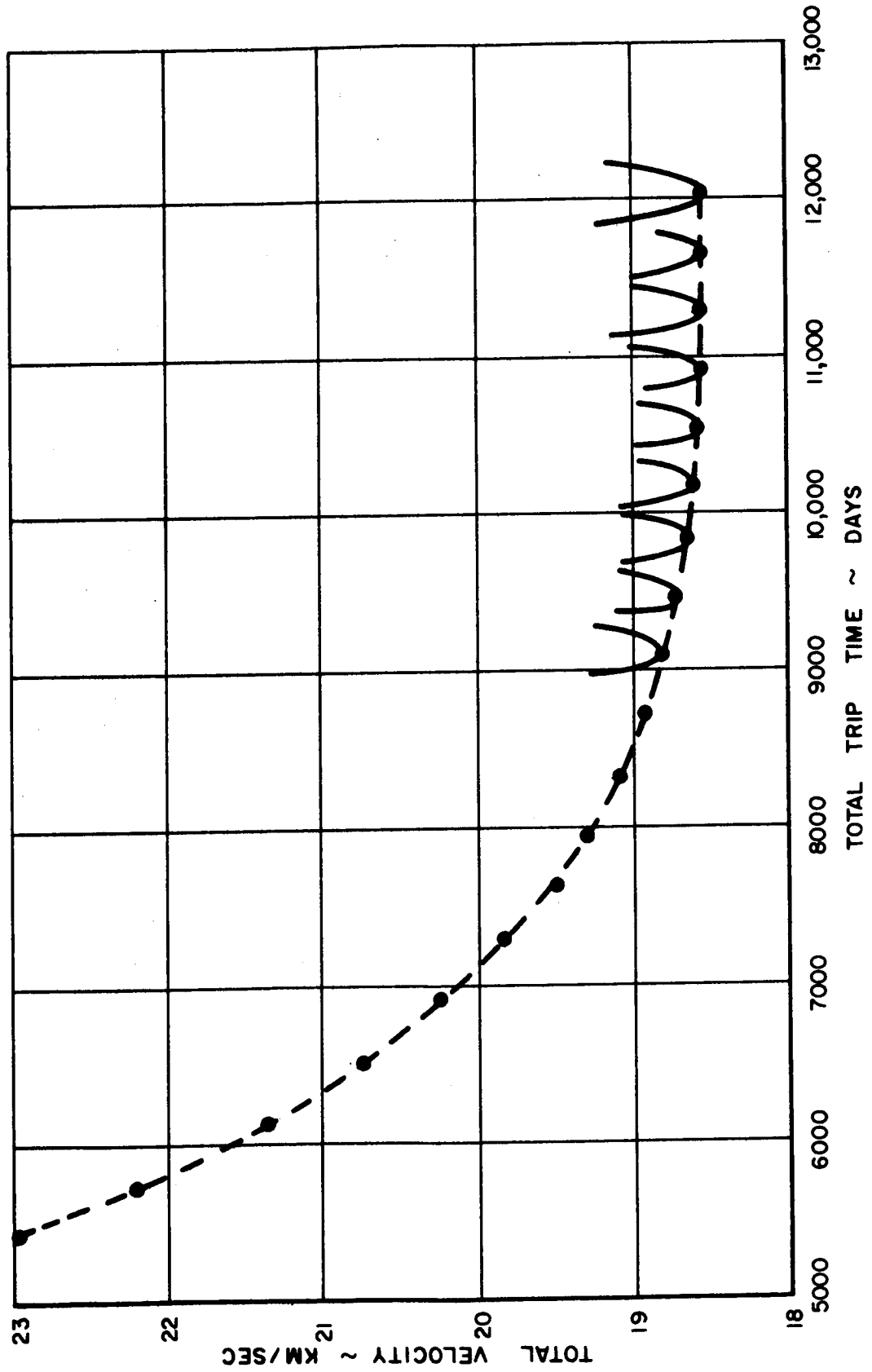


FIG. 22

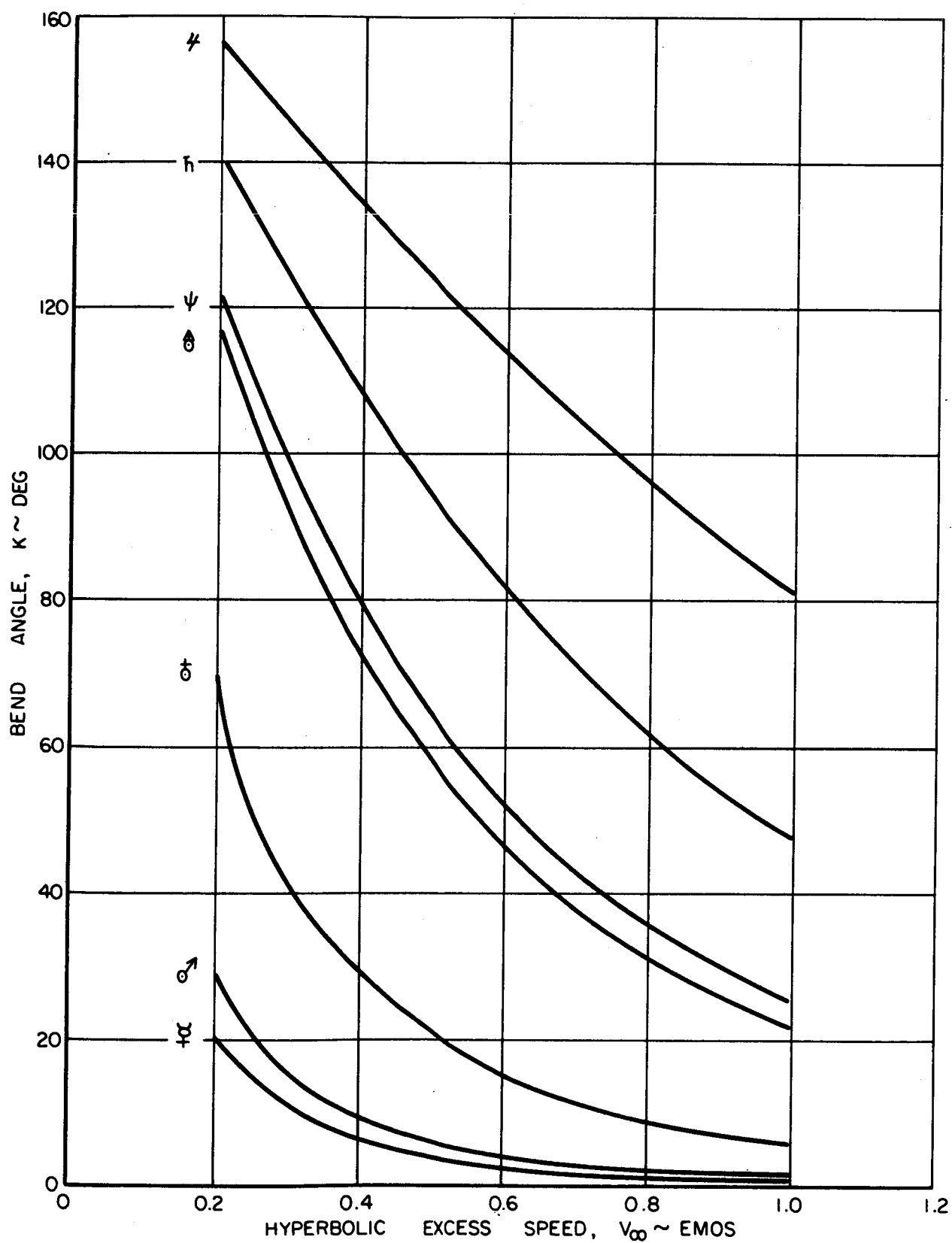
E-910352-9

URANUS ROUND TRIPS

ORBITAL ALTITUDE = 413,200 KM



FLYBY BEND ANGLE

 $r_p = 1.1 \times$ (PLANETARY RADIUS)

PLANETARY FLYBY GEOMETRY

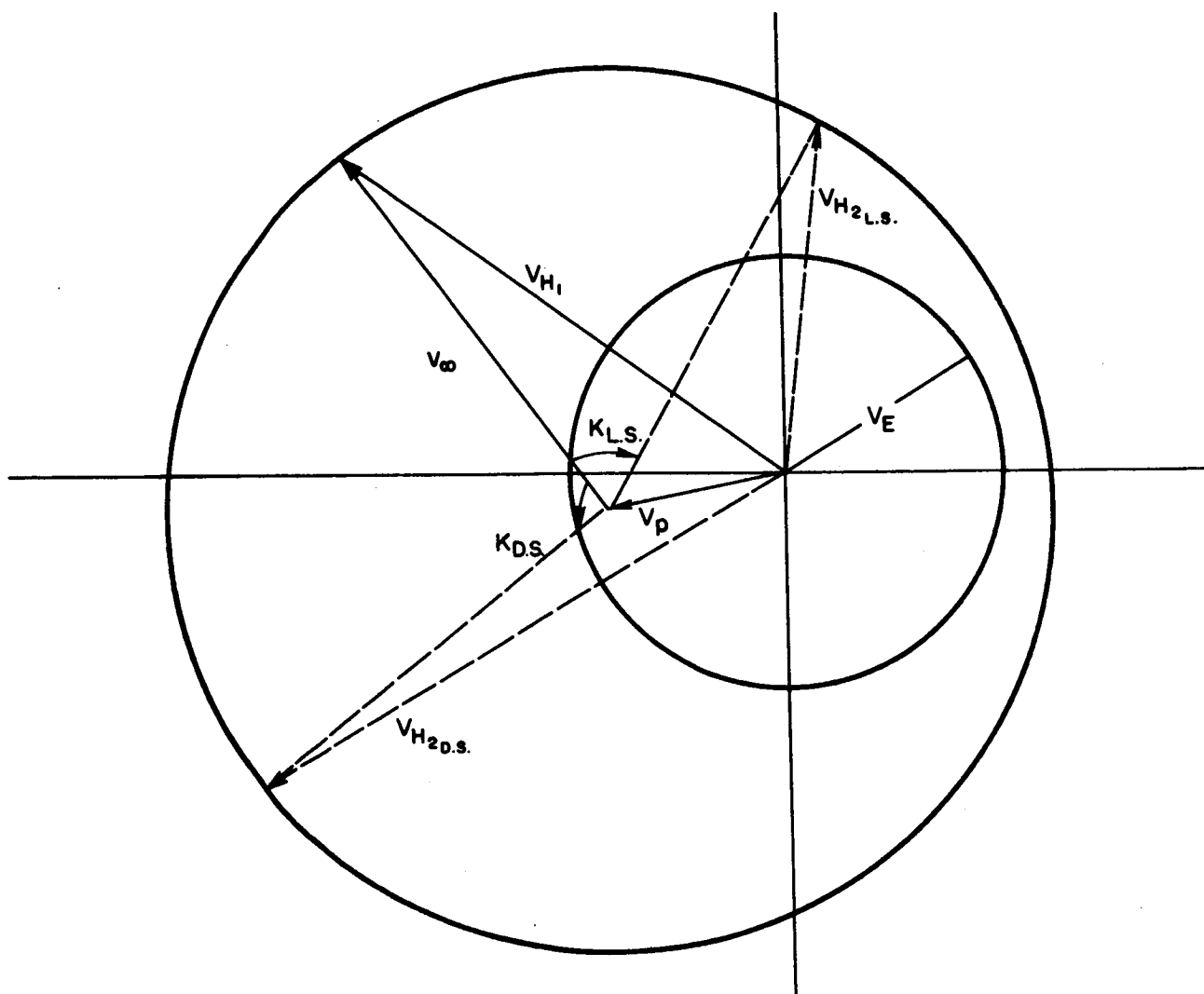
NO ELLIPTIC RETURN TO EARTH

TYPICAL FOR:

CERES 150-DAY
JUPITER 200-DAY
URANUS 1000-DAY
NEPTUNE 1500-DAY

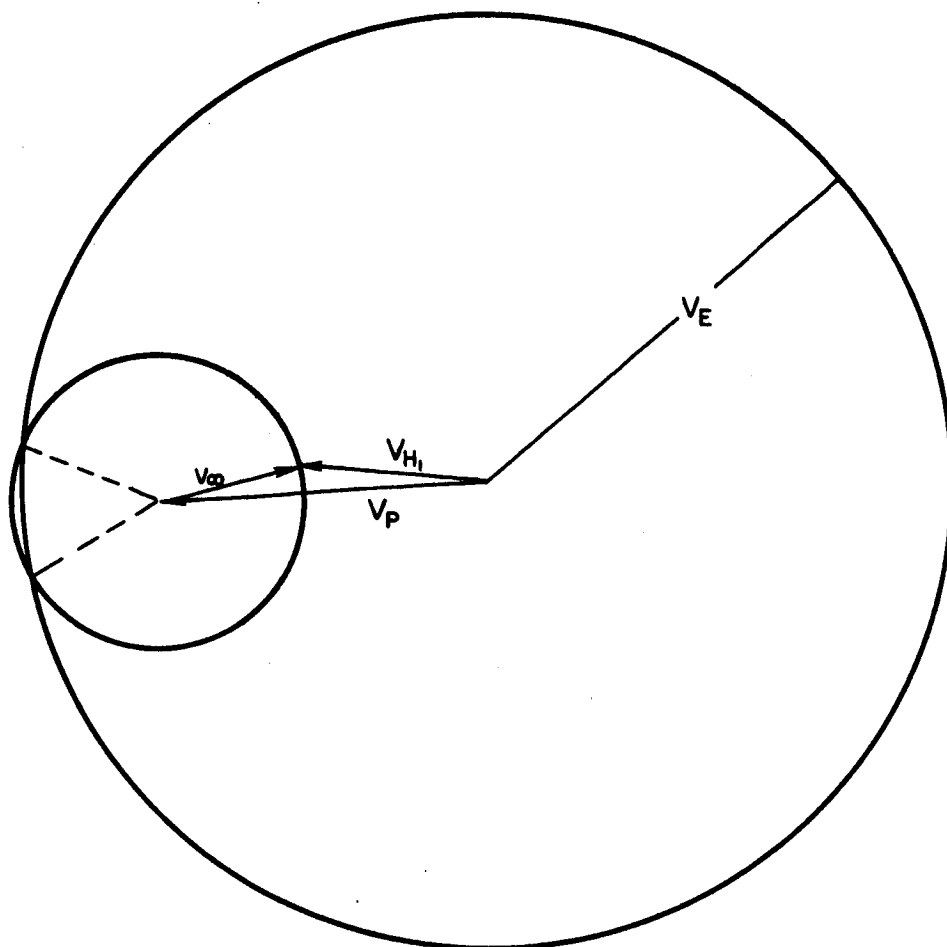
$$V_{H2L.S.} > V_E$$

$$V_{H2D.S.} > V_E$$



JUPITER FLYBY
TRIP TIME = 950 DAYS
LEAVE EARTH ~ 2444970

$$r_{p\gamma=0} = 531.2$$
$$R_{\pi} = 2.25$$

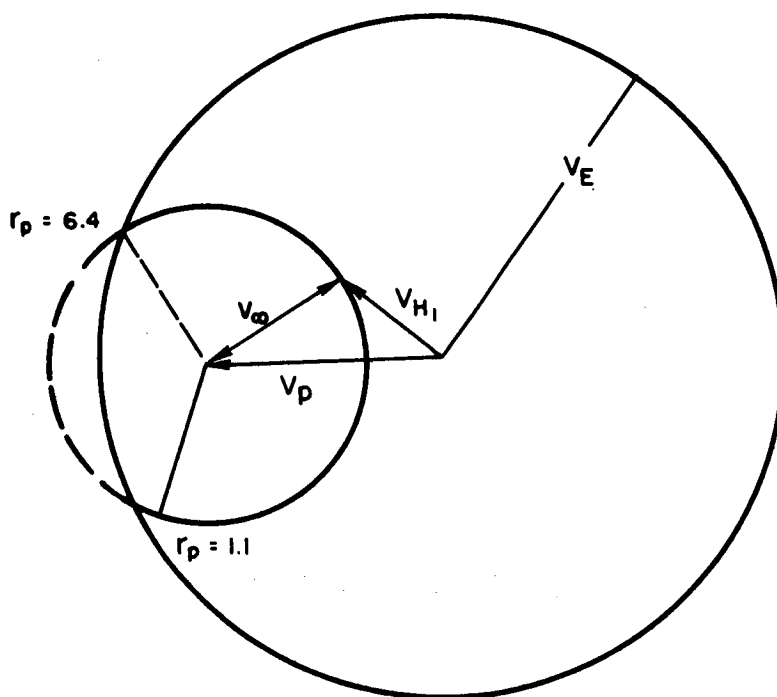


SATURN FLYBY

TRIP TIME = 1780 DAYS
LEAVE EARTH ~ 244 6840

$$r_{p_{\gamma=0}} = 46.17$$

$$R_{\pi} = 0.524$$

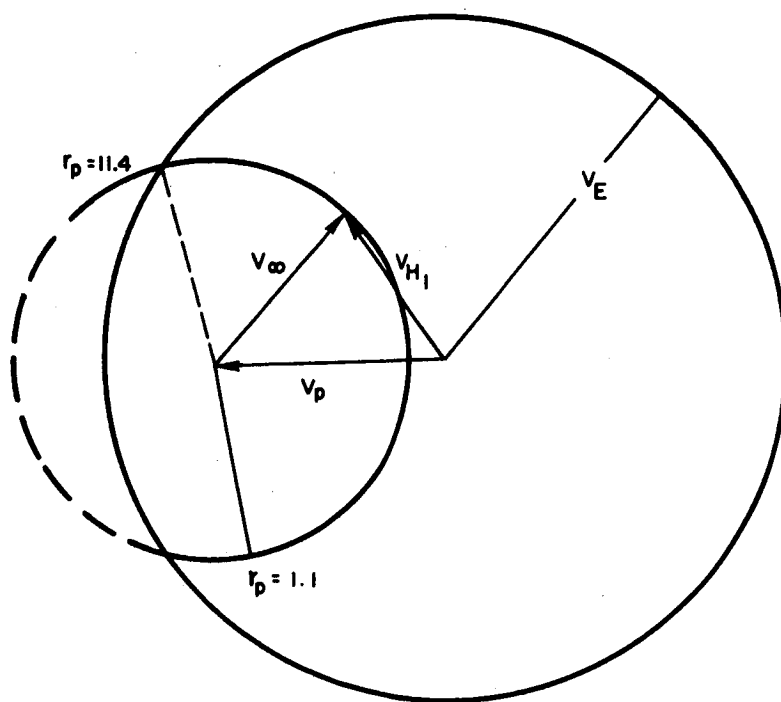


SATURN FLYBY

TRIP TIME = 1460 DAYS
LEAVE EARTH ~ 2446830

$$r_{p\gamma=0} = 15.45$$

$$R_{\pi} = 0.091$$

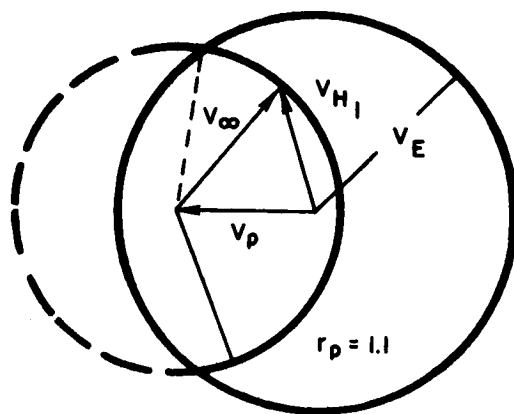


NEPTUNE FLYBY

TRIP TIME = 6000 DAYS
LEAVE EARTH ~ 2450520

$$r_{p \gamma=0} = 9.945$$

$$R_T = 0.410$$

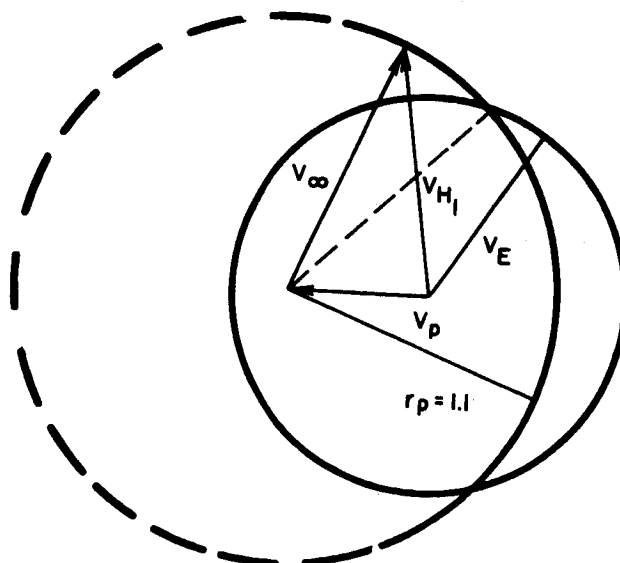


NEPTUNE FLYBY

TRIP TIME = 4000 DAYS
LEAVE EARTH ~ 2450520

$$r_{p\gamma=0} = 2.00$$

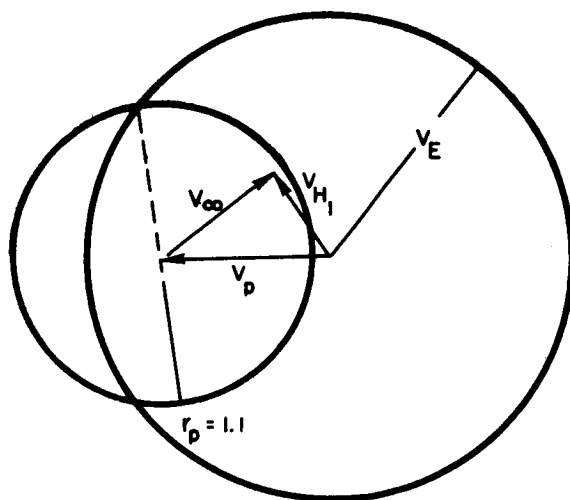
$$R_{\pi} = 23.02$$



URANUS FLYBY

TRIP TIME = 4200 DAYS
LEAVE EARTH ~ 2447210

$$r_{p\gamma=0} = 16.49$$
$$R_{\pi} = 0.07$$

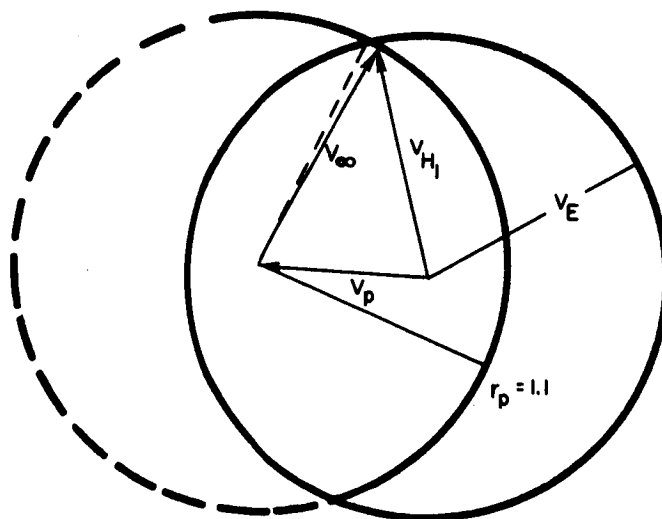


URANUS FLYBY

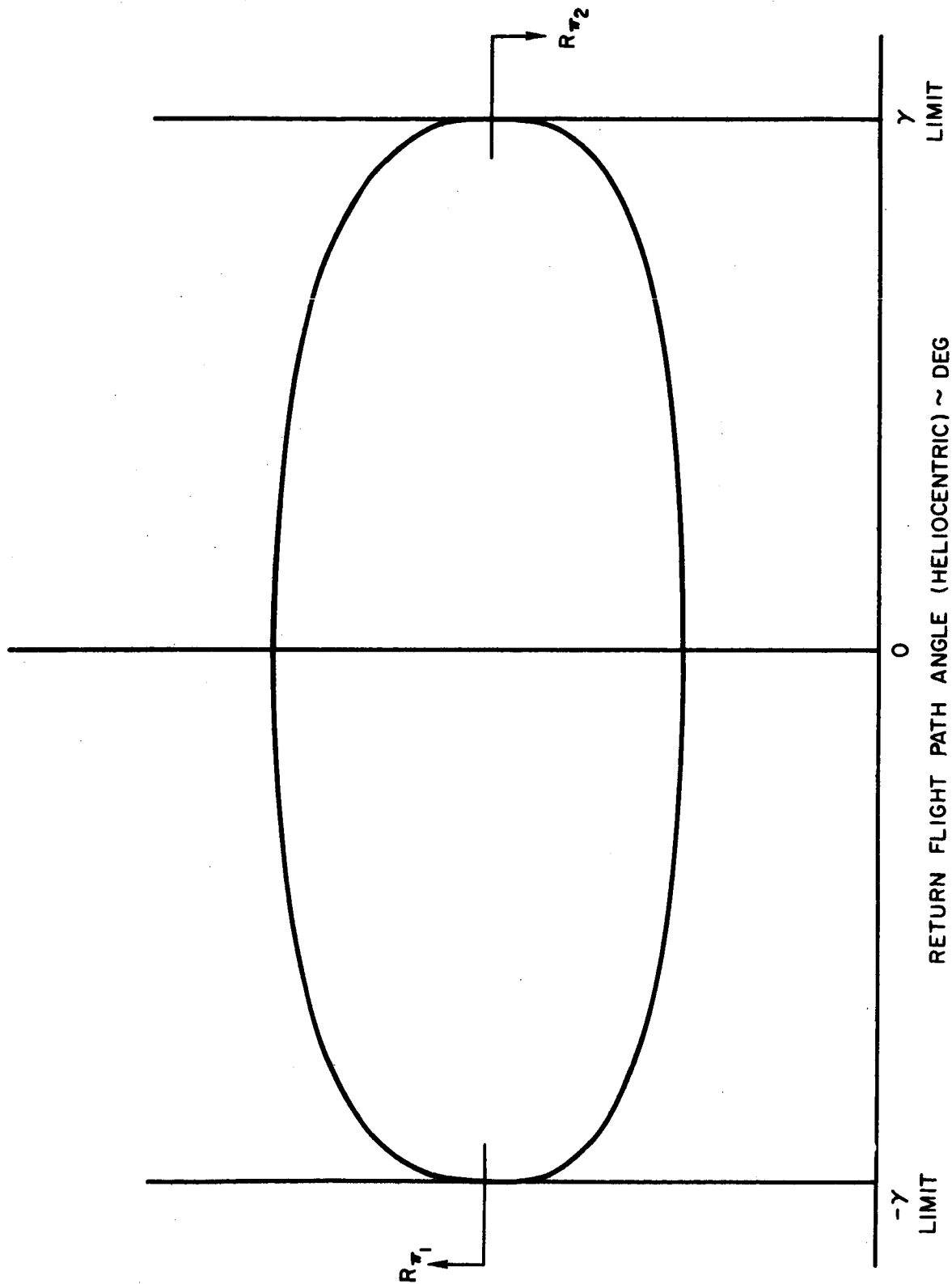
TRIP TIME = 2600 DAYS
LEAVE EARTH ~ 2447200

$$r_{p\gamma=0} = 16.90$$

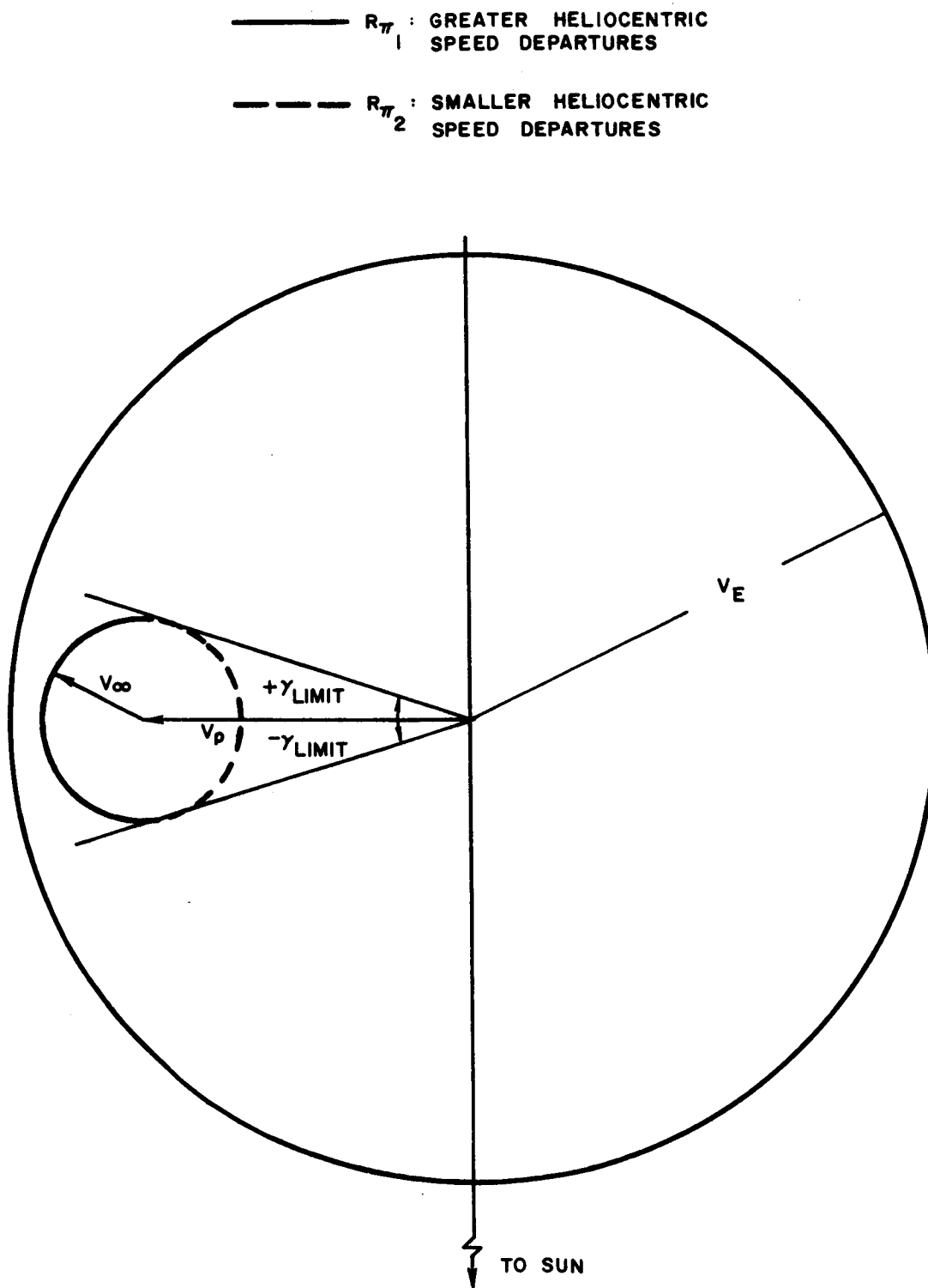
$$R_{\pi} = 2.74$$



HELIOCENTRIC PERIPOINT DISTANCE ~ A.U.



PLANETARY FLYBY PROBE GEOMETRY



RETURN SEMI-MAJOR AXIS AND RETURN ECCENTRICITY
AS A FUNCTION OF HELIOCENTRIC DEPARTURE
FLIGHT PATH ANGLE

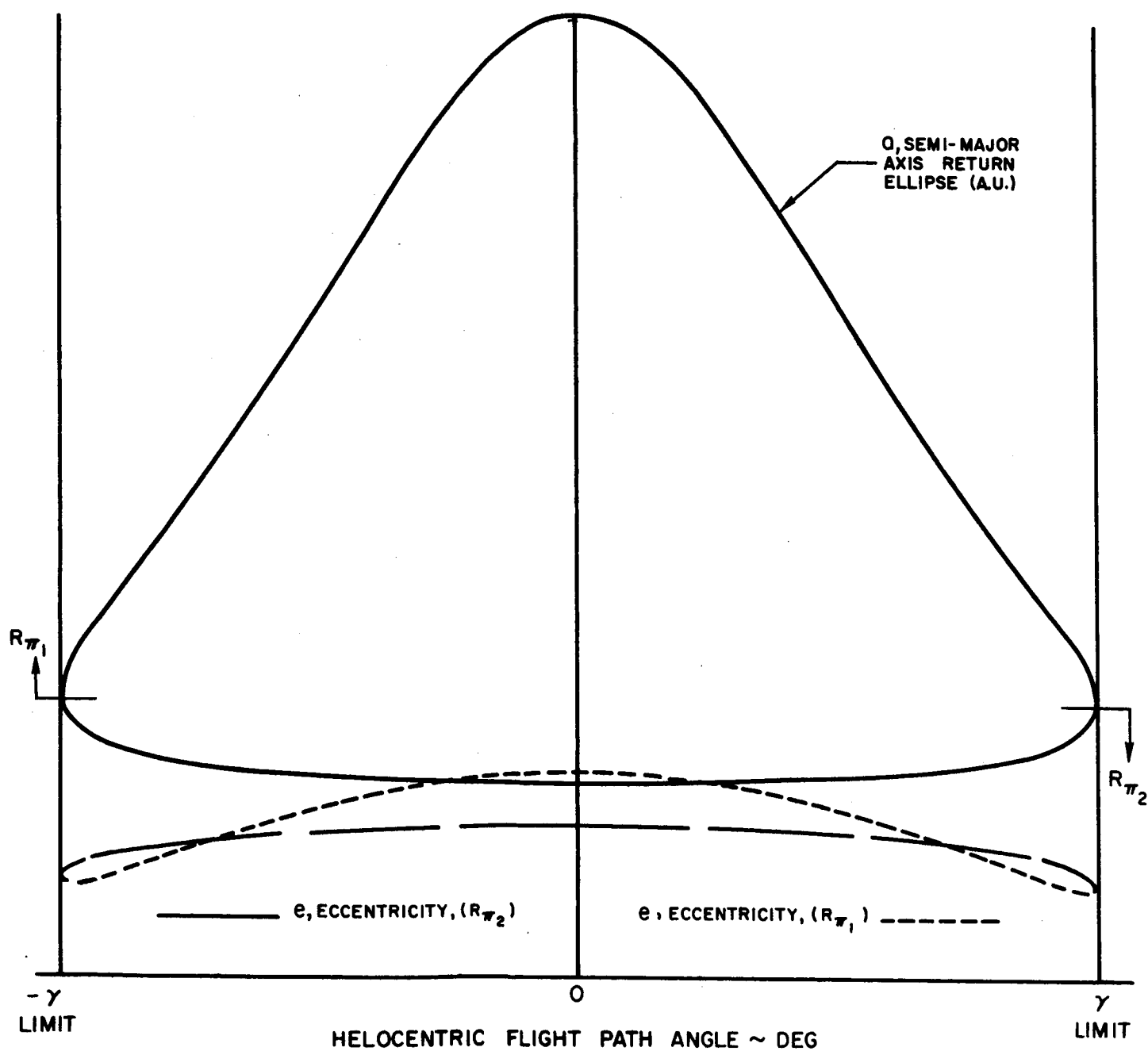


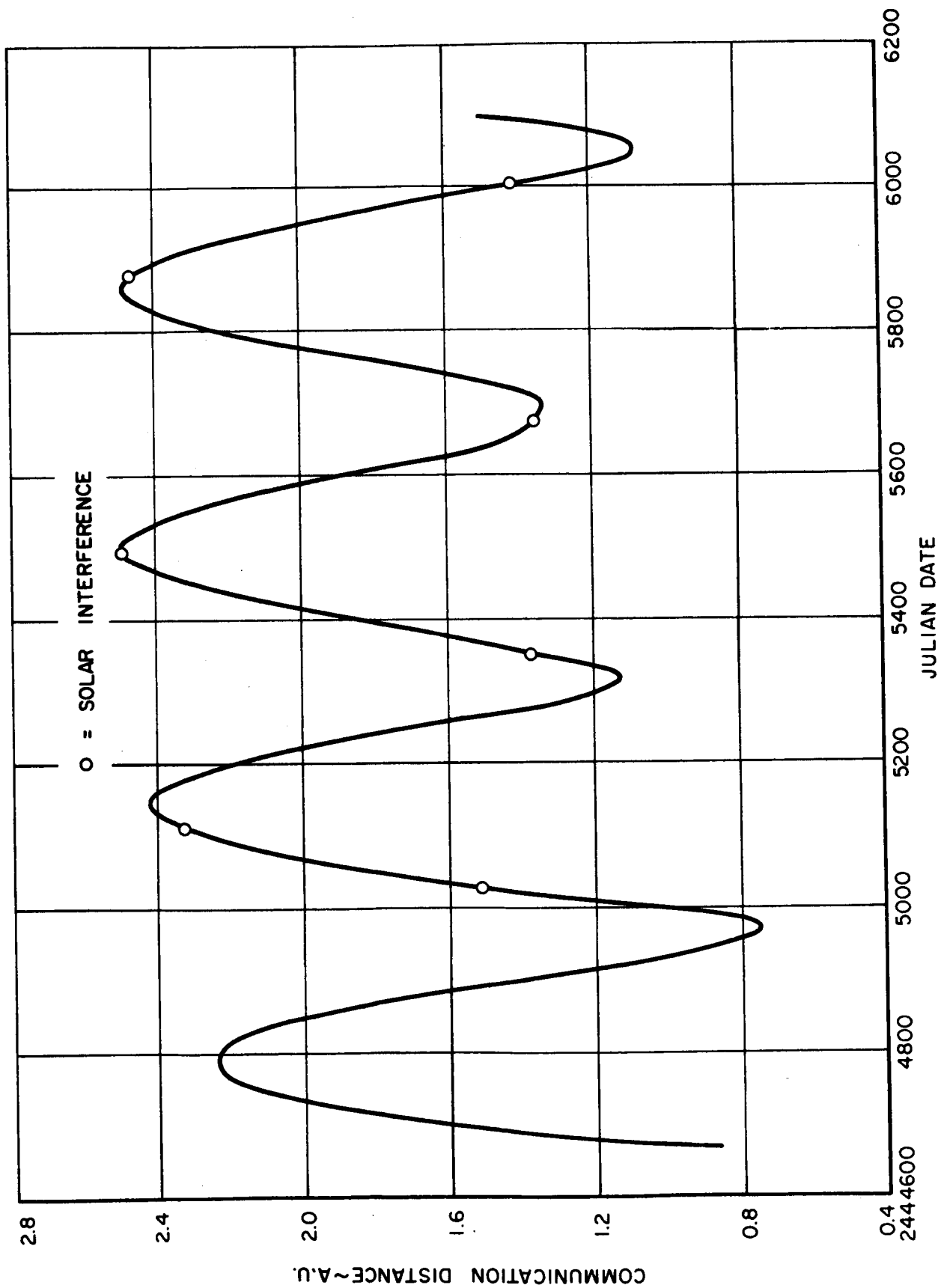
FIG. 36

E-910352-9

MERCURY FLYBY

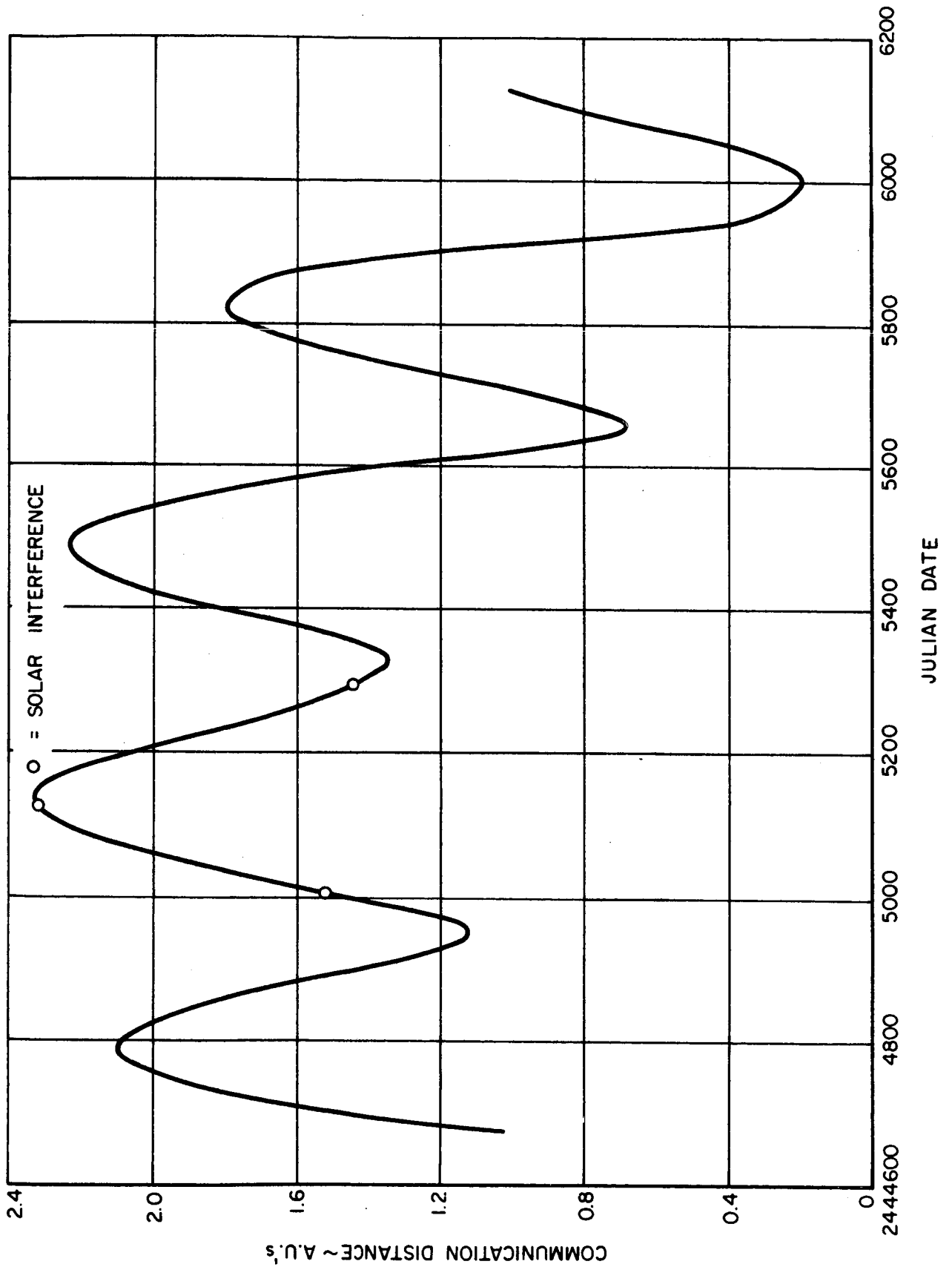
LEAVE EARTH ~ 2444605

TRIP TIME = 60 DAYS



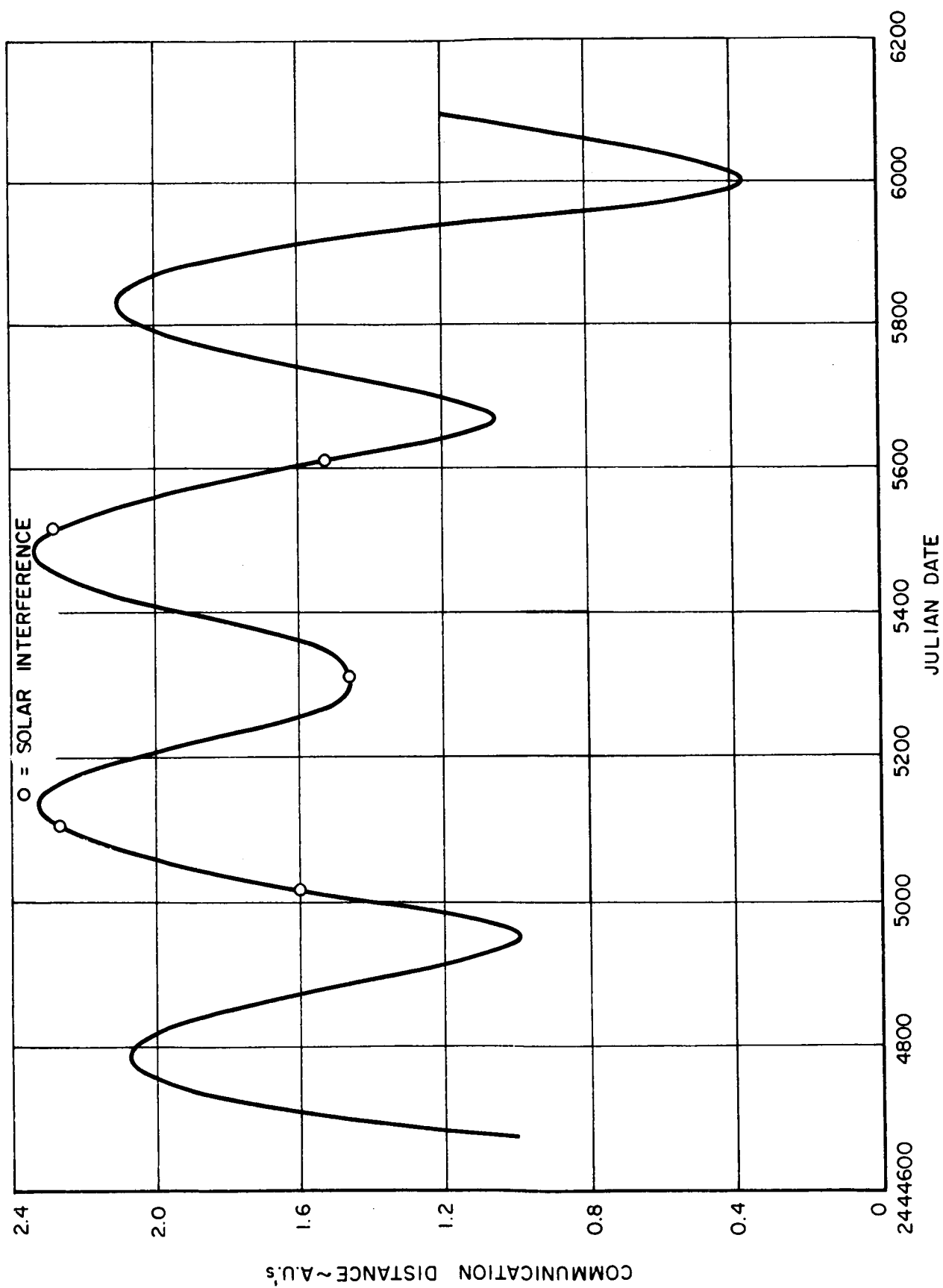
MERCURY FLYBY

LEAVE EARTH ~ 2444585
TRIP TIME = 90 DAYS



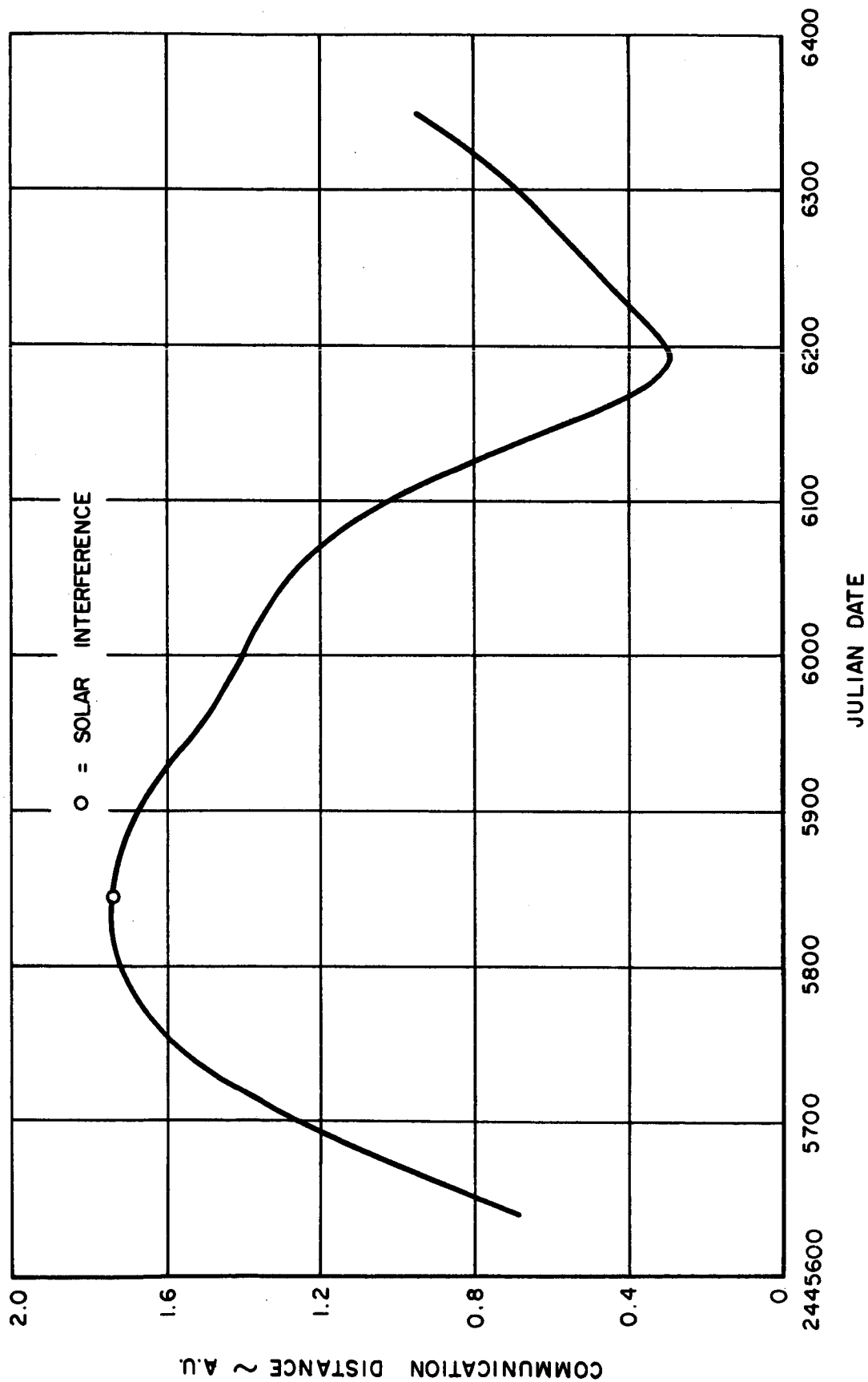
MERCURY FLYBY

LEAVE EARTH ~ 2444565
TRIP TIME = 110 DAYS



VENUS FLYBY

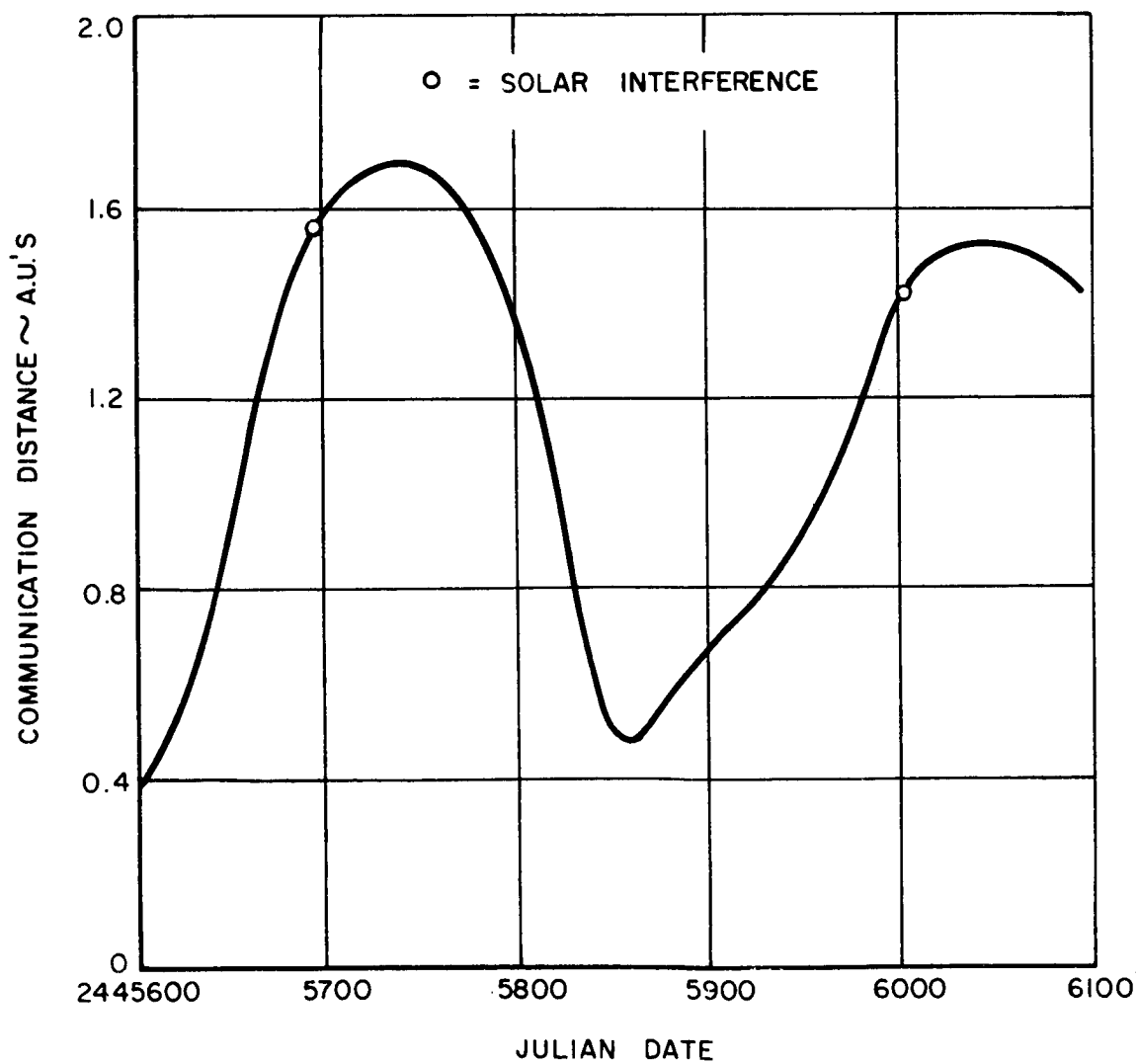
LEAVE EARTH ~ 2445480
TRIP TIME = 160 DAYS



VENUS FLYBY

LEAVE EARTH ~ 2445500

TRIP TIME = 100 DAYS



VENUS FLYBY

LEAVE EARTH ~ 2445540

TRIP TIME = 40 DAYS

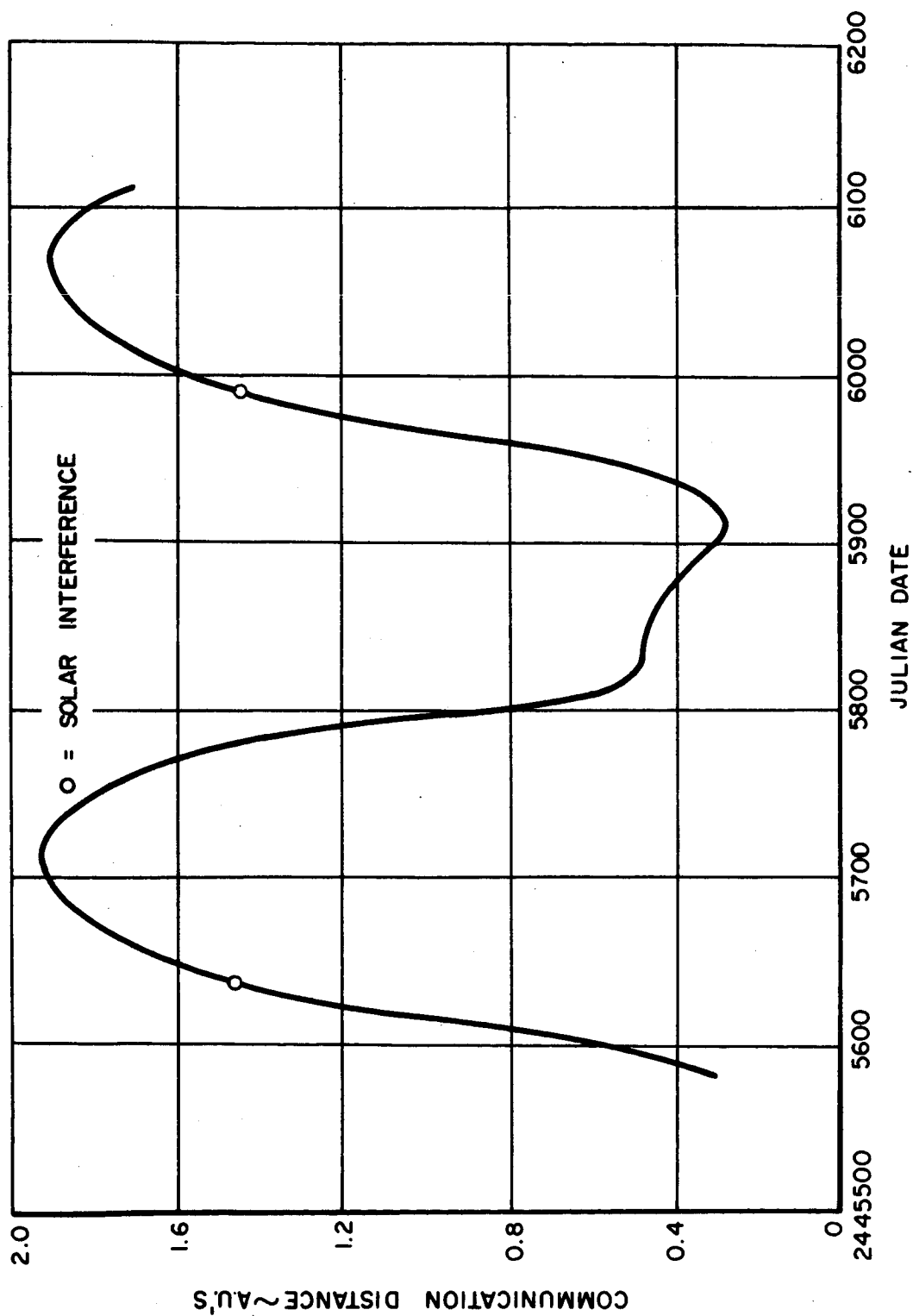
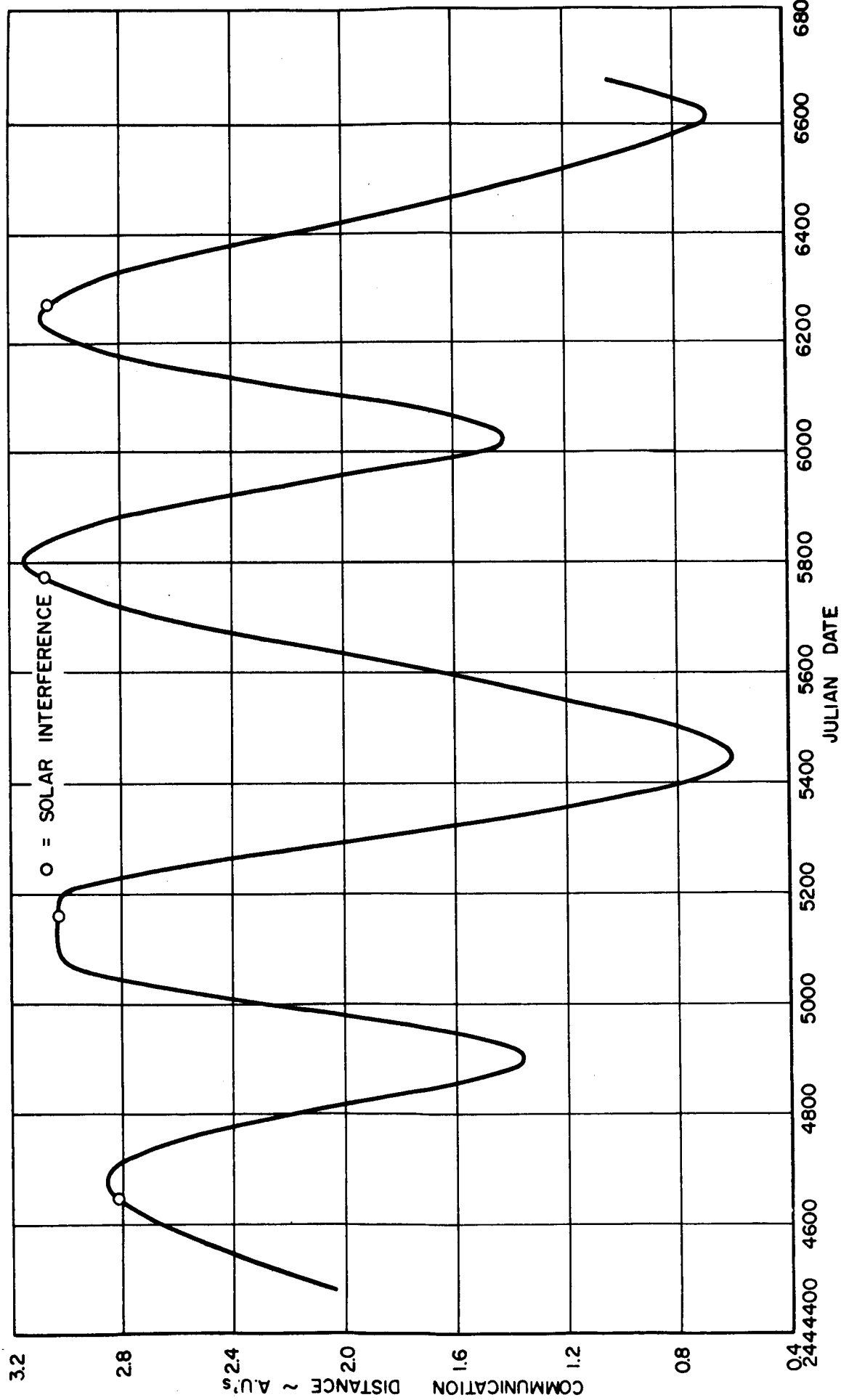


FIG. 42

MARS FLYBY

LEAVE EARTH ~ 2444480
TRIP TIME = 290 DAYS



E-910352-5

MARS FLYBY

LEAVE EARTH ~ 2444200

TRIP TIME = 200 DAYS

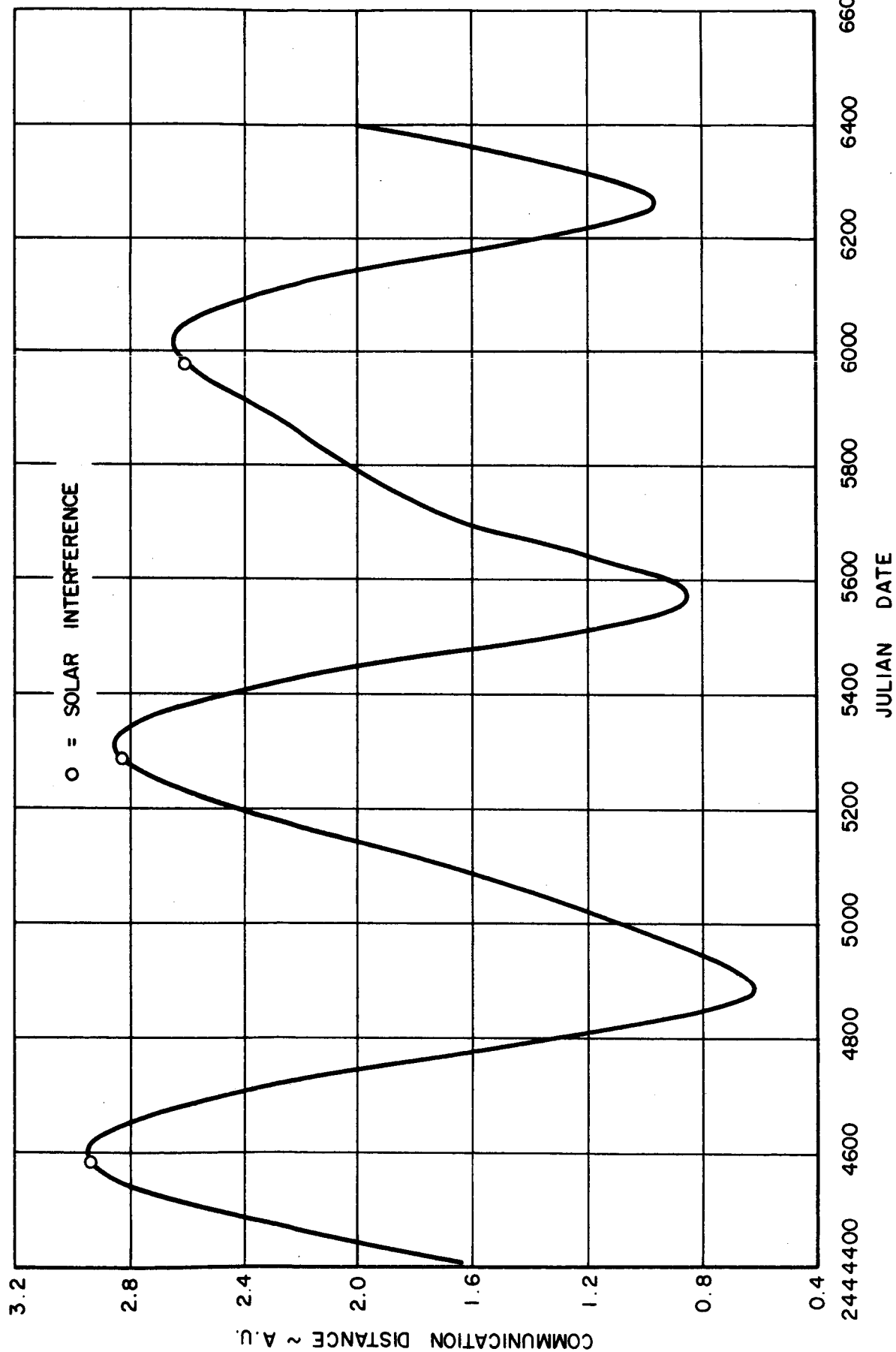


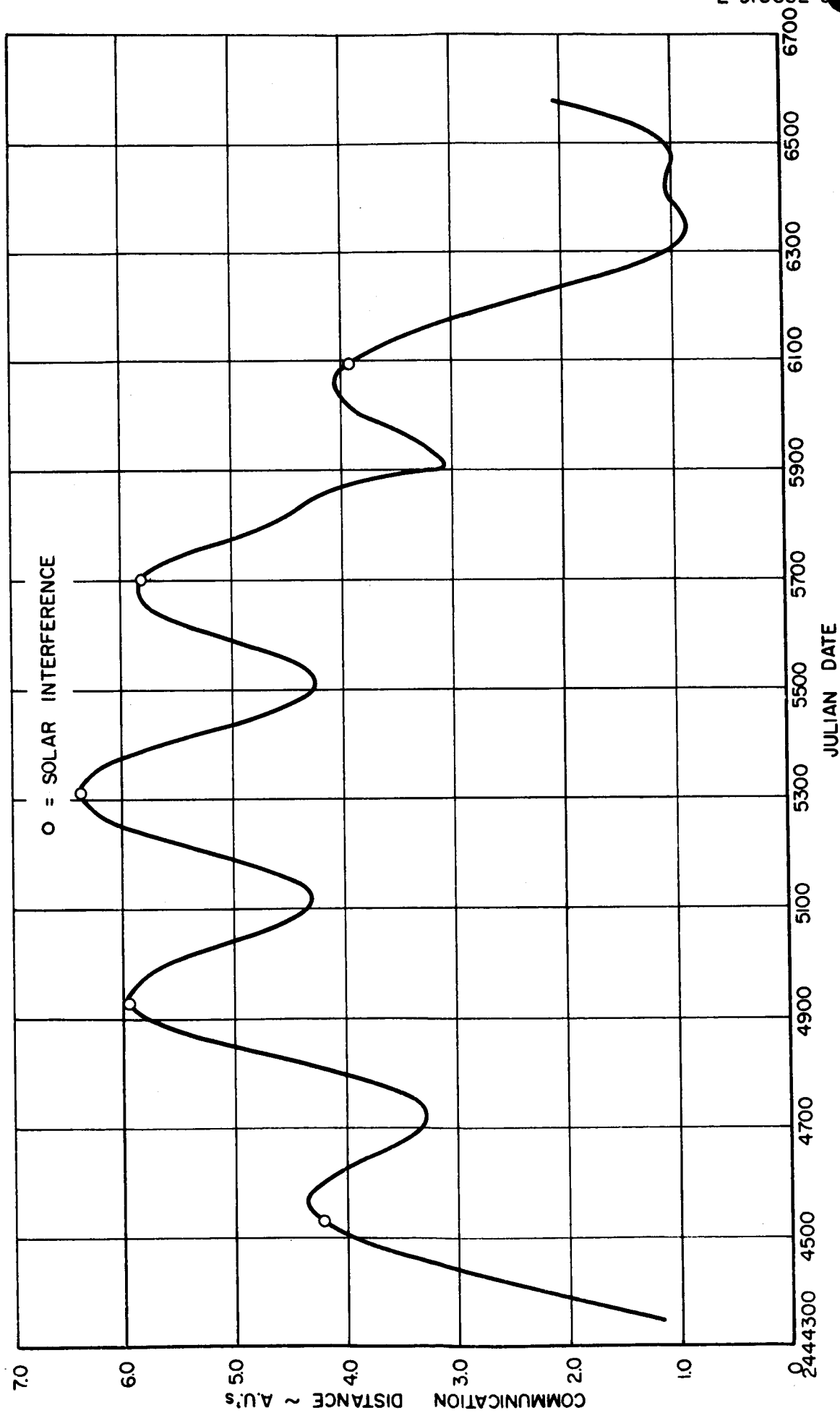
FIG. 43

FIG. 44

E-910352-S

MARS FLYBYS

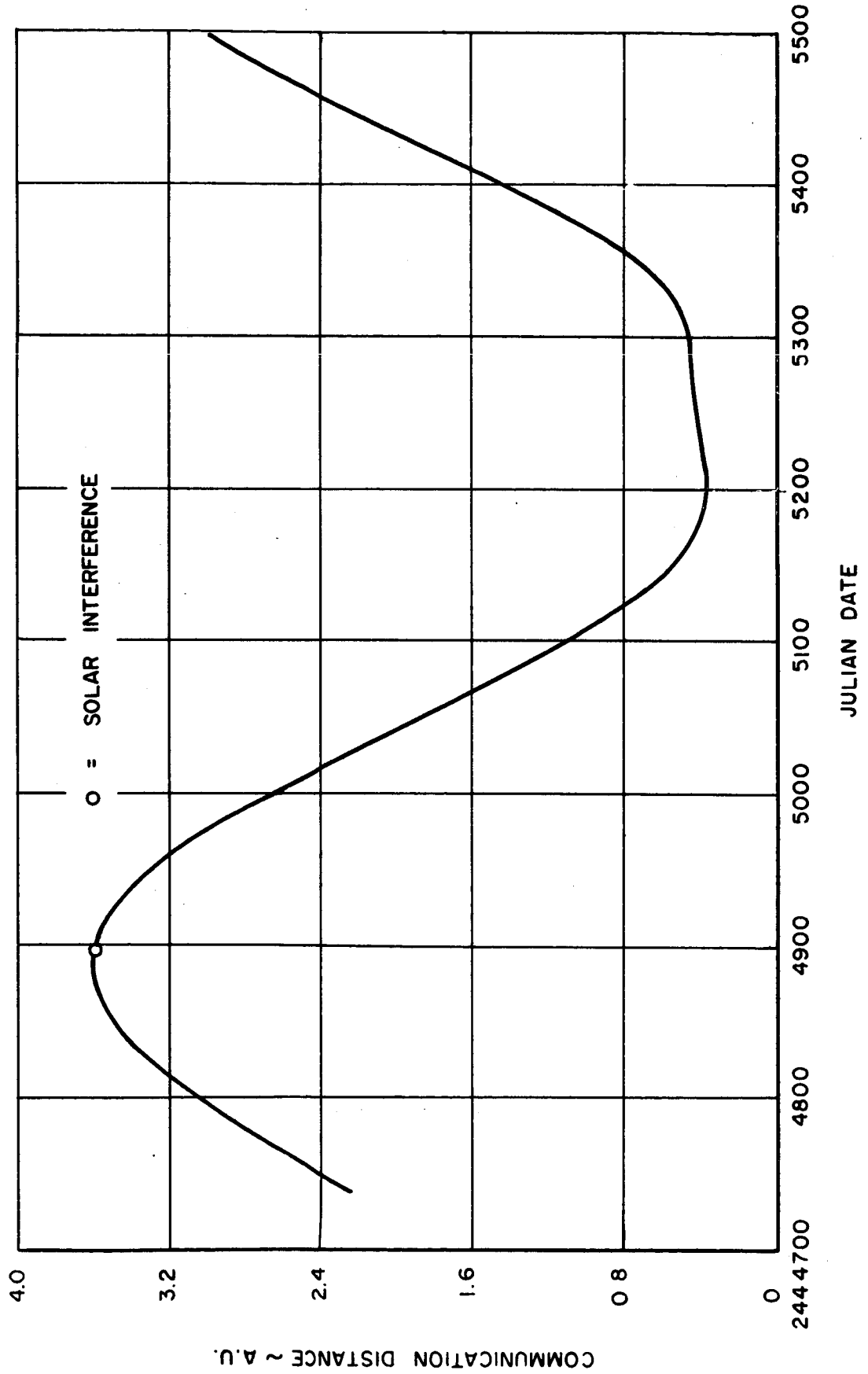
LEAVE EARTH ~ 2444230
TRIP TIME = 100 DAYS



SCHWASSMANN—WACHMANN II FLYBY

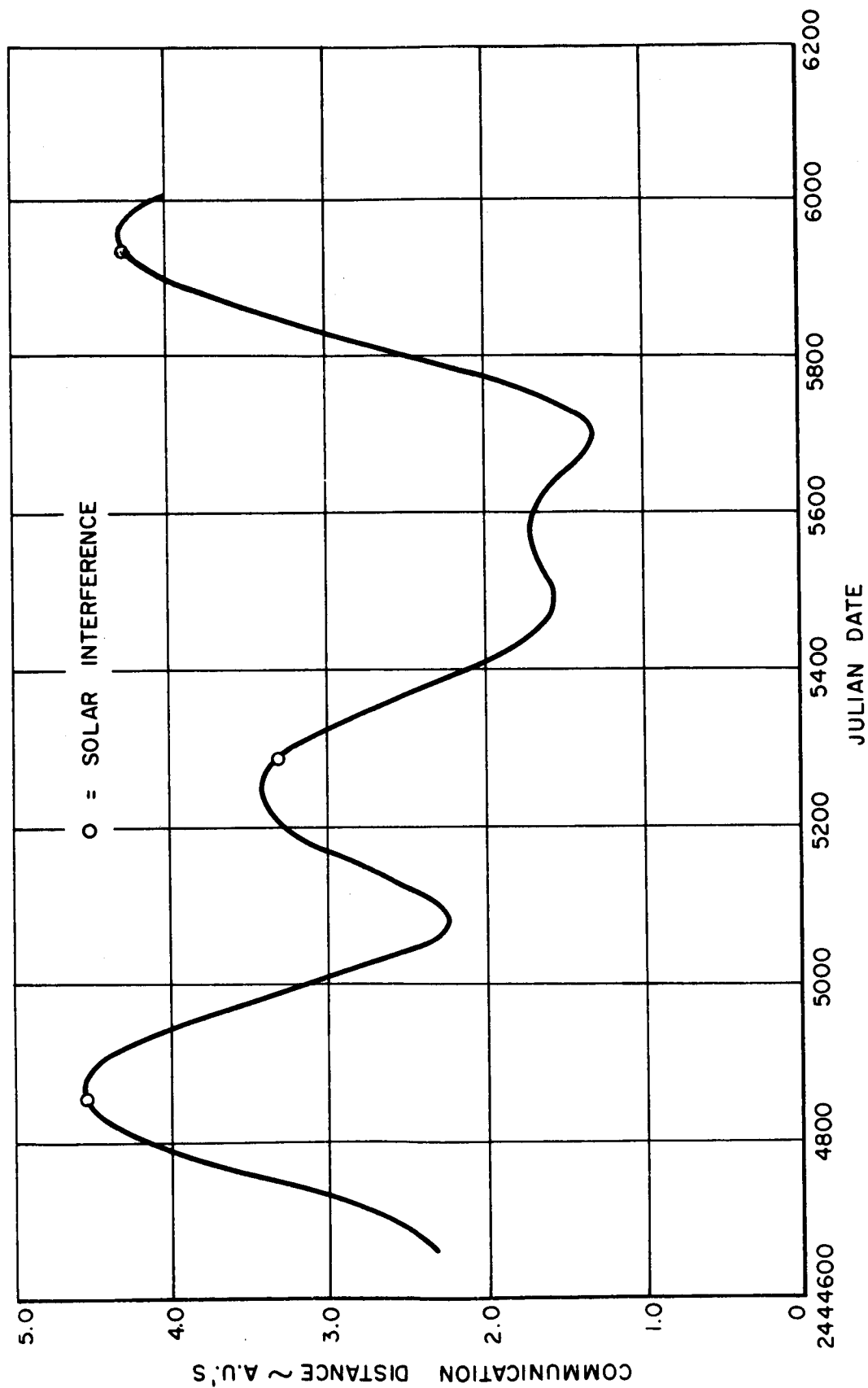
LEAVE EARTH ~ 2444220

TRIP TIME=500 DAYS



SCHWASSMANN - WACHMANN II FLYBY

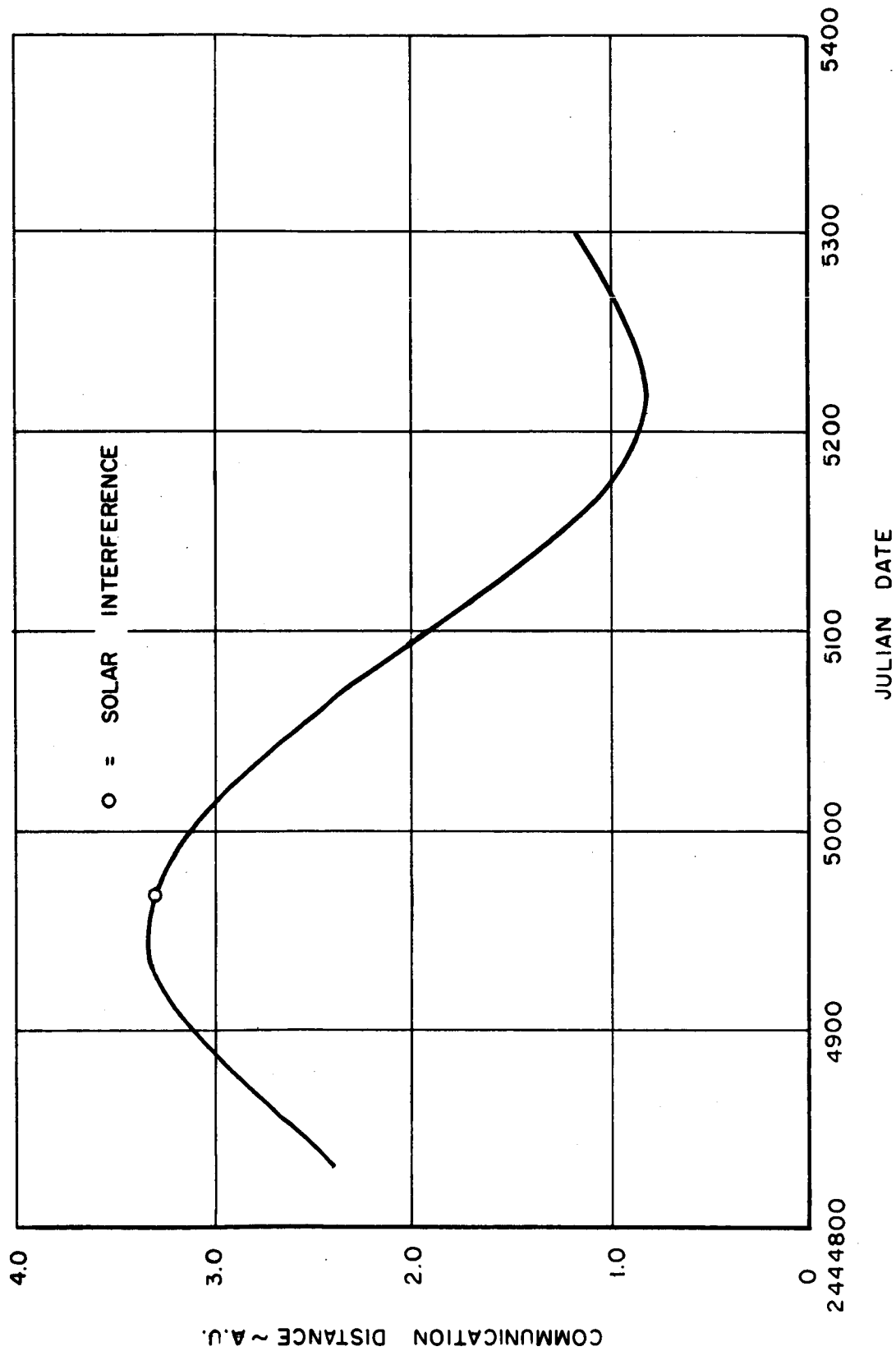
LEAVE EARTH ~ 2444260
TRIP TIME = 380 DAYS



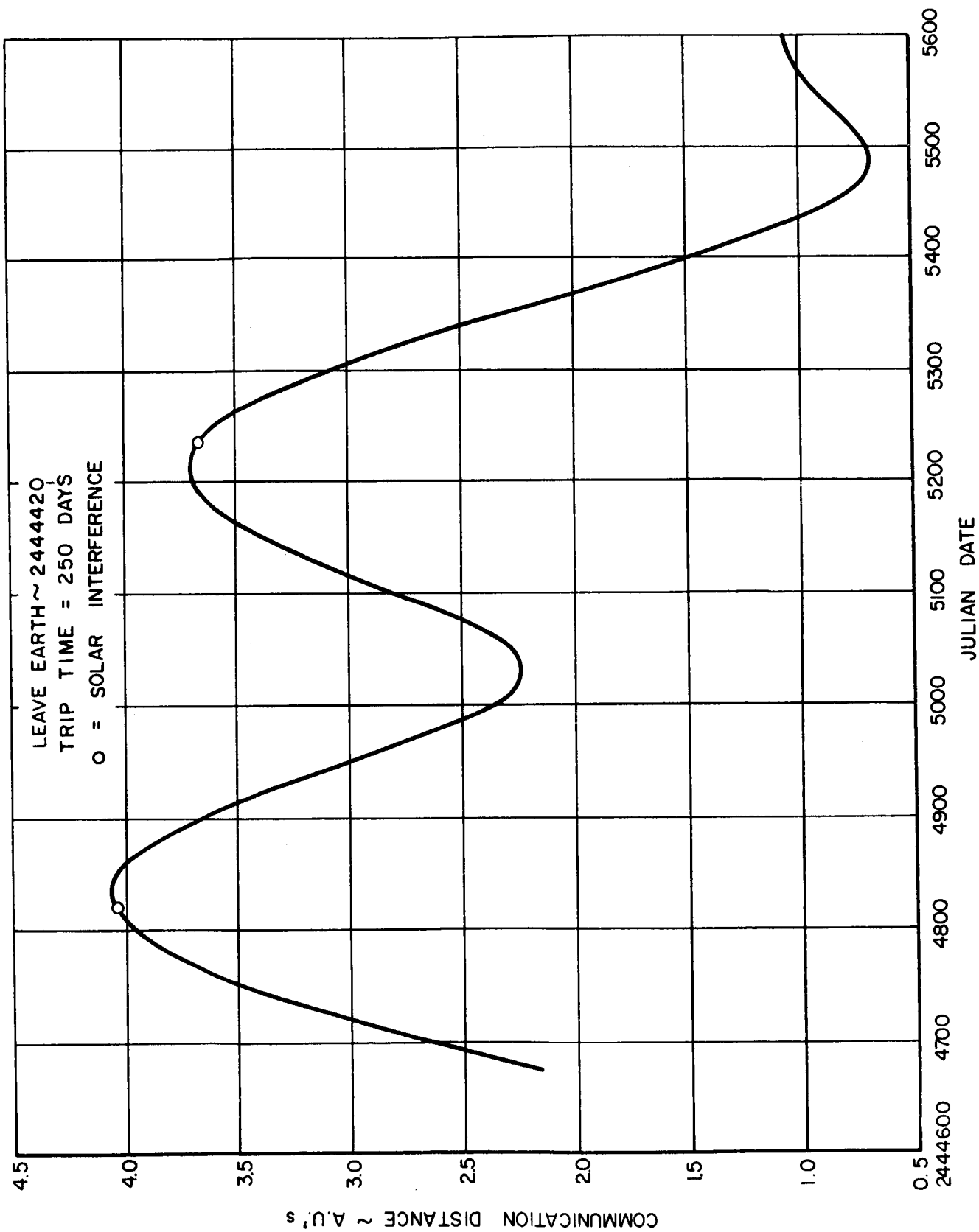
CERES FLYBY

LEAVE EARTH ~ 244400

TRIP TIME = 430 DAYS



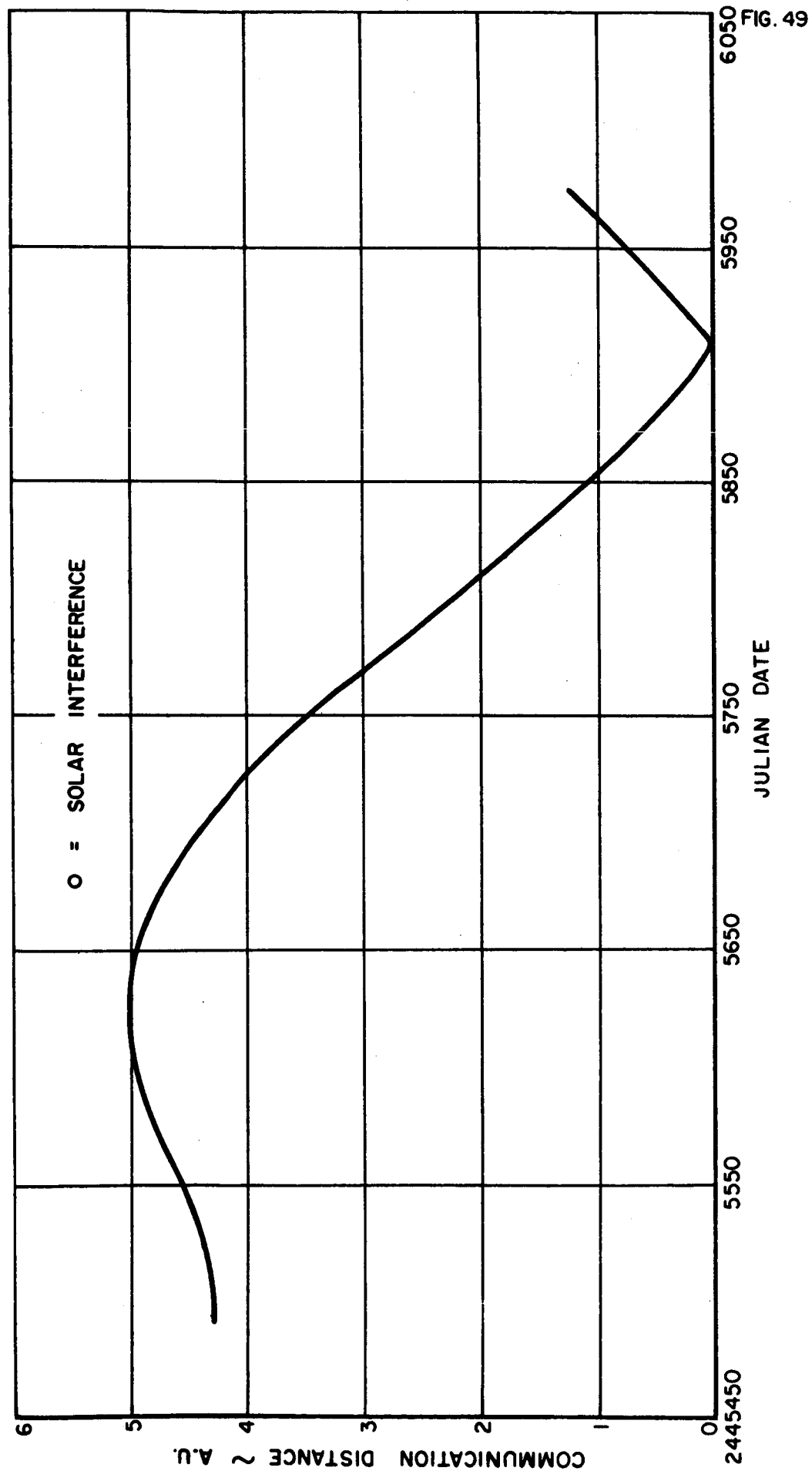
CERES FLYBY



JUPITER FLYBY

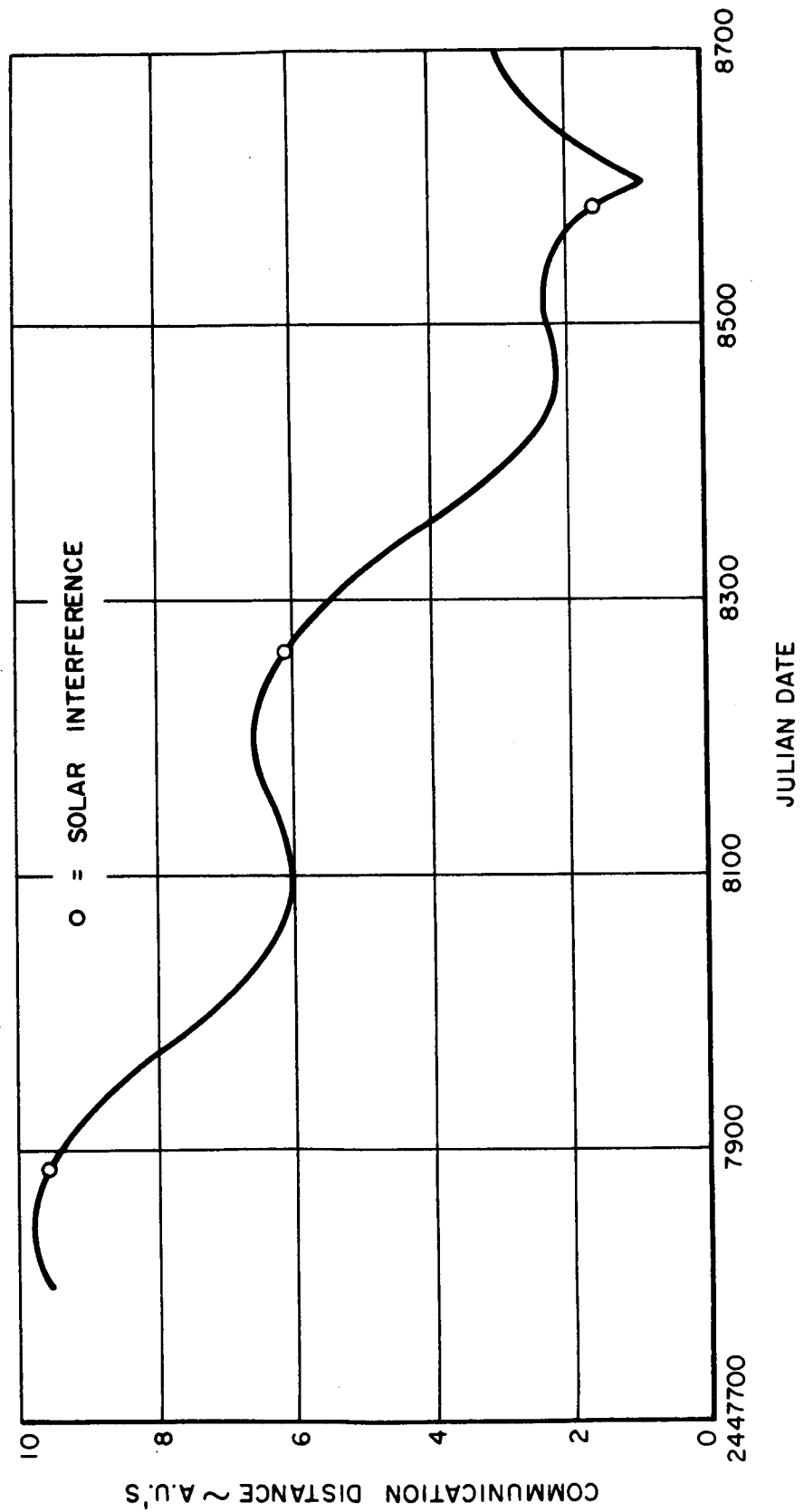
LEAVE EARTH ~ 2444980

TRIP TIME = 500 DAYS



SATURN FLYBY

TRIP TIME = 860 DAYS



MERCURY FLYBY

CHEMICAL

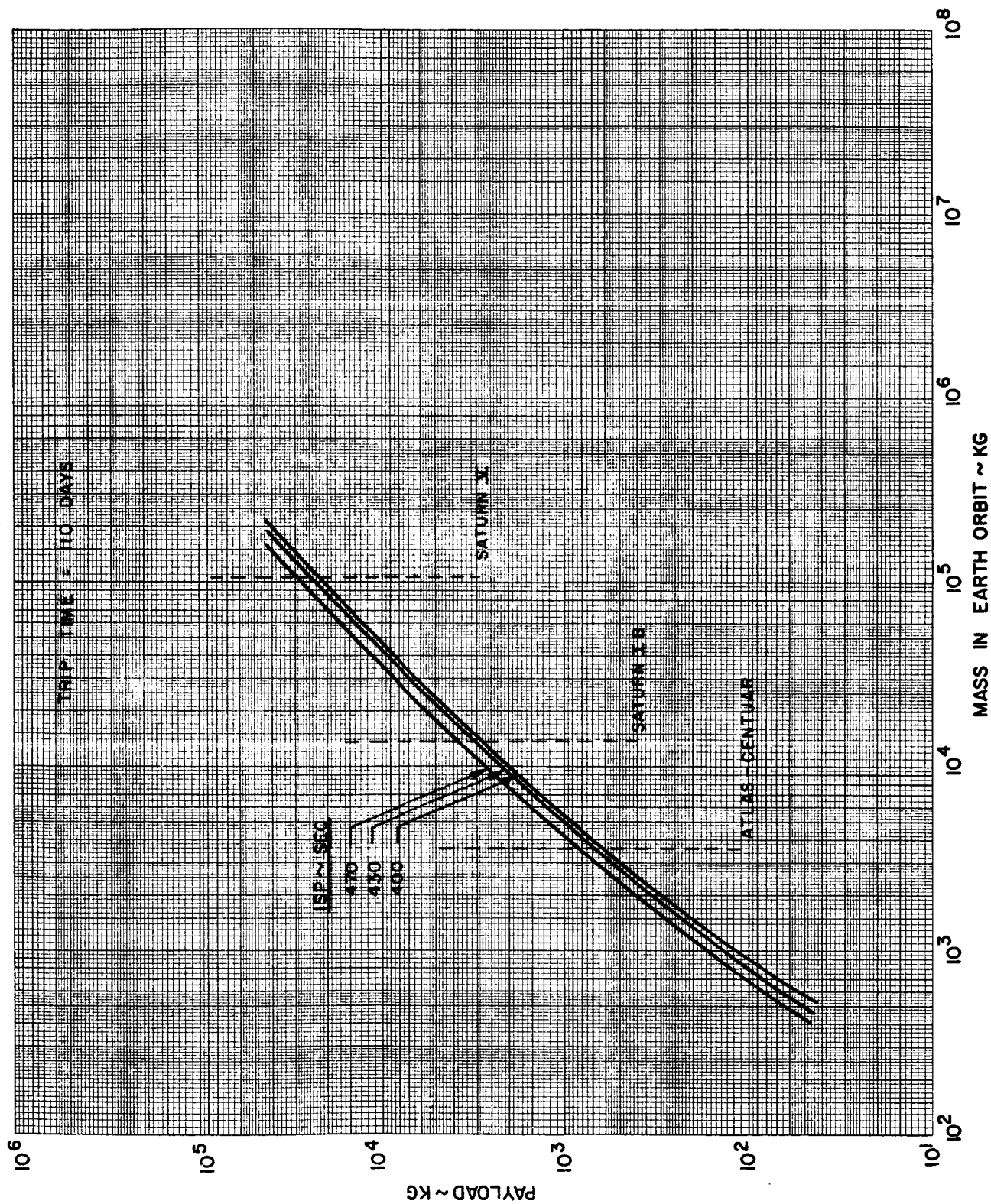
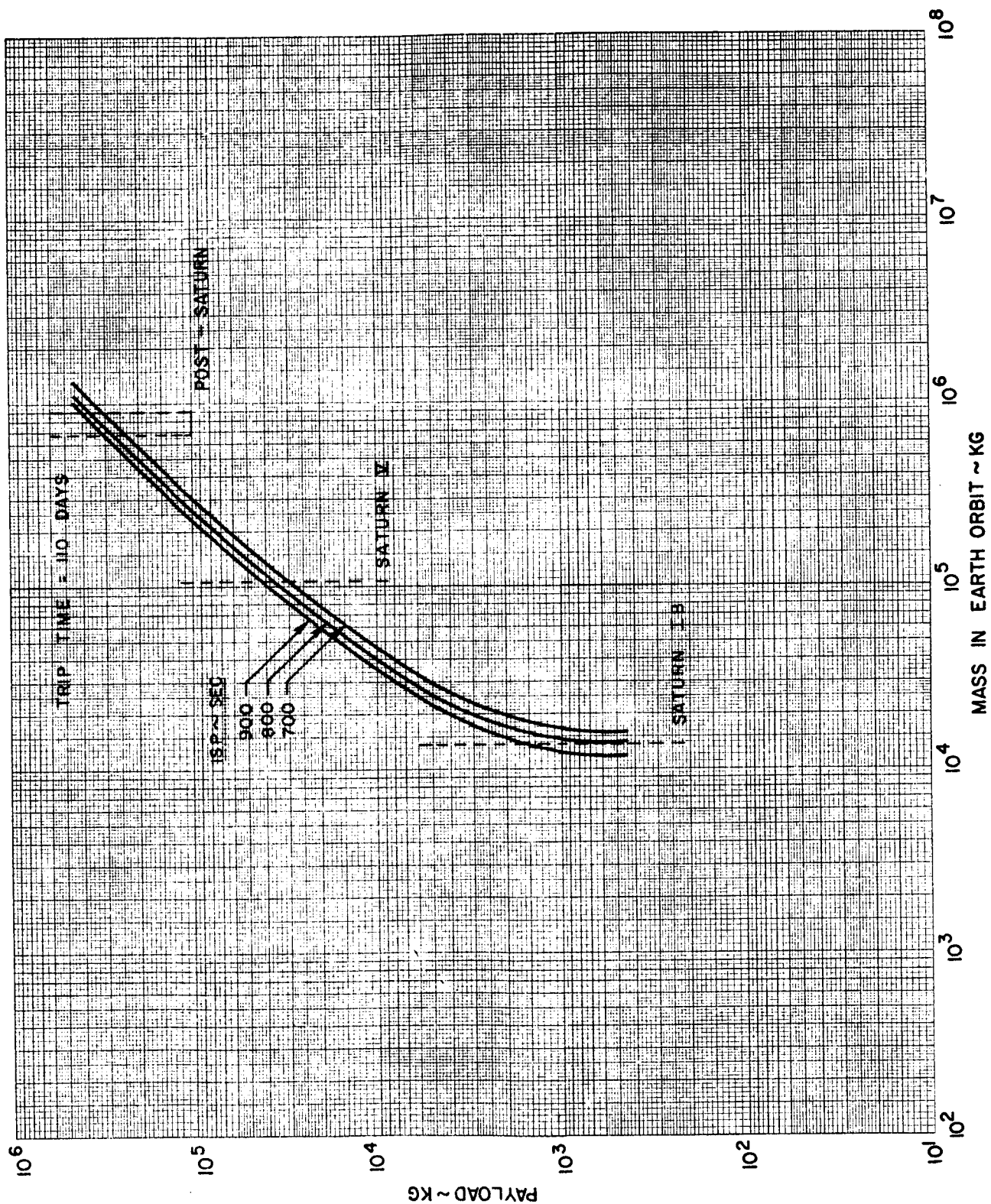


FIG. 52

E - 910352-9

MERCURY FLYBY

SOLID - CORE NUCLEAR



MERCURY FLYBY

LIGHTWEIGHT ADVANCED NUCLEAR

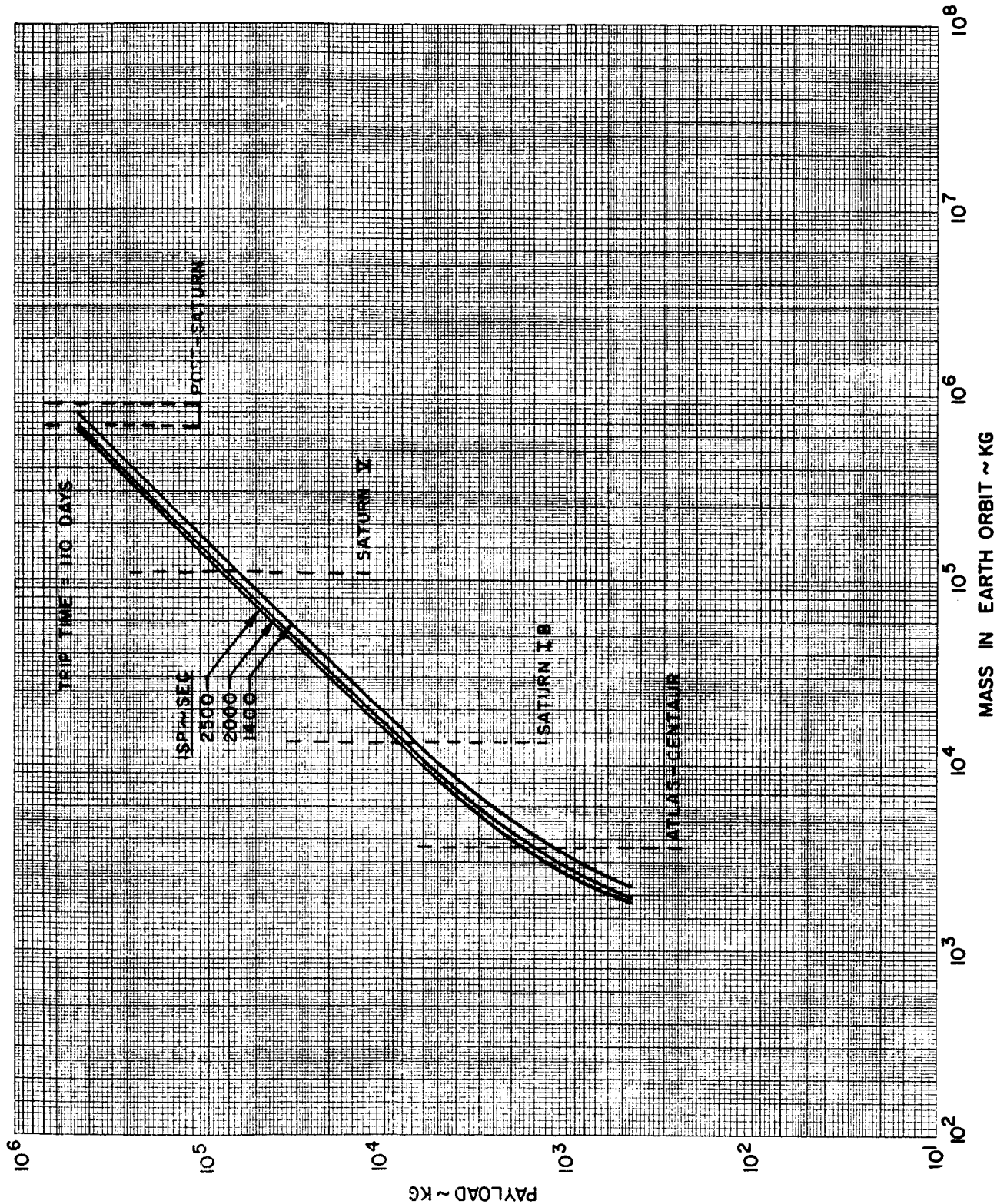
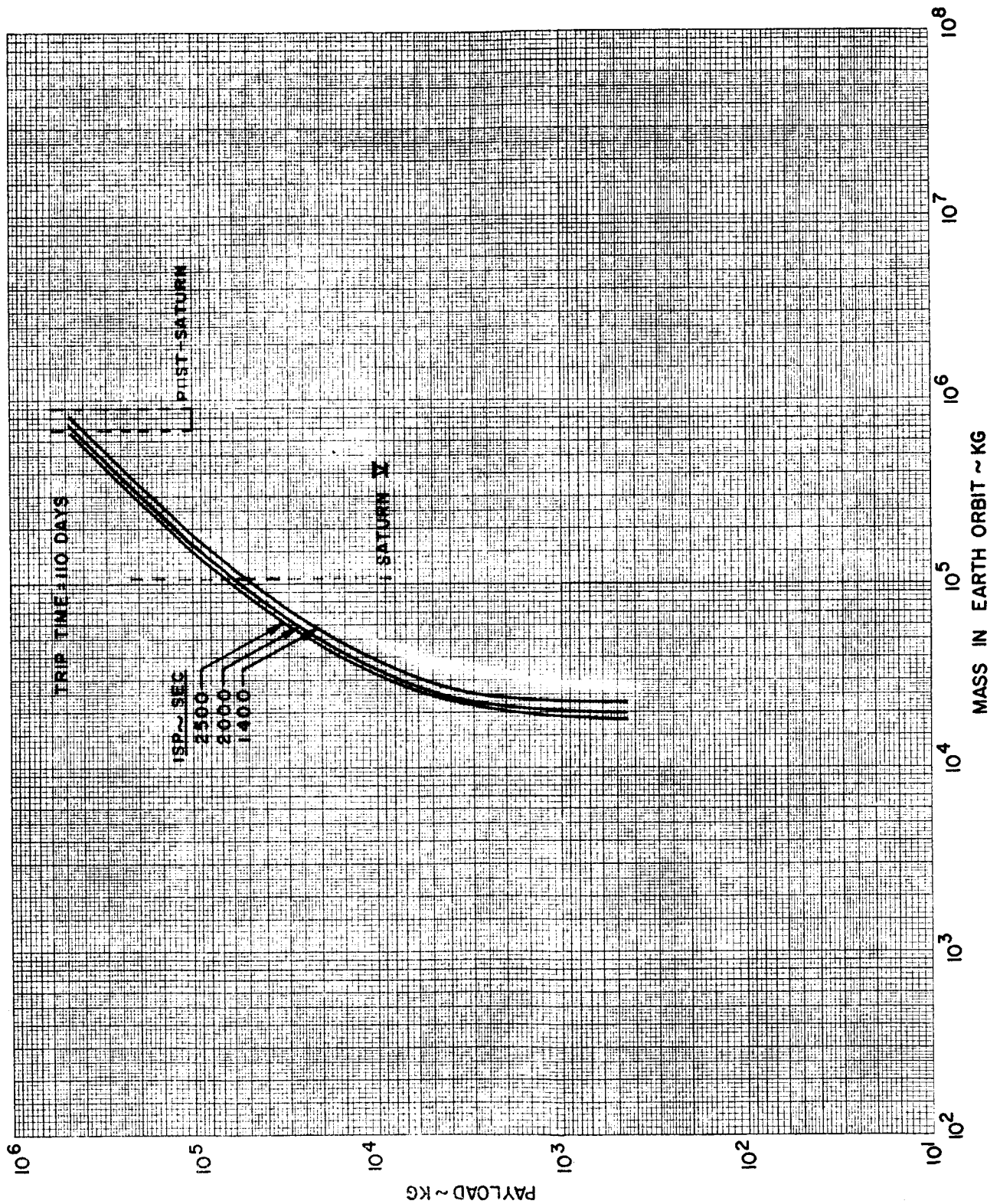


FIG. 54

E-910352-9

MERCURY FLYBY

GASEOUS-CORE NUCLEAR



MERCURY FLYBY

GASEOUS-CORE NUCLEAR

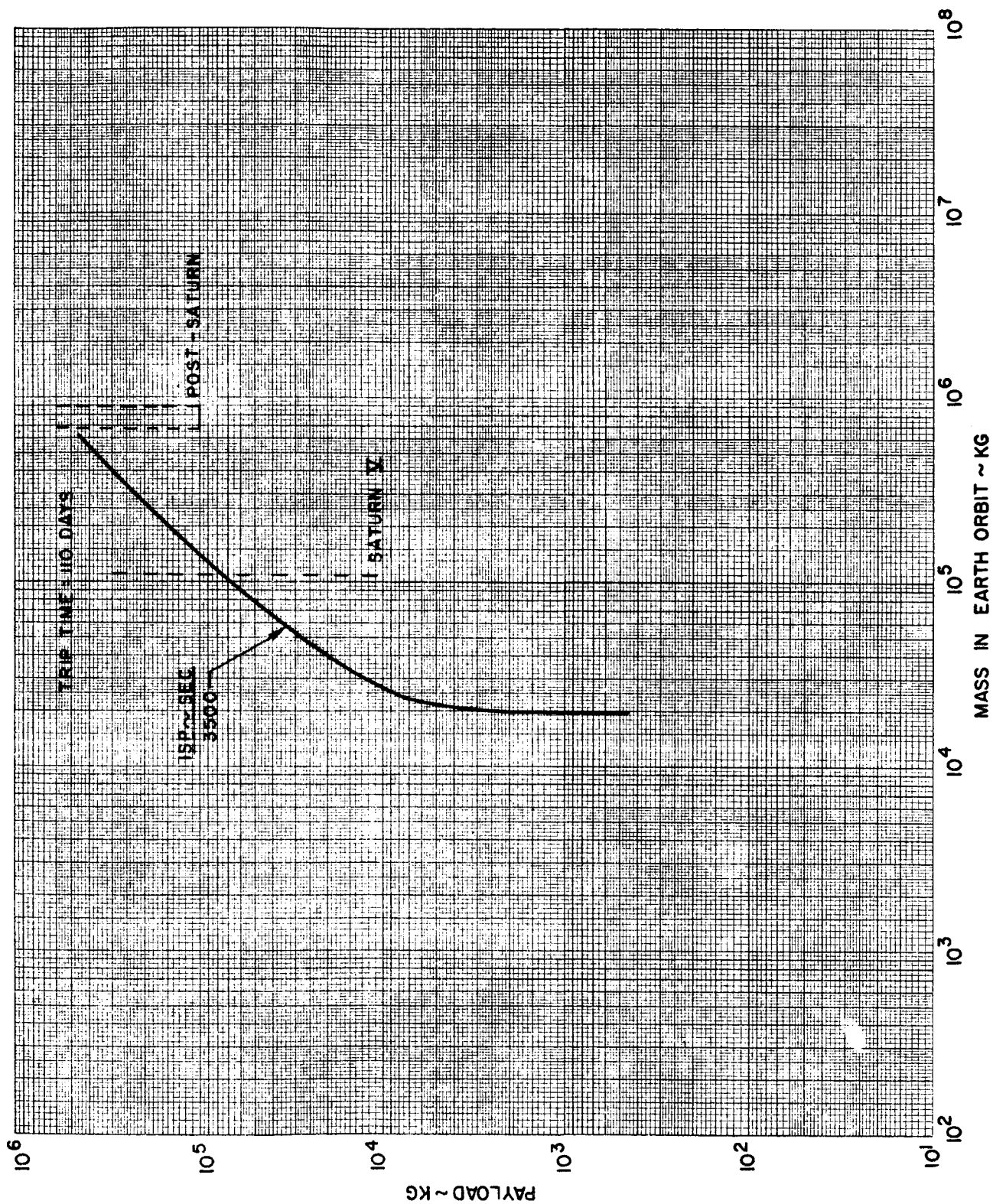
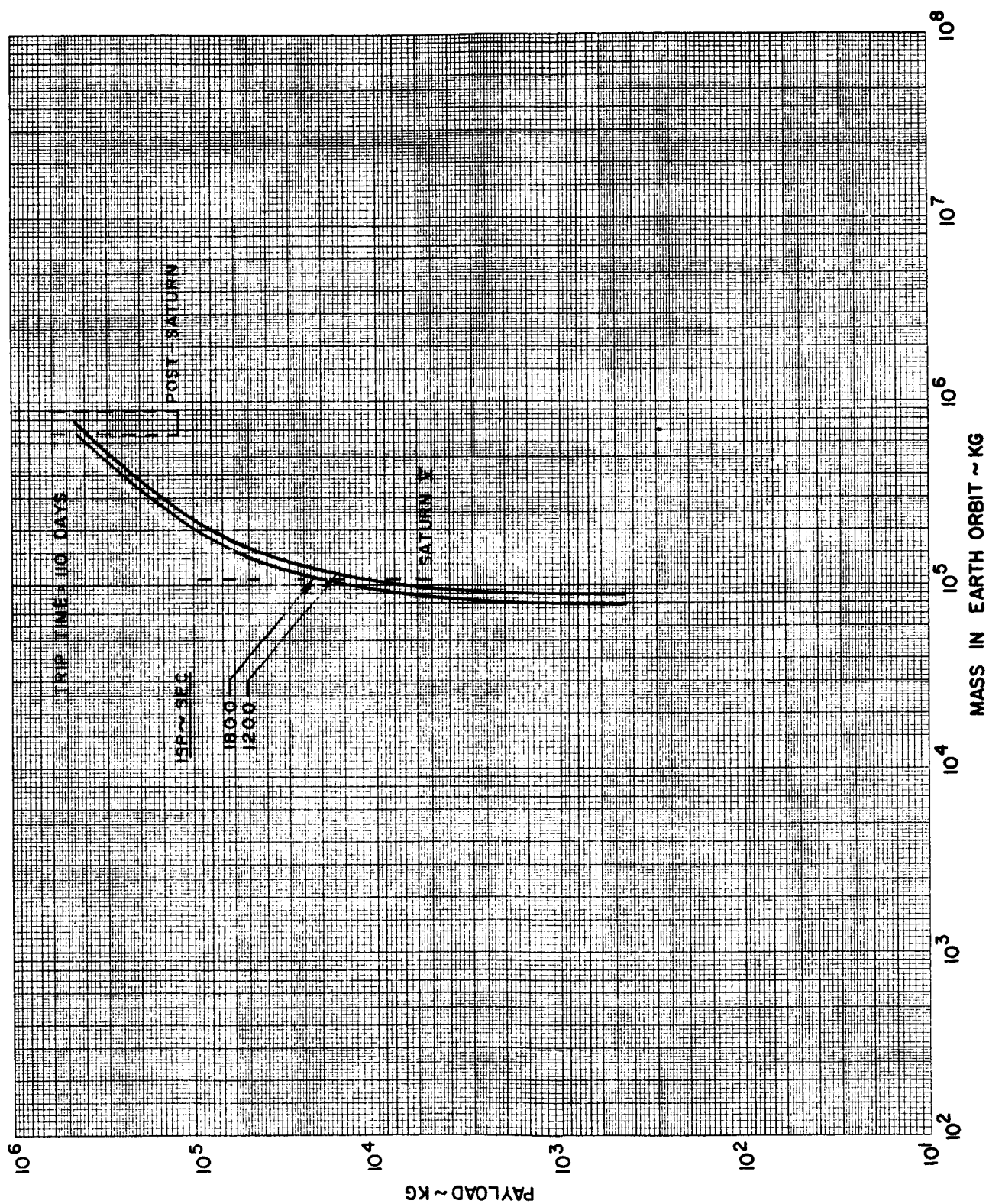


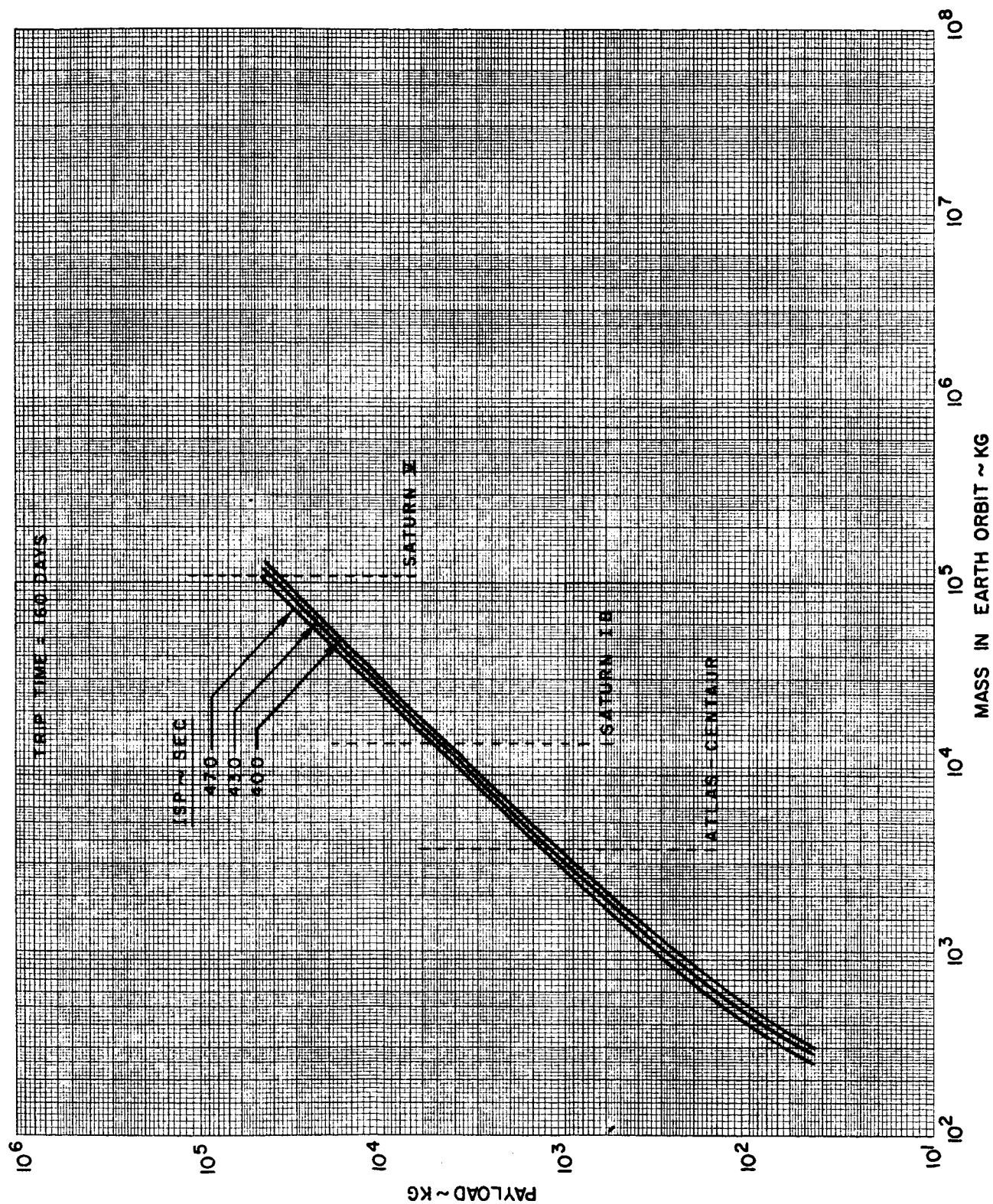
FIG. 56

E-910352-9

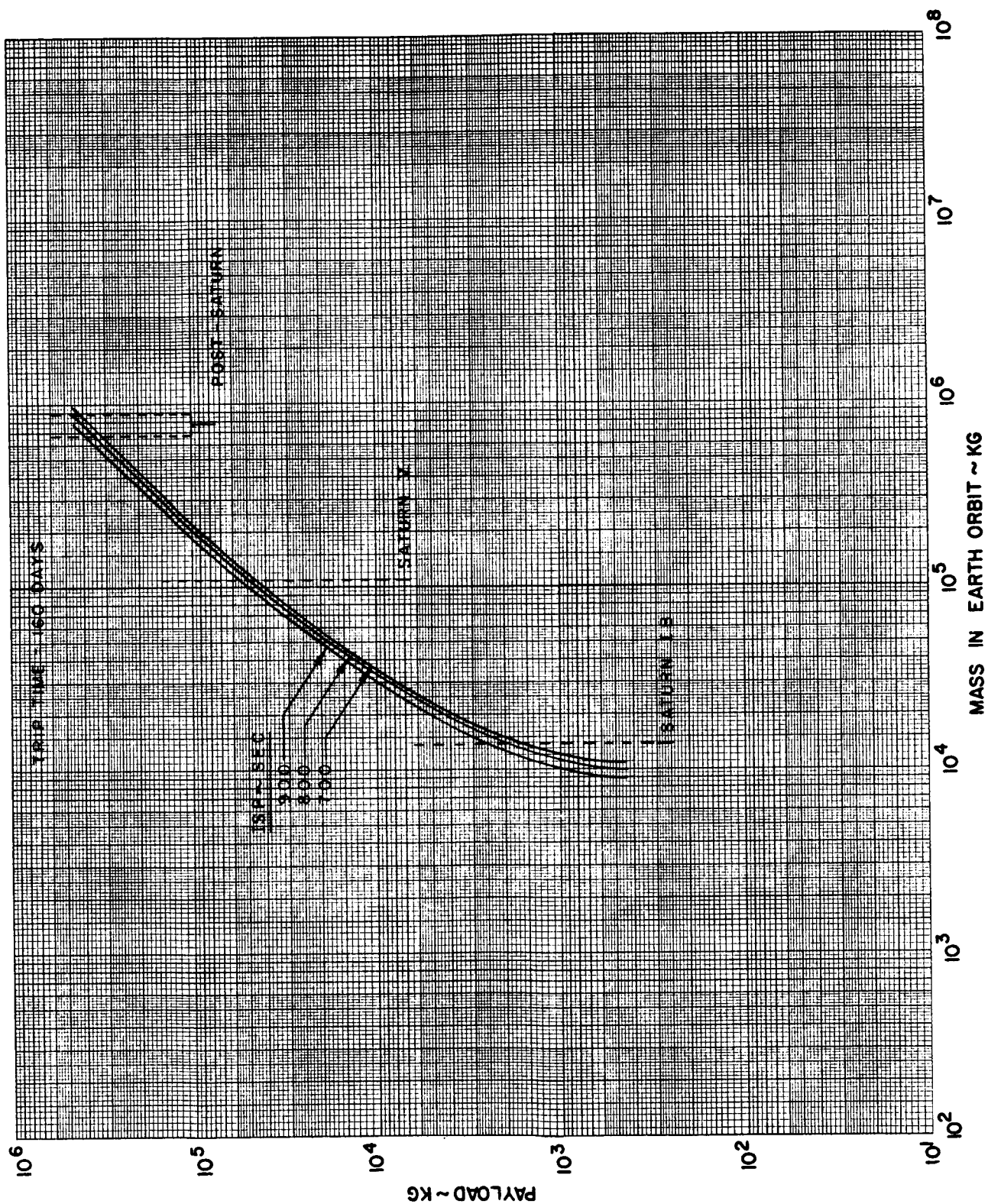
MERCURY FLYBY PULSED NUCLEAR



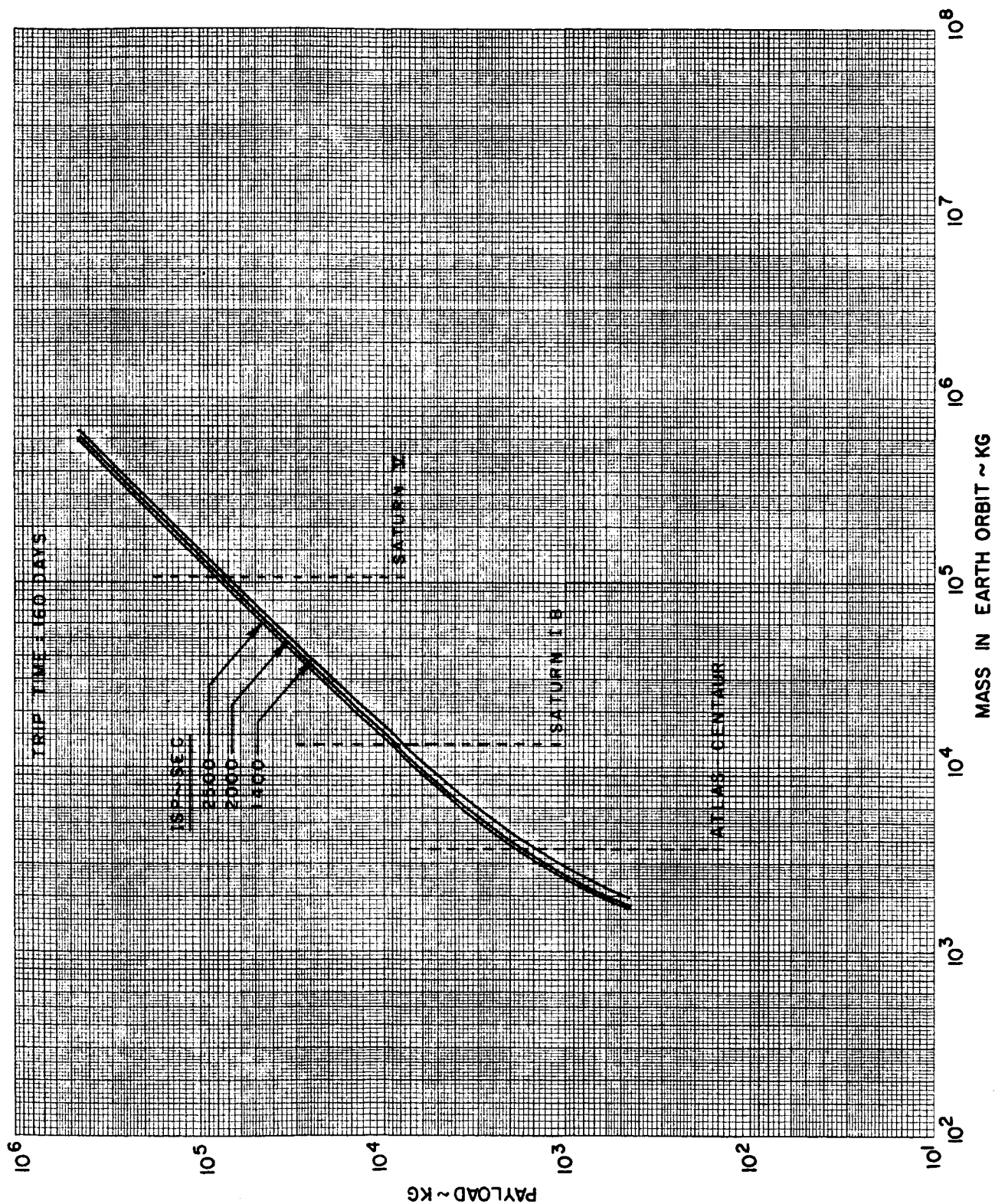
VENUS FLYBY CHEMICAL



VENUS FLYBY SOLID-CORE NUCLEAR

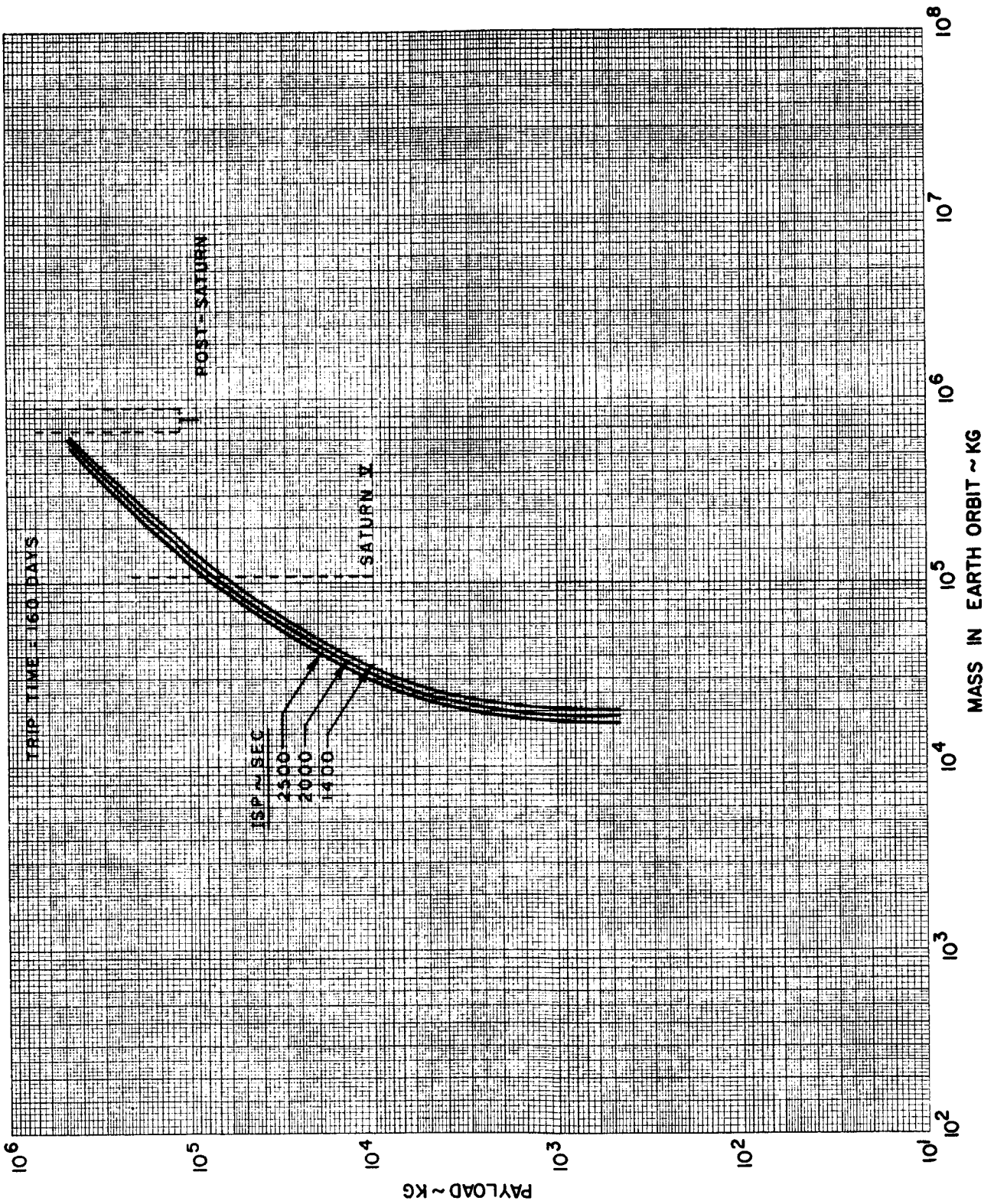


VENUS FLYBY LIGHTWEIGHT ADVANCED NUCLEAR



VENUS FLYBY

GASEOUS - CORE NUCLEAR



VENUS FLYBY
GASEOUS-CORE NUCLEAR

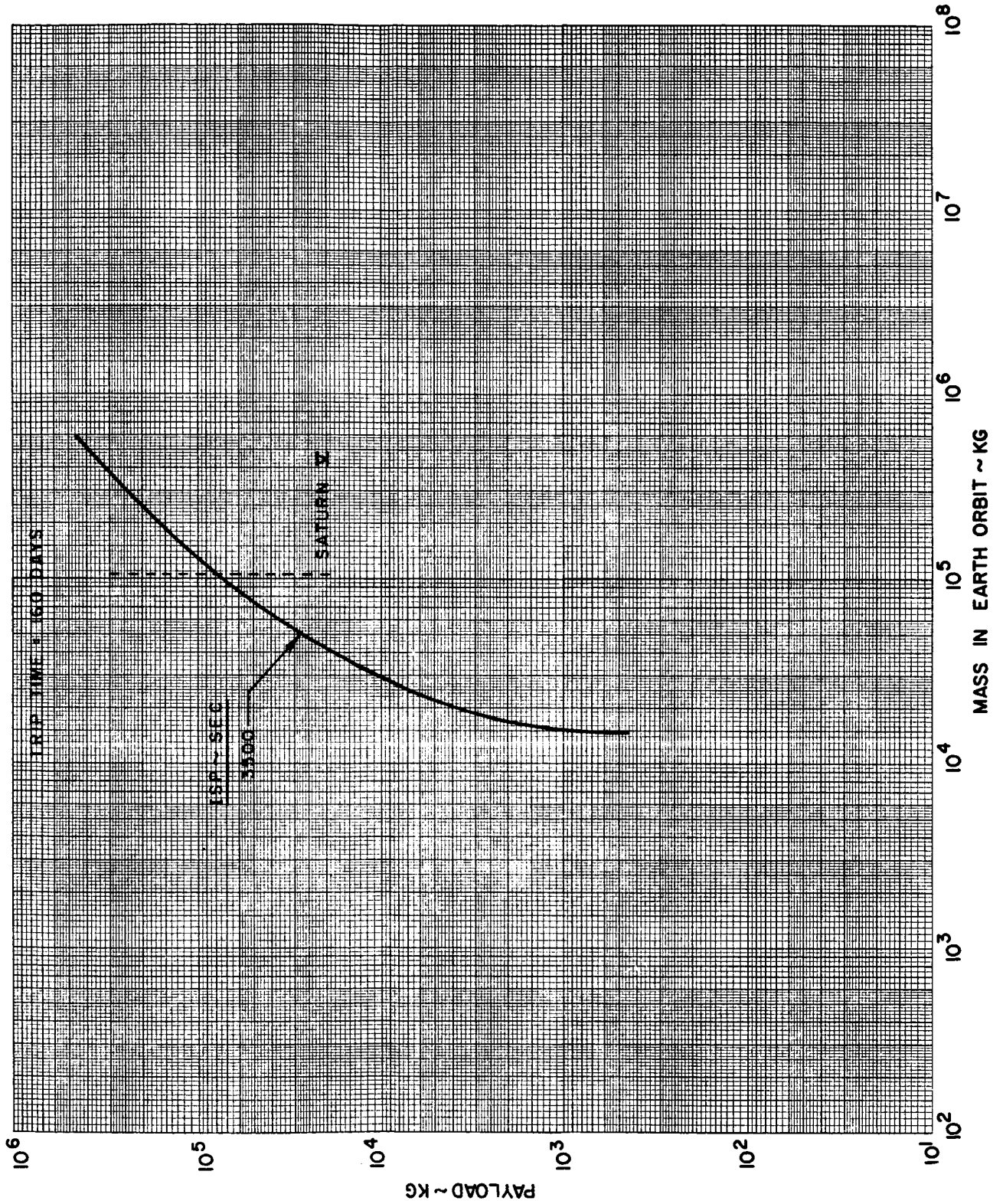
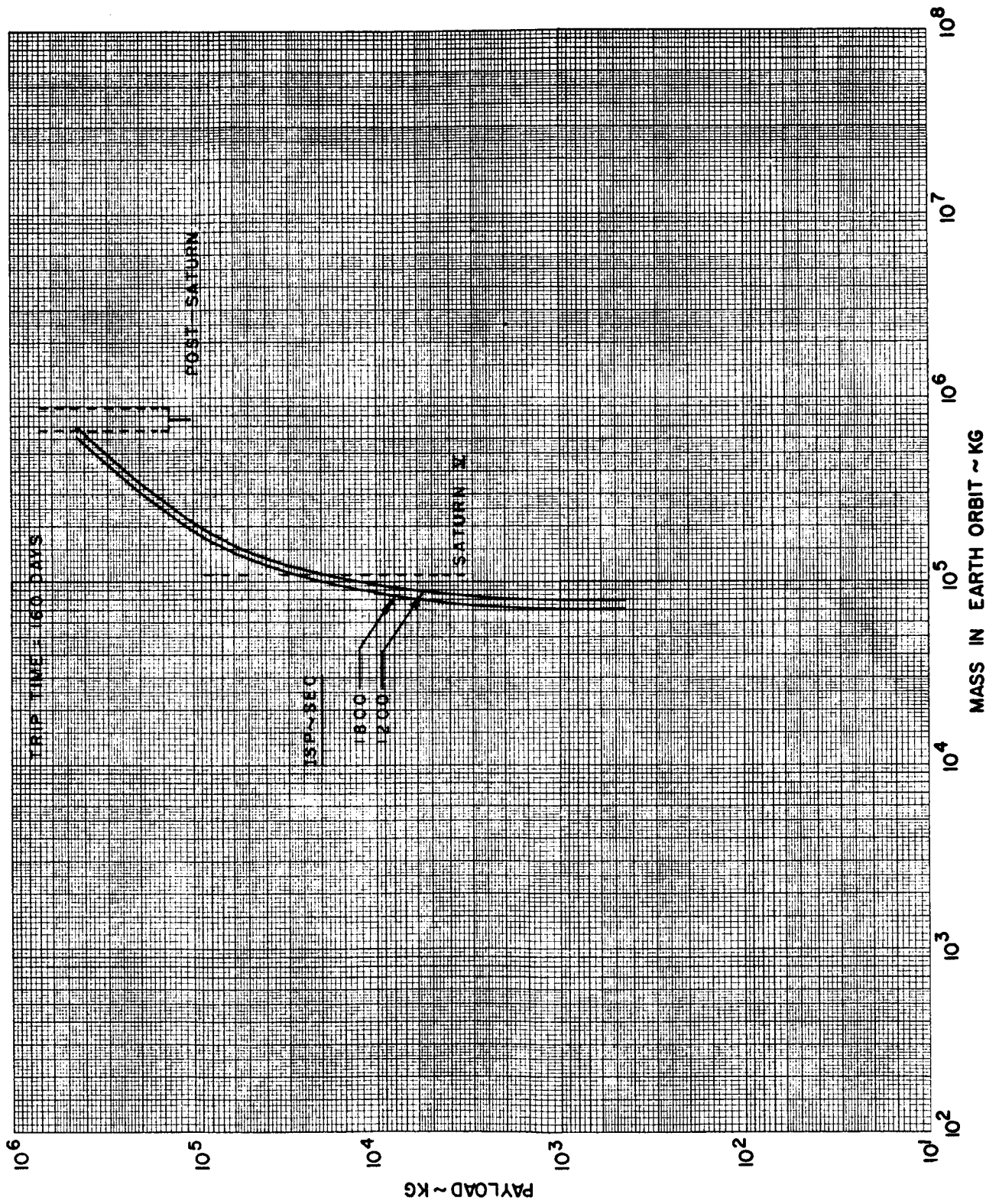


FIG. 62

E-910352-9

VENUS FLYBY
PULSED NUCLEAR



MARS FLYBY

CHEMICAL

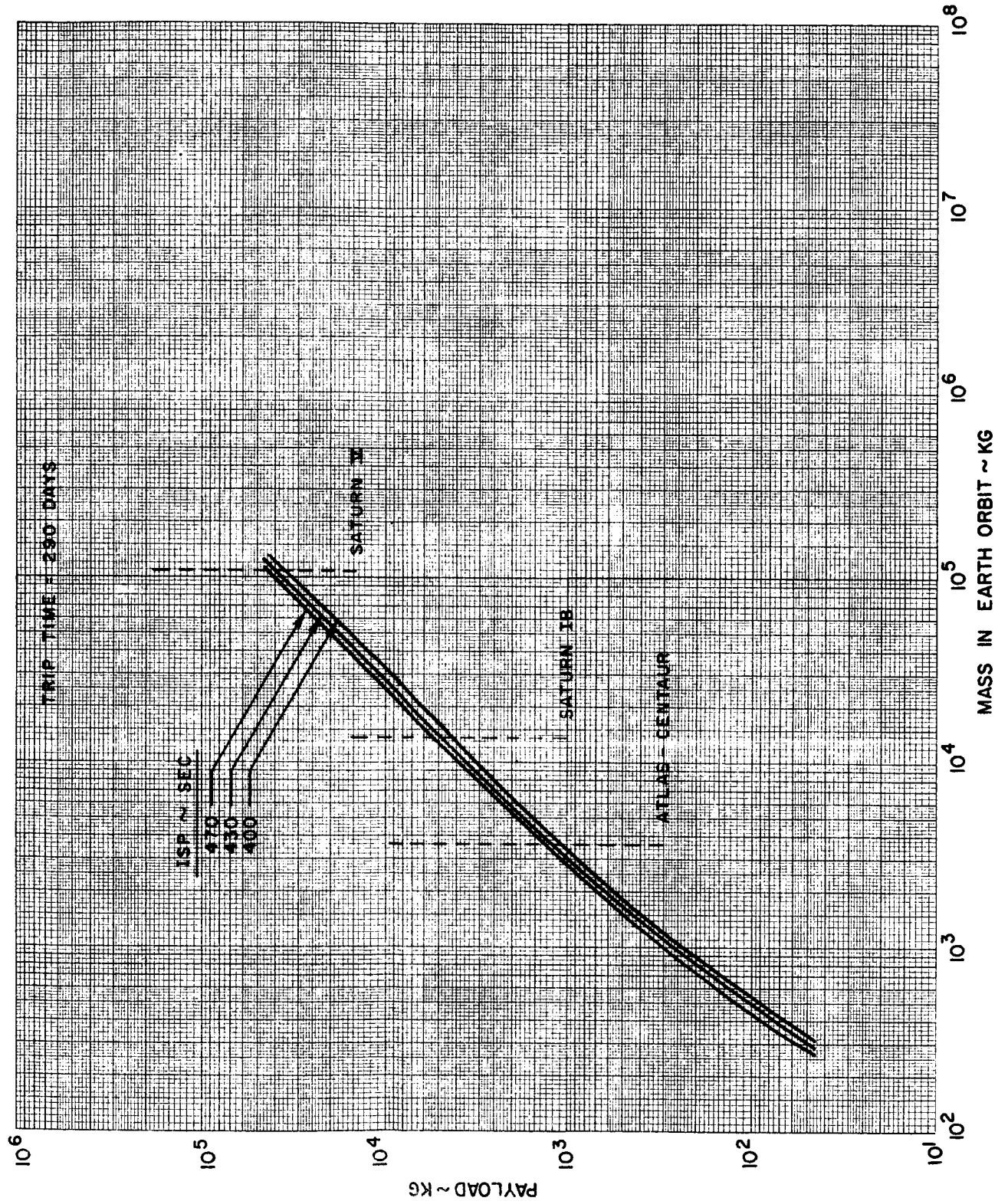
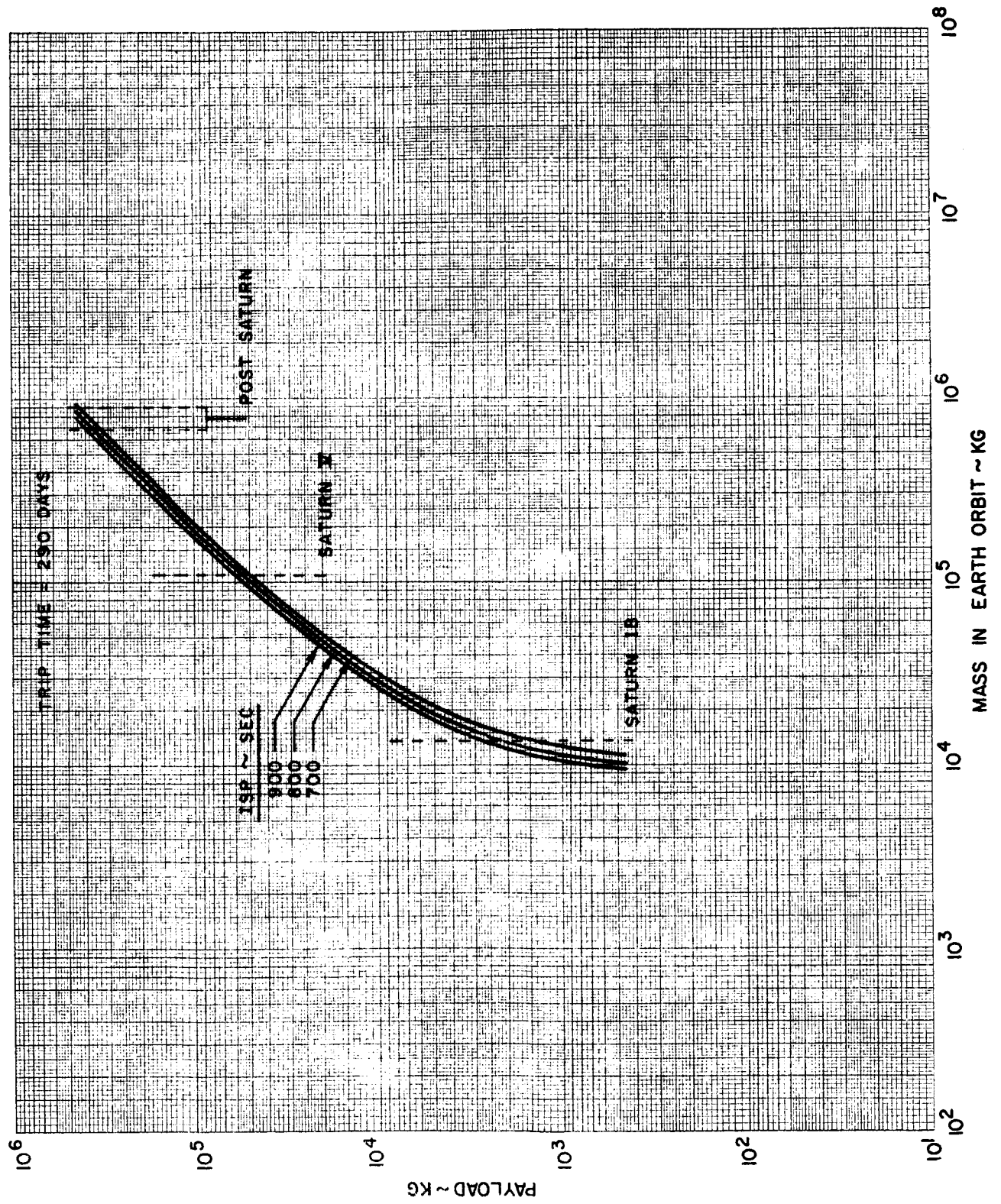


FIG. 64

E-910352-9

MARS FLYBY

SOLID - CORE NUCLEAR



MARS FLYBY

LIGHTWEIGHT ADVANCED NUCLEAR

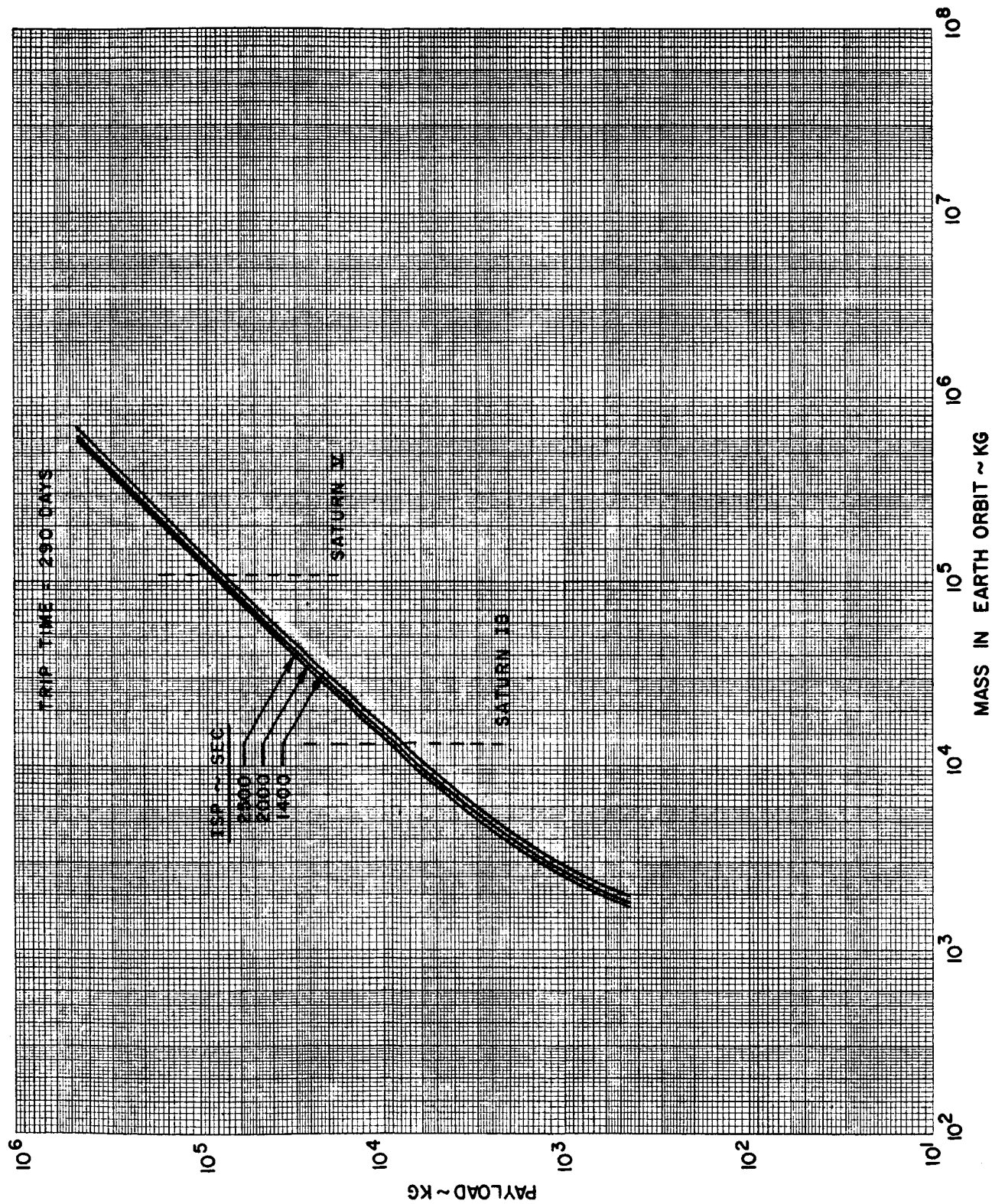
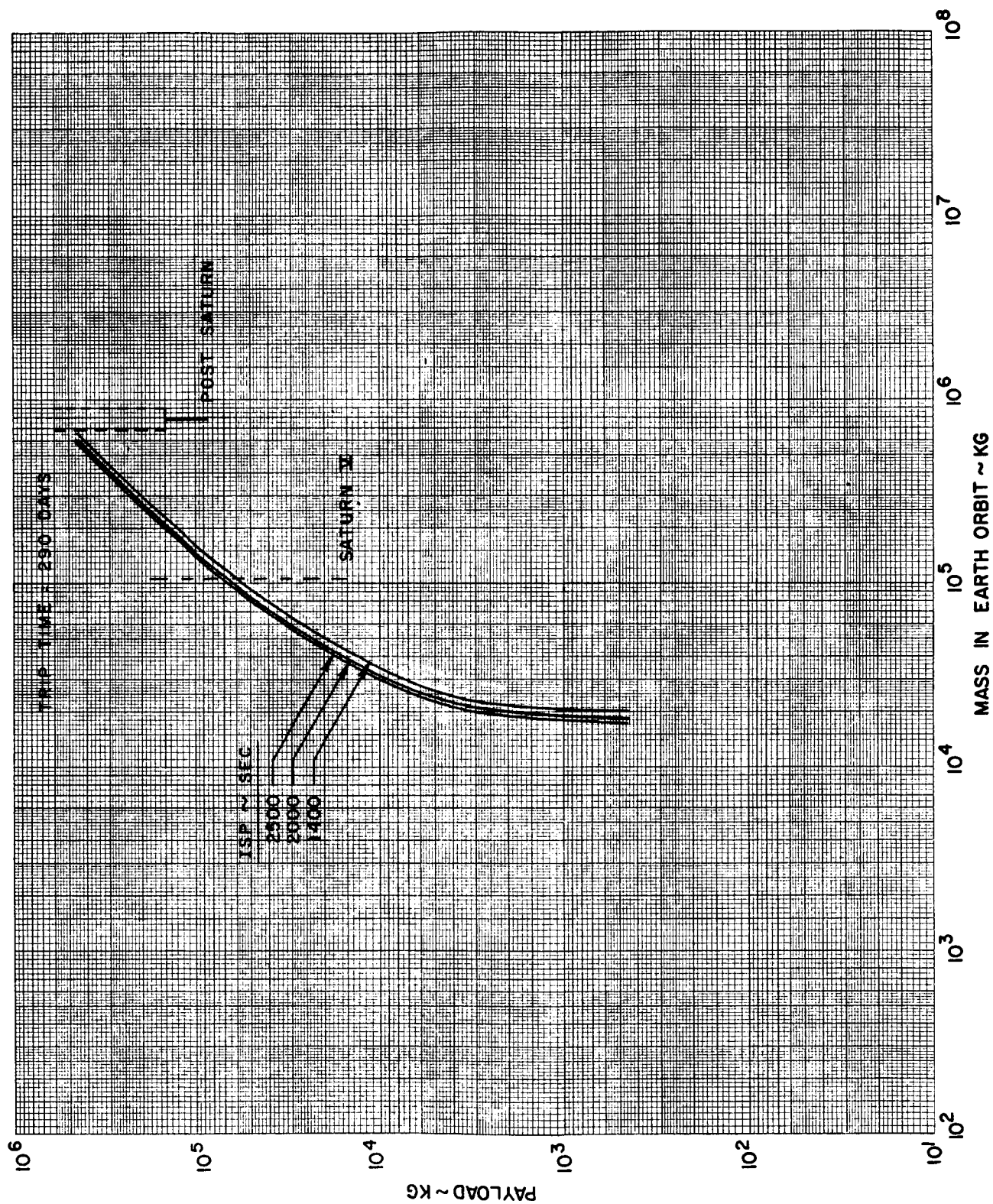


FIG. 66

E-910352-9

MARS FLYBY

GASEOUS - CORE NUCLEAR



MARS FLYBY

GASEOUS - CORE NUCLEAR

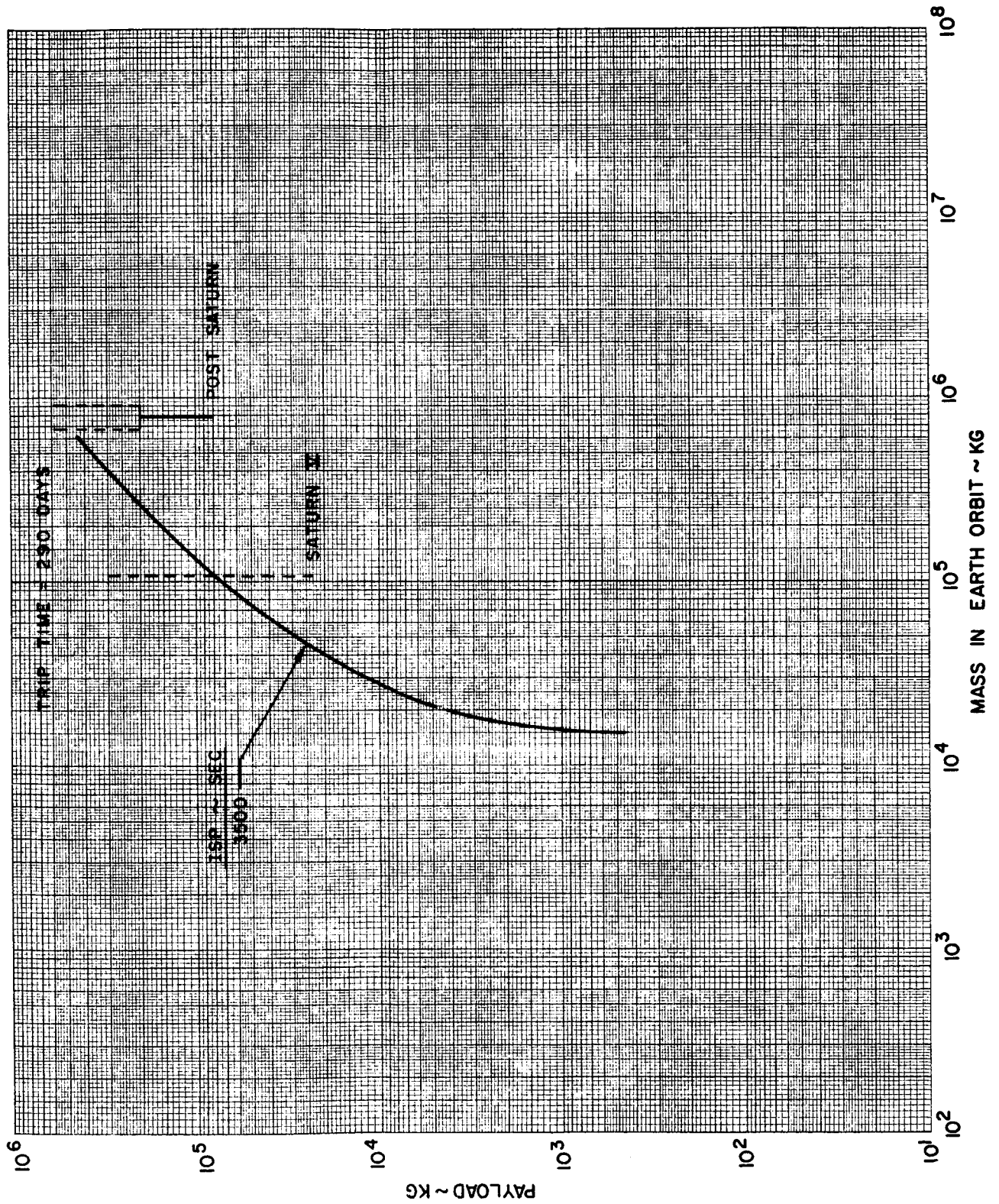
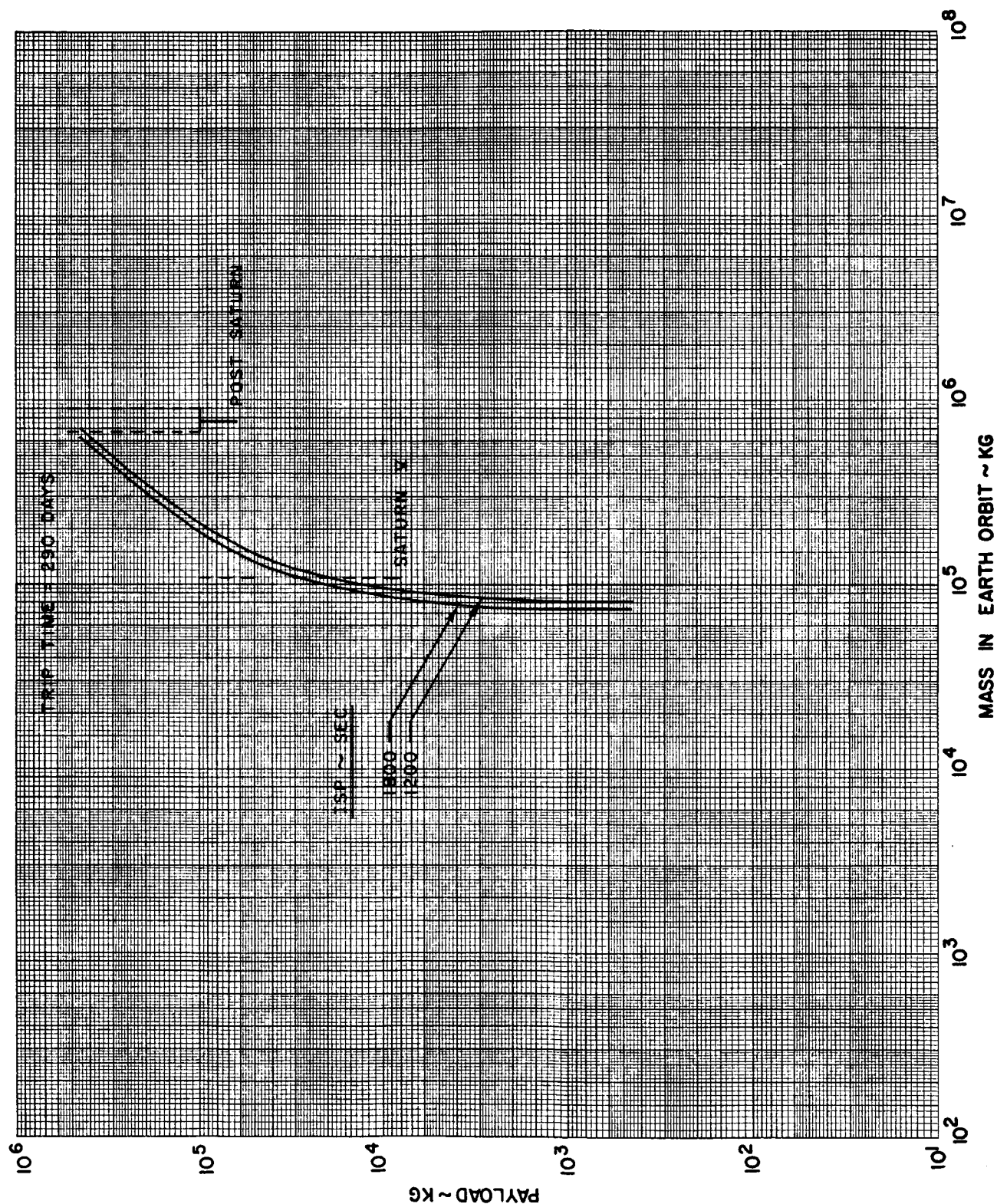


FIG. 68

E-910352-9

MARS FLYBY

PULSED NUCLEAR



JUPITER FLYBY

CHEMICAL

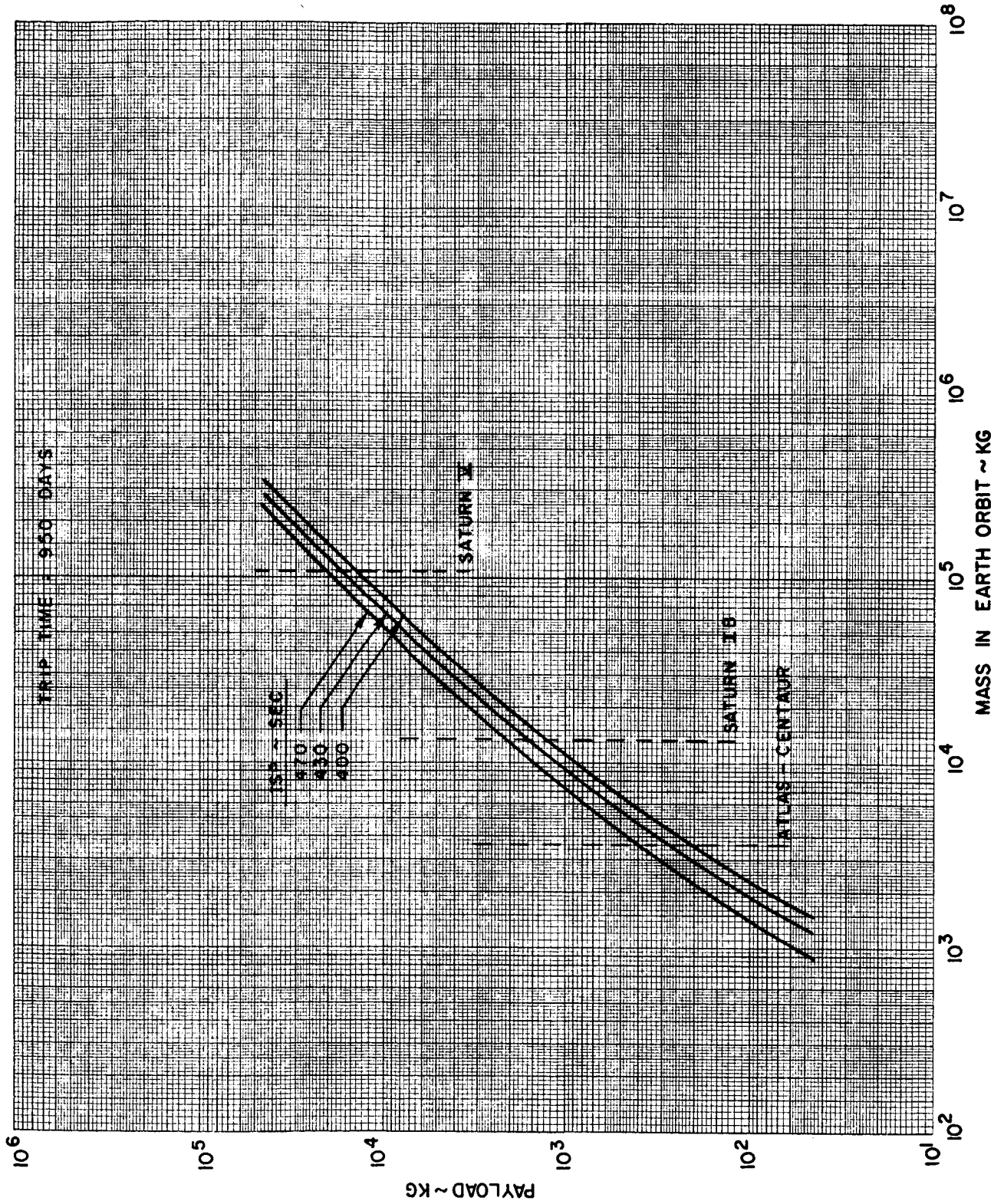
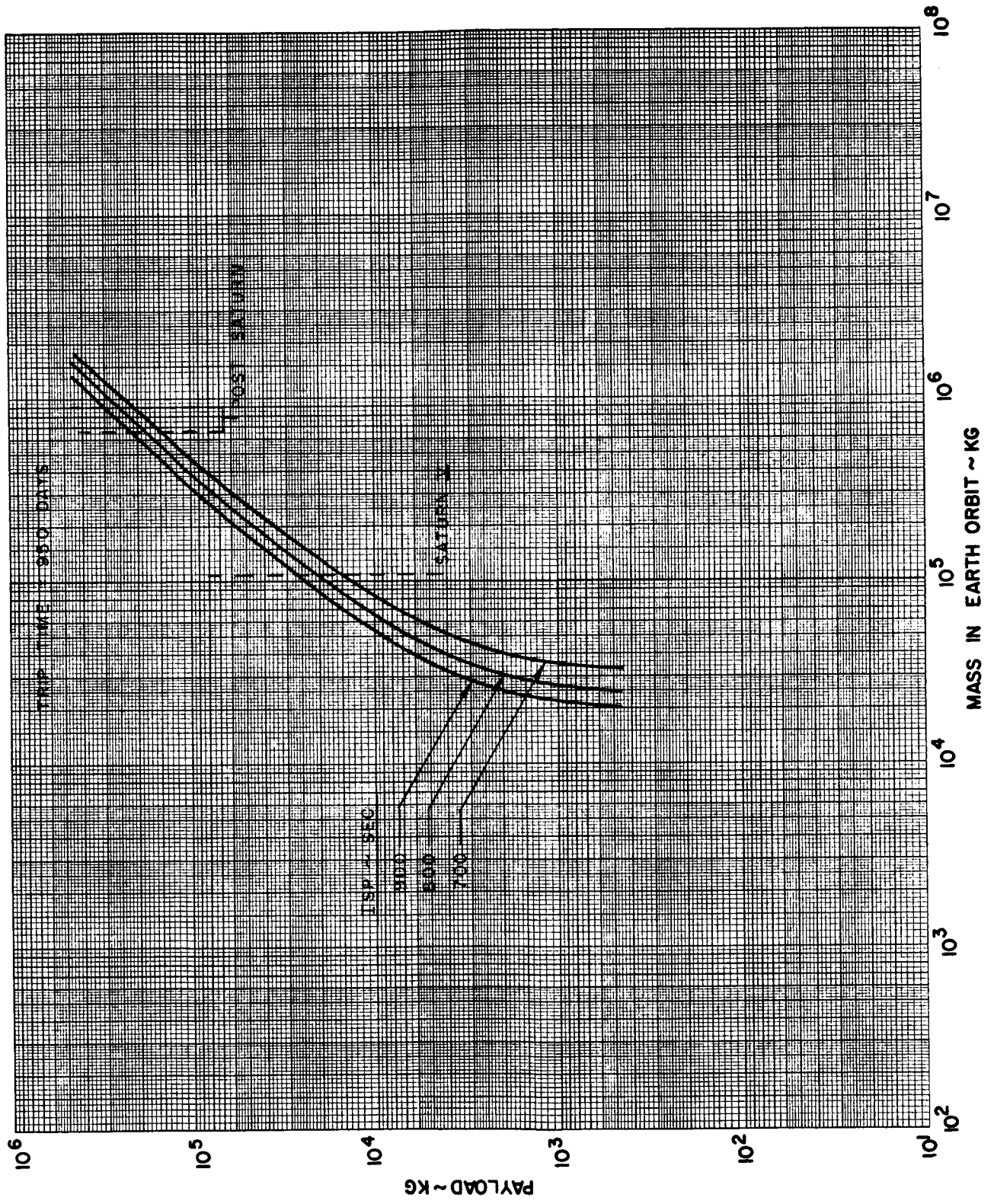


FIG. 70

E-910352-9

JUPITER FLYBY

SOLID - CORE NUCLEAR



JUPITER FLYBY

LIGHTWEIGHT ADVANCED NUCLEAR

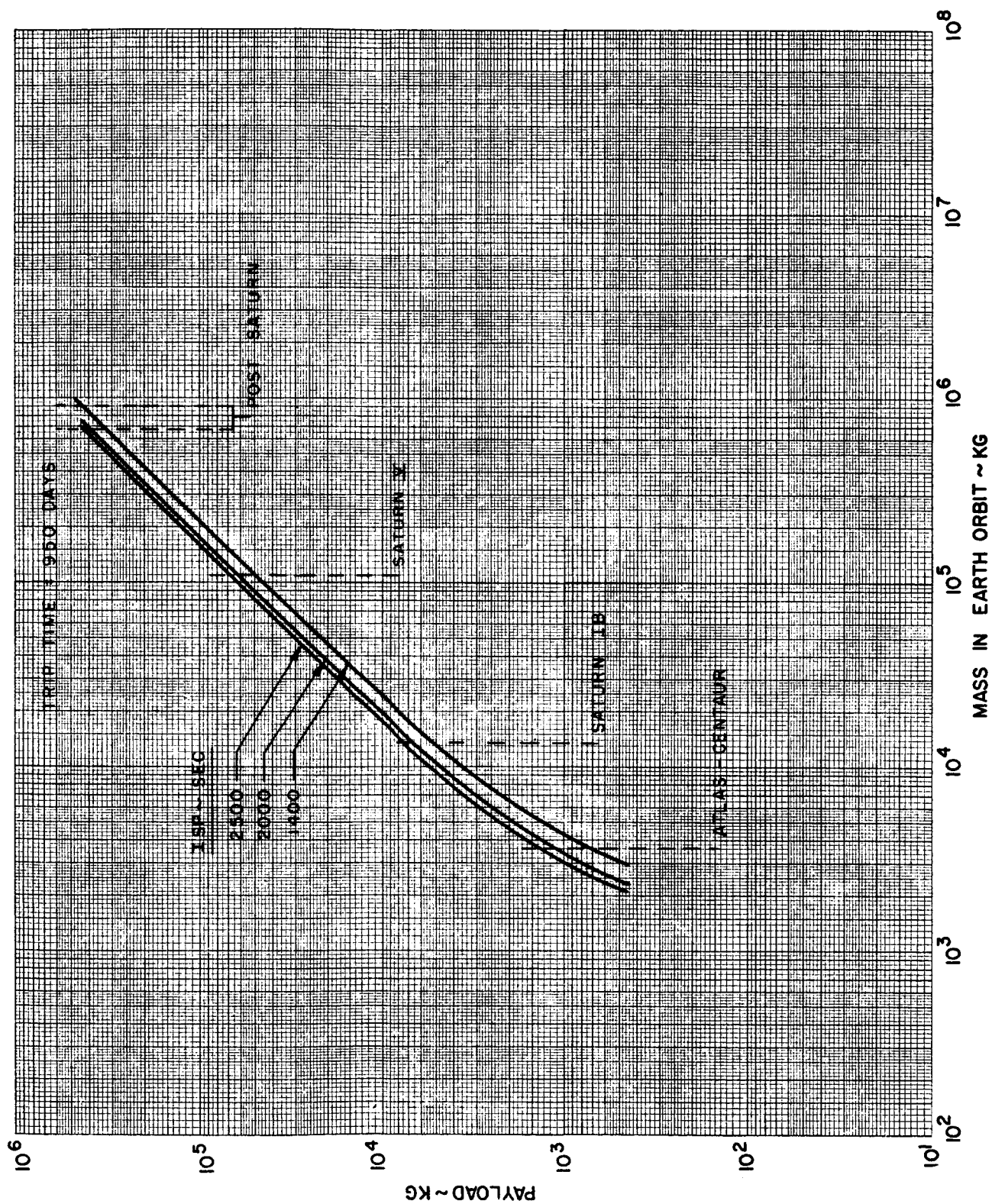
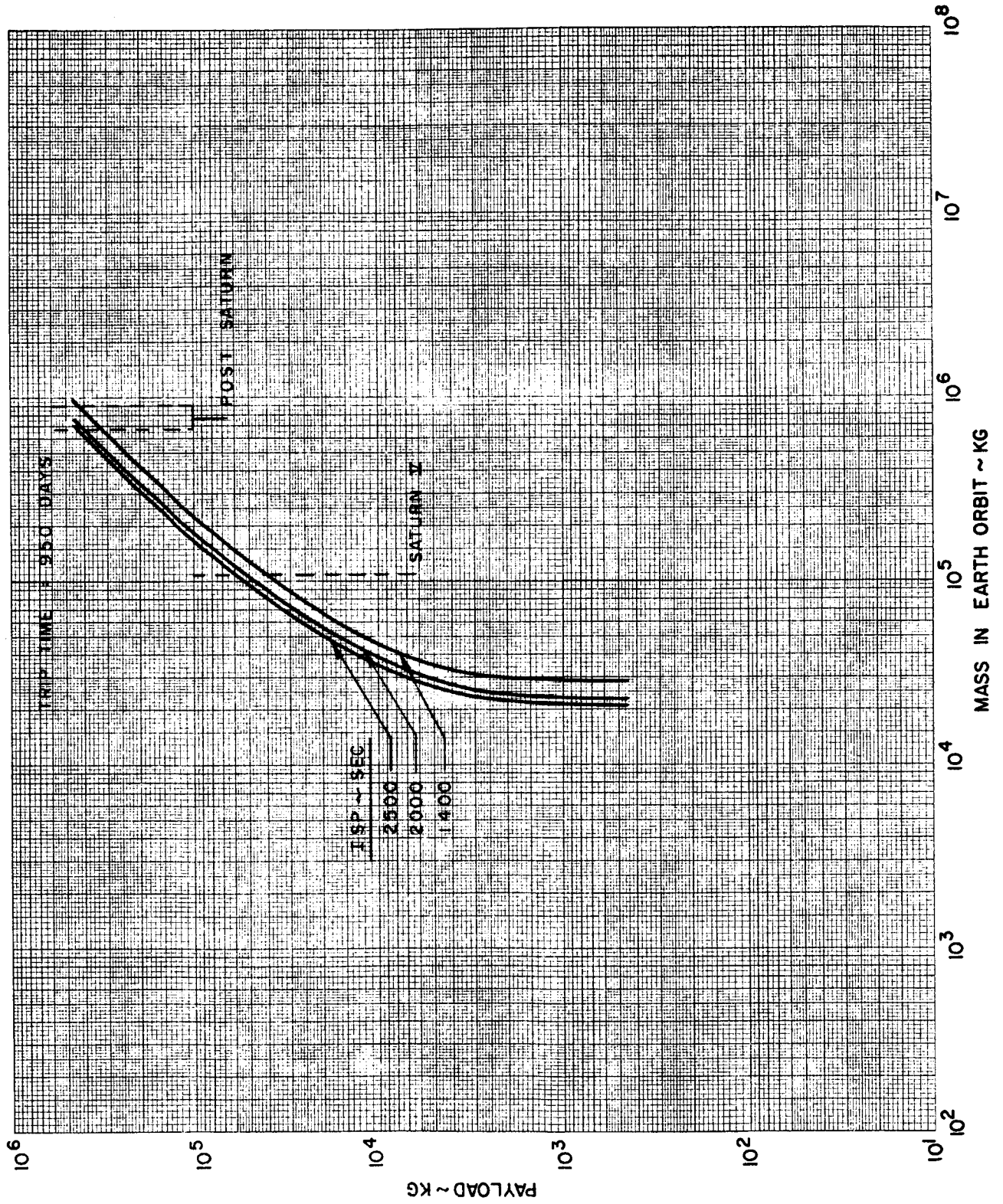


FIG. 72

E-910352-9

JUPITER FLYBY

GASEOUS - CORE NUCLEAR



JUPITER FLYBY
GASEOUS - CORE NUCLEAR

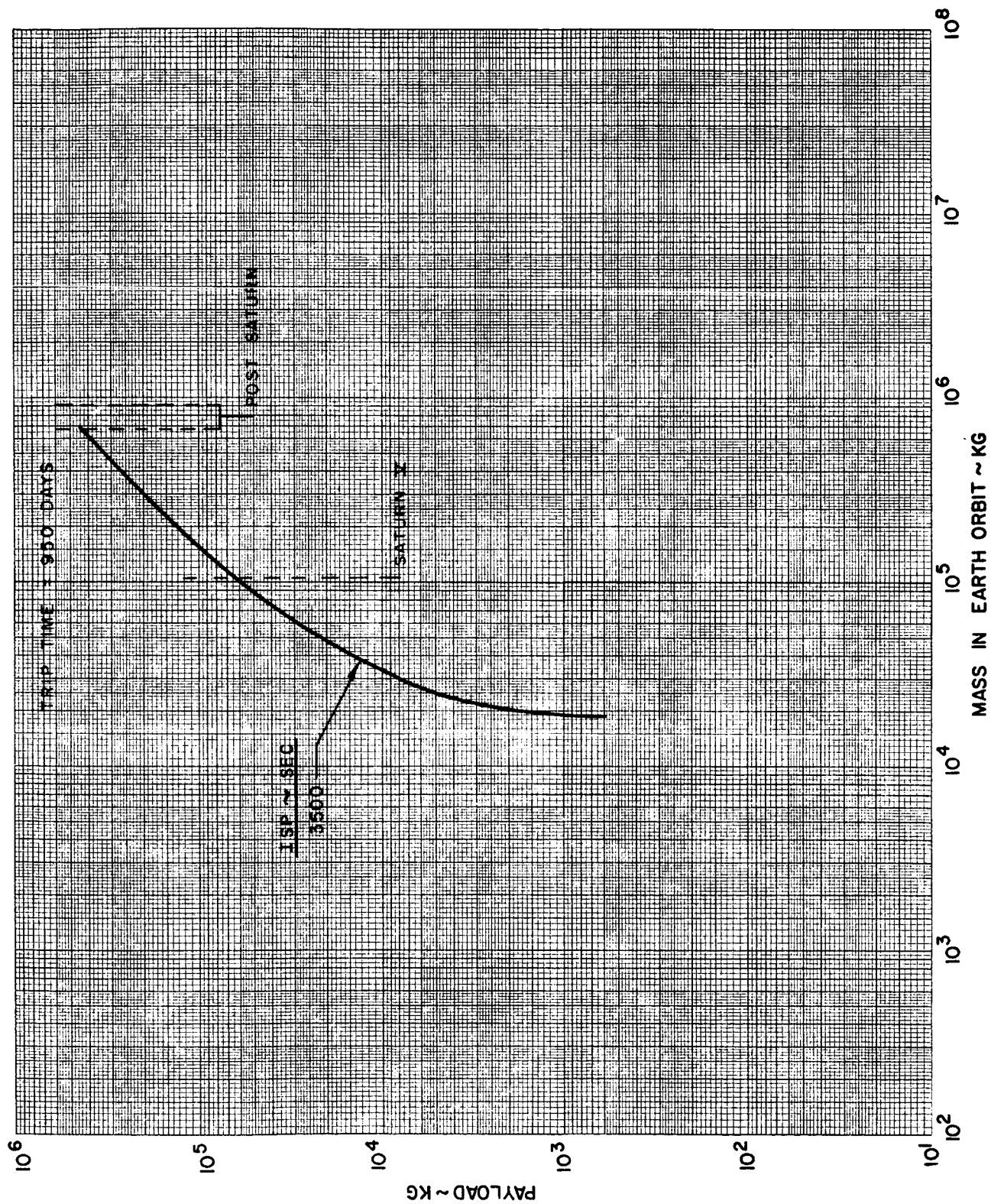
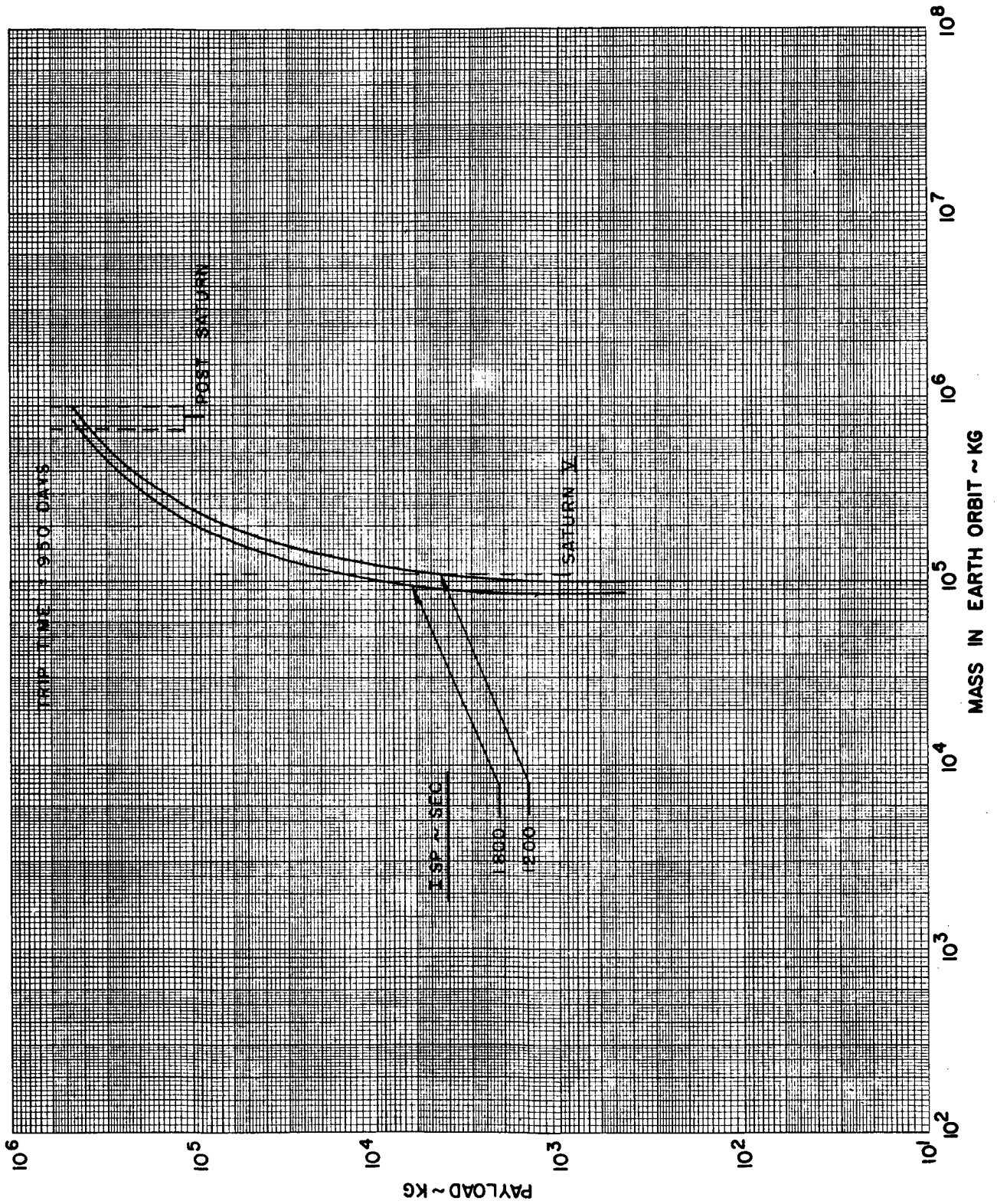


FIG. 74

E-910352-9

JUPITER FLYBY

PULSED NUCLEAR



SATURN FLYBY

CHEMICAL

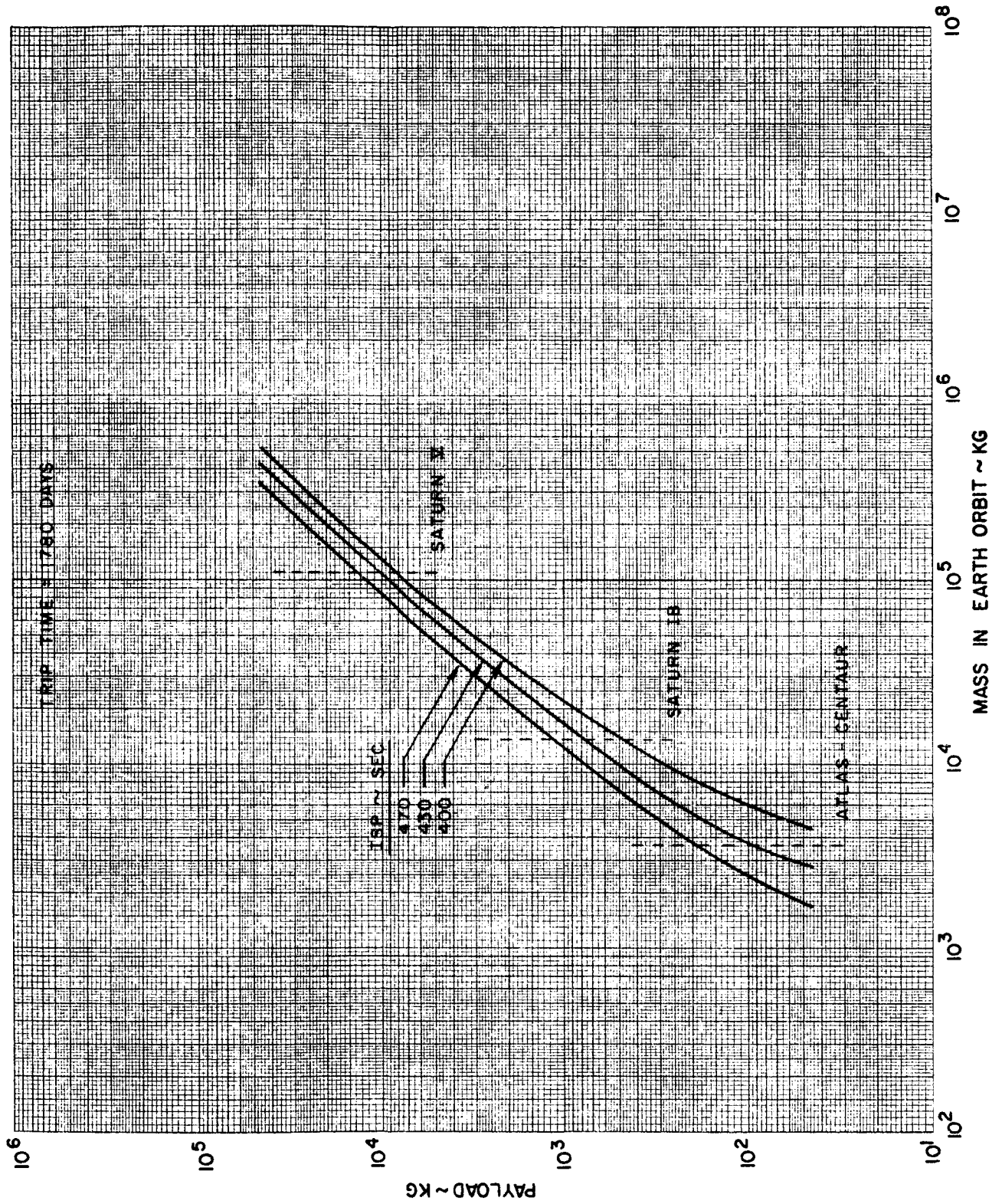
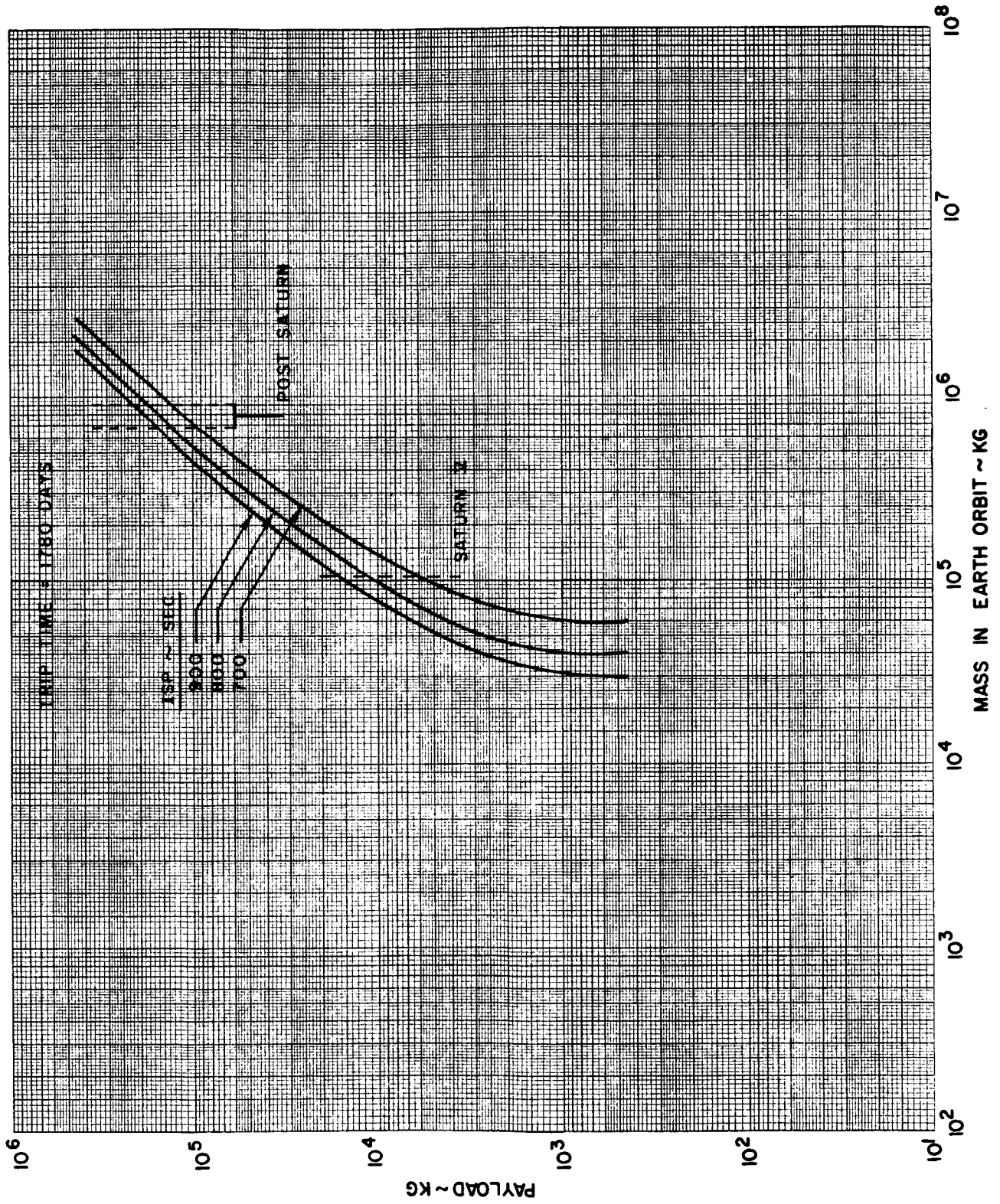


FIG. 76

E-910352-9

SATURN FLYBY

SOLID - CORE NUCLEAR



SATURN FLYBY

LIGHTWEIGHT ADVANCED NUCLEAR

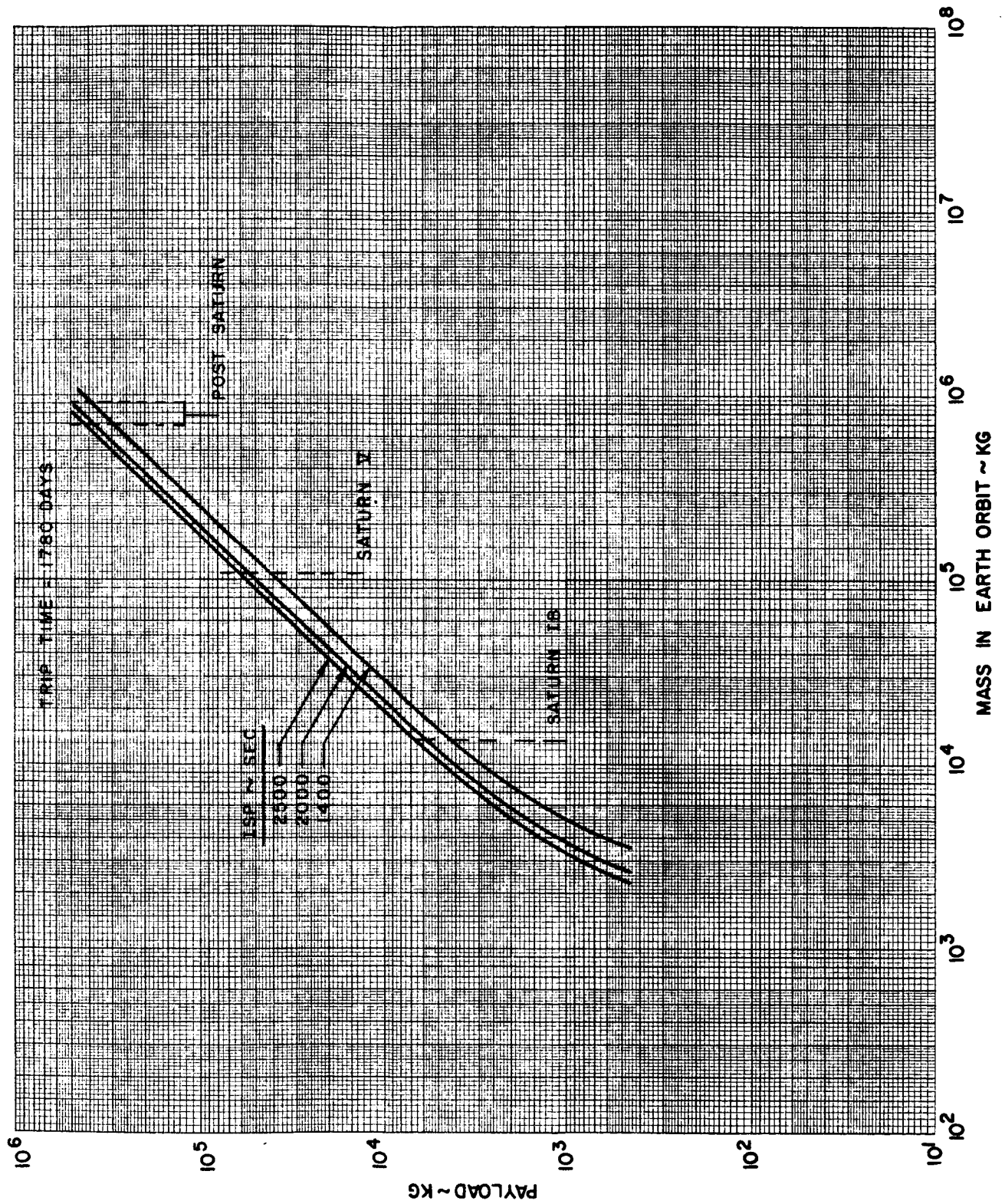
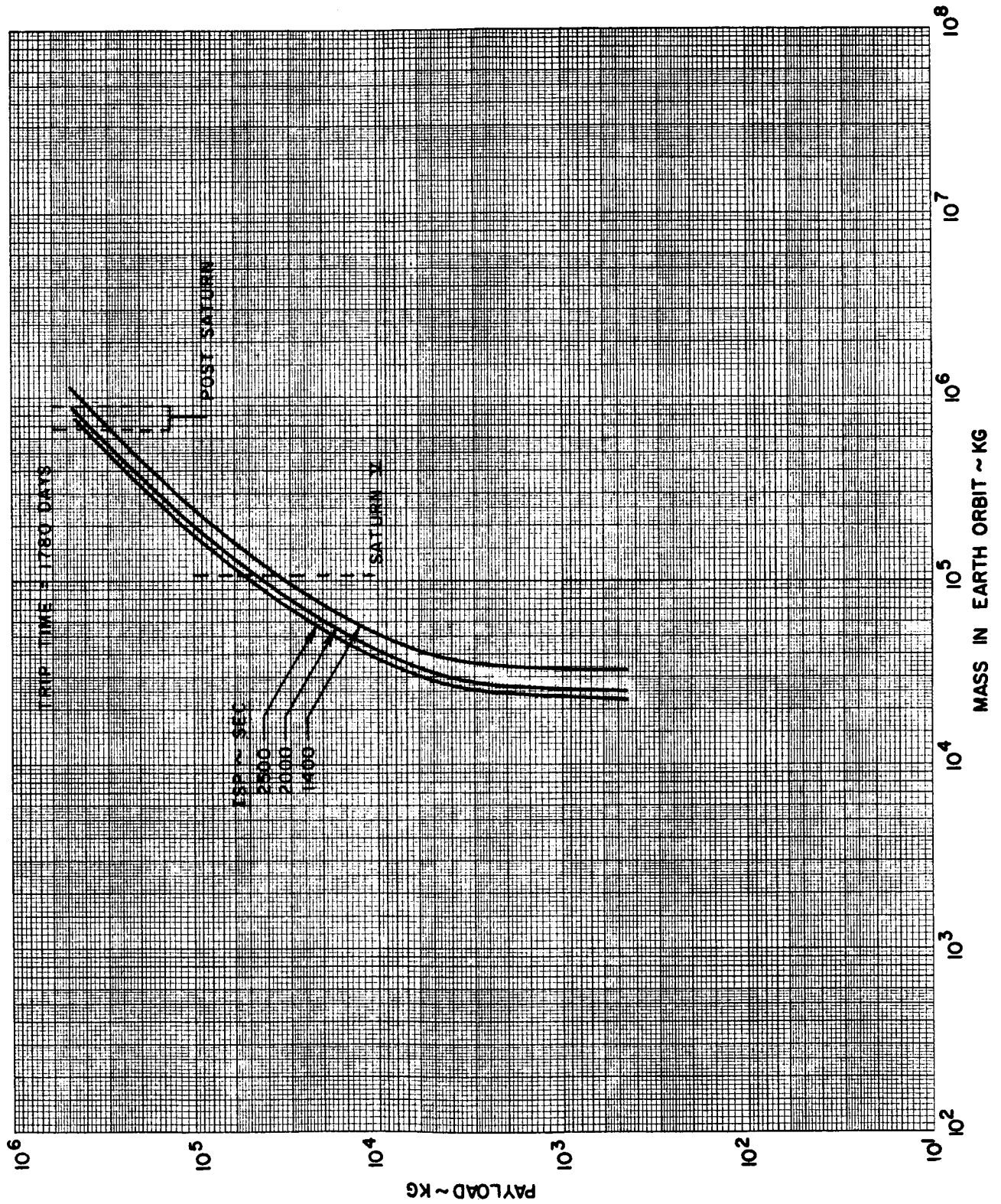


FIG. 78

E-910352-9

SATURN FLYBY

GASEOUS - CORE NUCLEAR



SATURN FLYBY

GASEOUS - CORE NUCLEAR

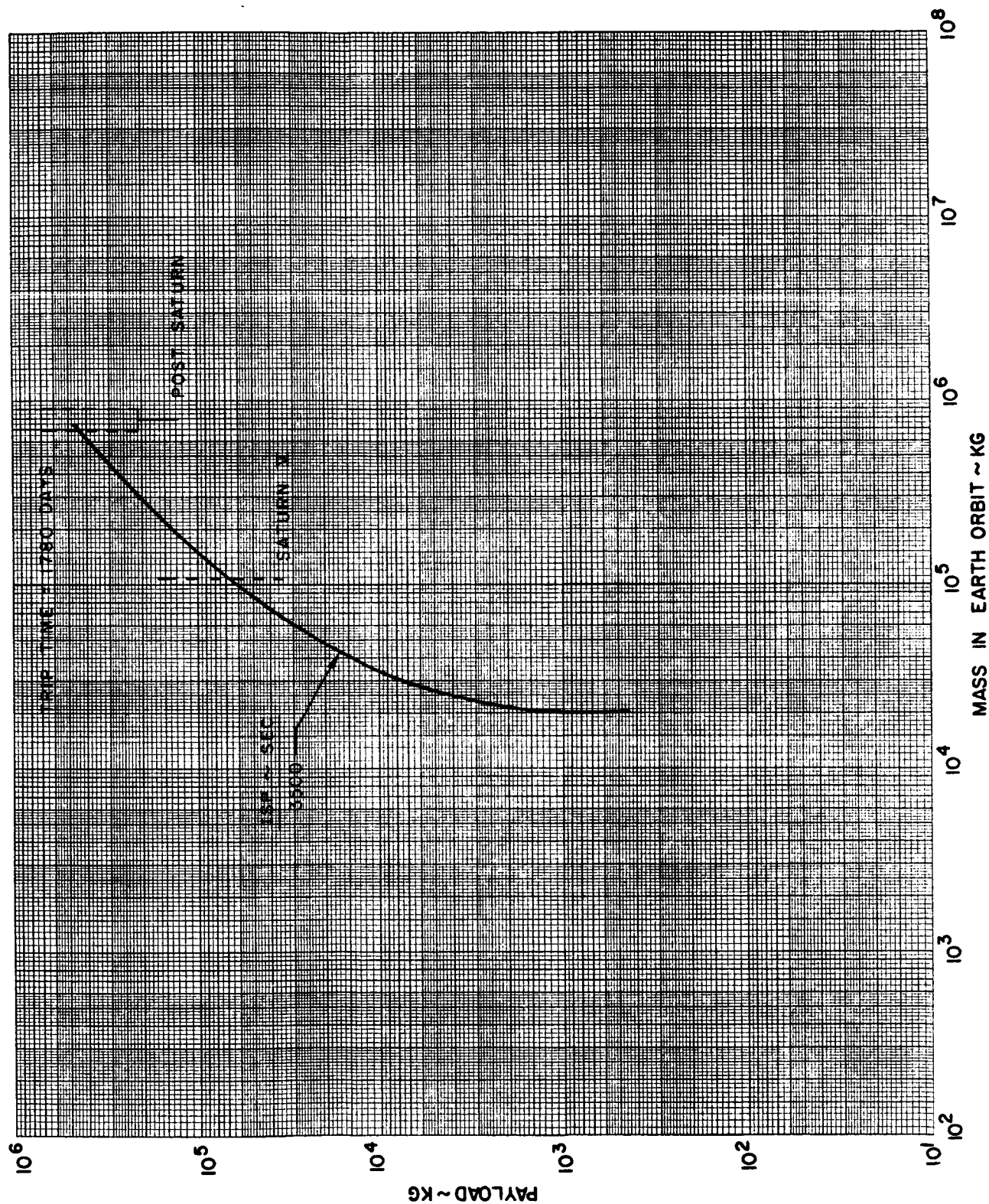
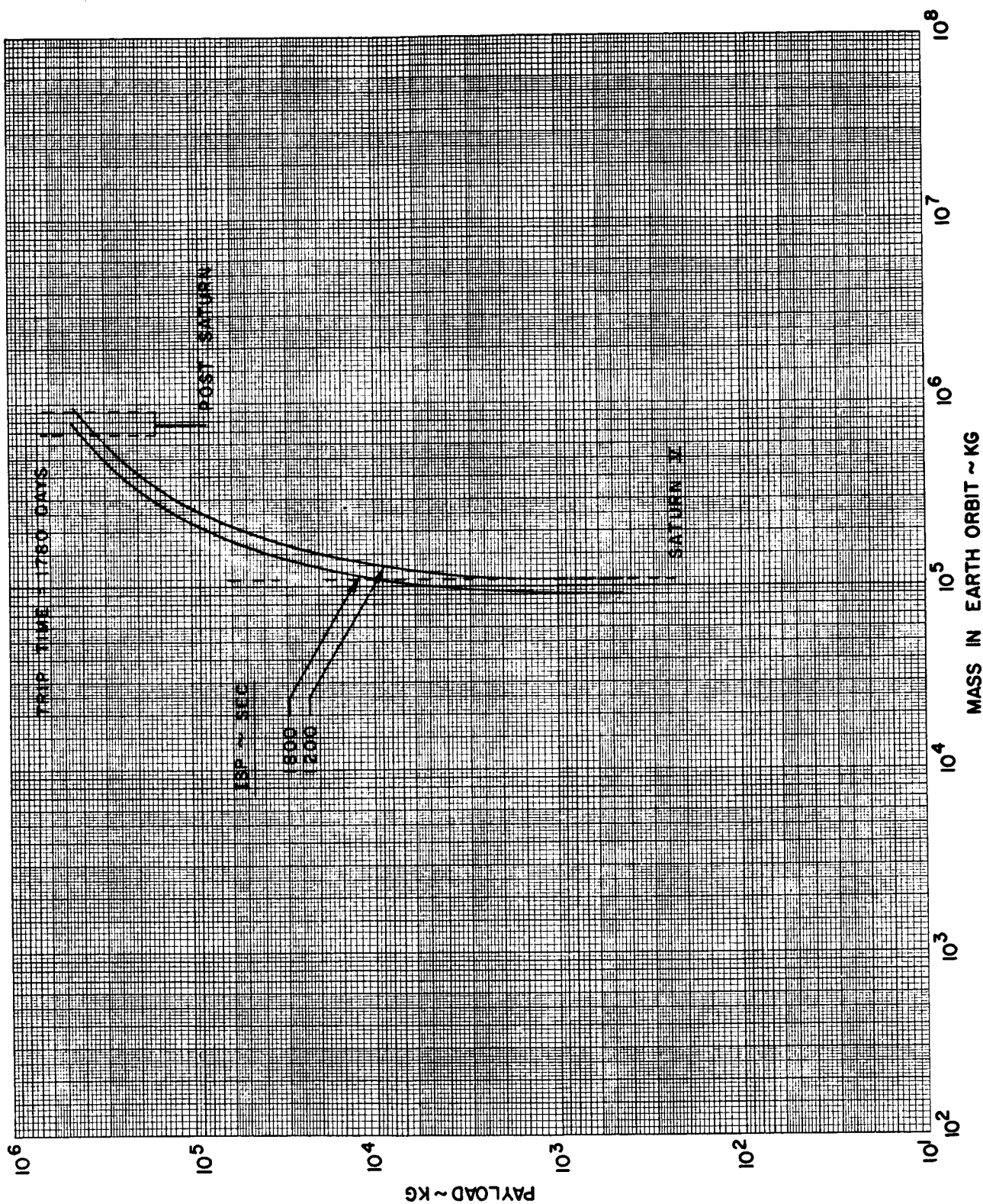


FIG. 80

E-910352-9

SATURN FLYBY

PULSED NUCLEAR



URANUS FLYBY

CHEMICAL

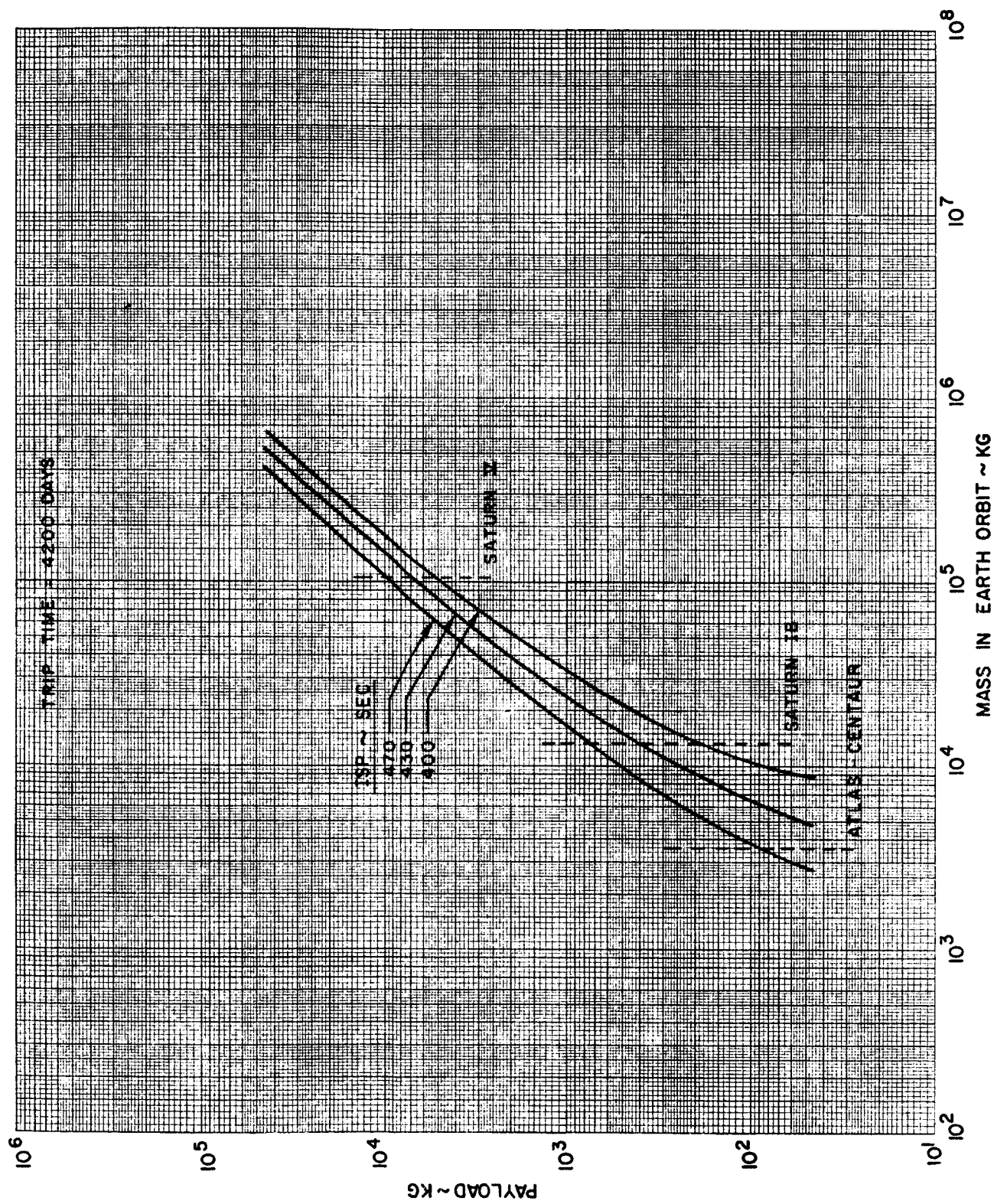
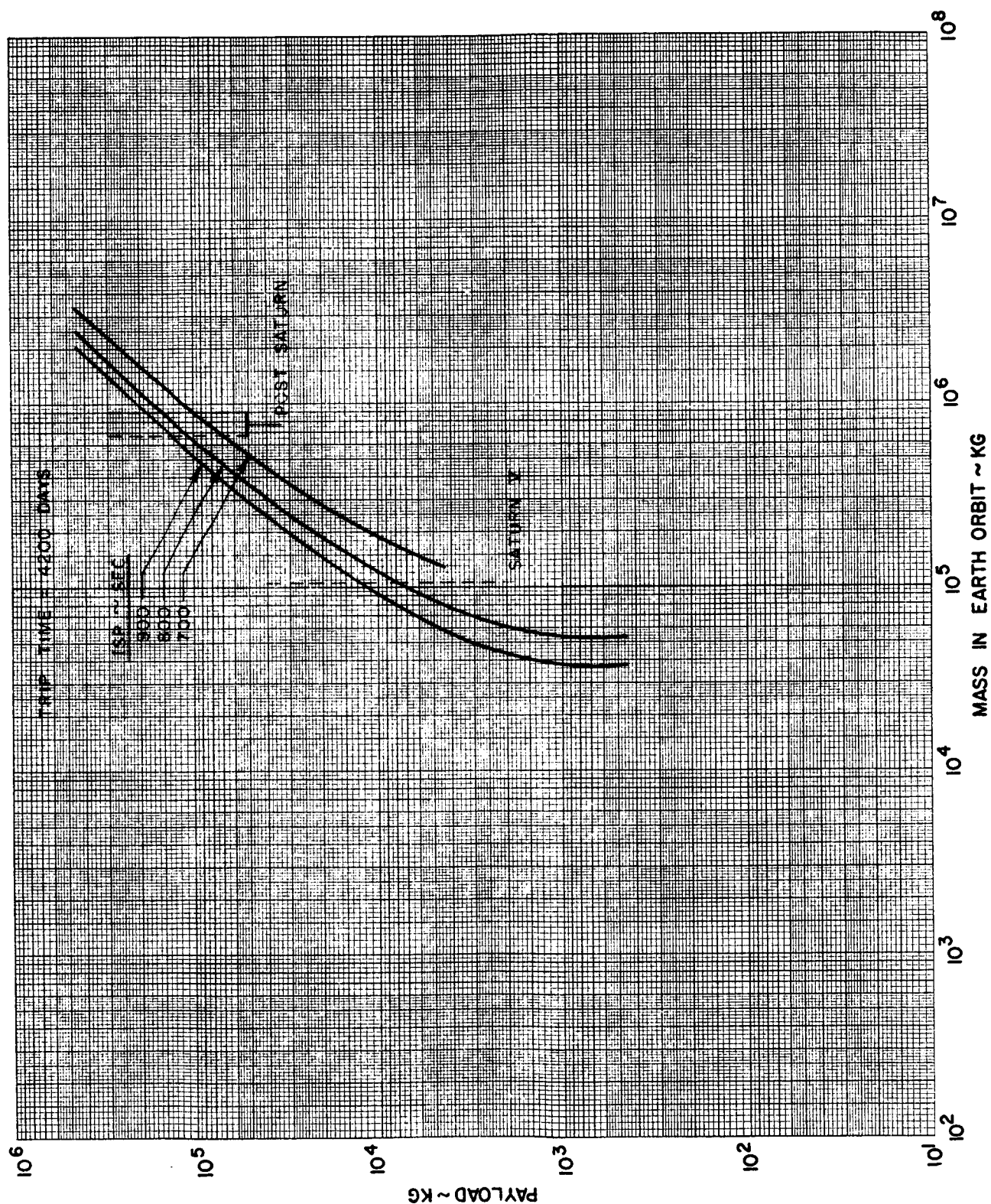


FIG. 82

E-910352-9

URANUS FLYBY

SOLID - CORE NUCLEAR



URANUS FLYBY

LIGHTWEIGHT ADVANCED NUCLEAR

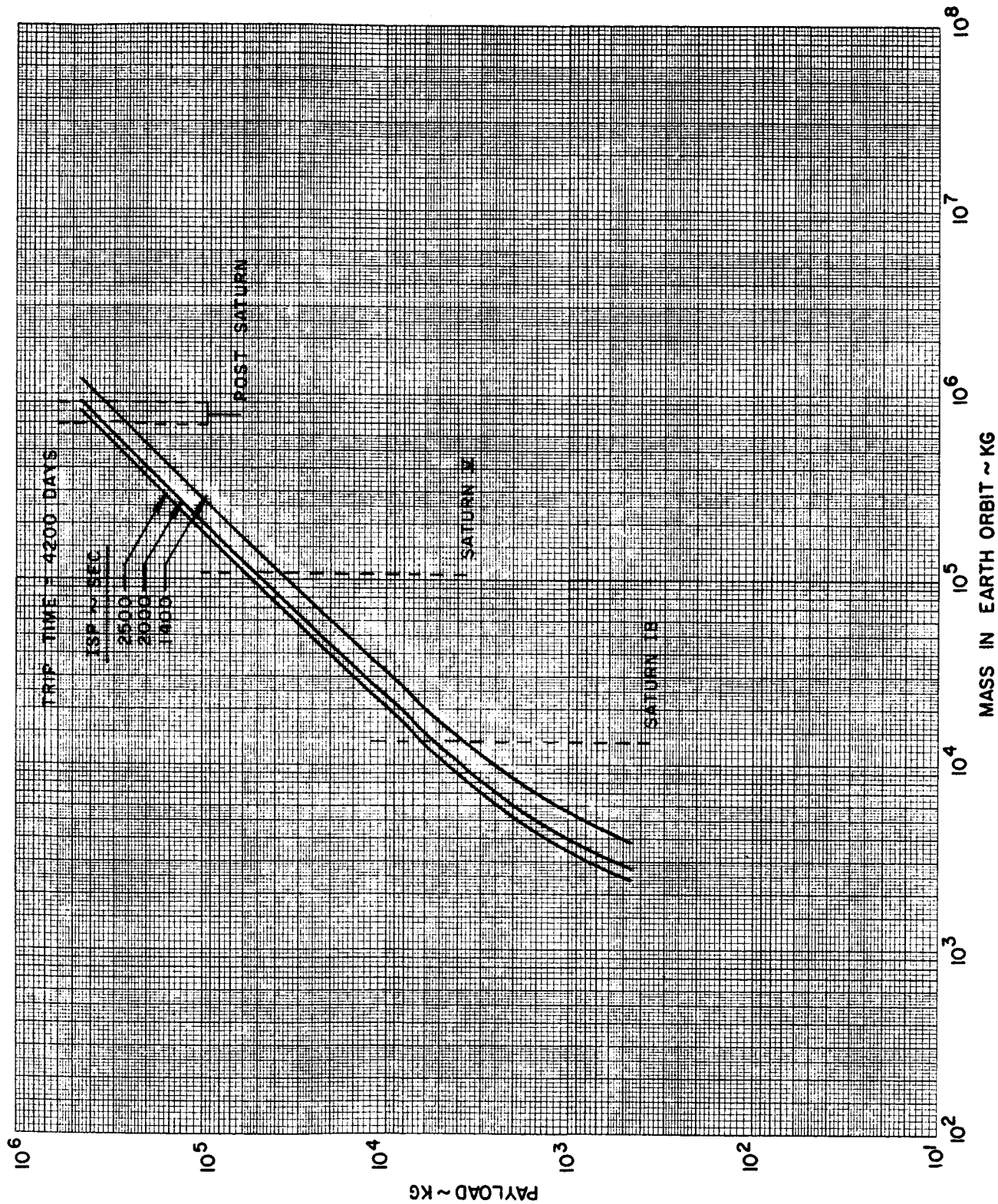
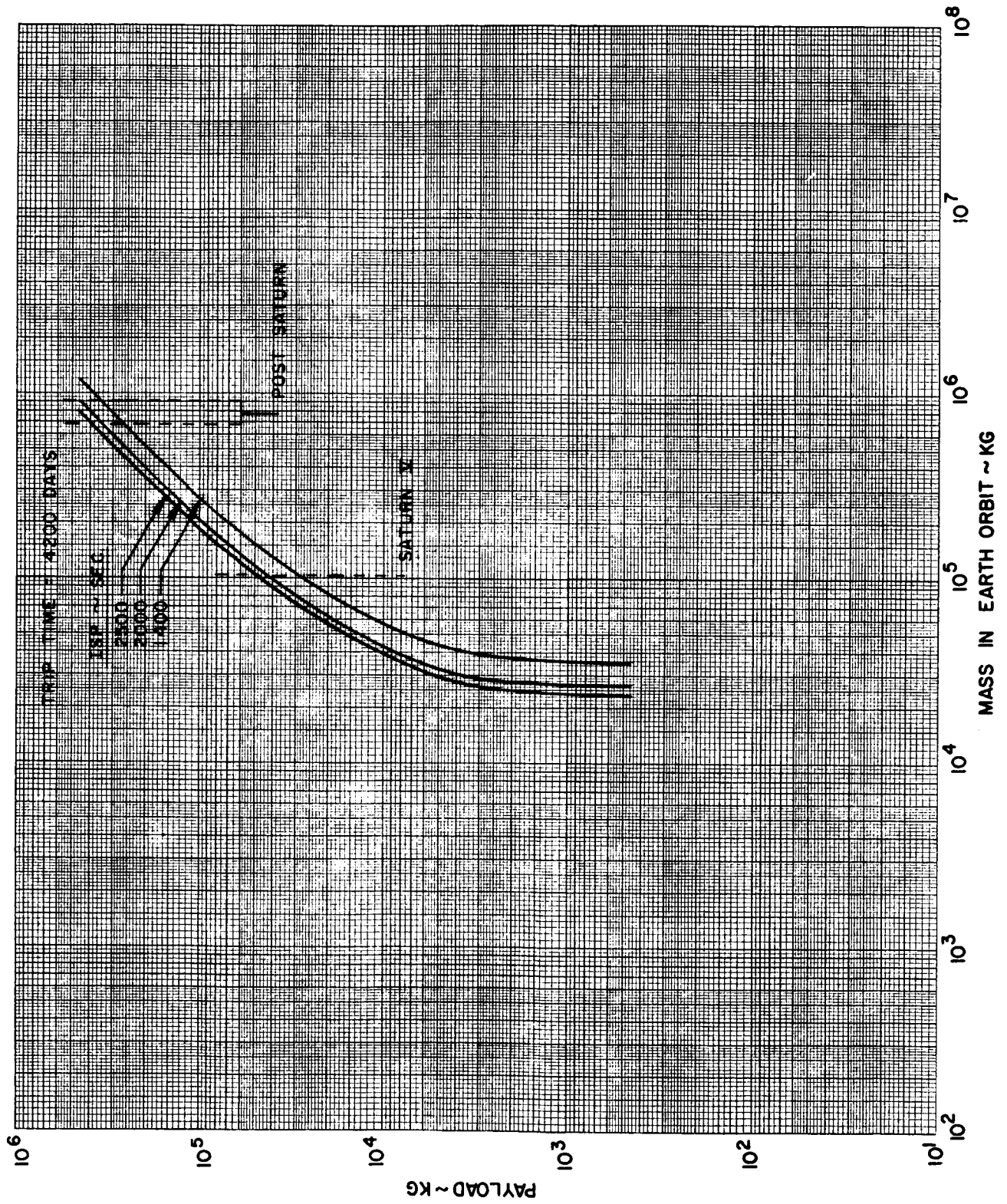


FIG. 84

E-910352-9

URANUS FLYBY

GASEOUS - CORE NUCLEAR



URANUS FLYBY

GASEOUS - CORE NUCLEAR

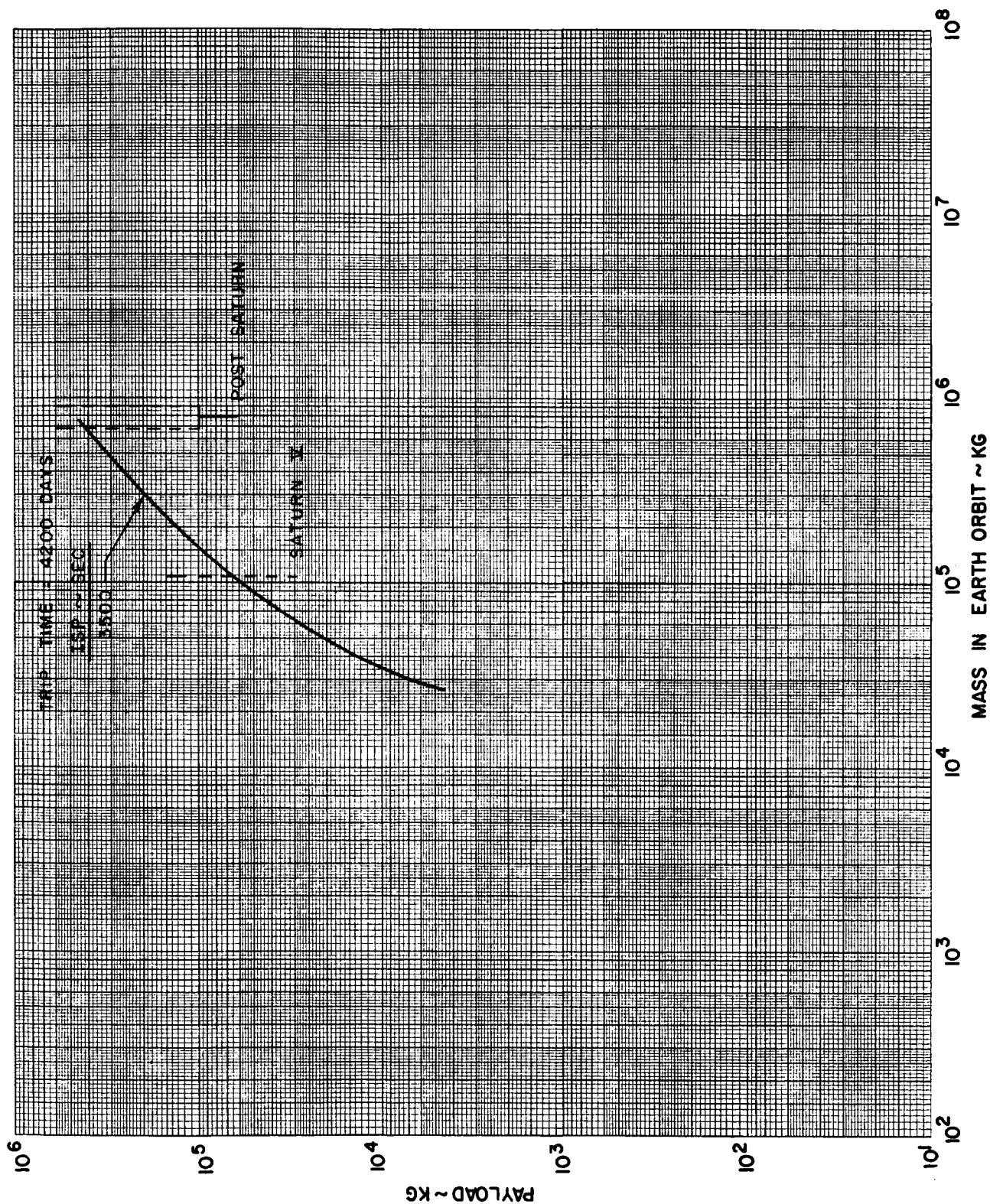
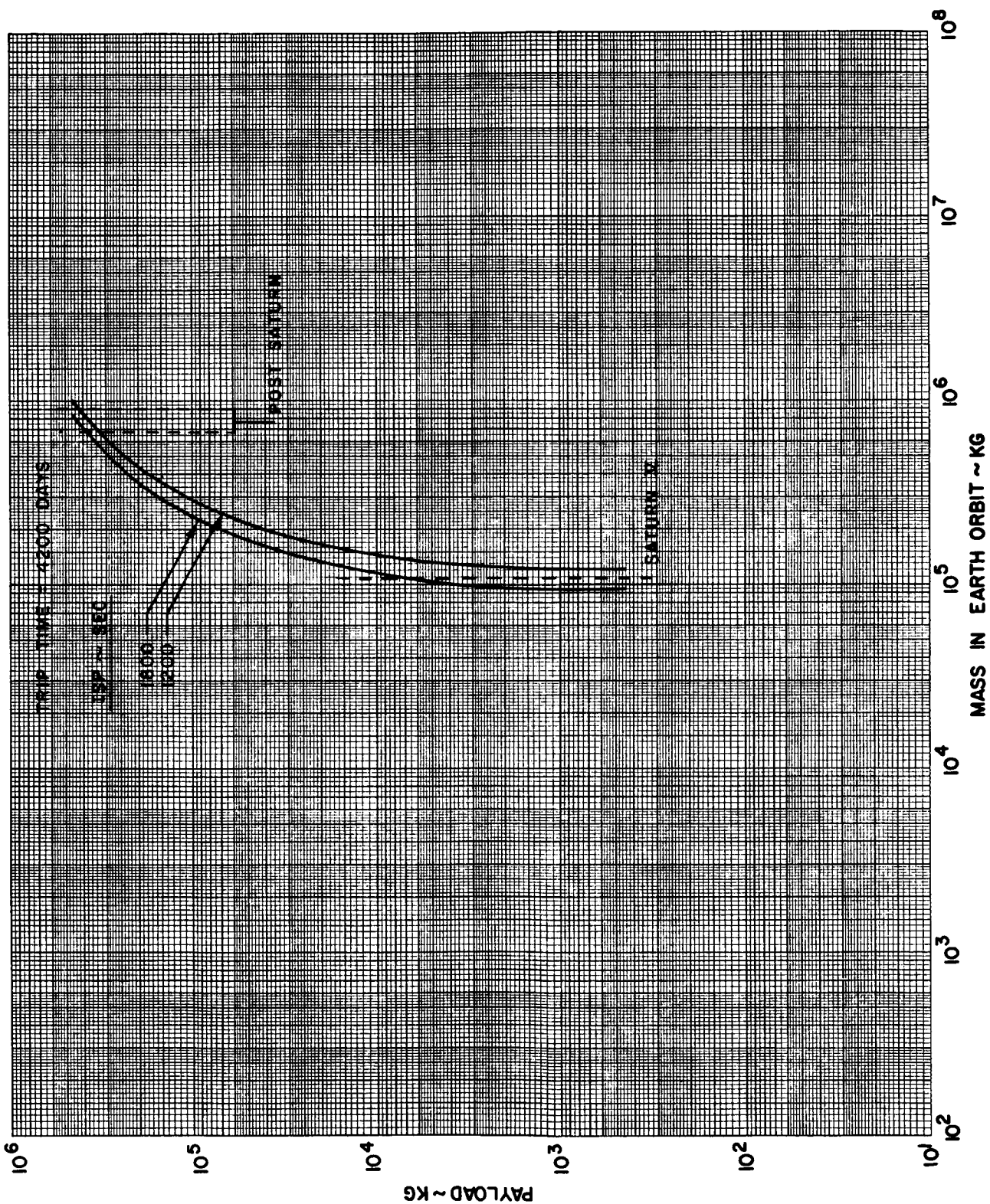


FIG. 86

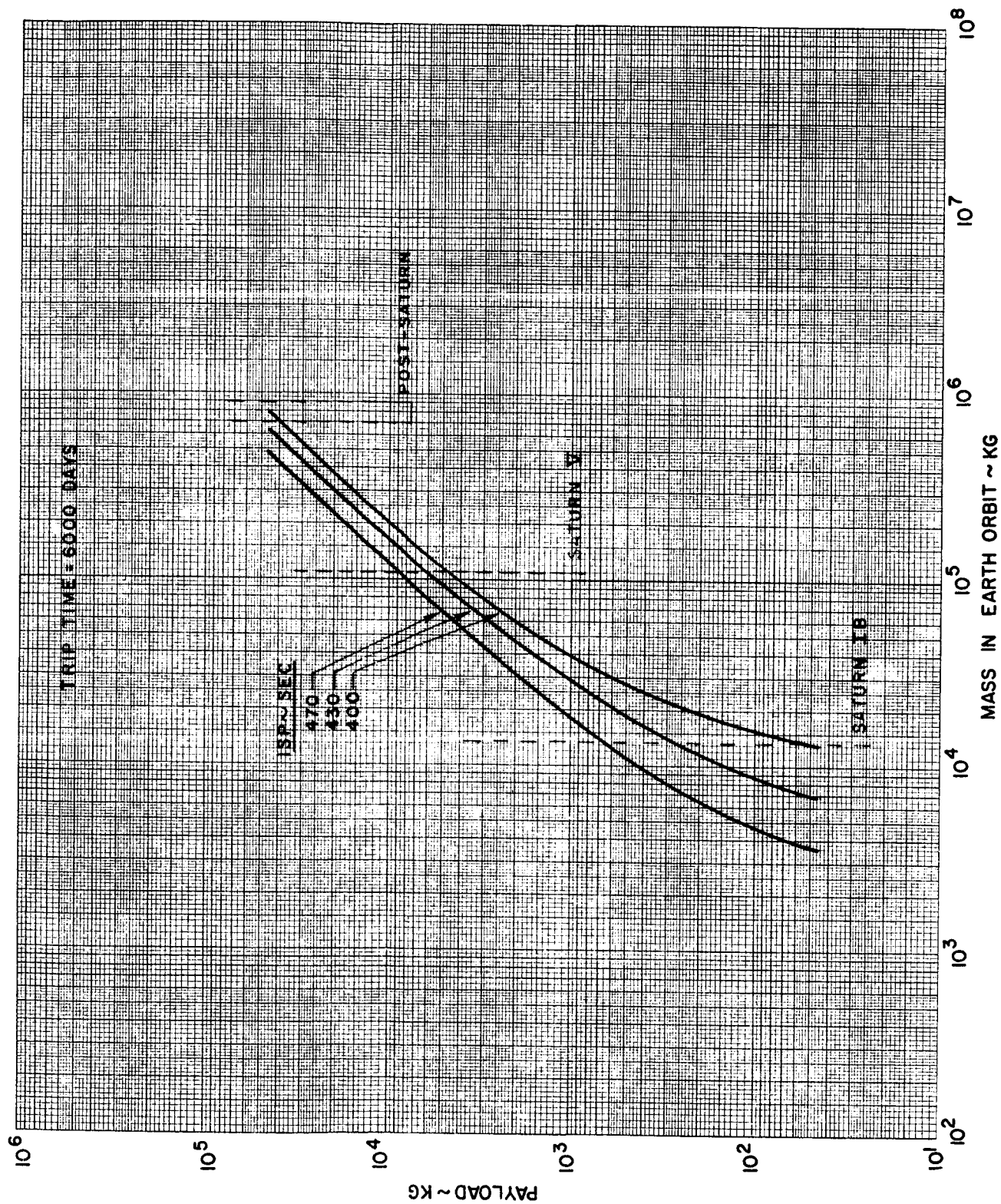
E-910352-9

URANUS FLYBY

PULSED NUCLEAR

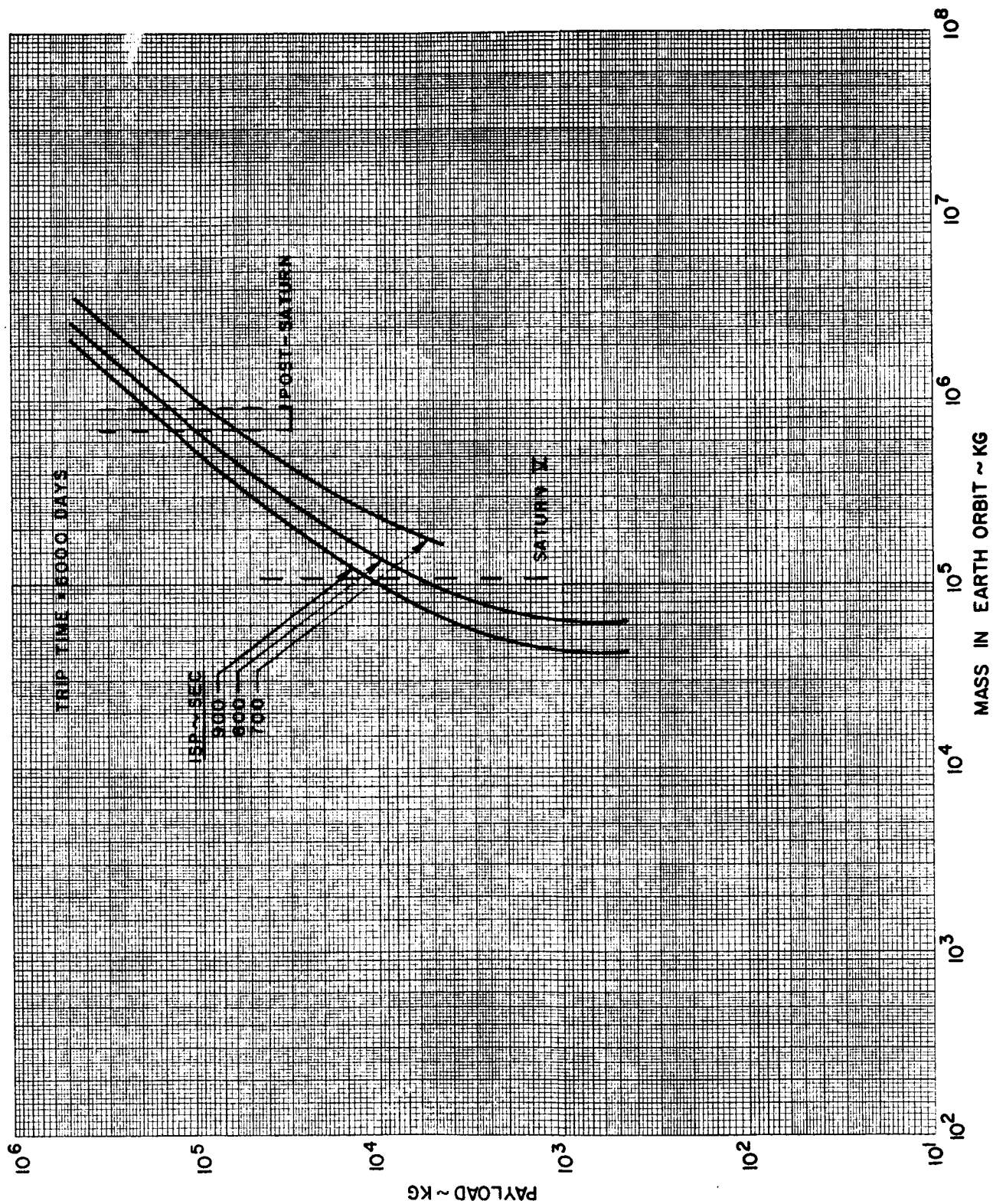


NEPTUNE FLYBY CHEMICAL



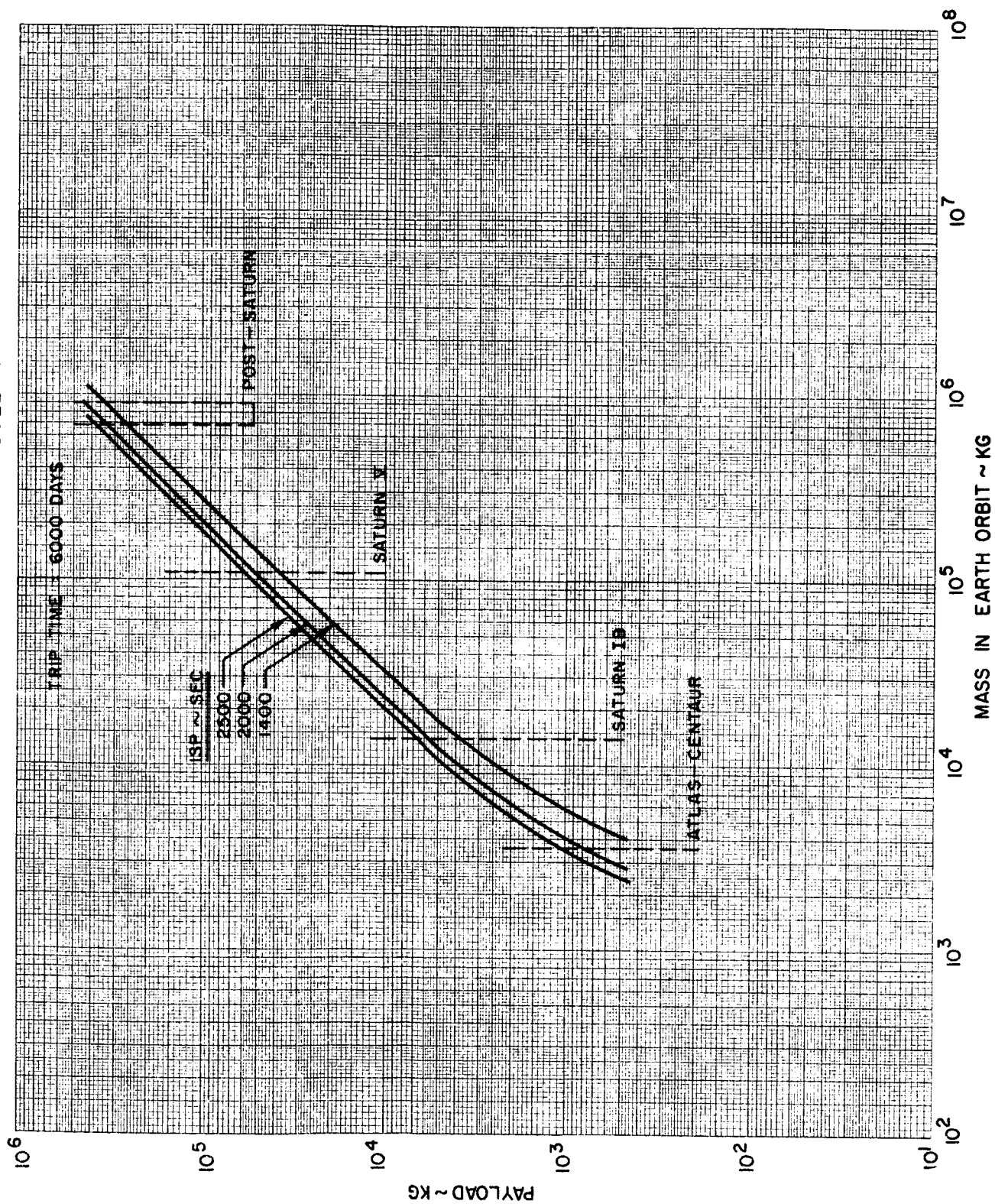
NEPTUNE FLYBY

SOLID-CORE NUCLEAR

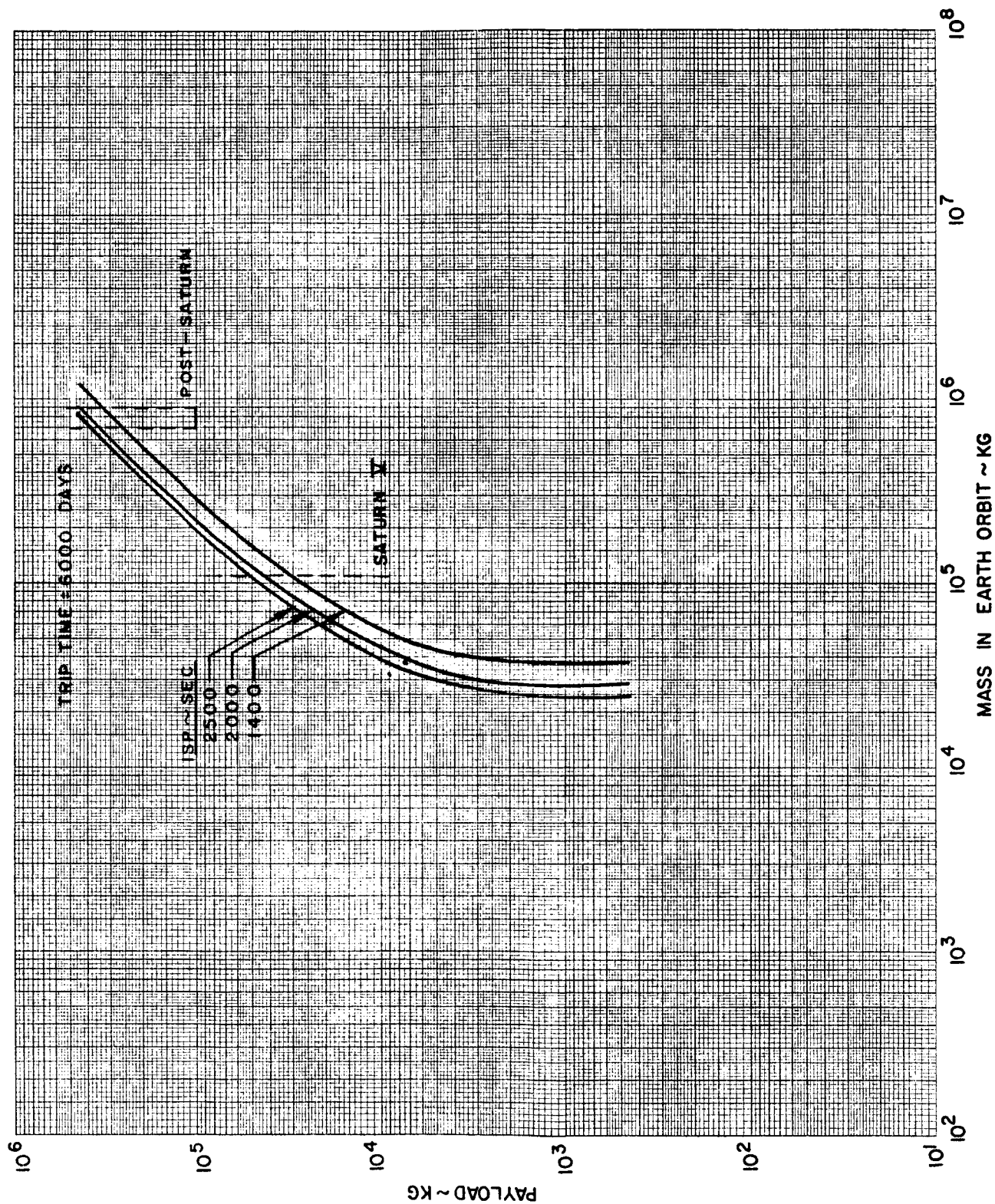


NEPTUNE FLYBY

LIGHTWEIGHT ADVANCED NUCLEAR



NEPTUNE FLYBY GASEOUS-CORE NUCLEAR



NEPTUNE FLYBY
GASEOUS-CORE NUCLEAR

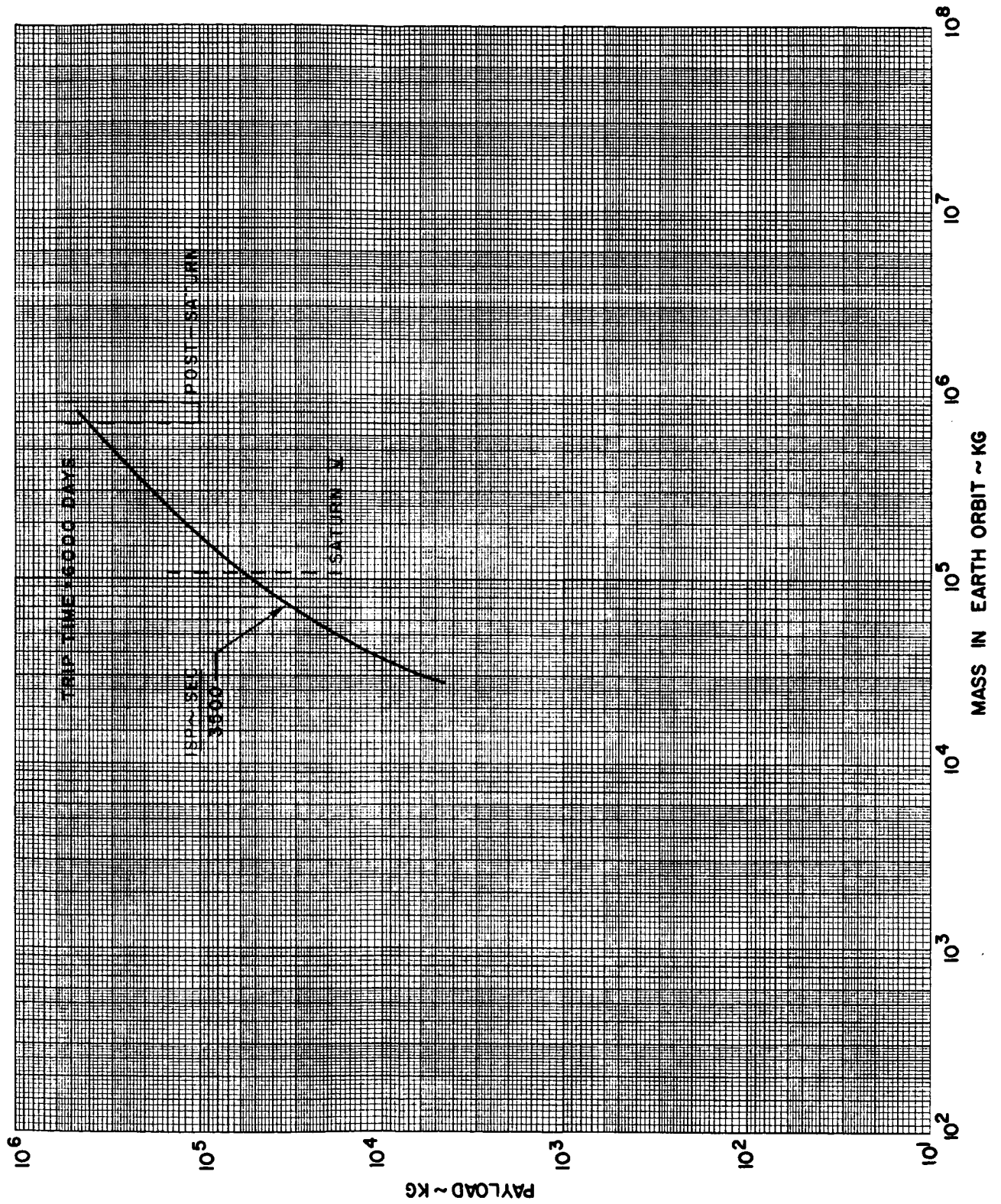
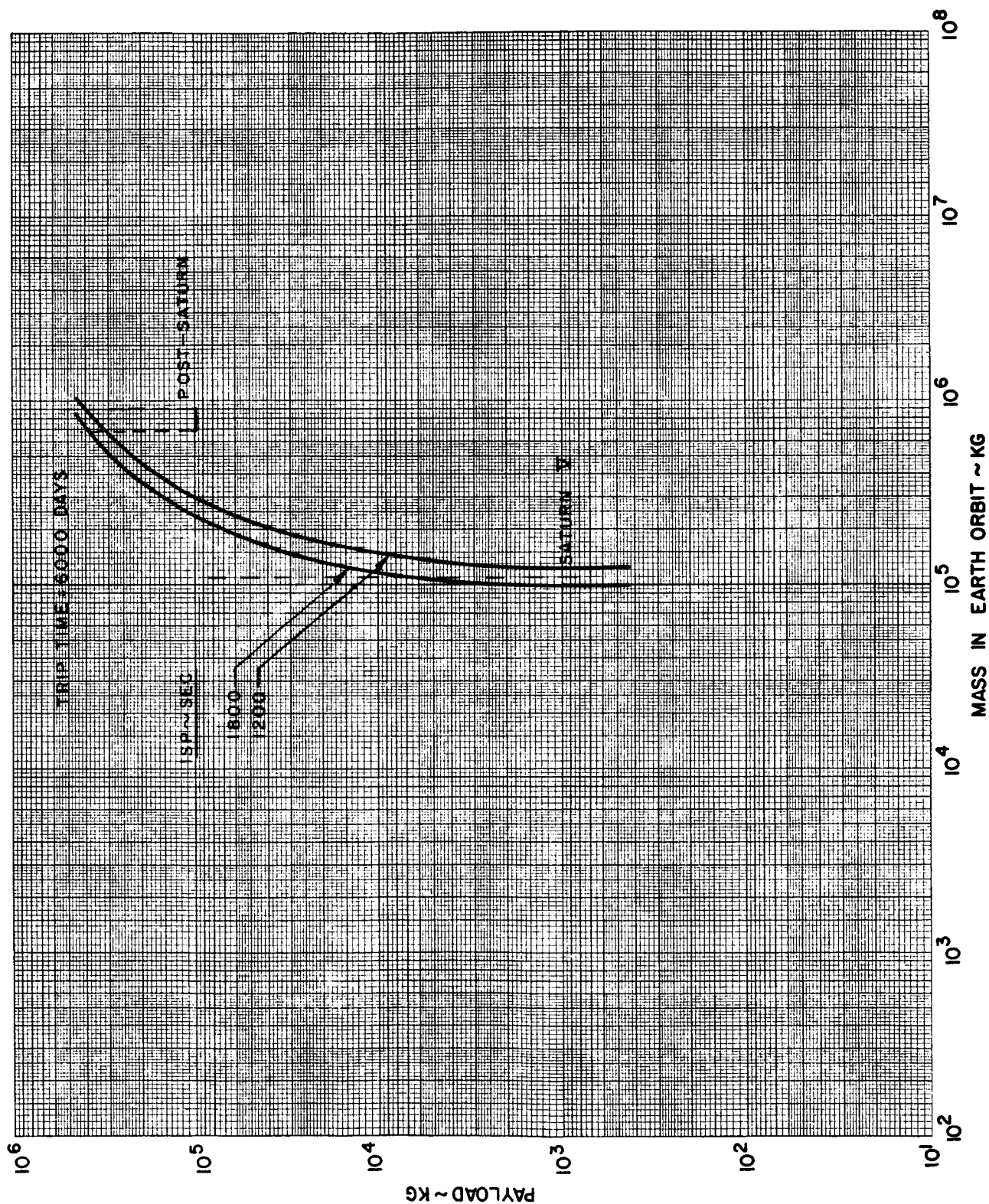


FIG.92

E-910352-9

NEPTUNE FLYBY PULSED NUCLEAR



PLUTO FLYBY

CHEMICAL

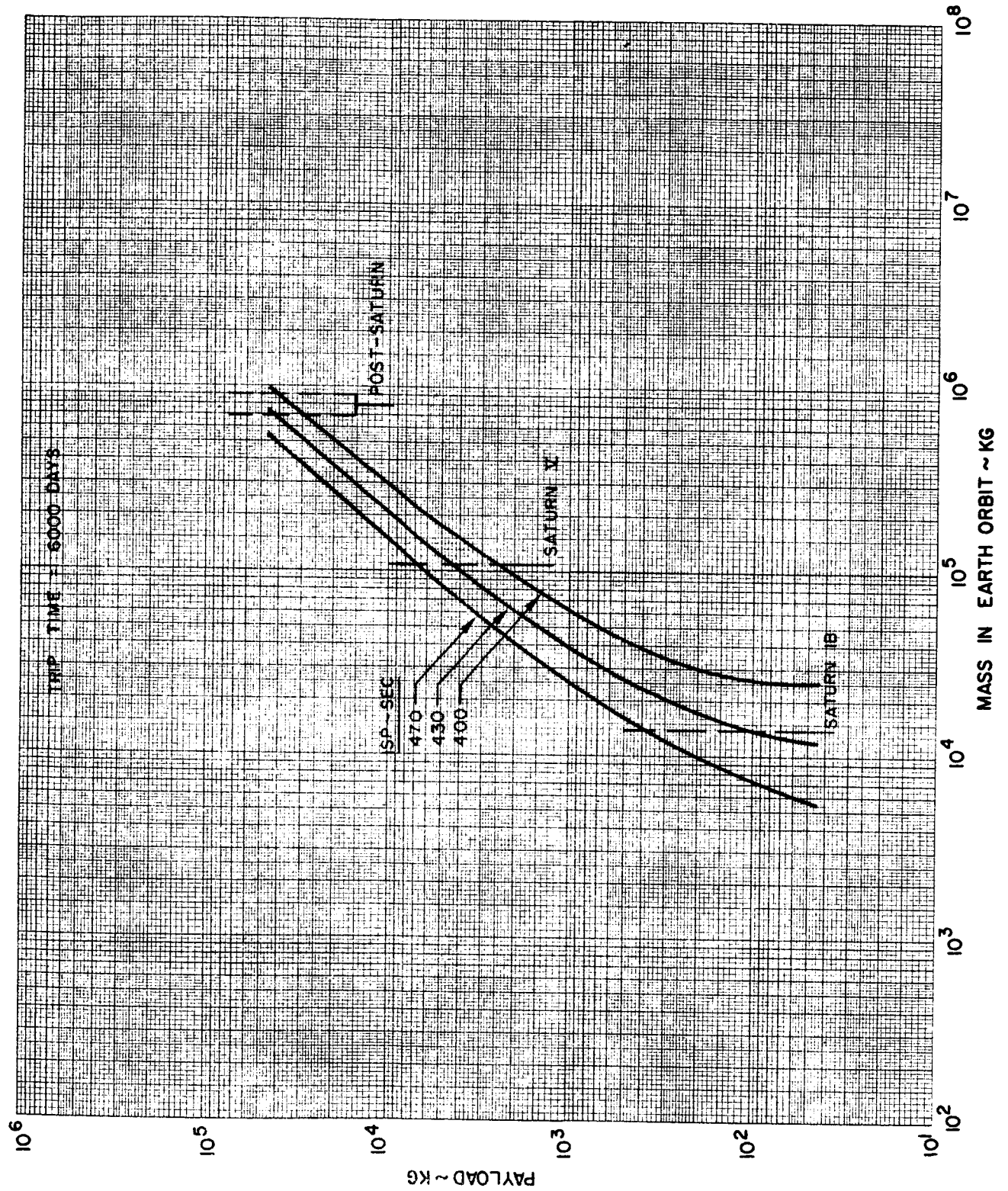
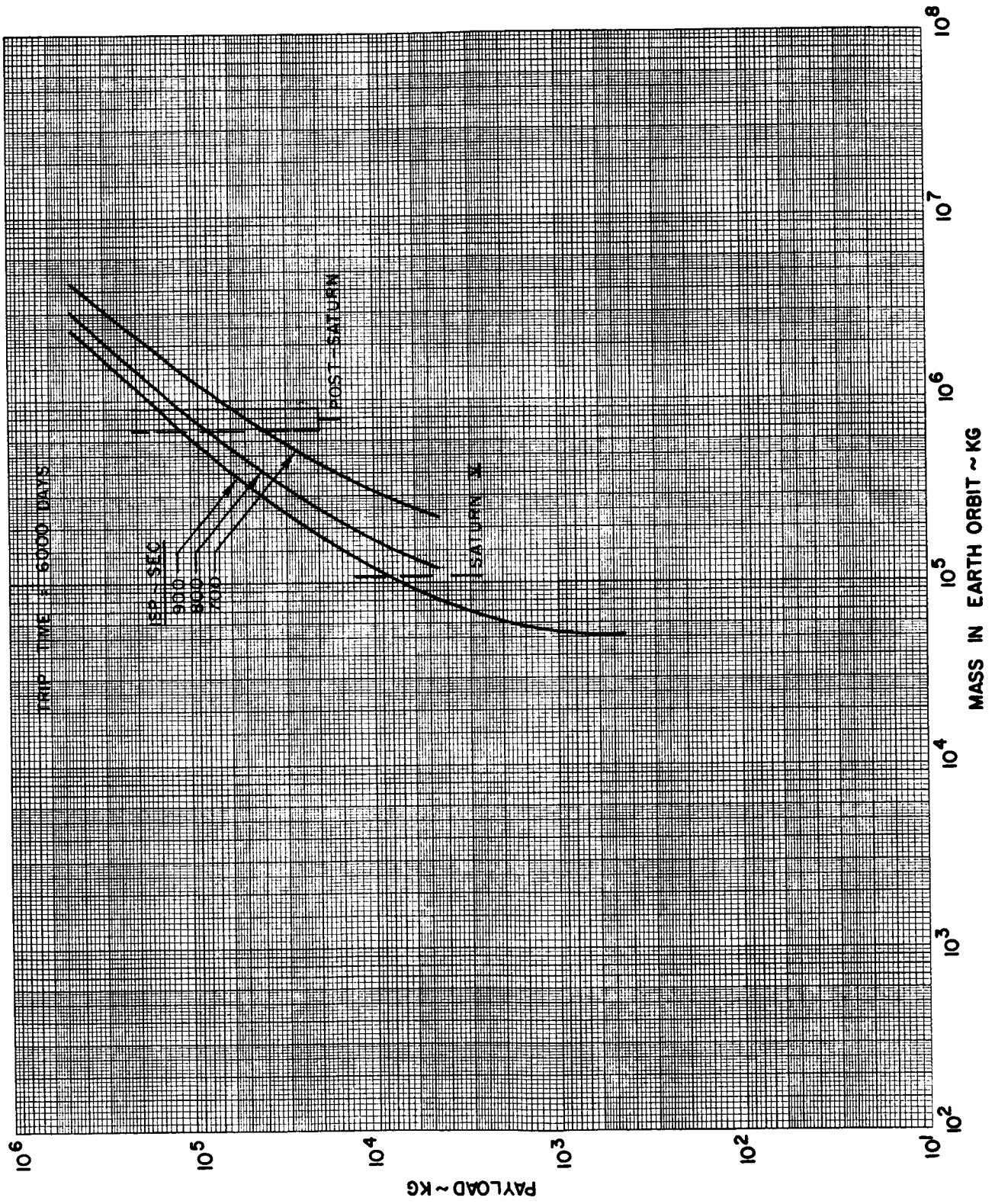


FIG. 94

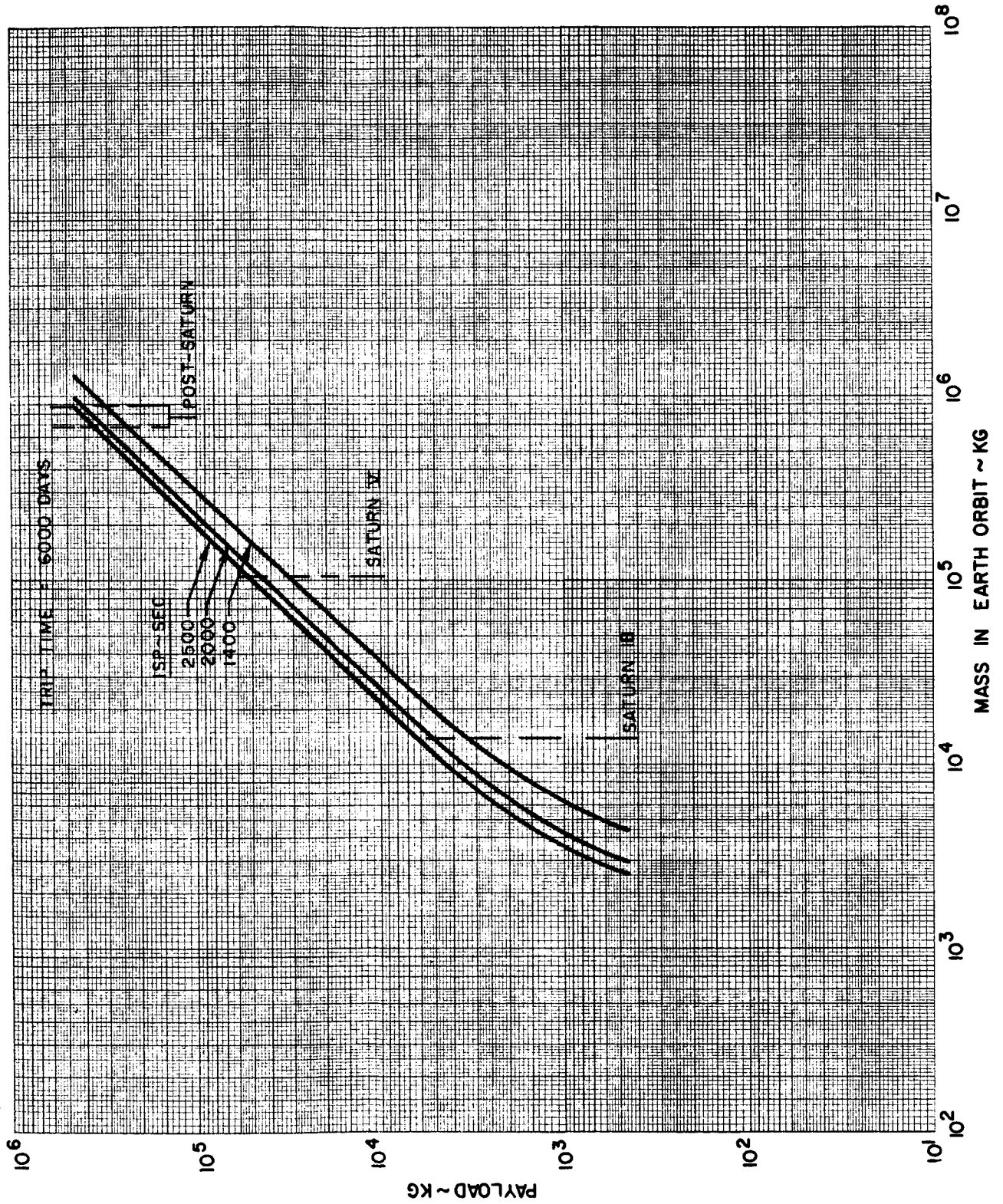
E-910352-9

PLUTO FLYBY
SOLID-CORE NUCLEAR

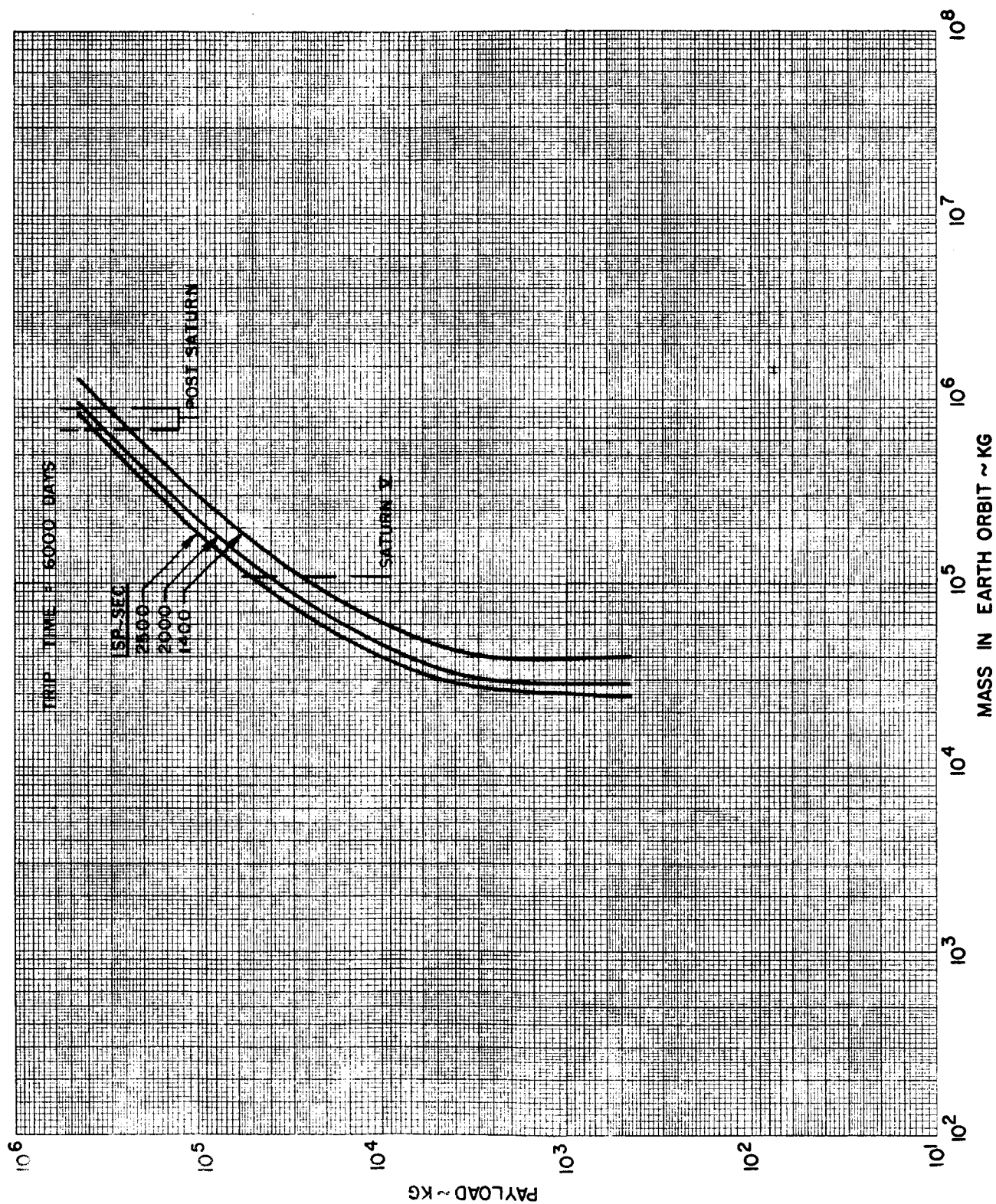


PLUTO FLYBY

LIGHTWEIGHT ADVANCED NUCLEAR

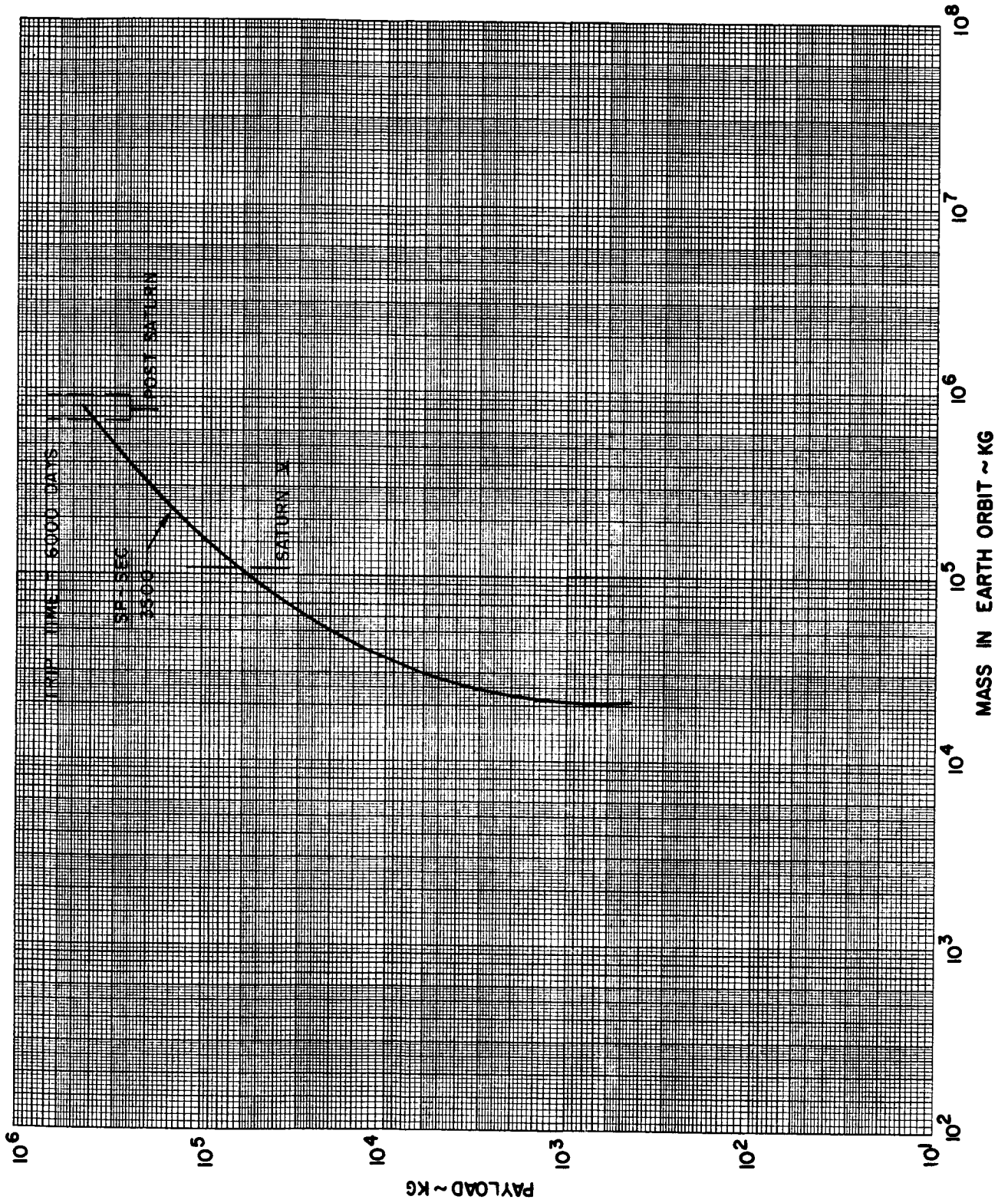


PLUTO FLYBY GASEOUS-CORE NUCLEAR



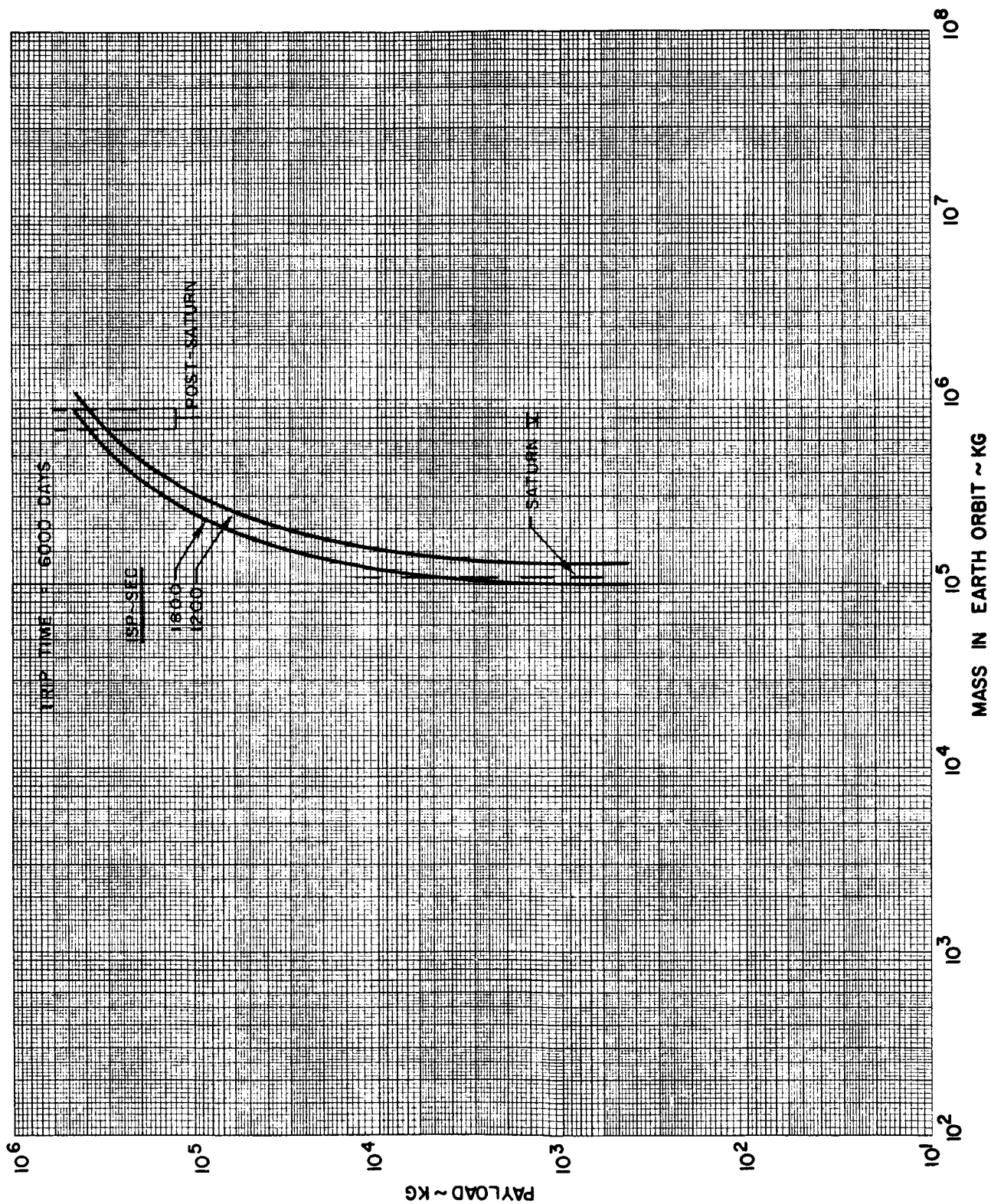
PLUTO FLYBY

GASEOUS - CORE NUCLEAR



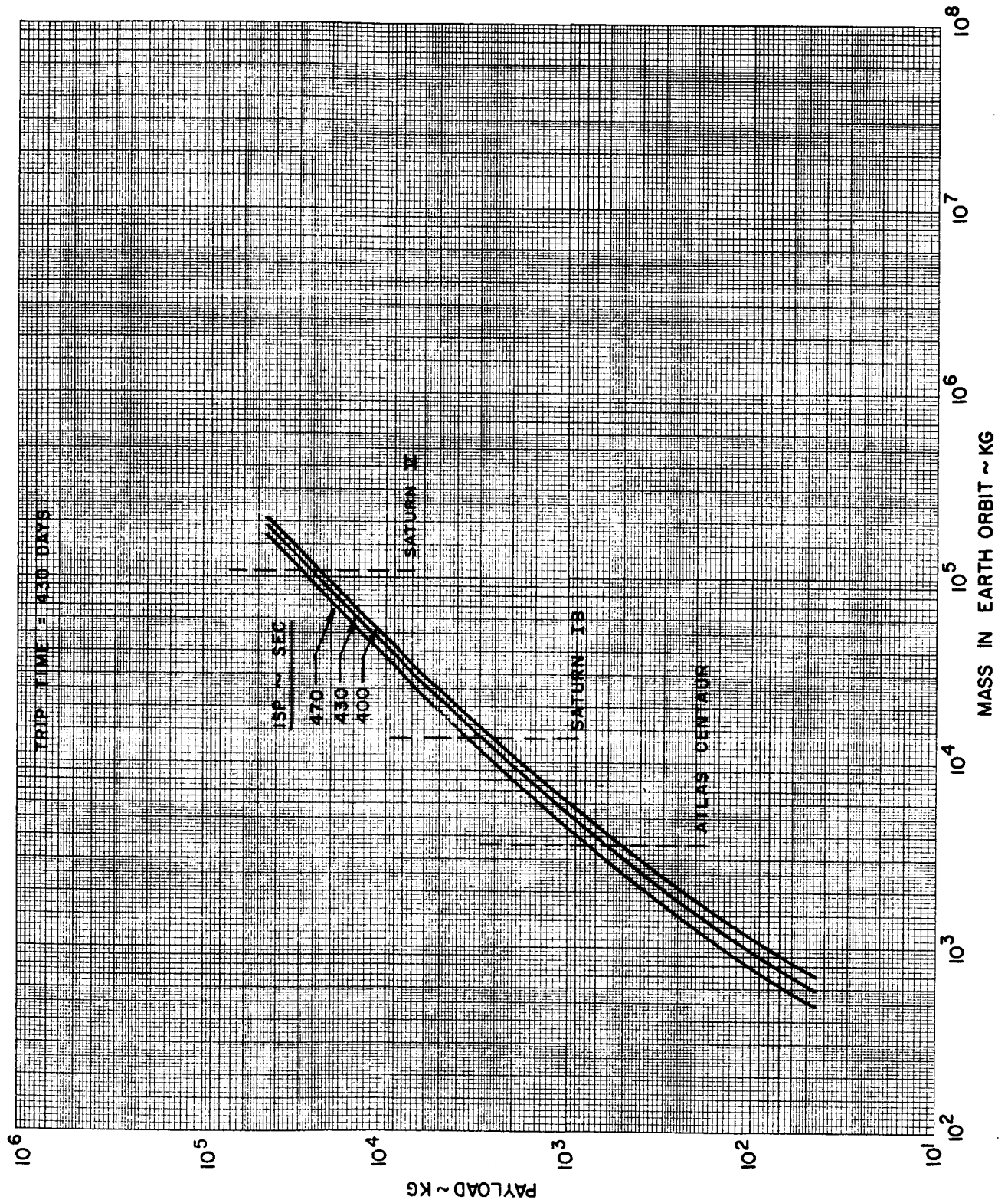
PLUTO FLYBY

PULSED NUCLEAR



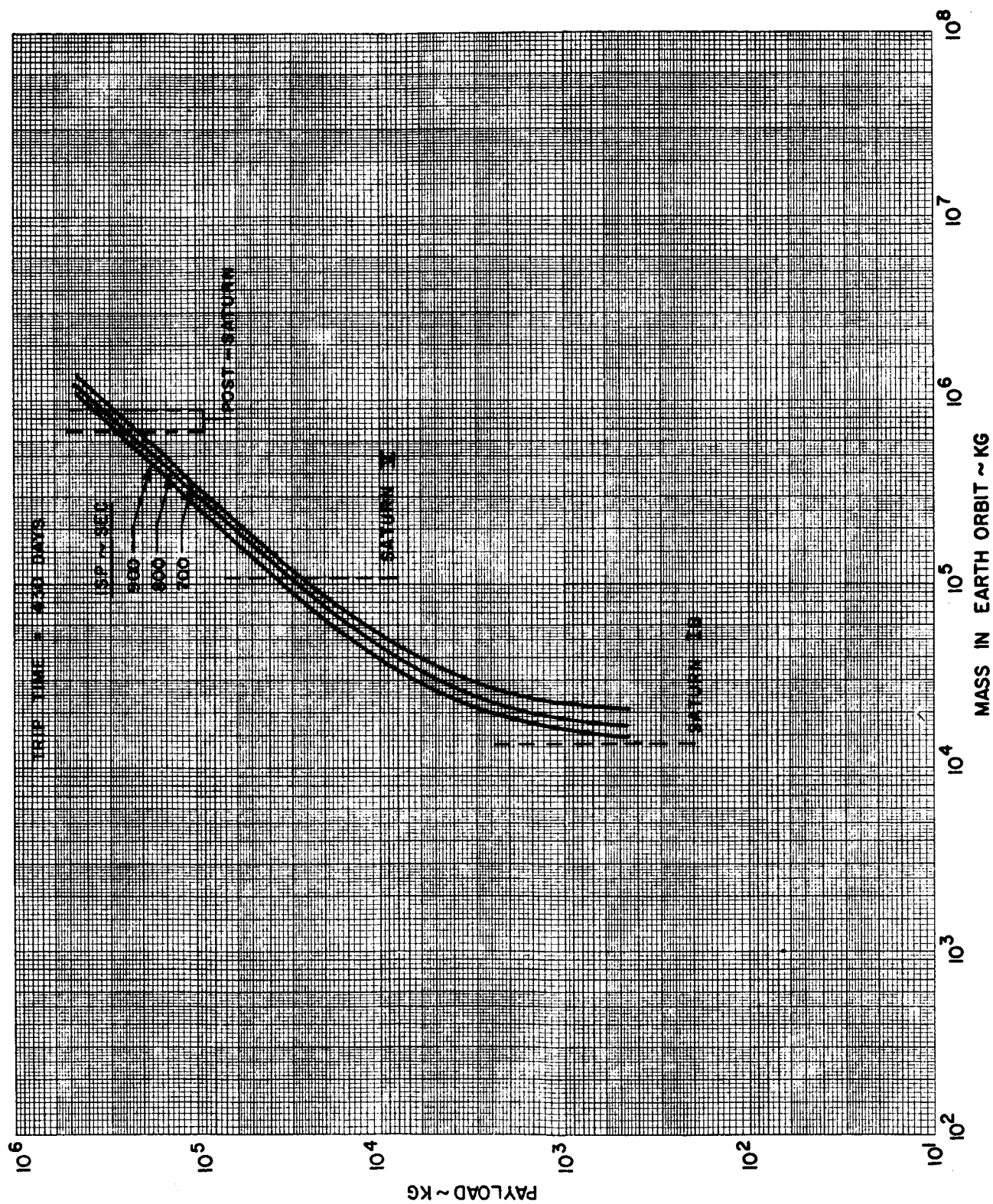
CERES FLYBY

CHEMICAL

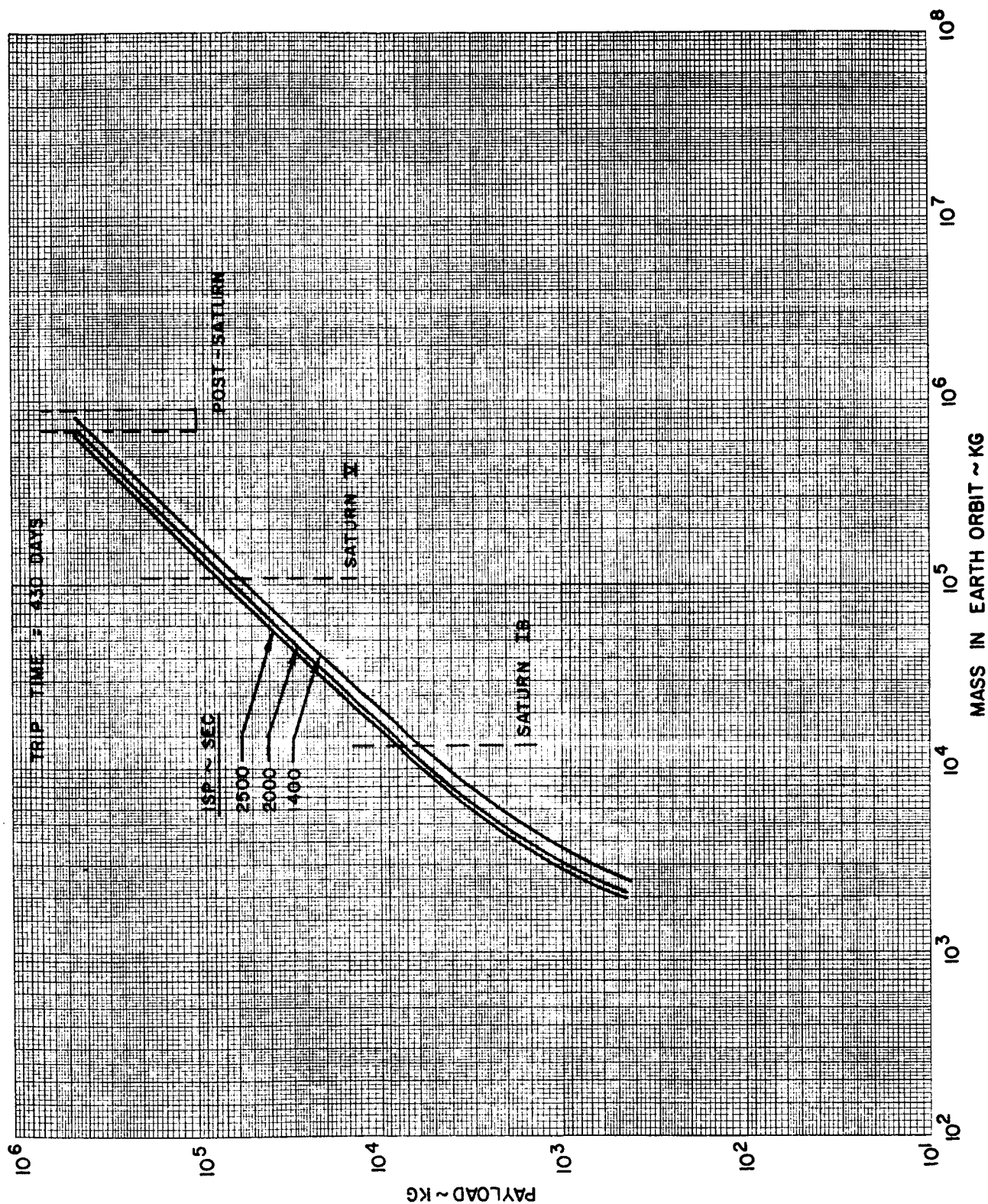


CERES FLYBY

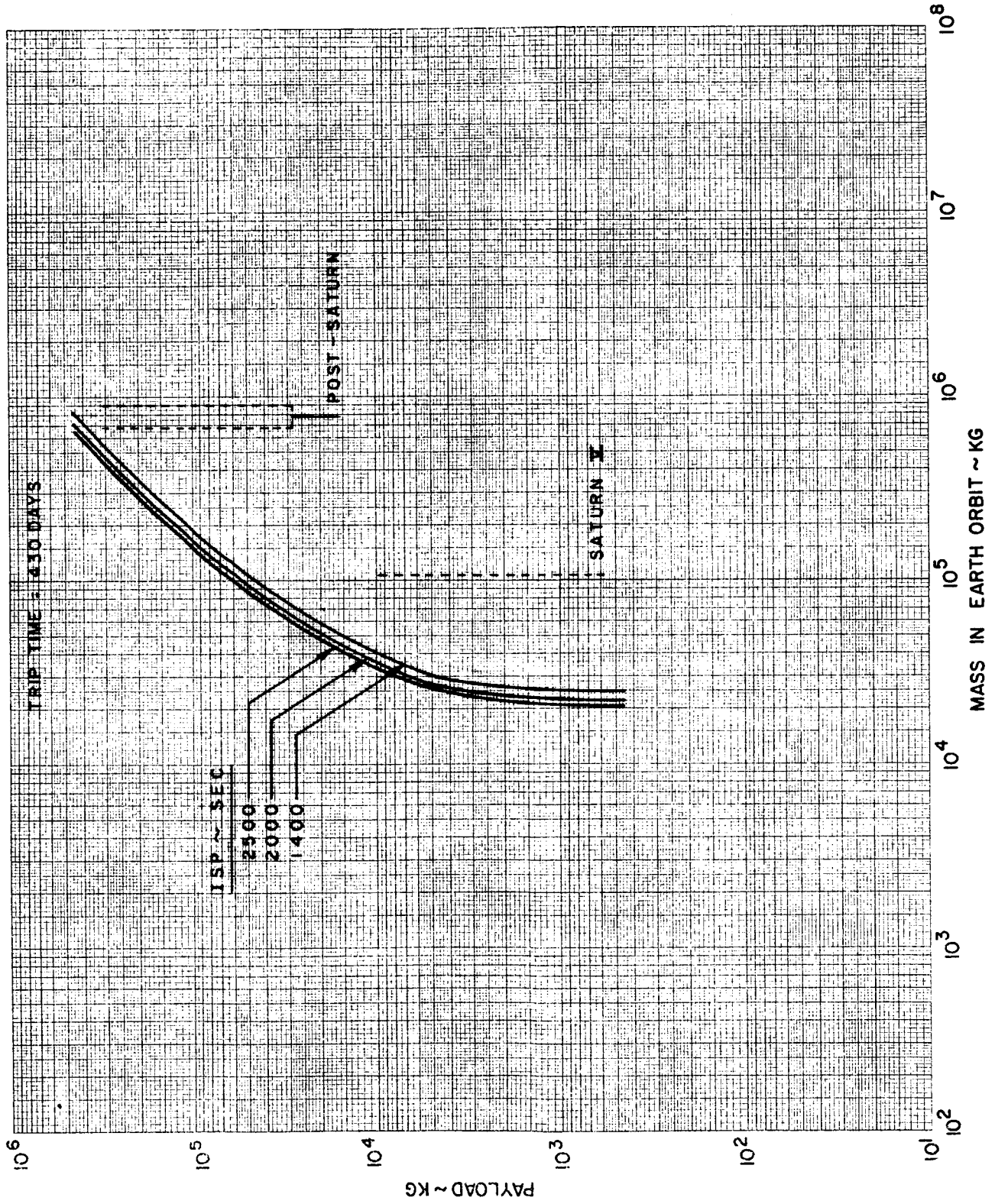
SOLID - CORE NUCLEAR



CERES FLYBY
LIGHTWEIGHT ADVANCED NUCLEAR



CERES FLYBY GASEOUS - CORE NUCLEAR



CERES FLYBY
GASEOUS-CORE NUCLEAR

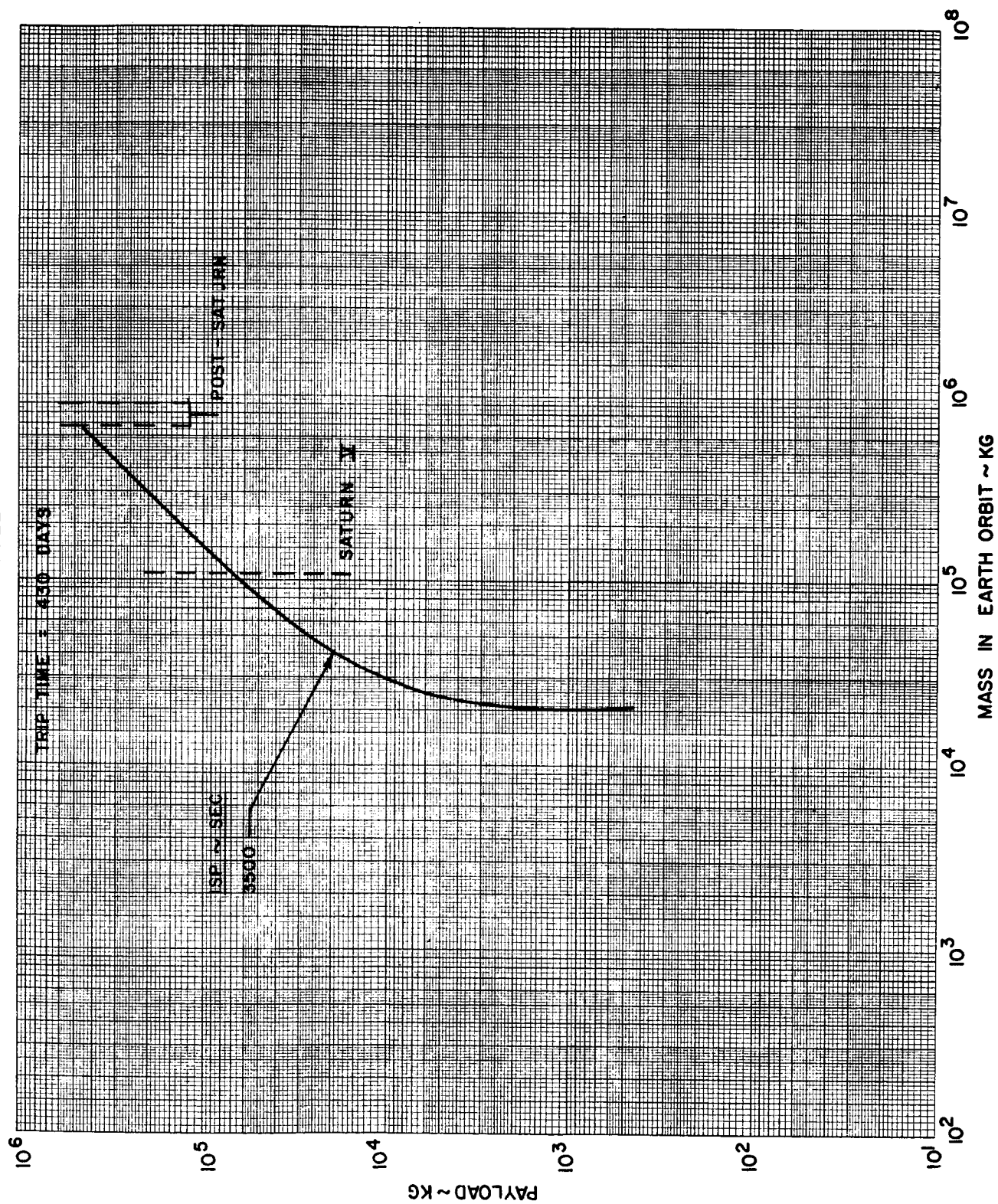
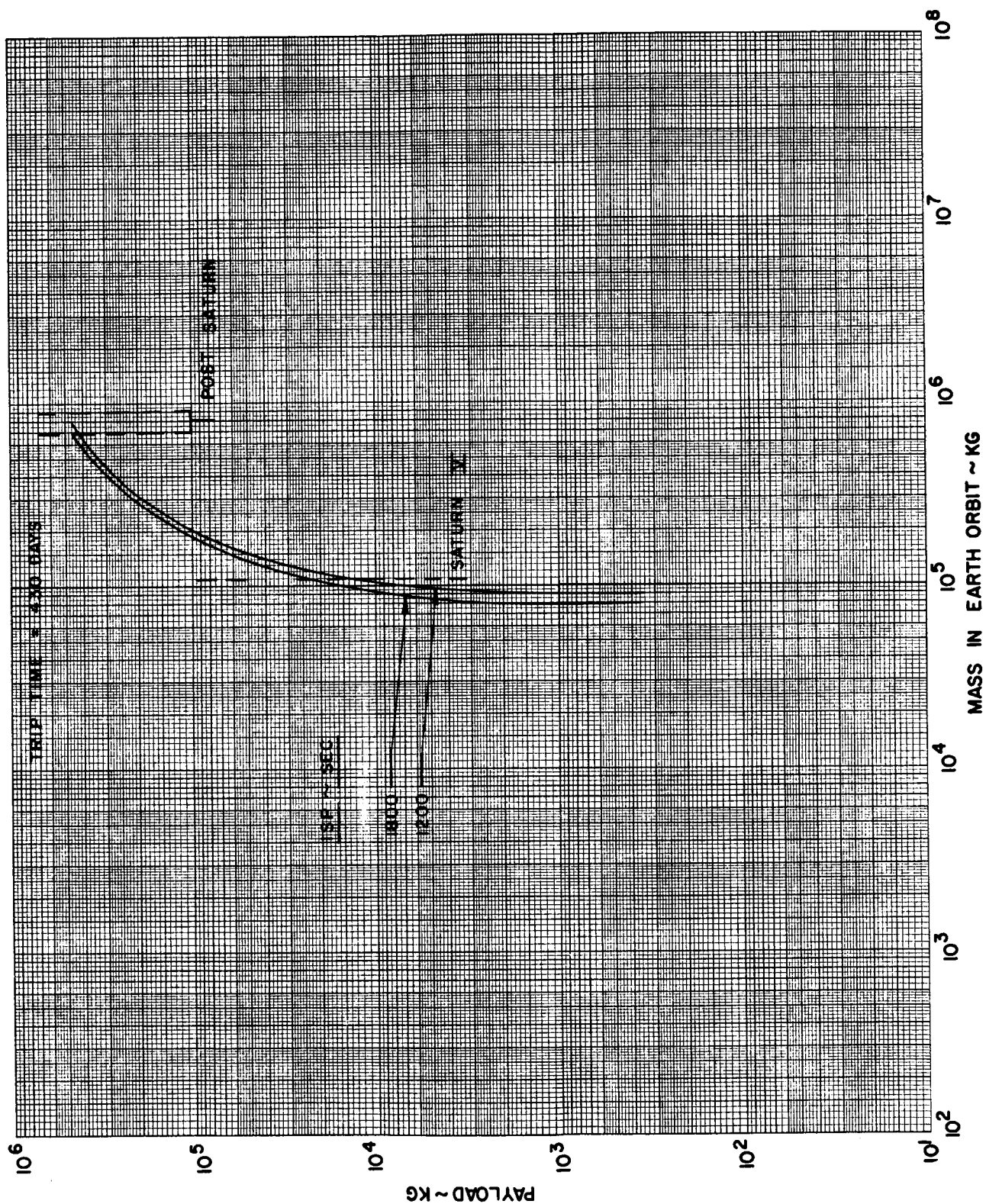


FIG. 104

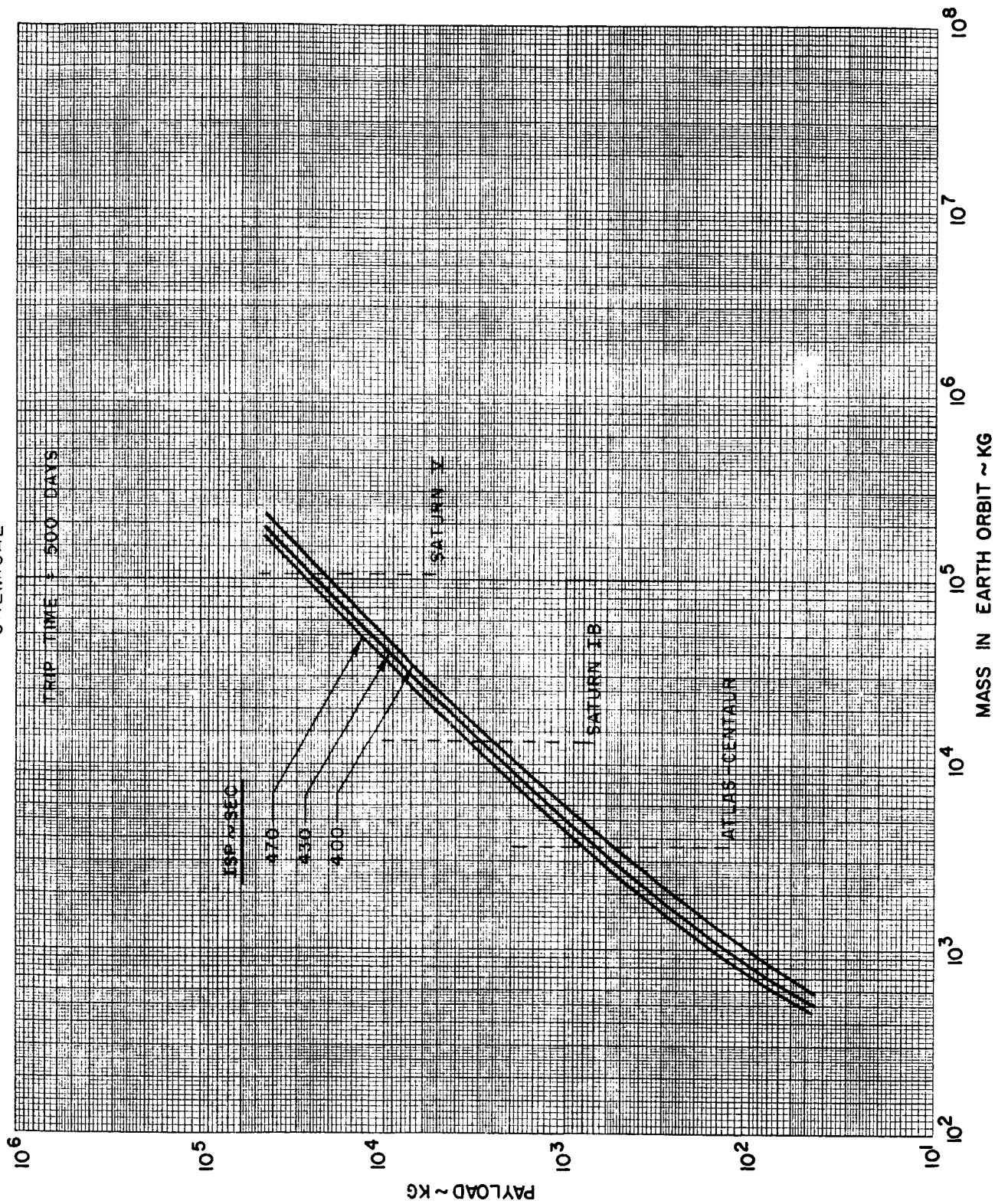
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CERES FLYBY PULSED NUCLEAR



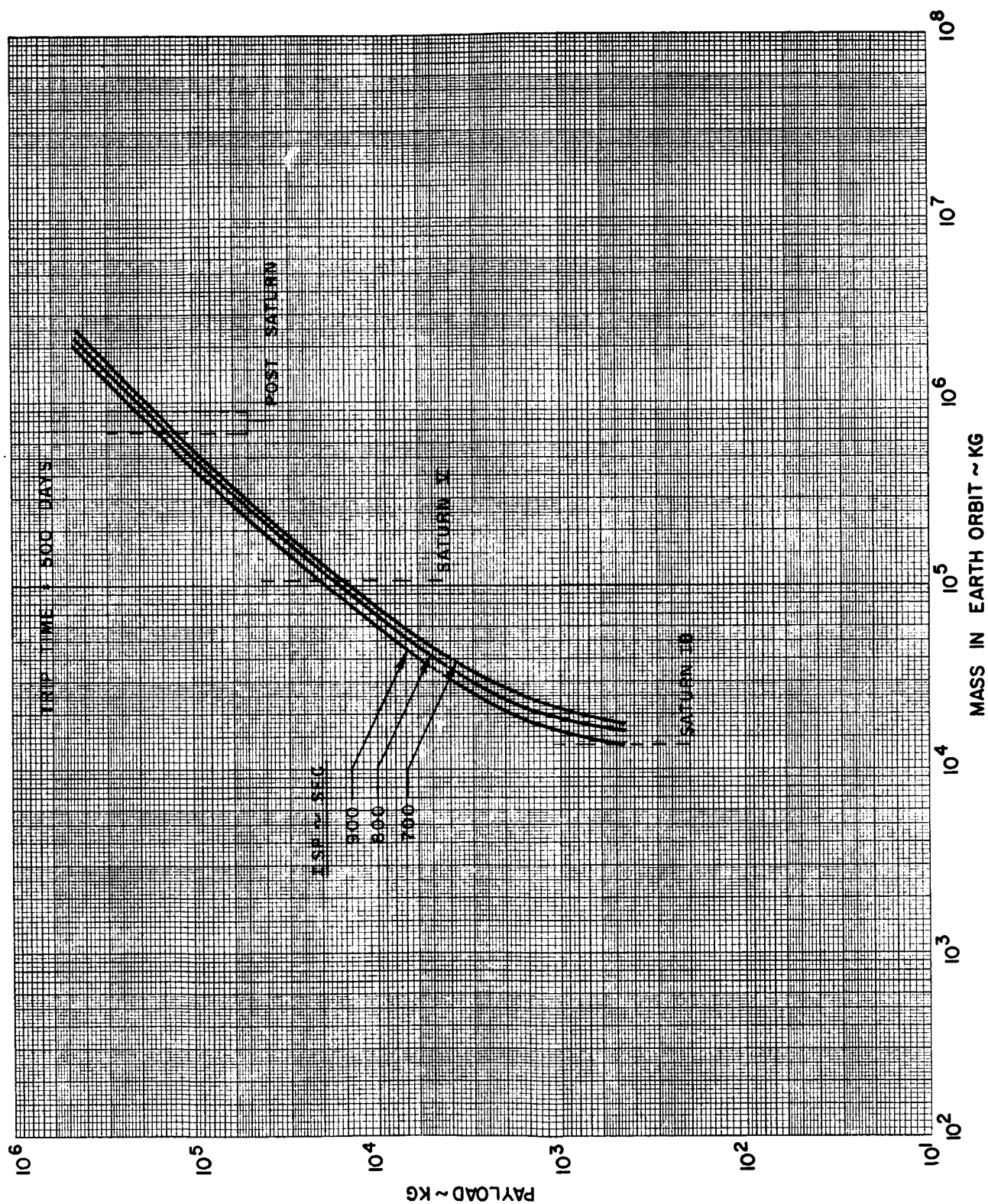
SCHWASSMANN-WACHMANN II FLYBY

CHEMICAL



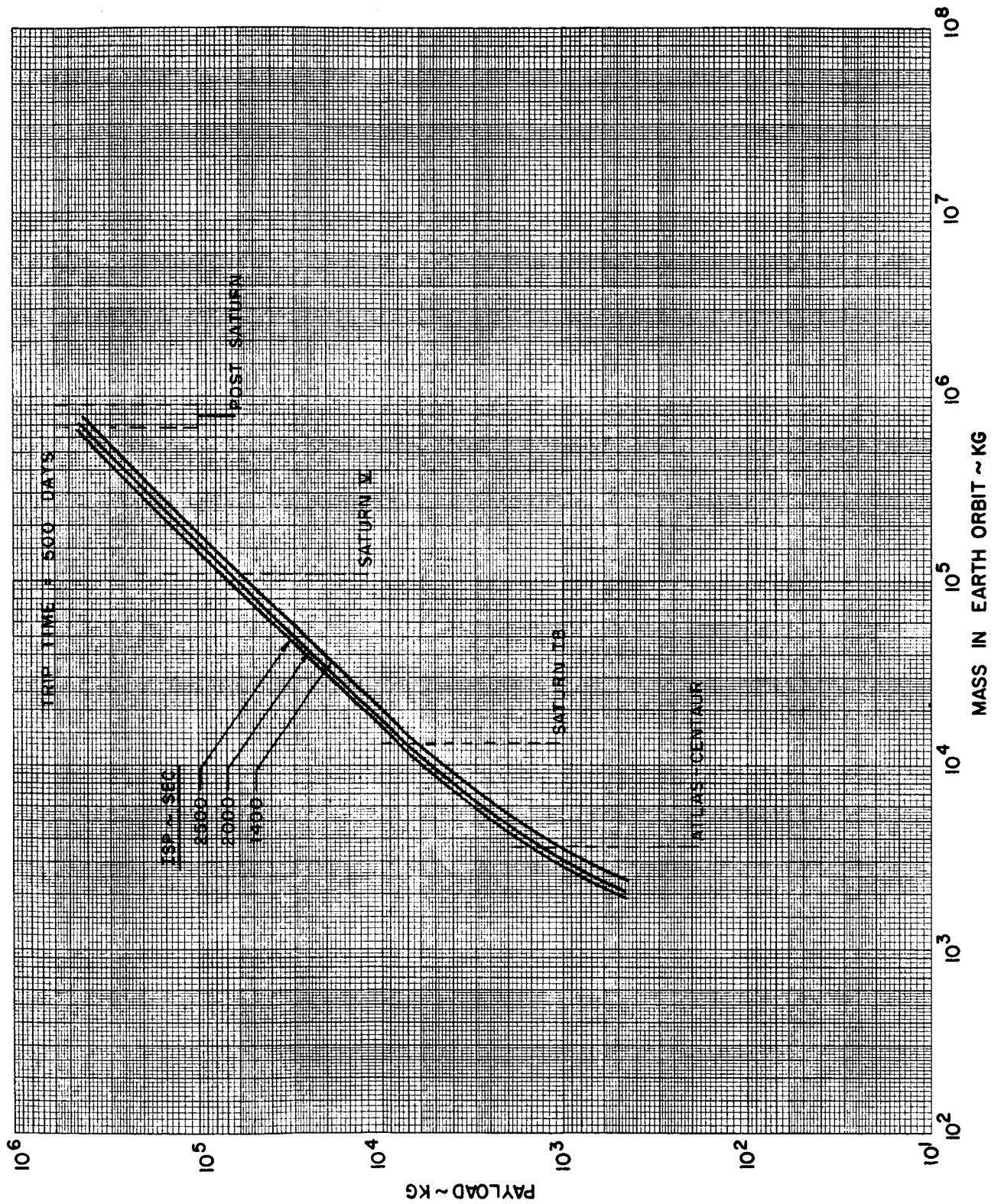
SCHWASSMANN-WACHMANN II FLYBY

SOLID - CORE NUCLEAR



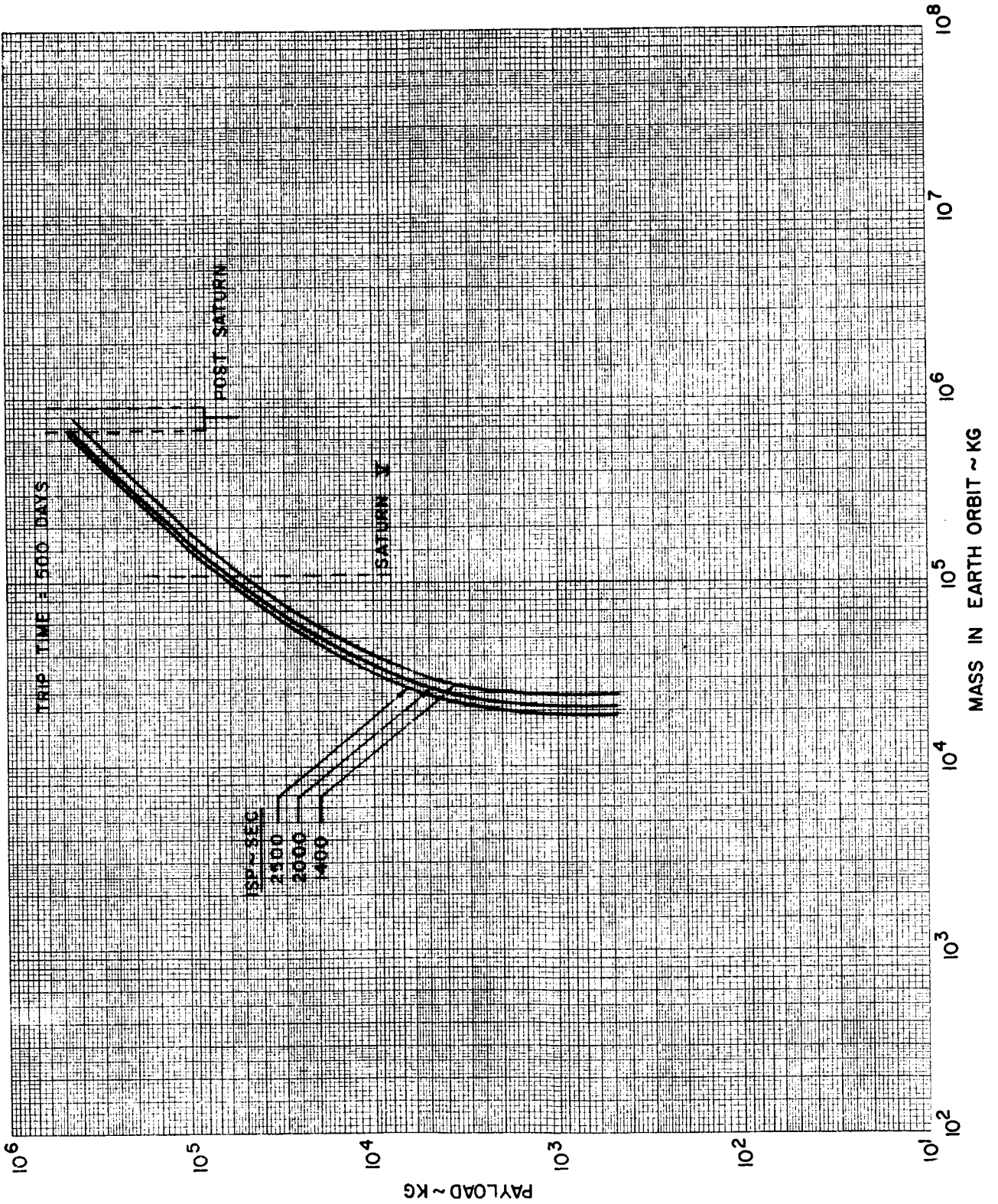
SCHWASSMANN-WACHMANN II FLYBY

LIGHTWEIGHT ADVANCED NUCLEAR



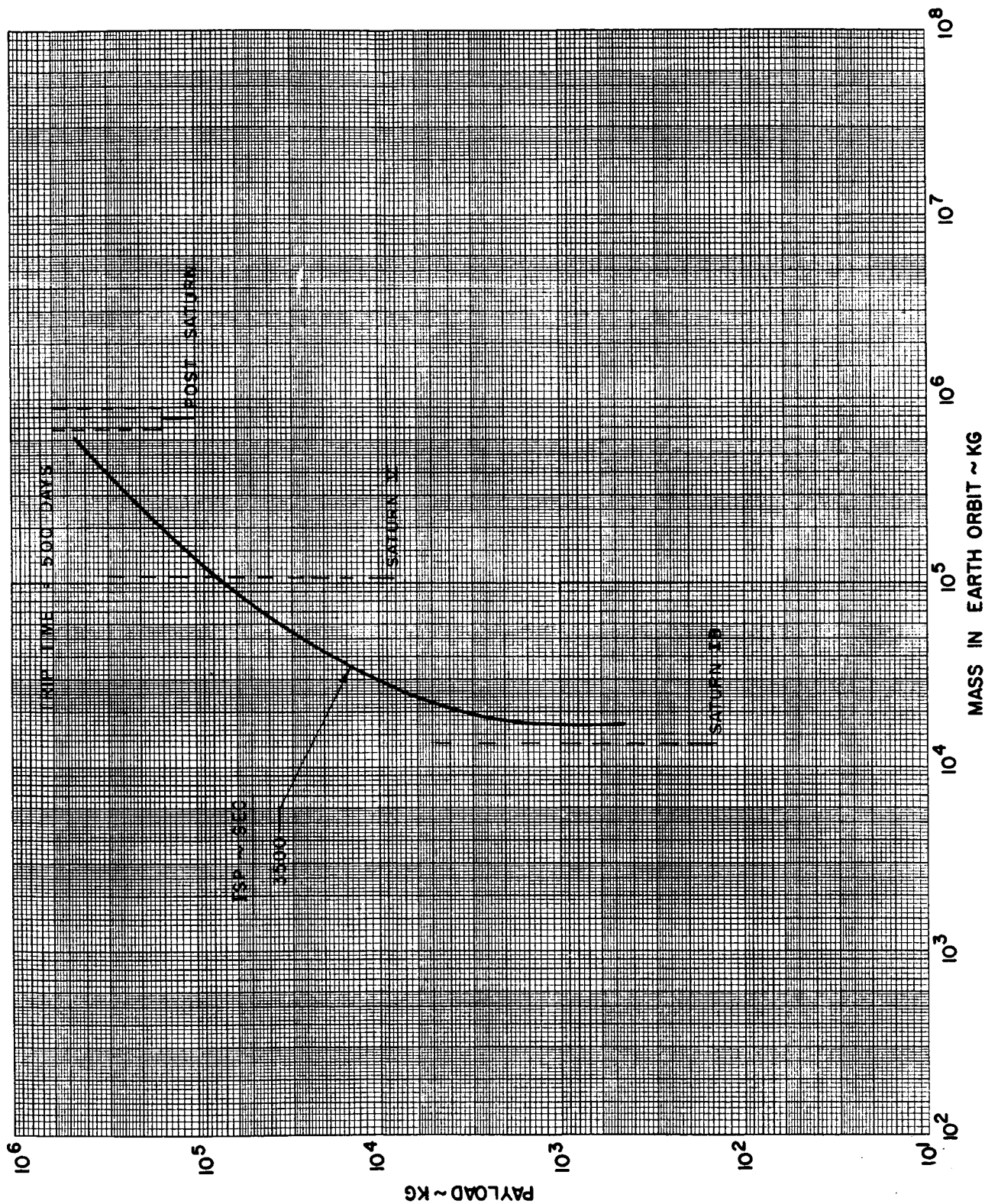
SCHWASSMANN-WACHMANN II FLYBY

GASEOUS - CORE NUCLEAR



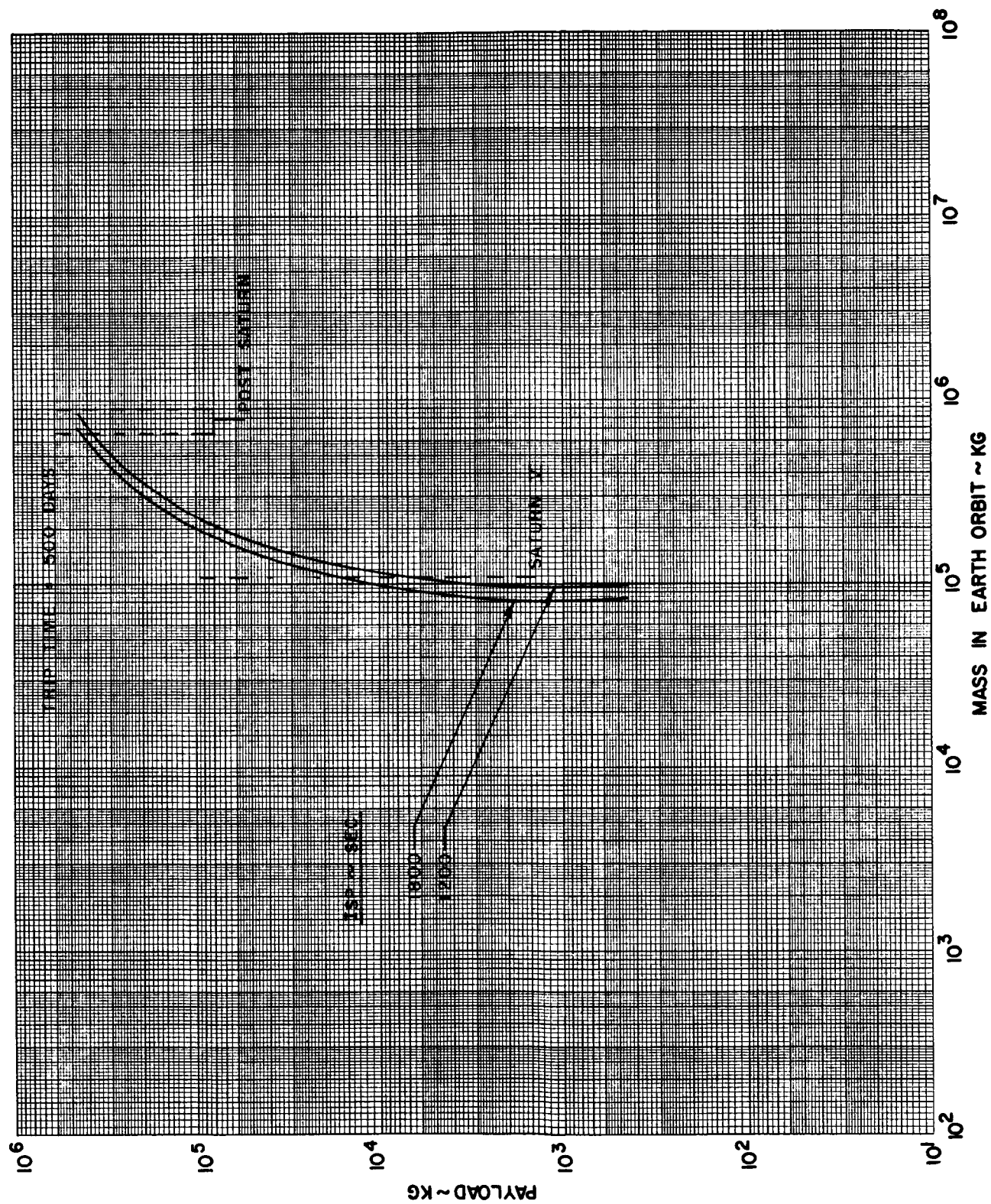
SCHWASSMANN-WACHMANN II FLYBY

GASEOUS - CORE NUCLEAR



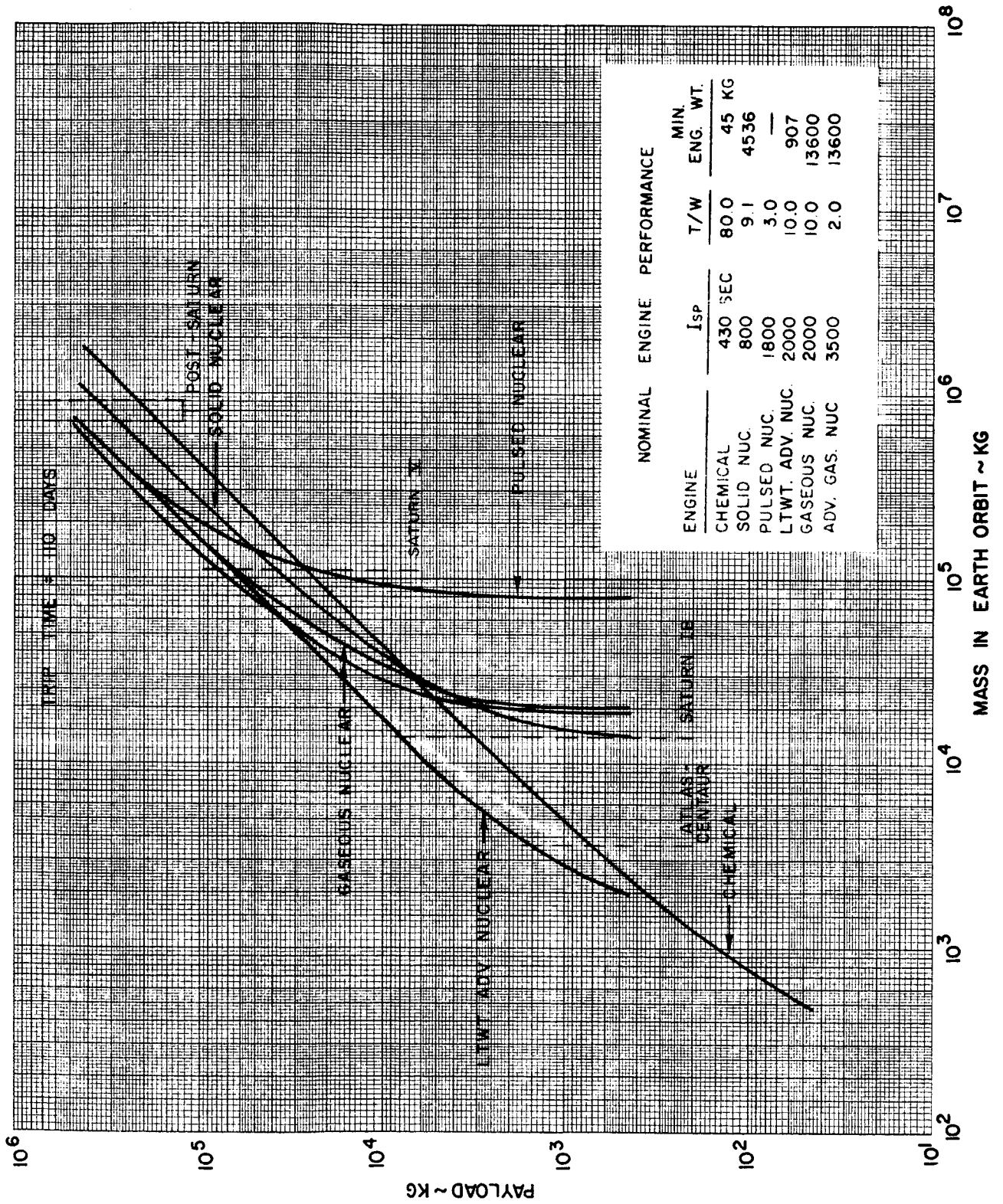
SCHWASSMANN-WACHMANN II FLYBY

PULSED NUCLEAR



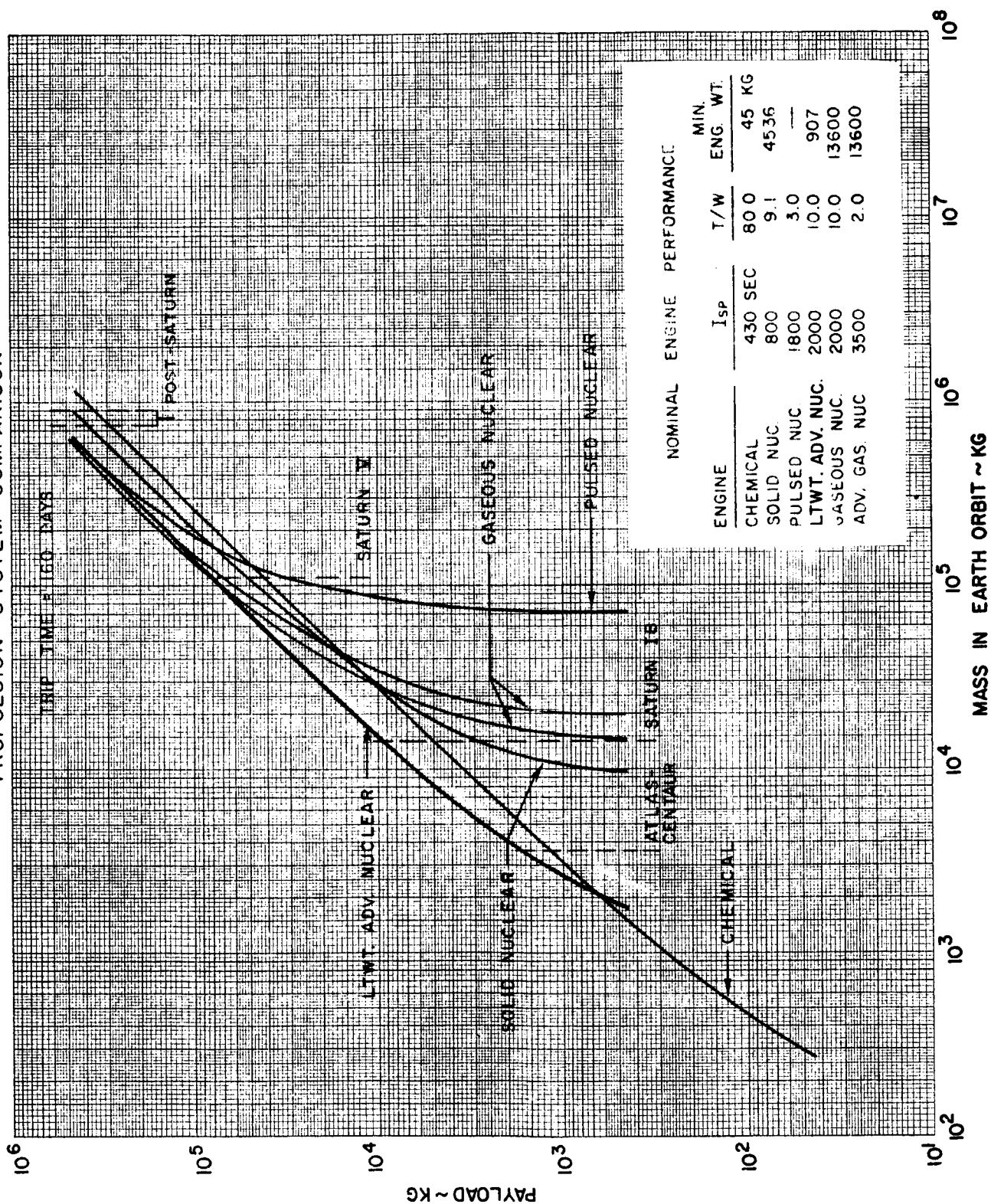
MERCURY FLYBYS

PROPULSION SYSTEM COMPARISON



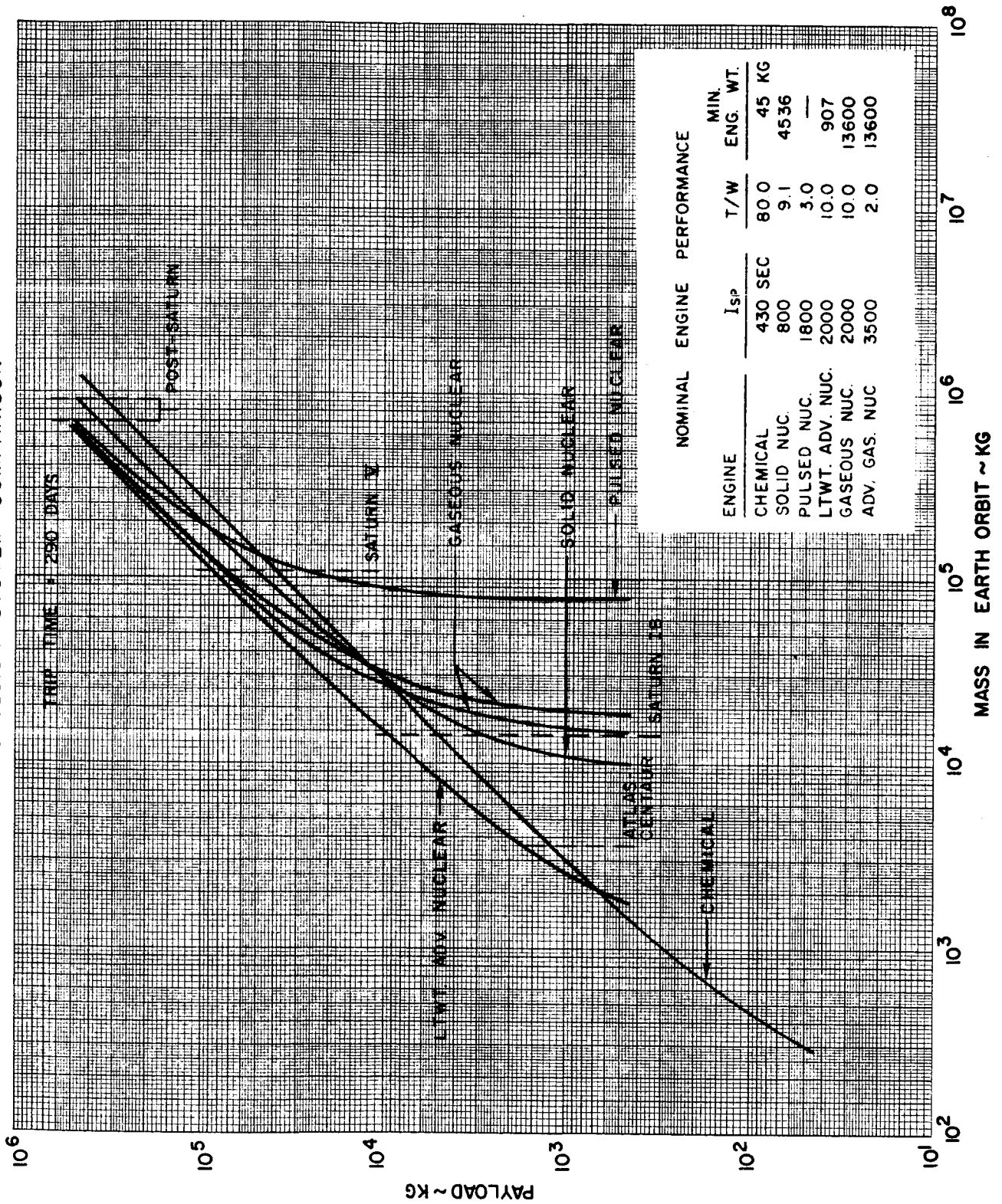
VENUS FLYBYS

PROPULSION SYSTEM COMPARISON

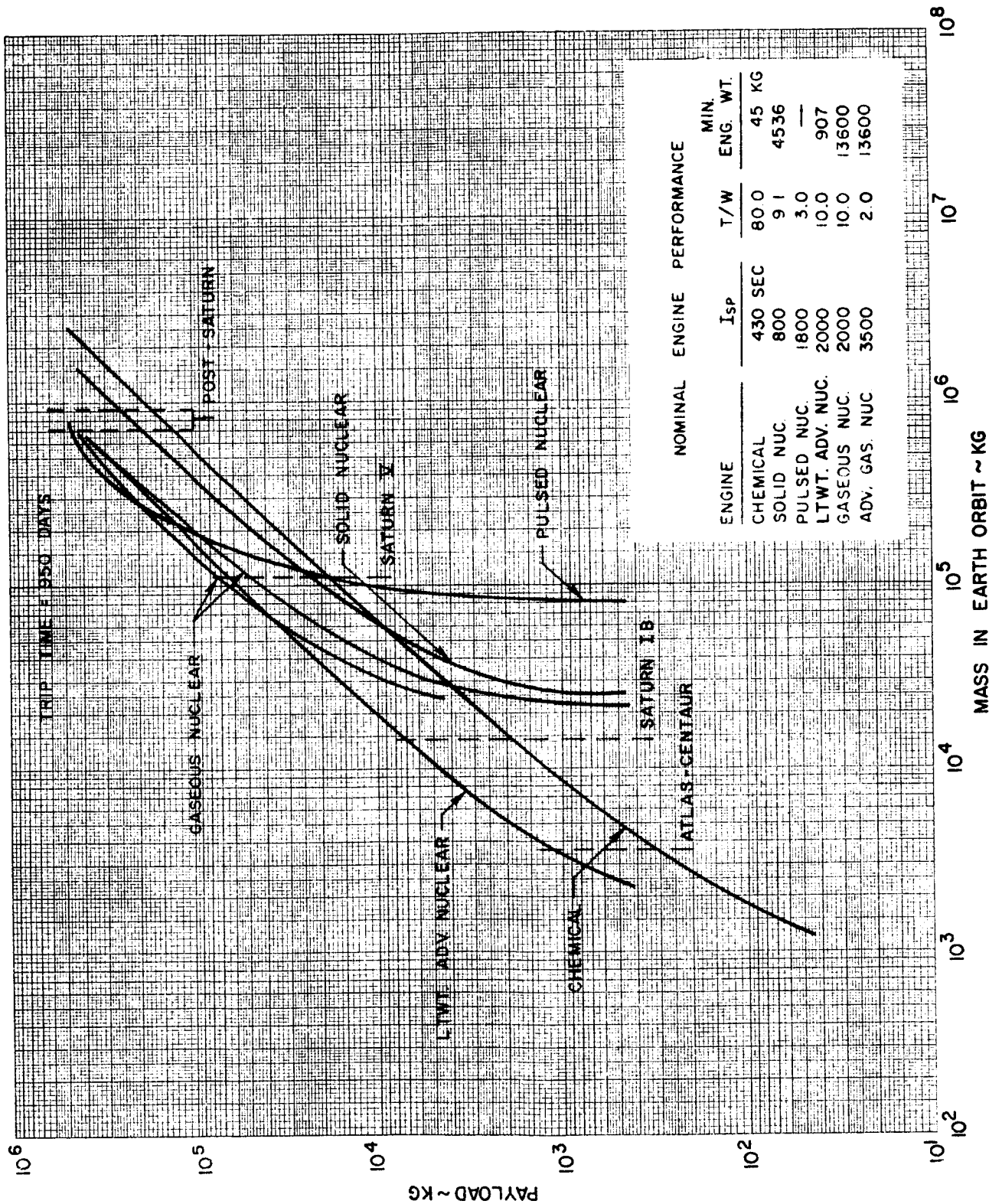


MARS FLYBYS

PROPULSION SYSTEM COMPARISON

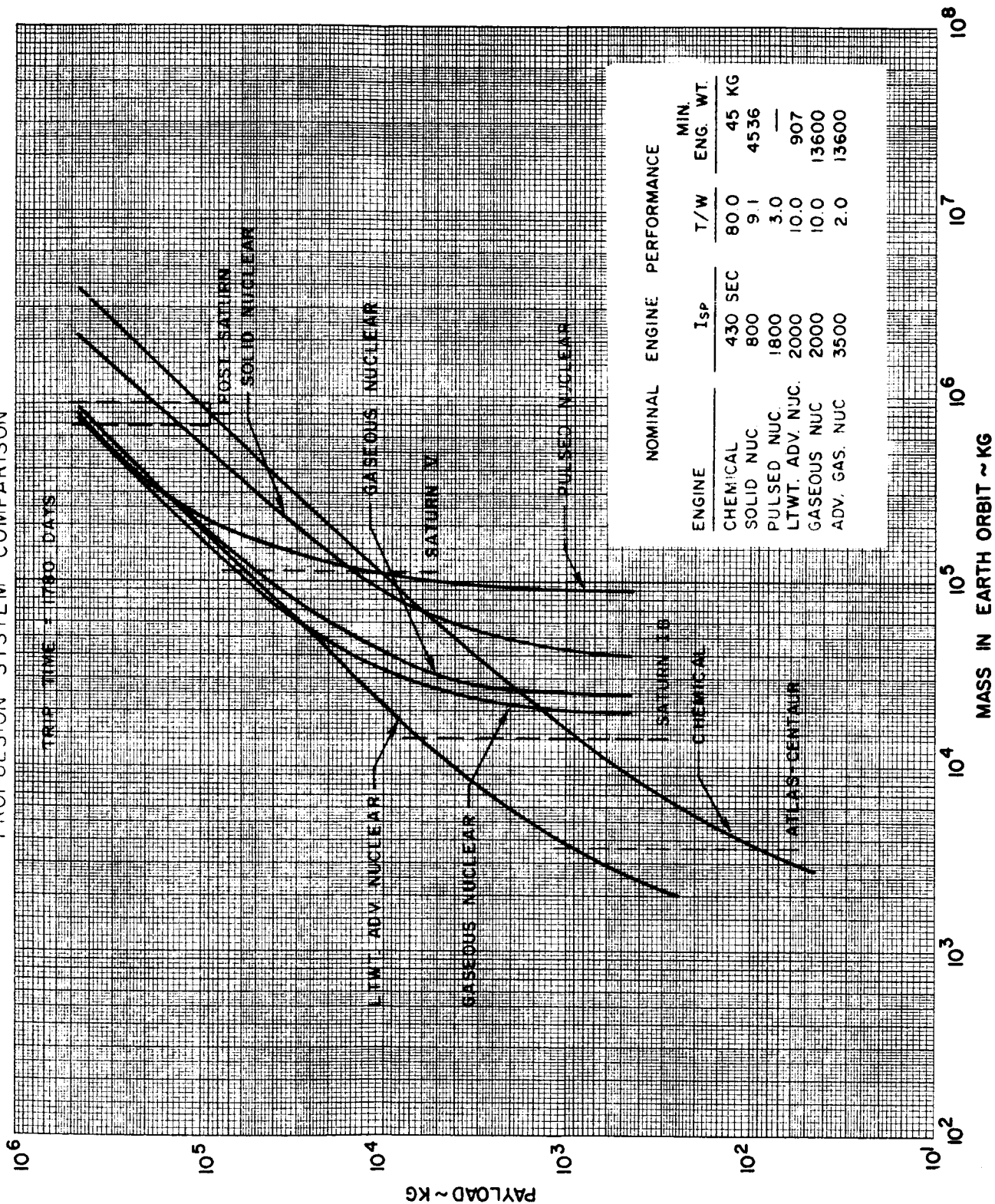


JUPITER FLYBYS PROPULSION SYSTEM COMPARISON



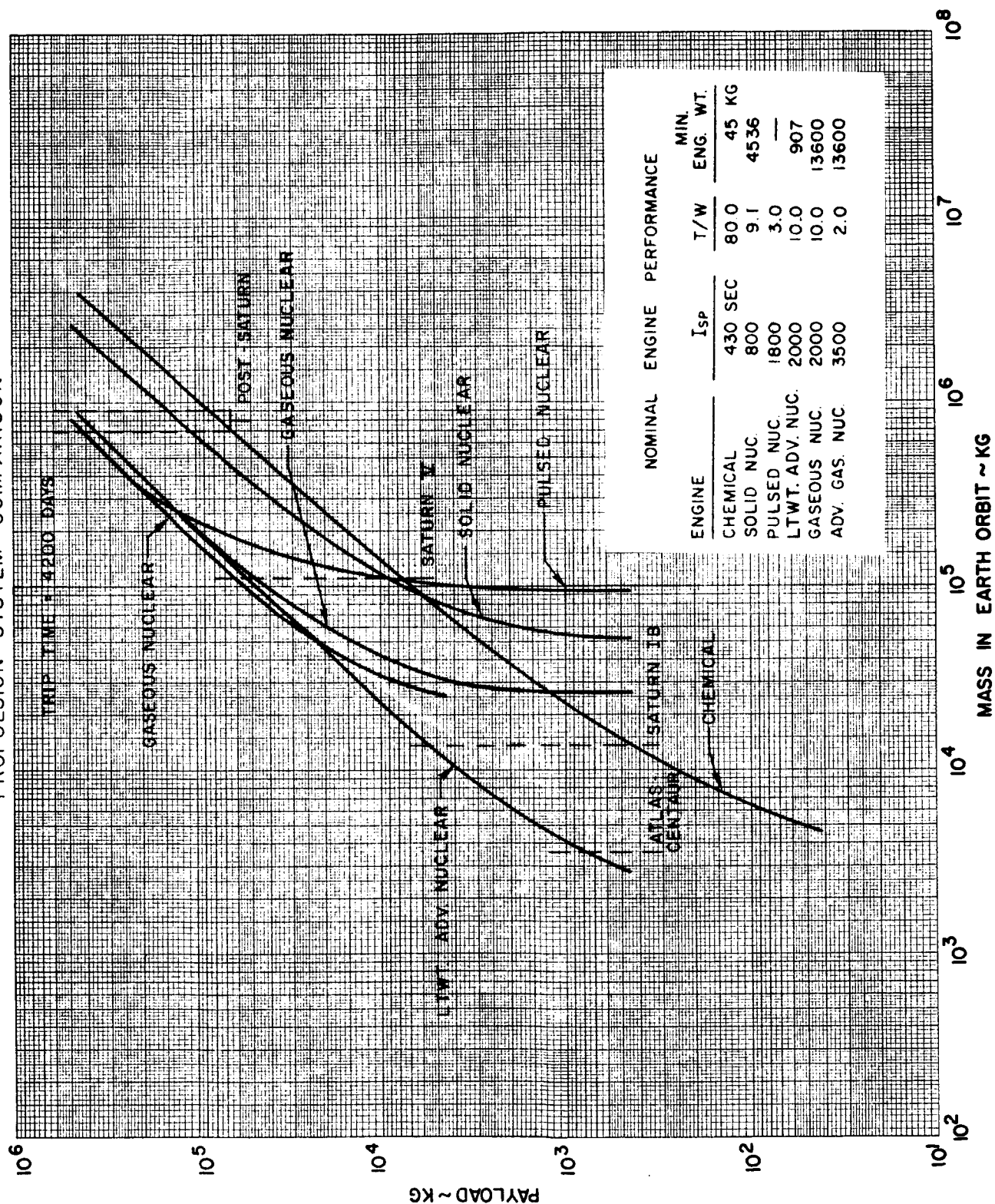
SATURN FLYBYS

PROPULSION SYSTEM COMPARISON

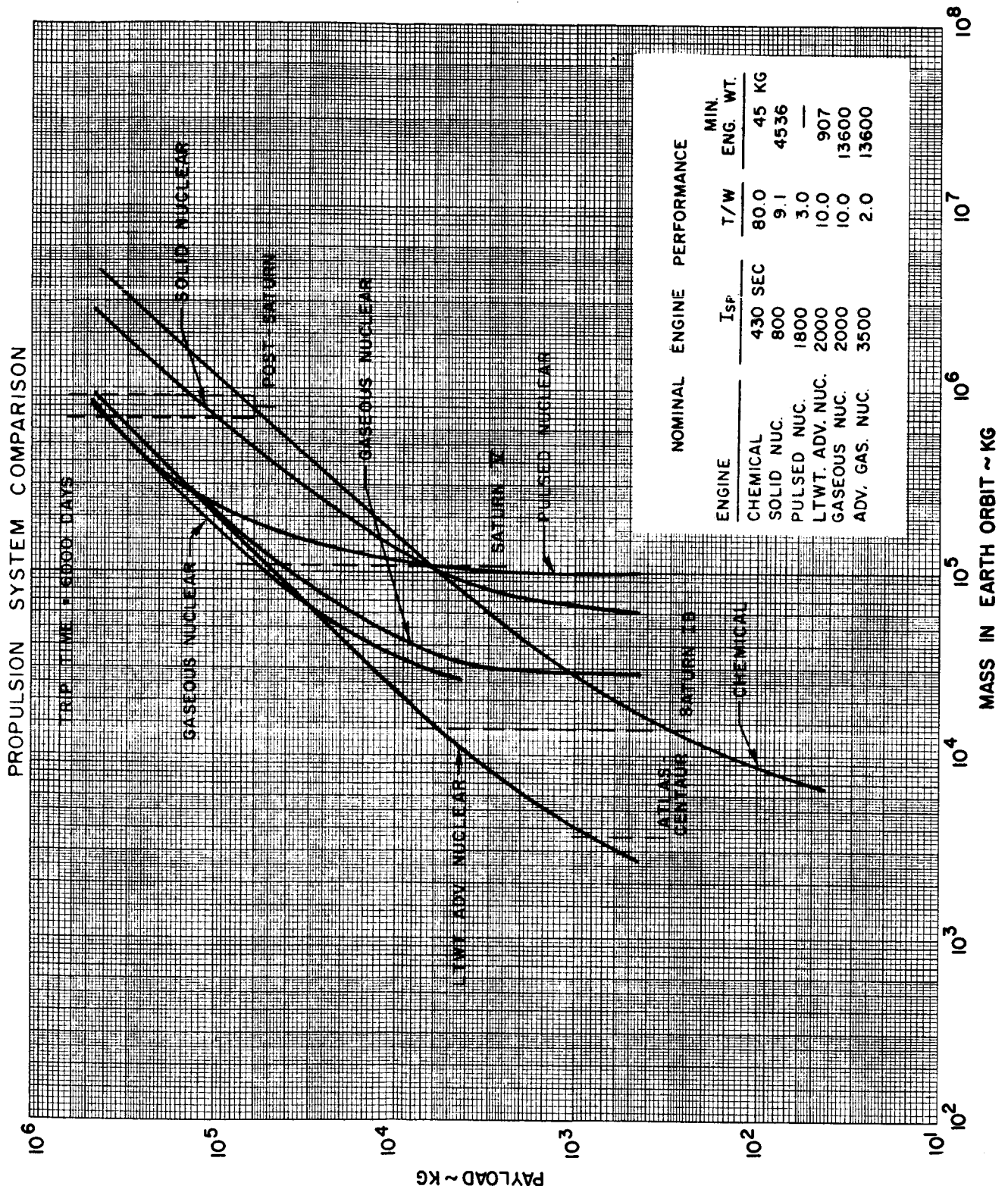


URANUS FLYBYS

PROPULSION SYSTEM COMPARISON

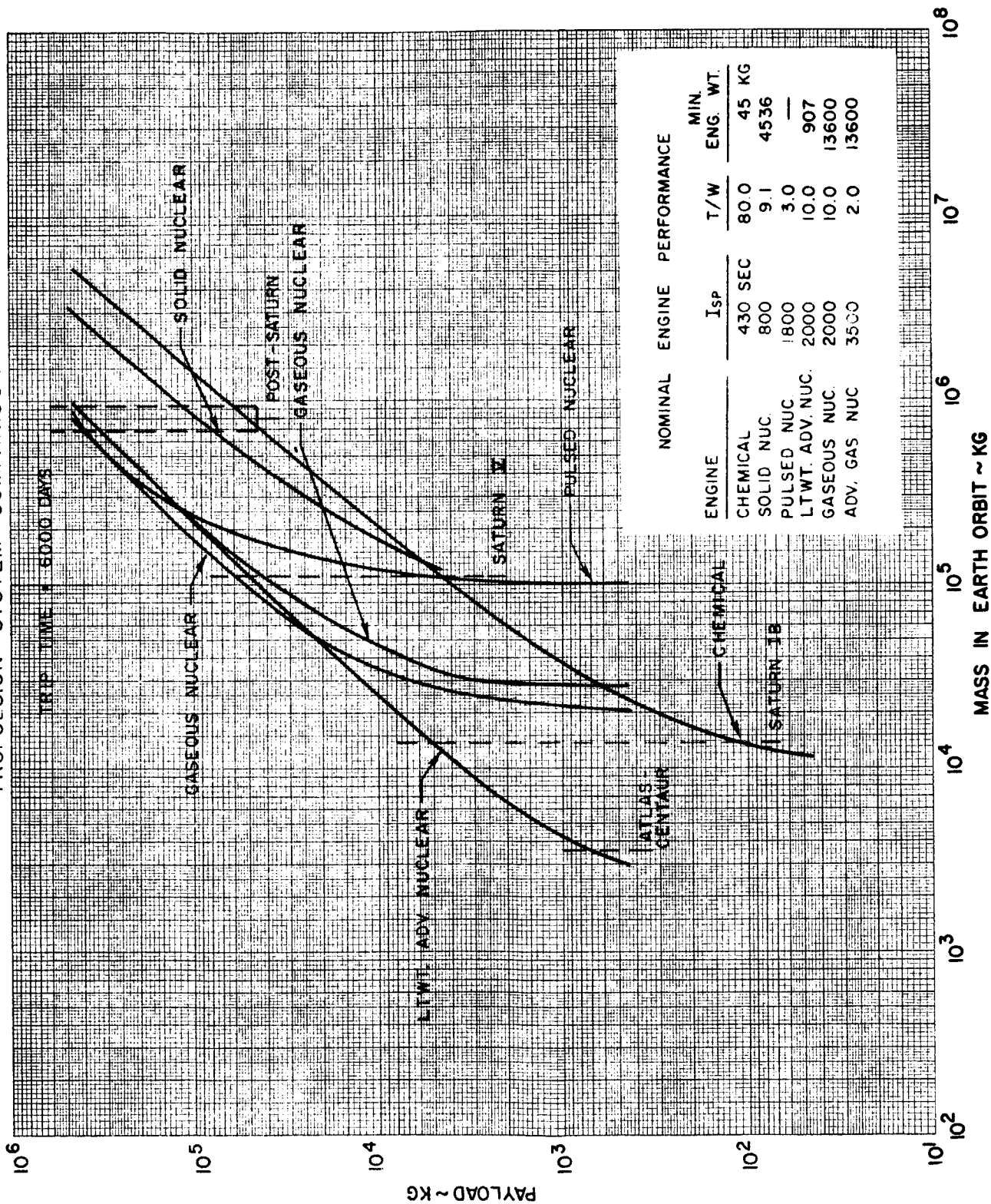


NEPTUNE FLYBYS



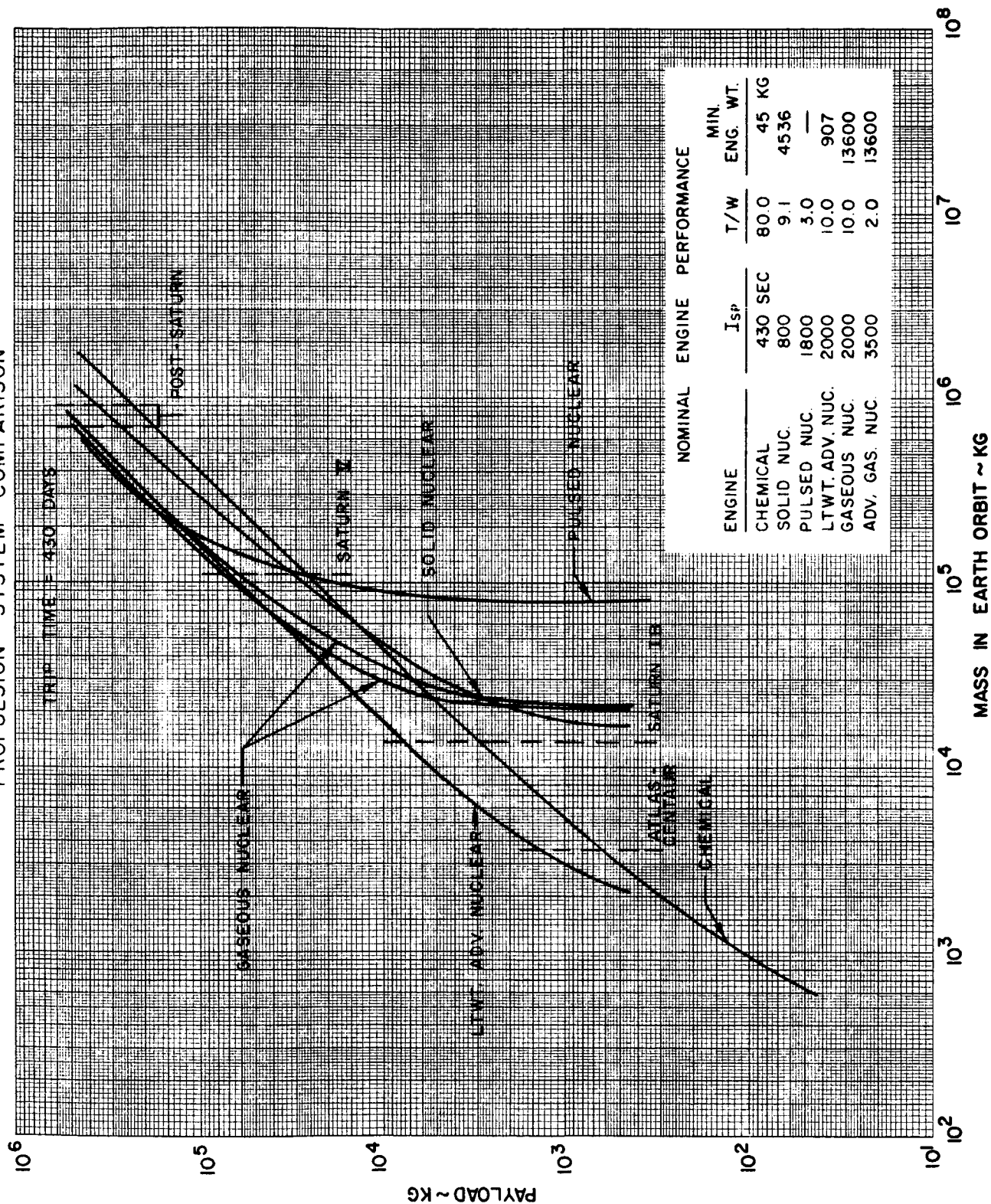
PLUTO FLYBYS

PROPULSION SYSTEM COMPARISON

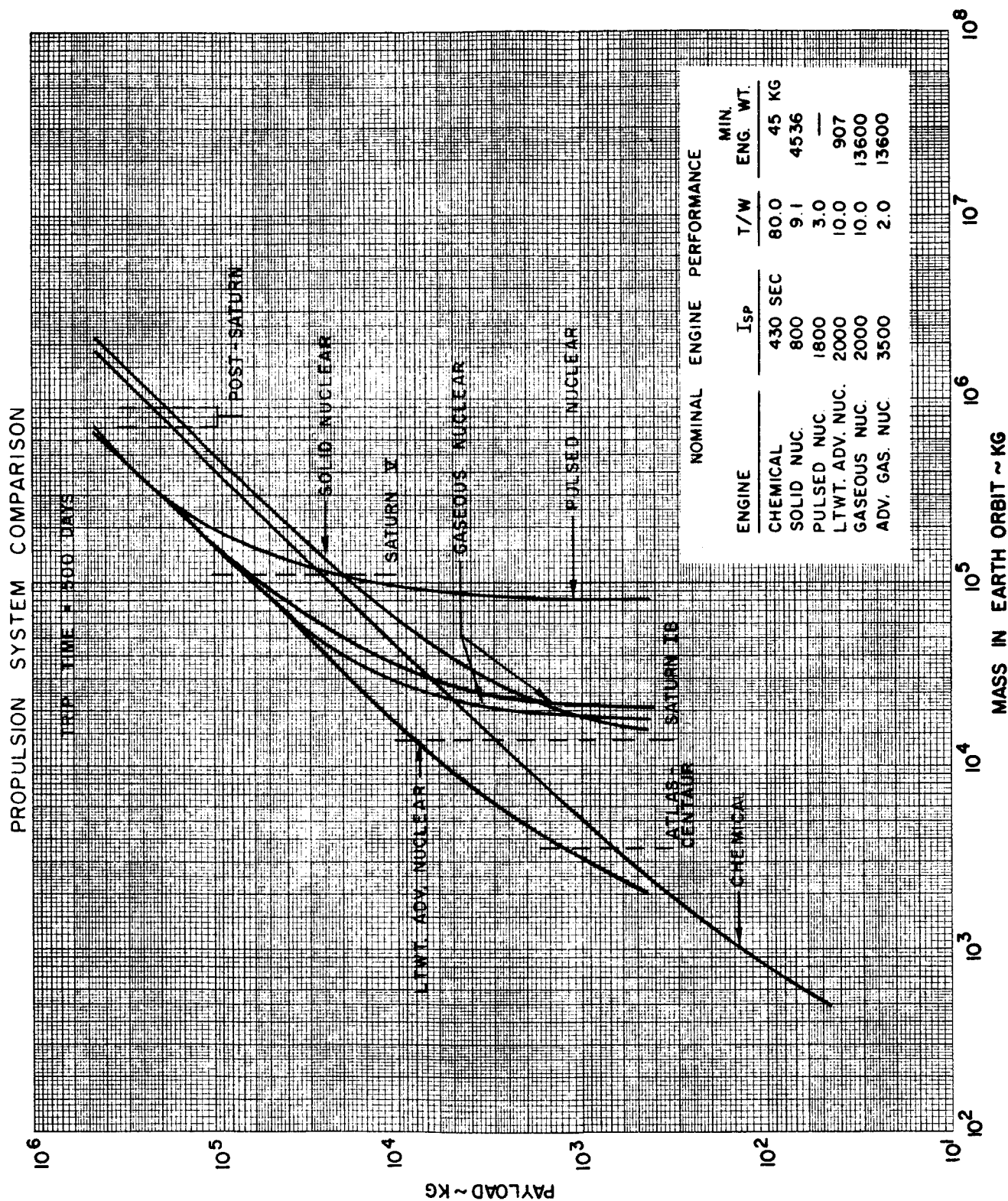


CERES FLYBYS

PROPULSION SYSTEM COMPARISON



SCHWASSMANN-WACHMANN II FLYBYS

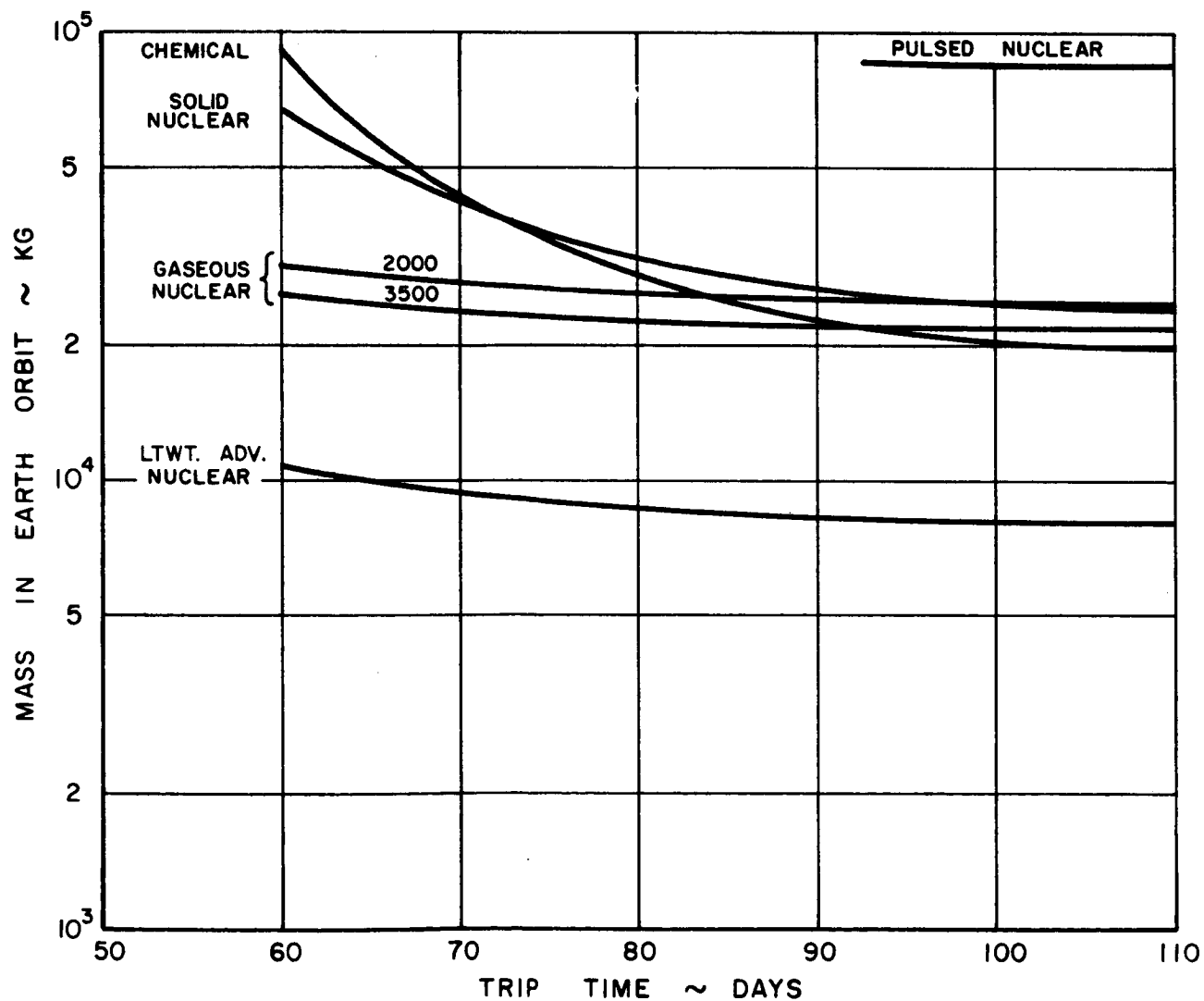


MERCURY FLYBYS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I _{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

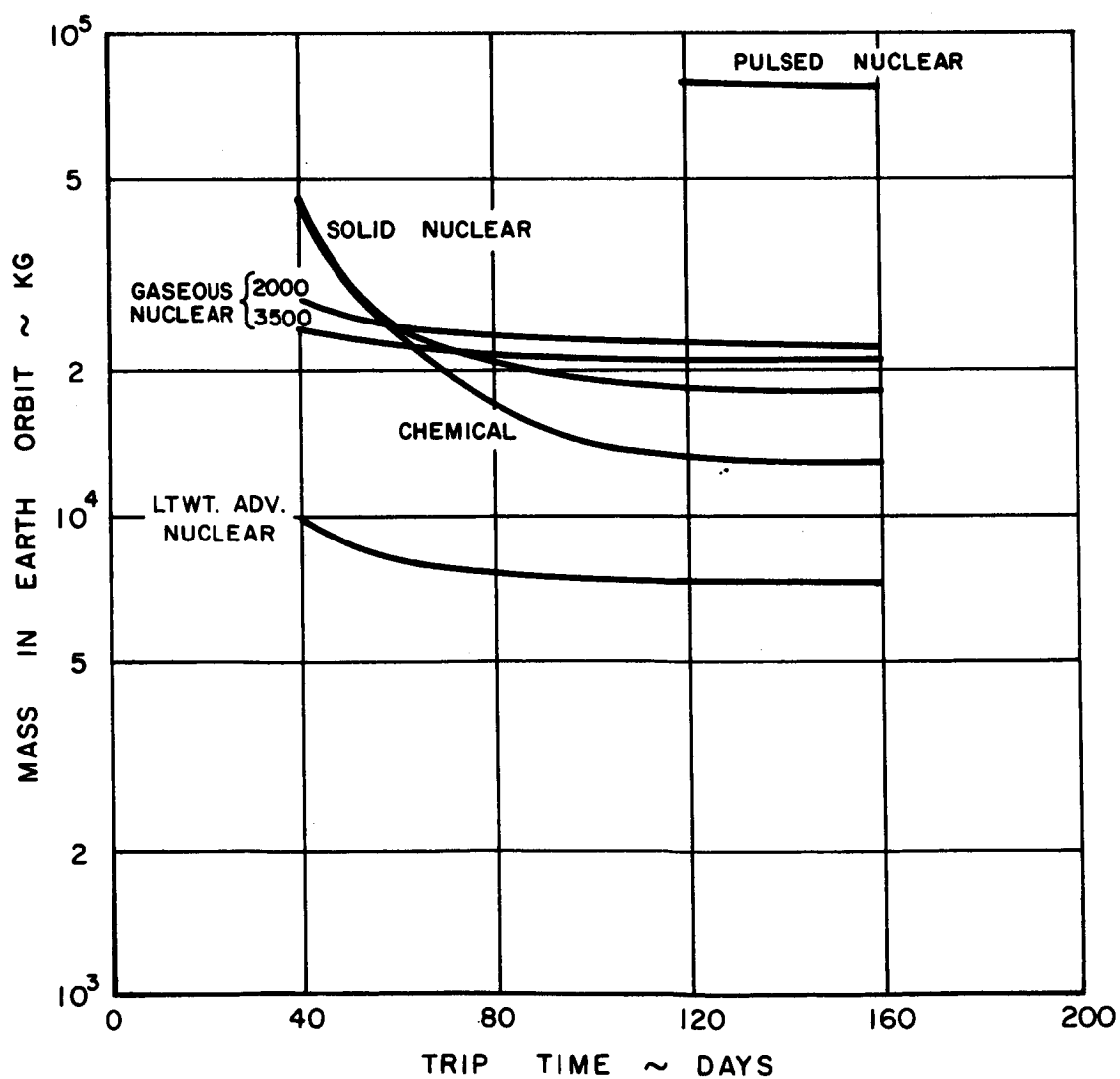


VENUS FLYBYS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	Isp	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

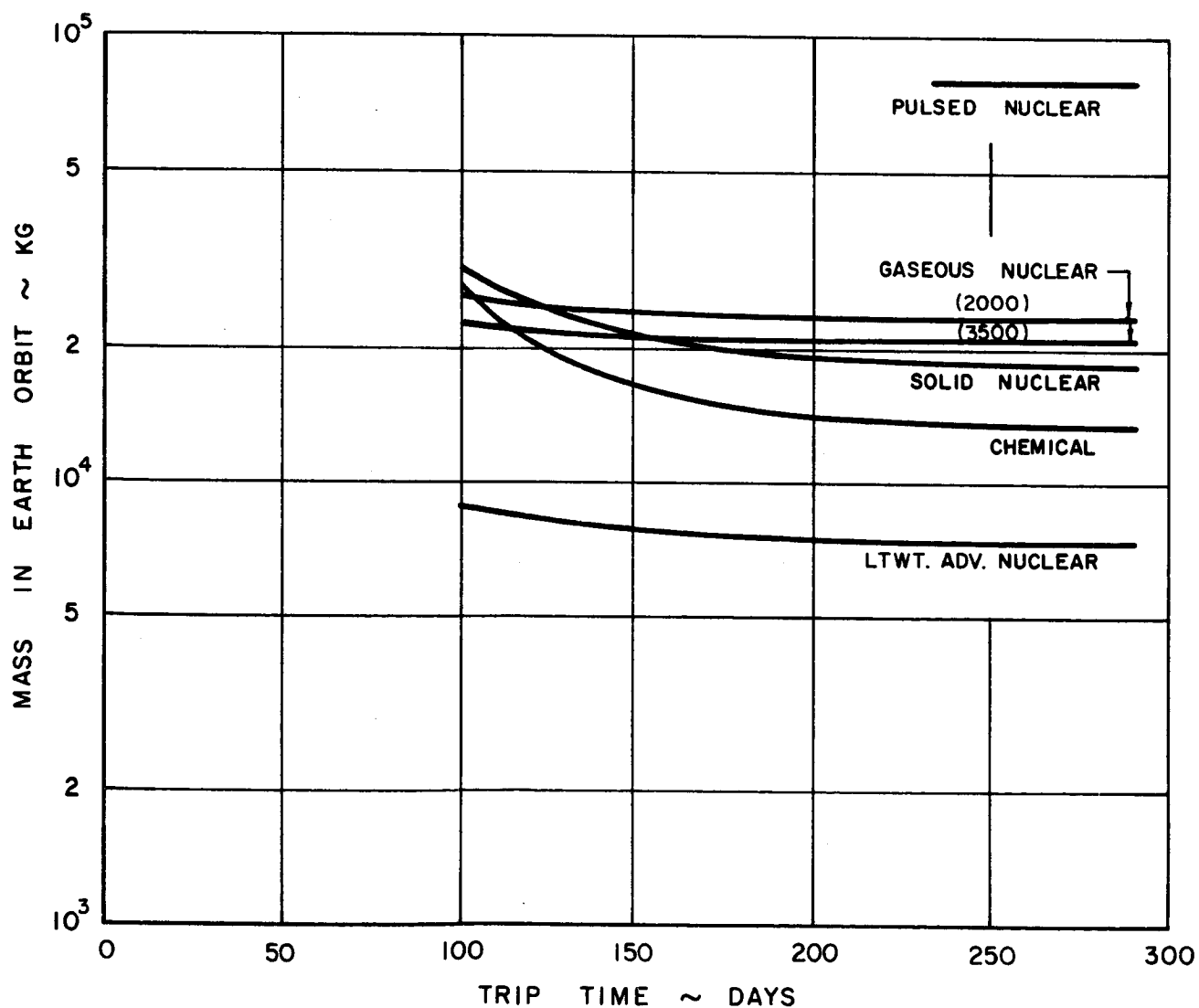


MARS FLYBYS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{SP}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600



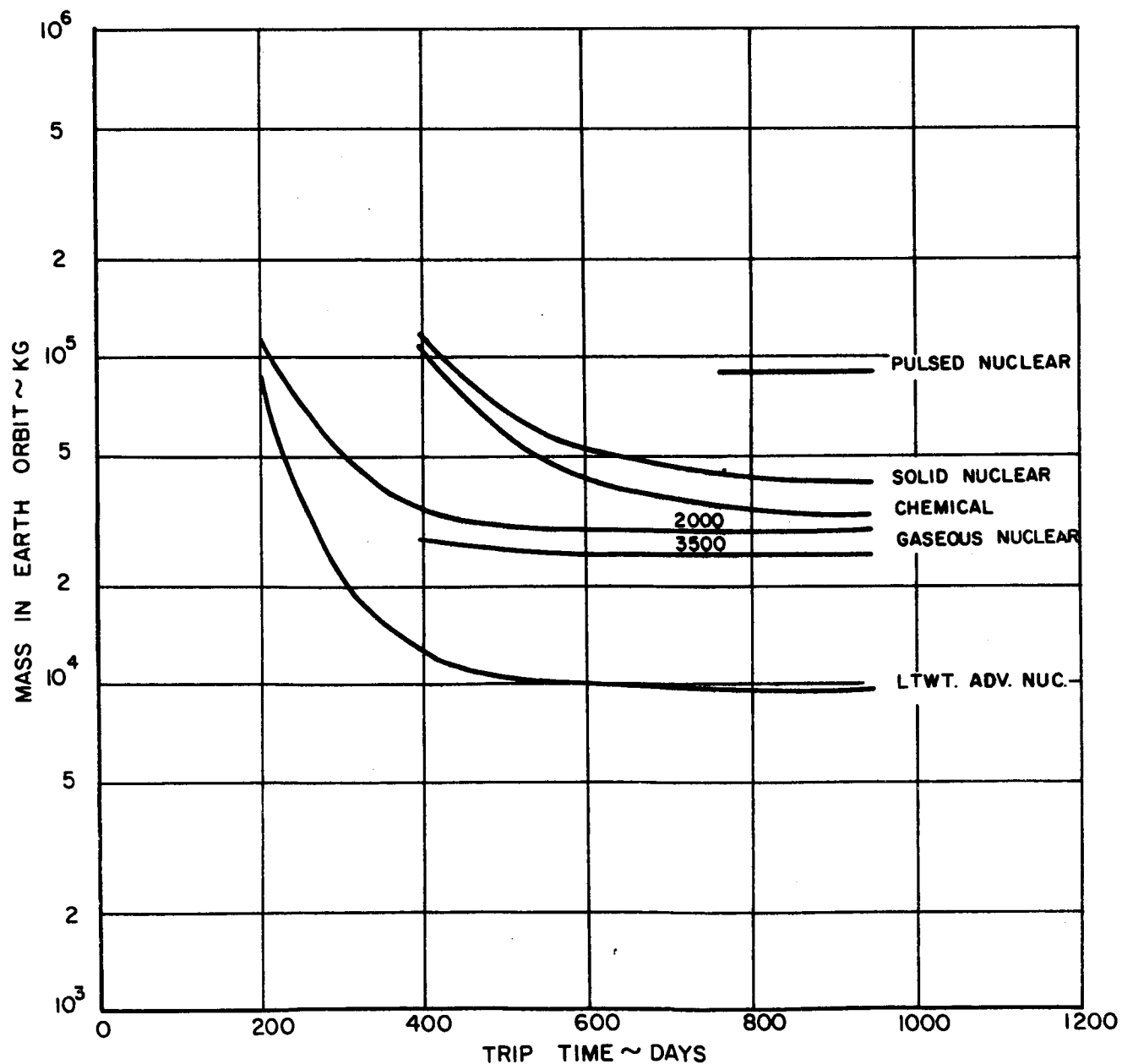
JUPITER FLYBYS

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$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

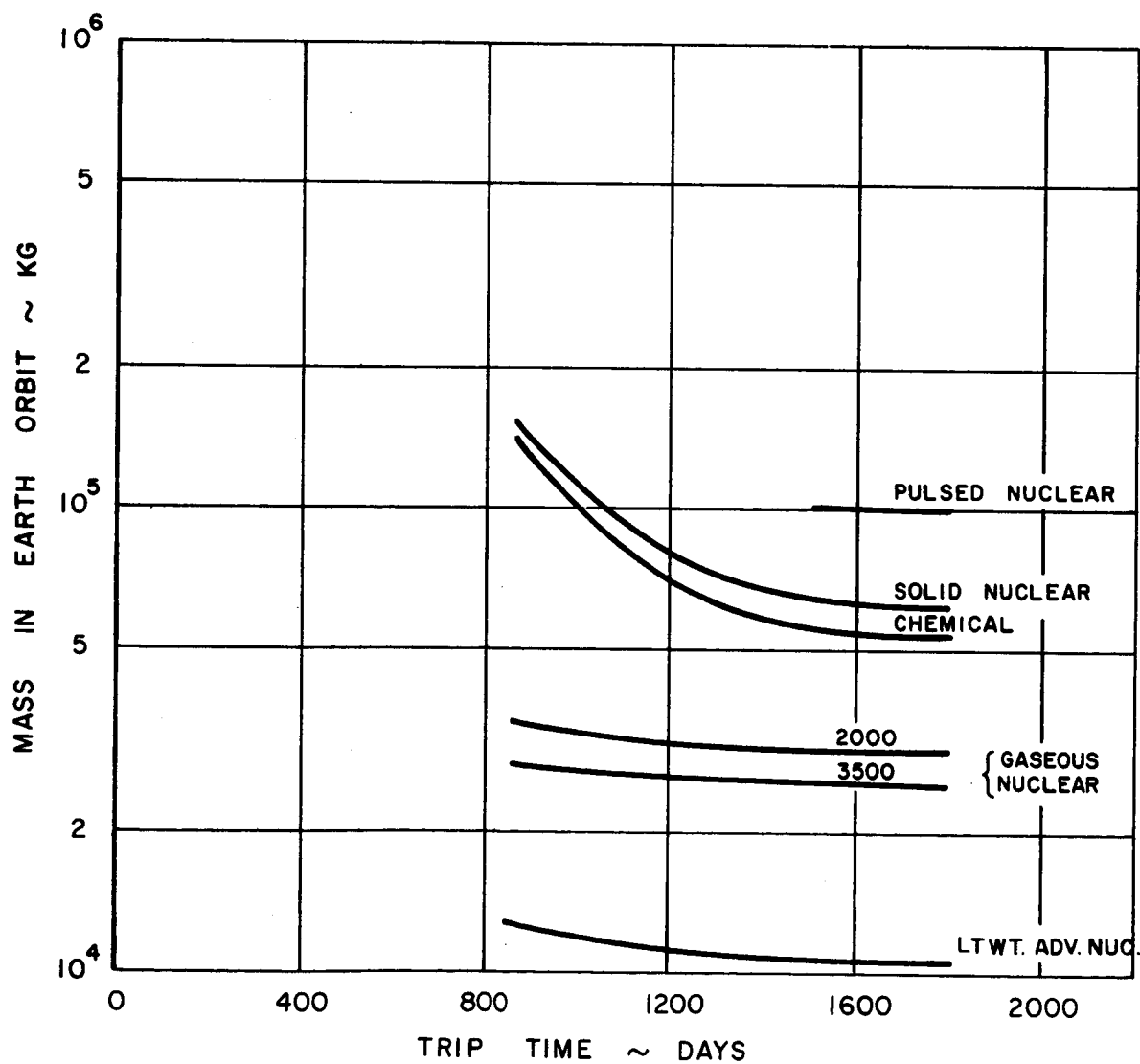


SATURN FLYBYS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{SP}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

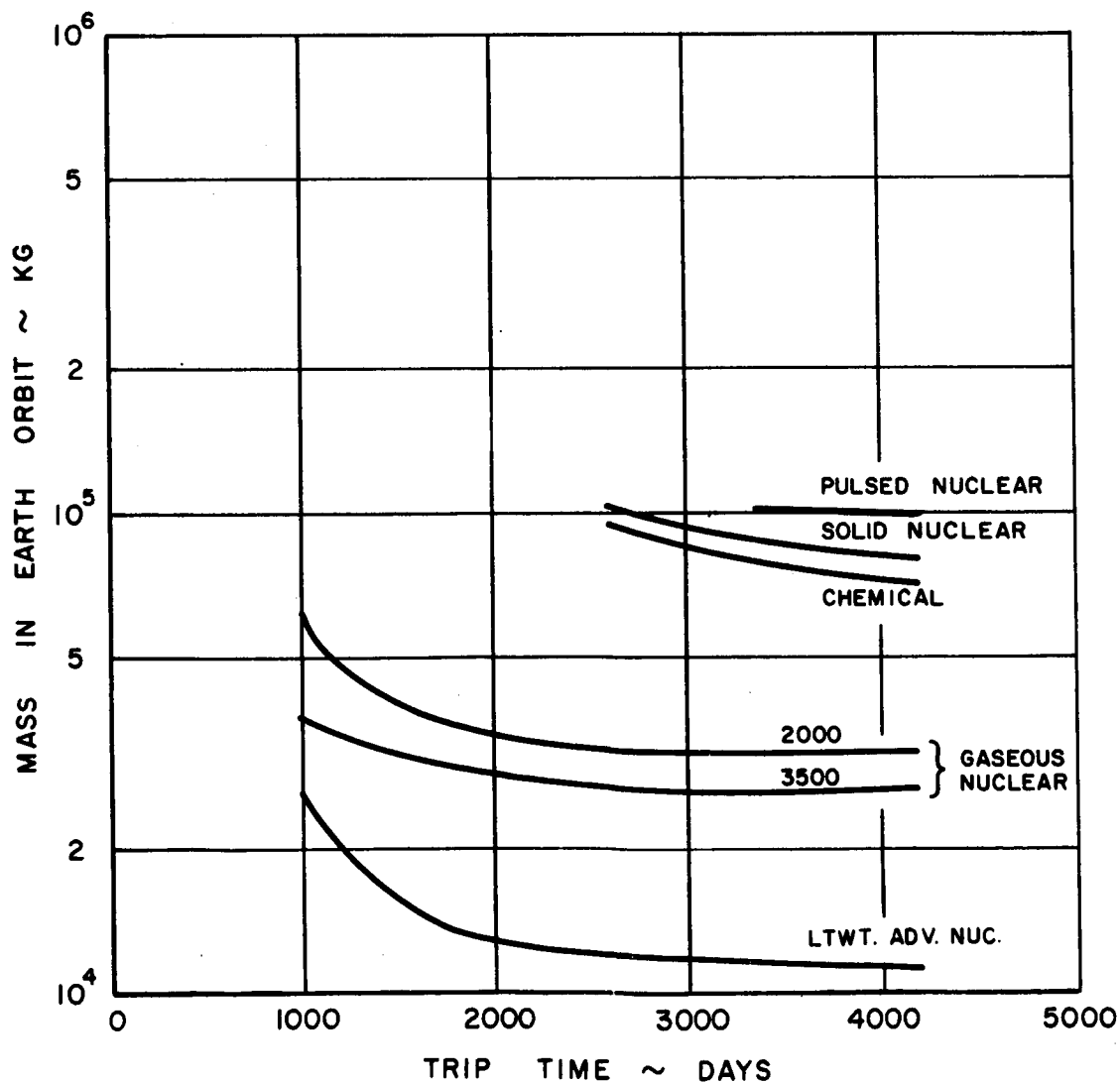


URANUS FLYBYS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I _{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

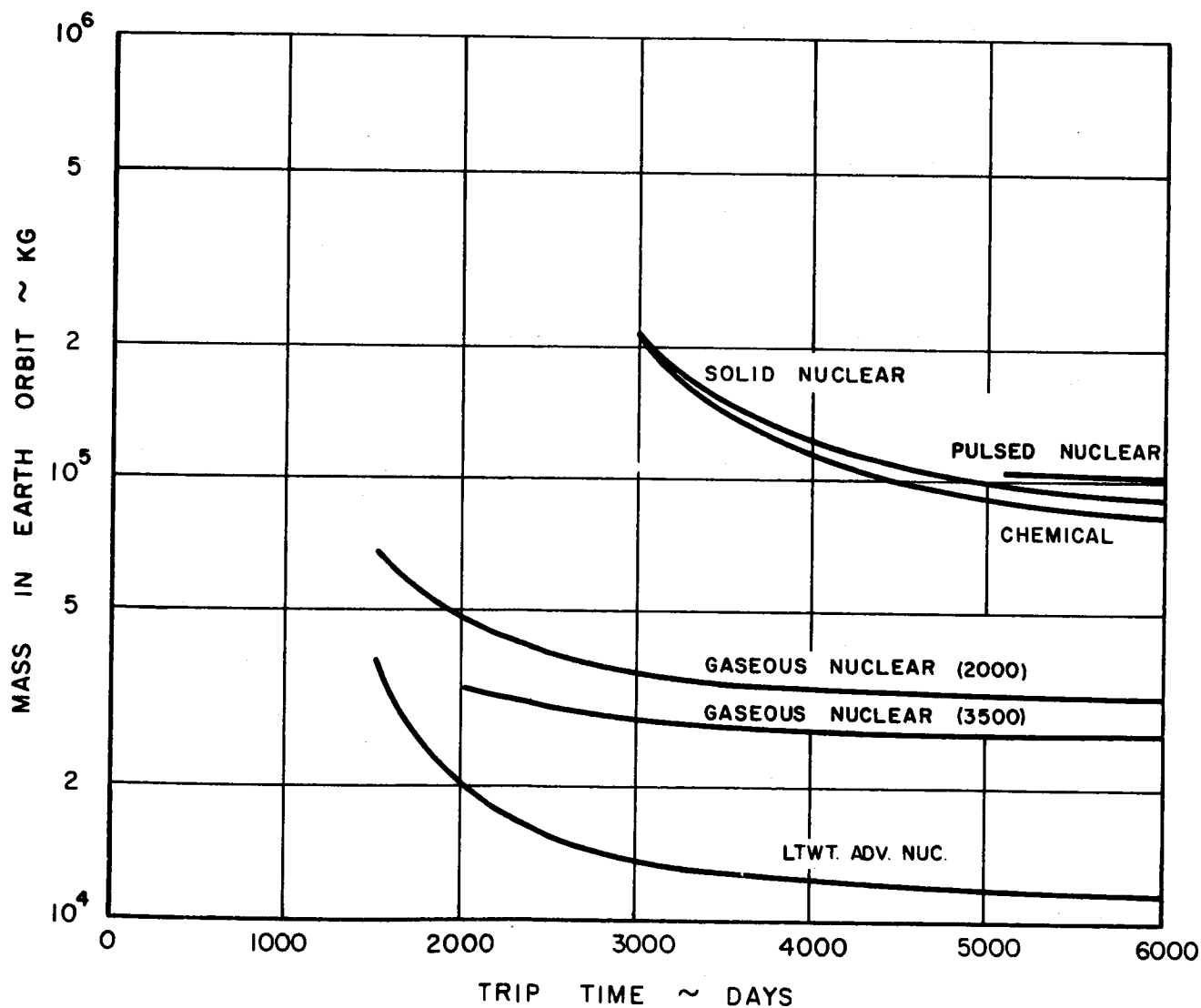


NEPTUNE FLYBYS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I _{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

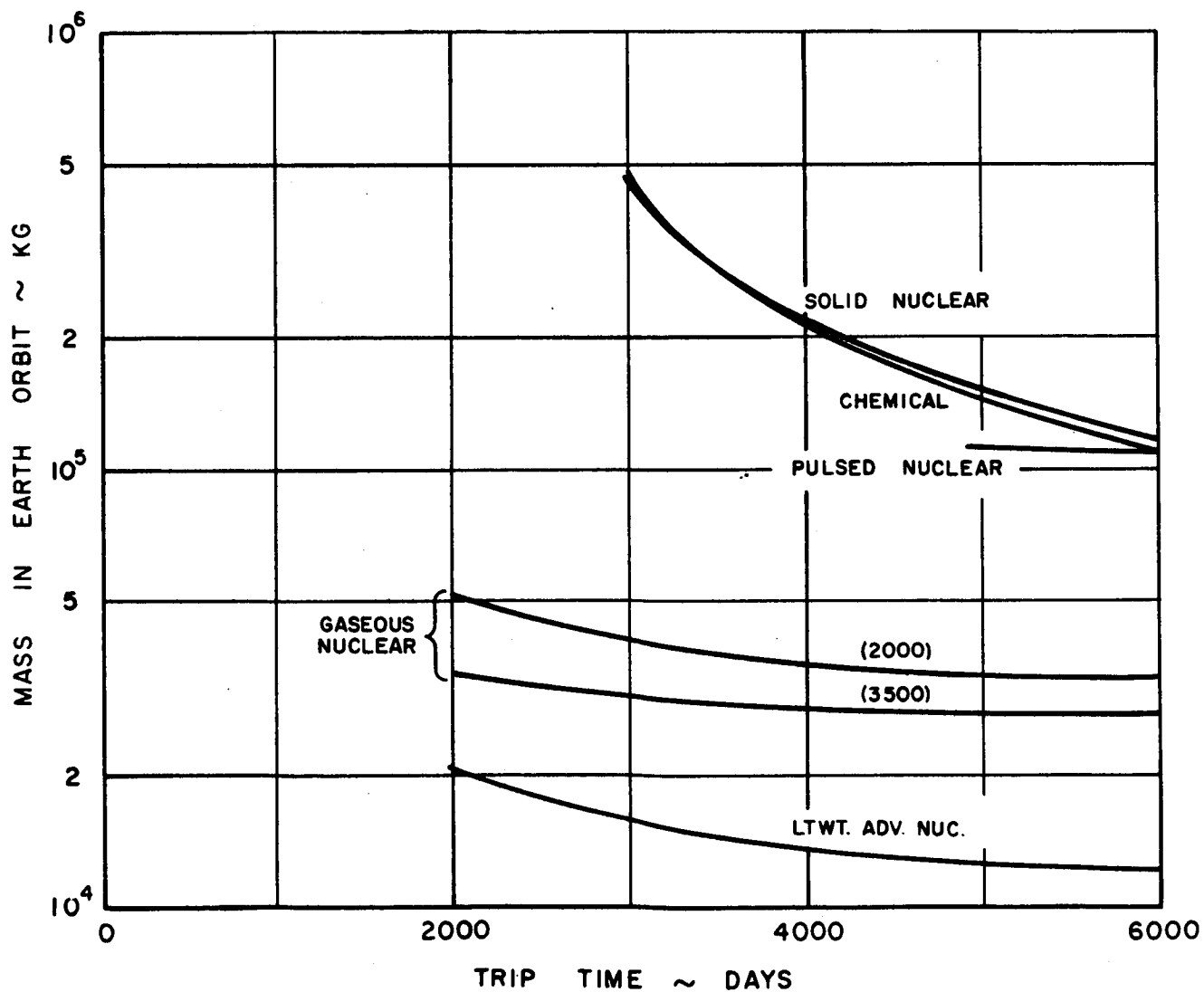


PLUTO FLYBYS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

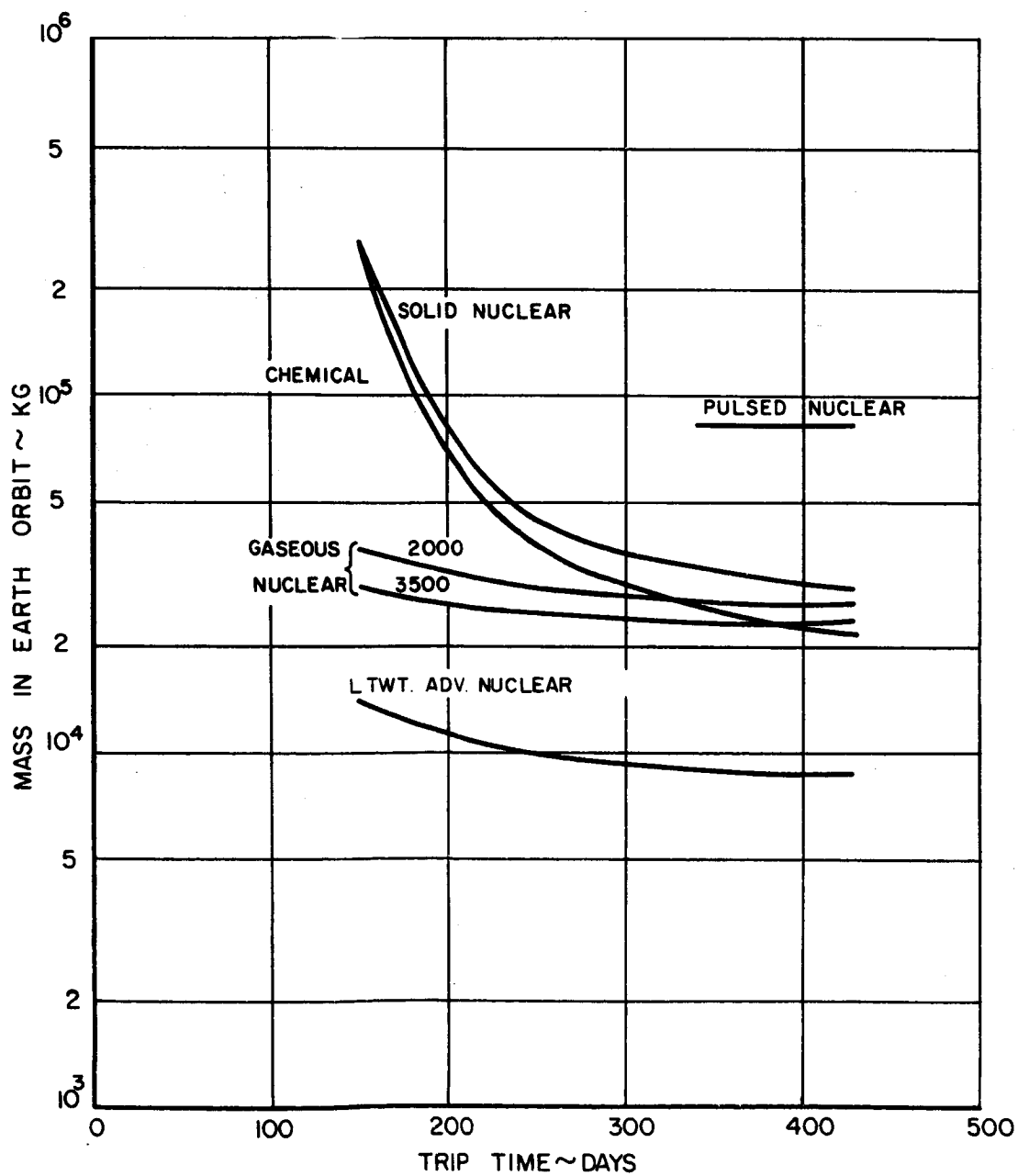


CERES FLYBY

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

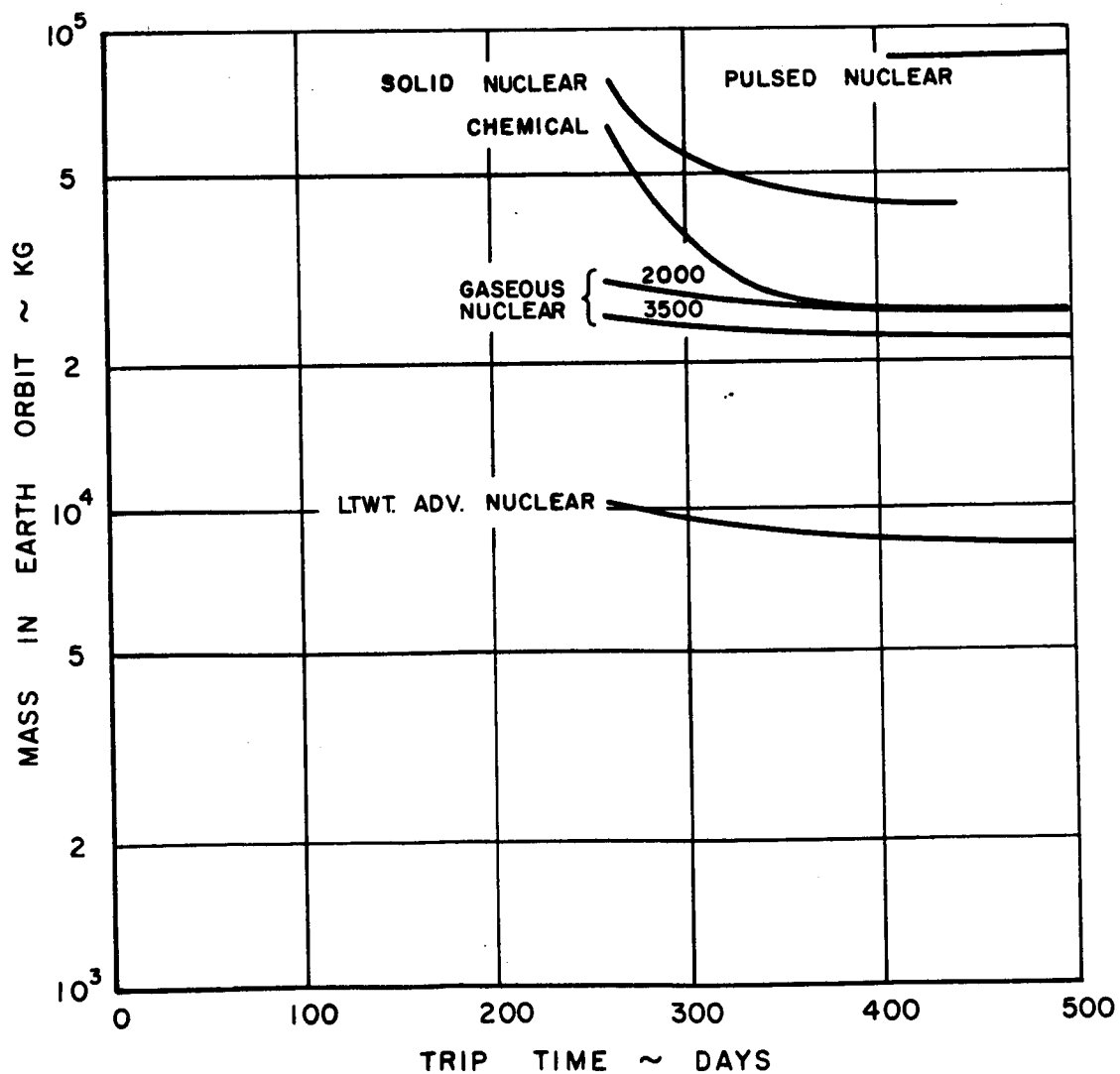


SCHWASSMANN — WACHMANN II FLYBYS

$$W_{PL} = 4536 \text{ KG}$$

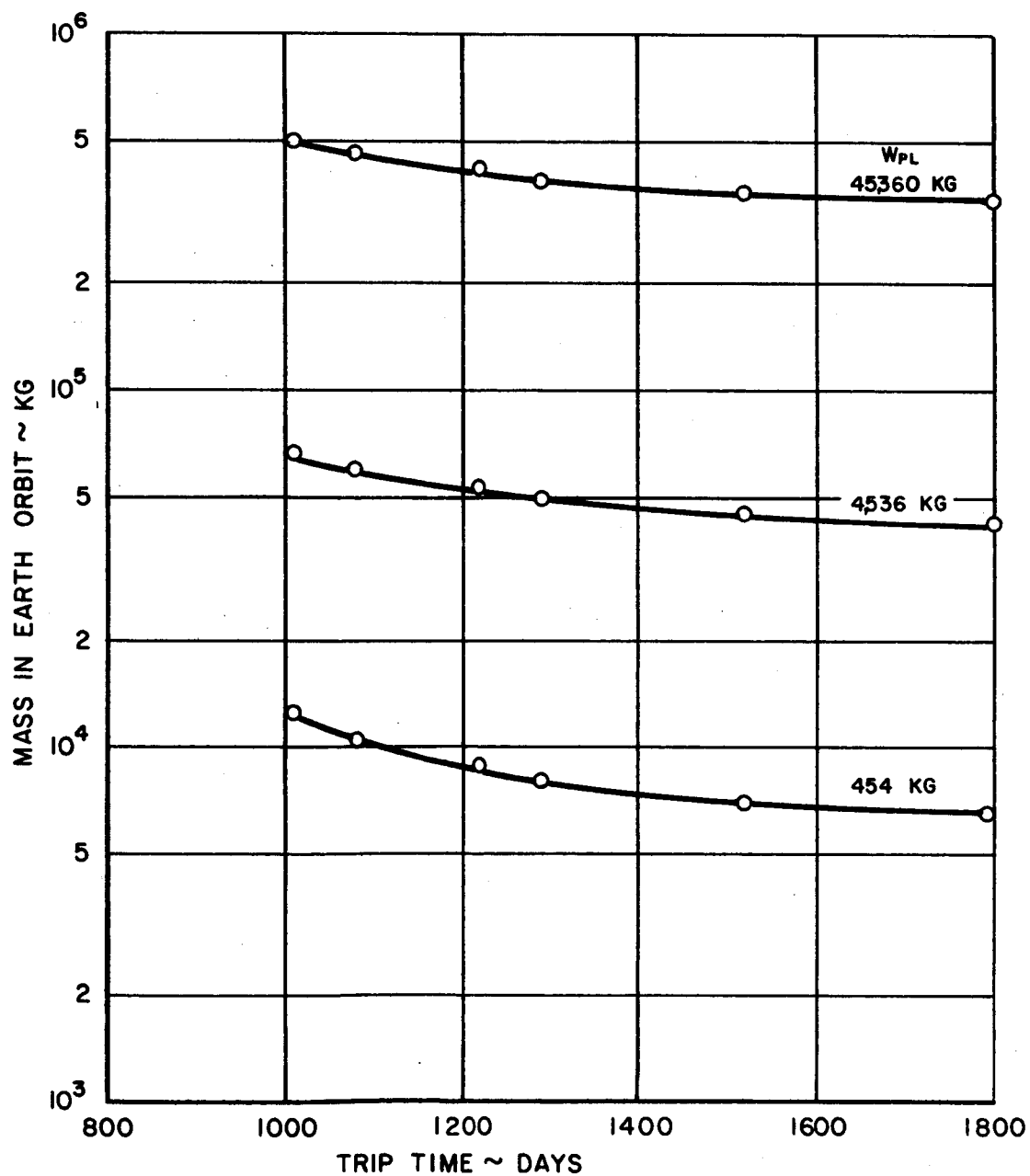
NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600



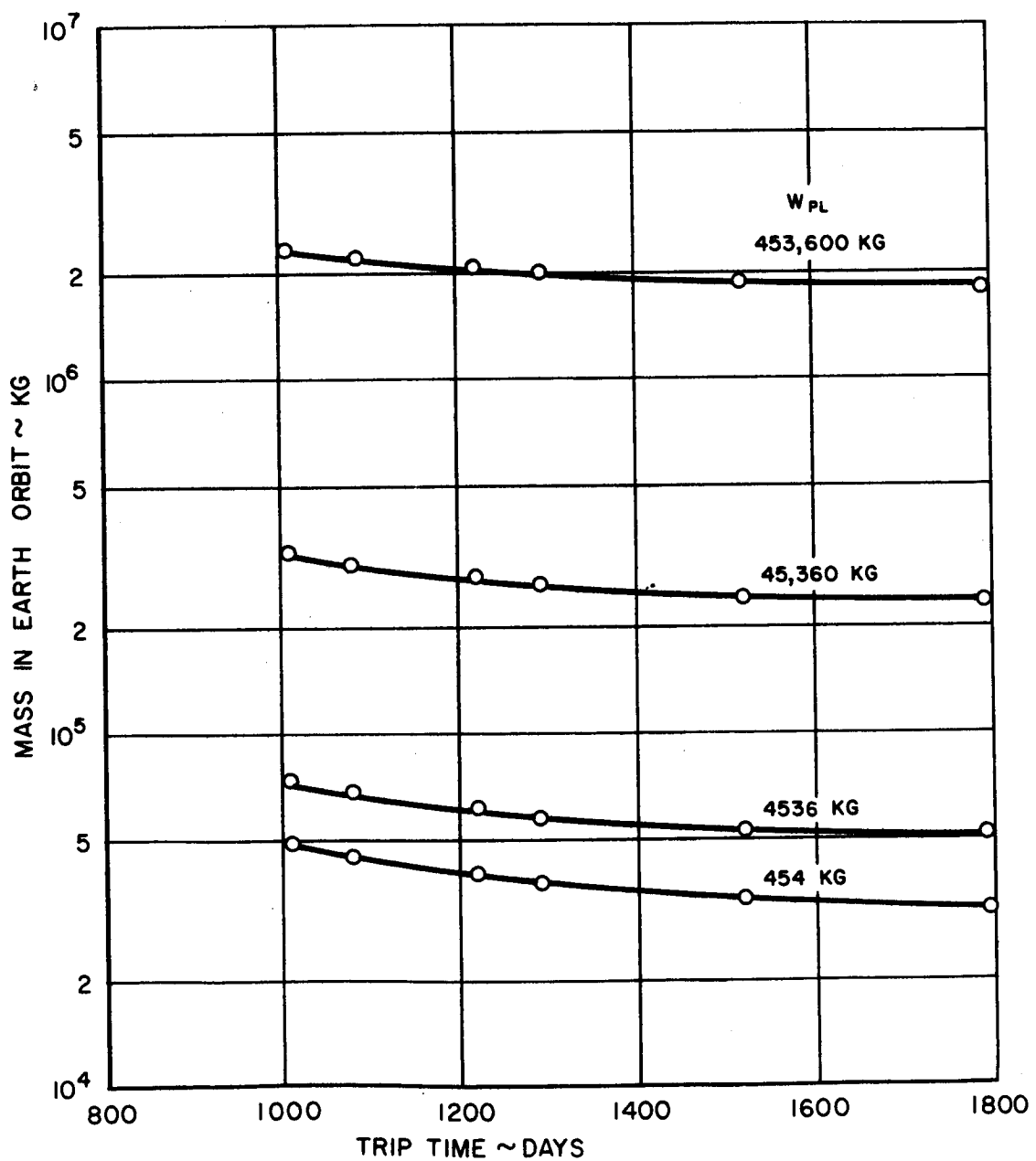
SATURN FLYBY MISSIONS WITH JUPITER SWINGBY

CHEMICAL

 $I_{sp} = 430 \text{ SEC}$ 

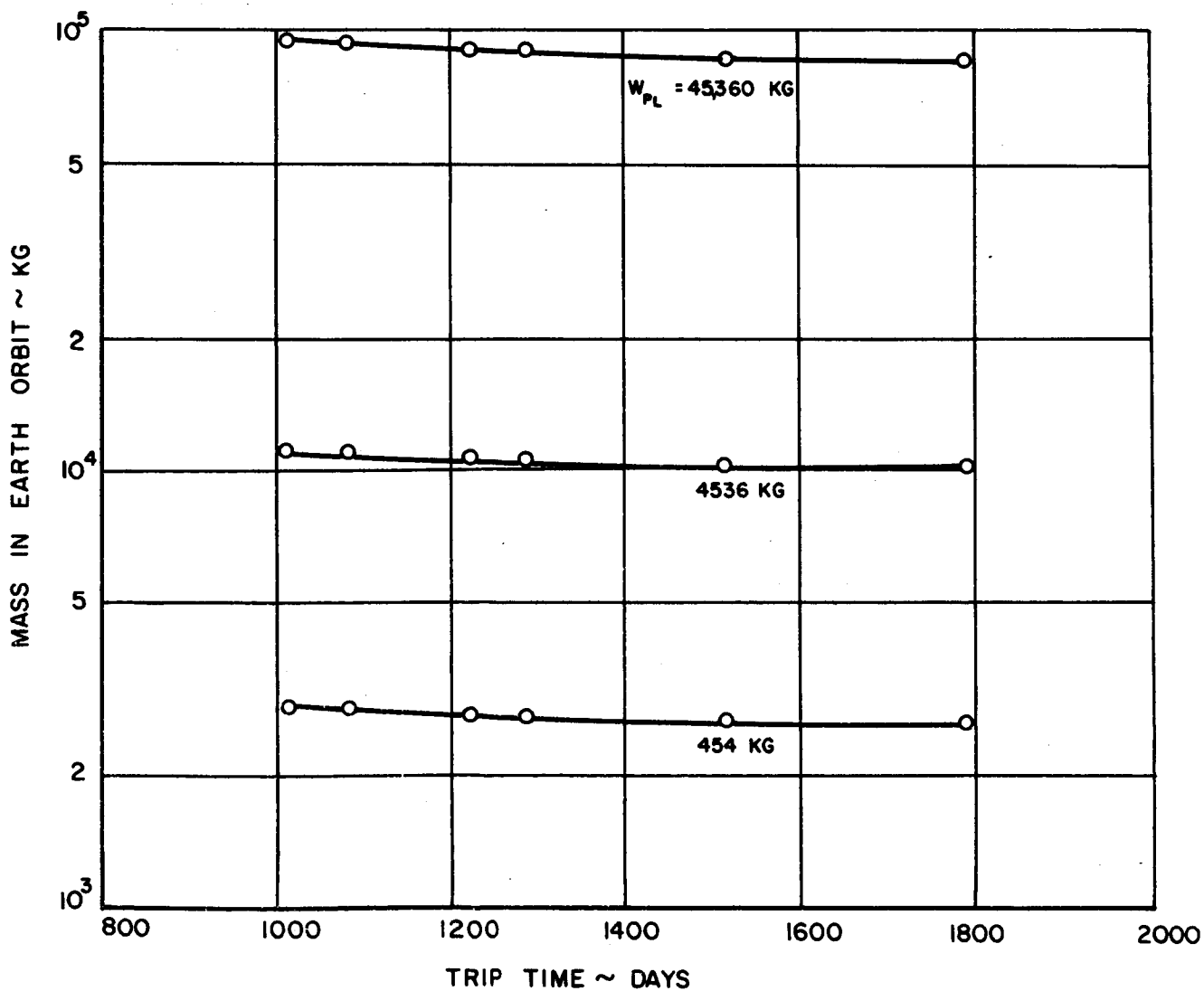
SATURN FLYBY MISSIONS WITH JUPITER SWINGBY

SOLID-CORE NUCLEAR

 $I_{SP} = 800 \text{ SEC}$ 

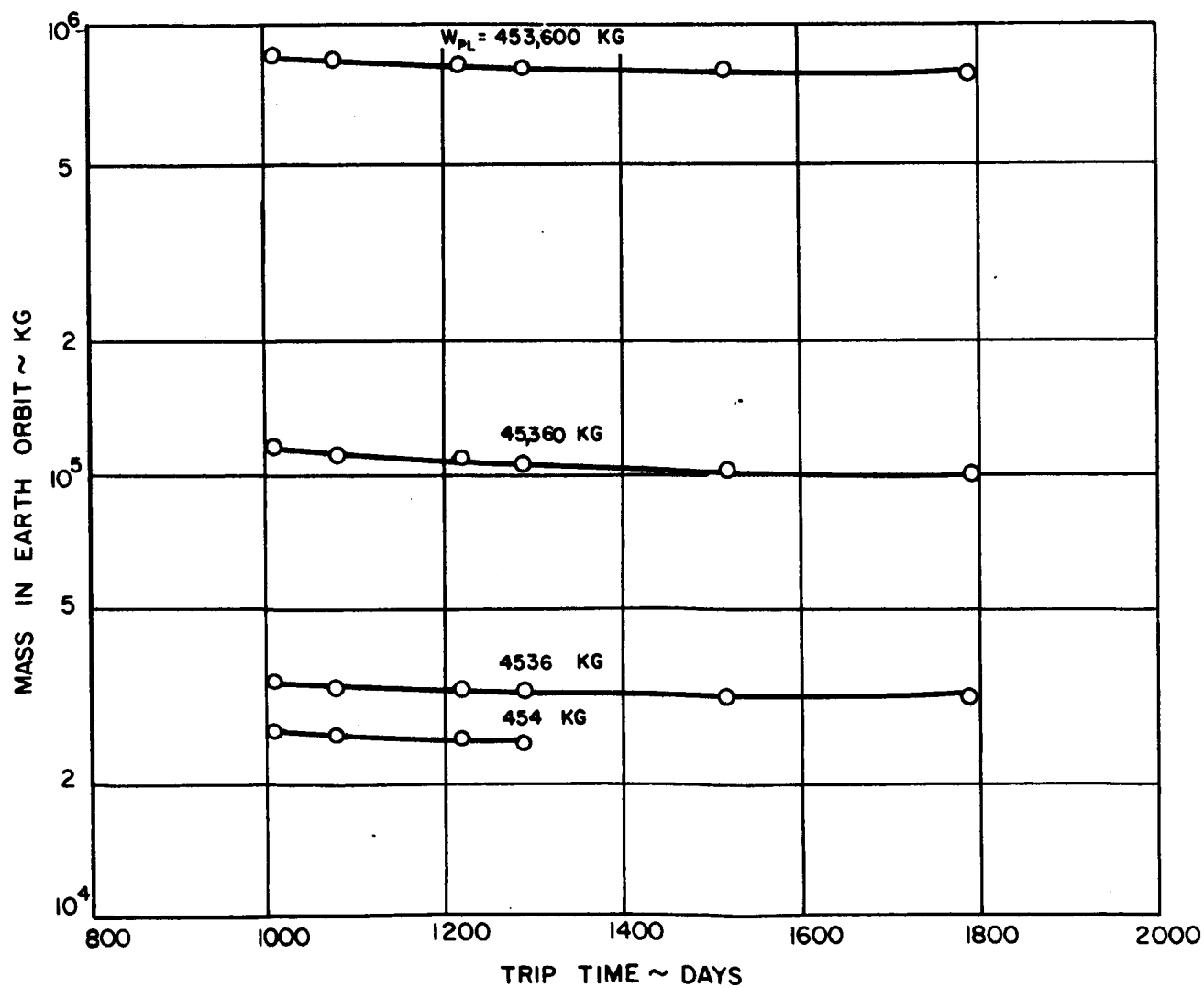
SATURN FLYBY MISSIONS WITH JUPITER SWINGBY

LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{sp} = 2000 \text{ SEC}$ 

SATURN FLYBY MISSIONS WITH JUPITER SWINGBYS

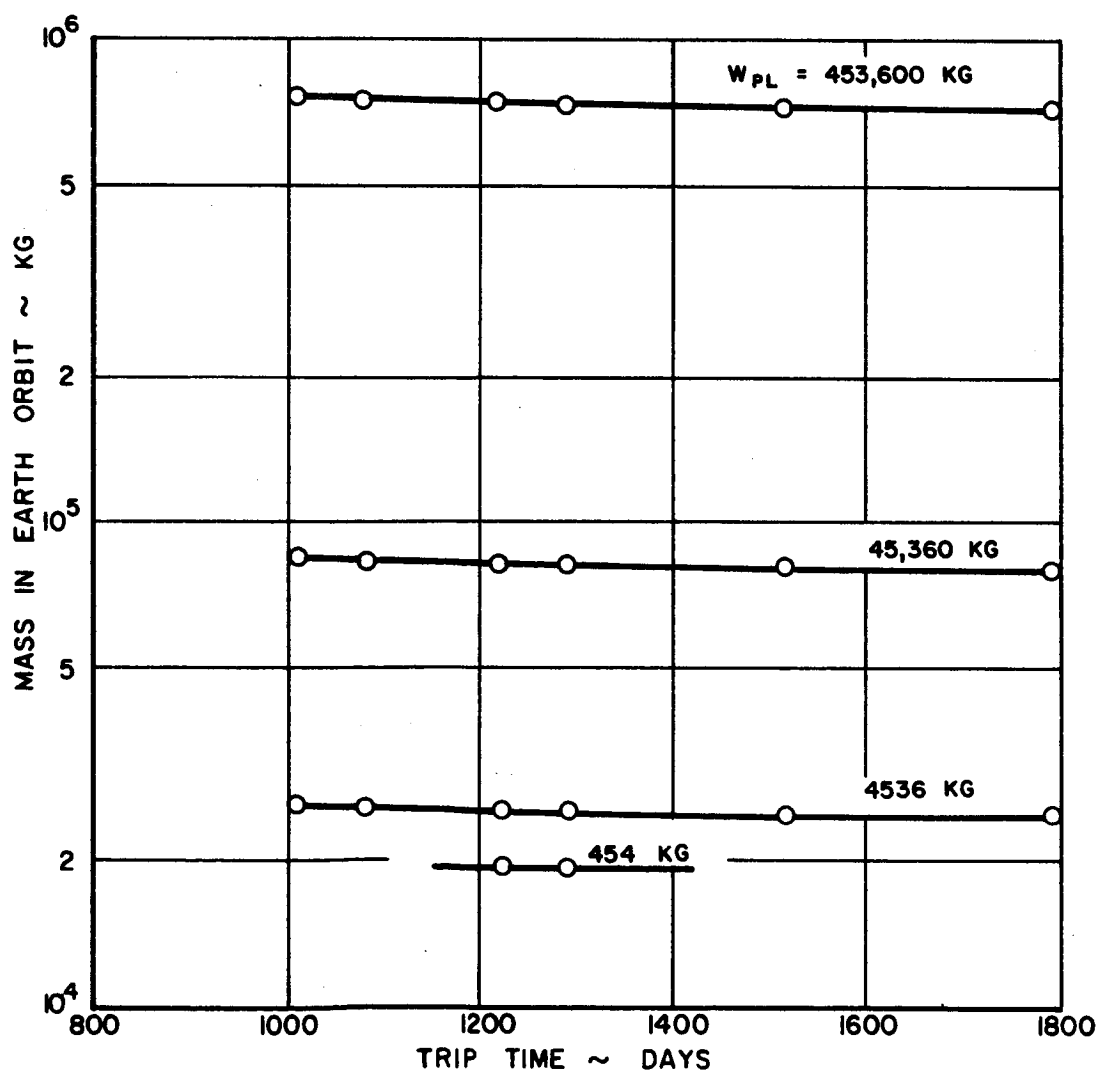
GASEOUS - CORE NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ 

SATURN FLYBY MISSIONS WITH JUPITER SWINGBY

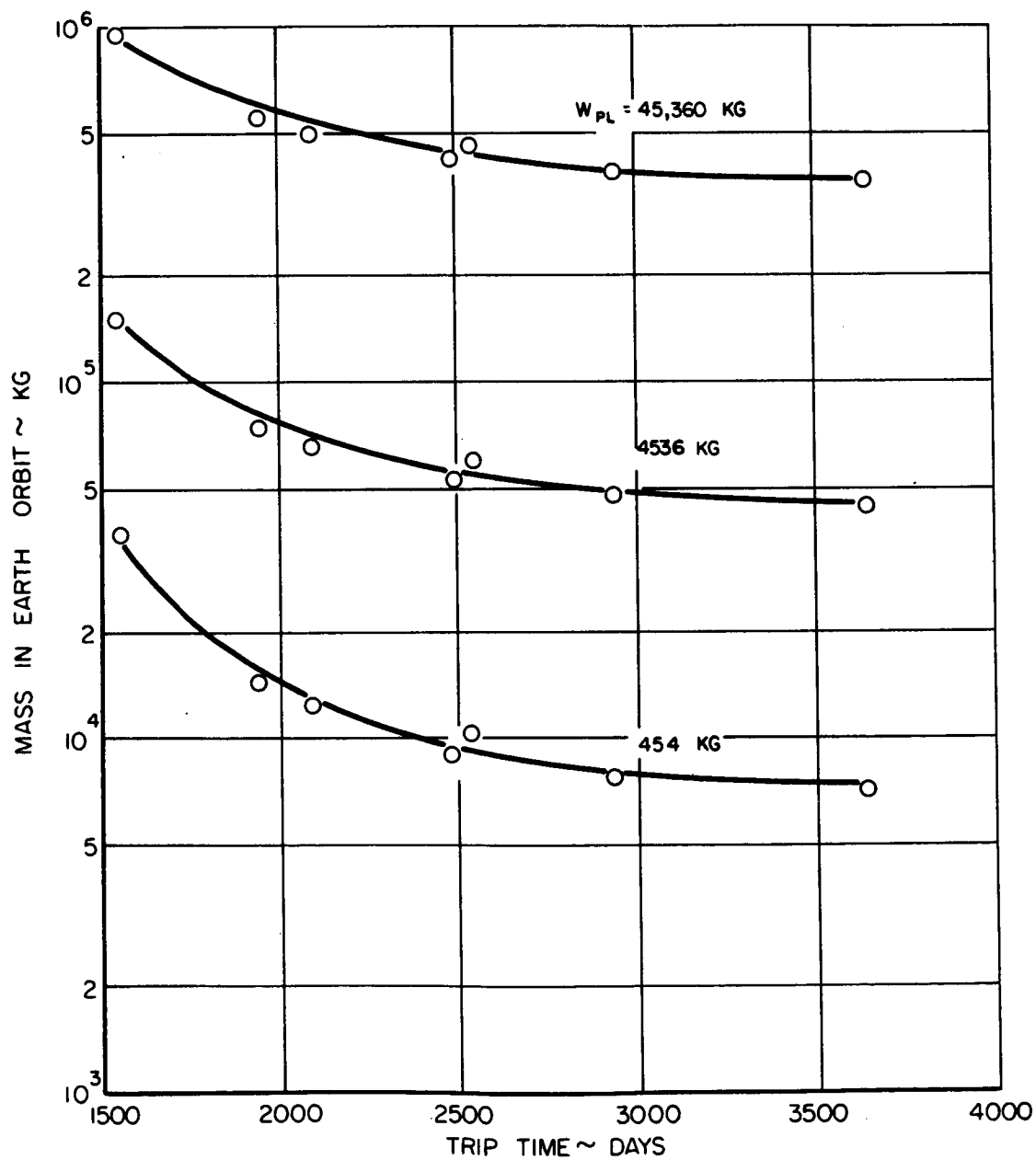
GASEOUS-CORE NUCLEAR

$$I_{SP} = 3500 \text{ SEC}$$



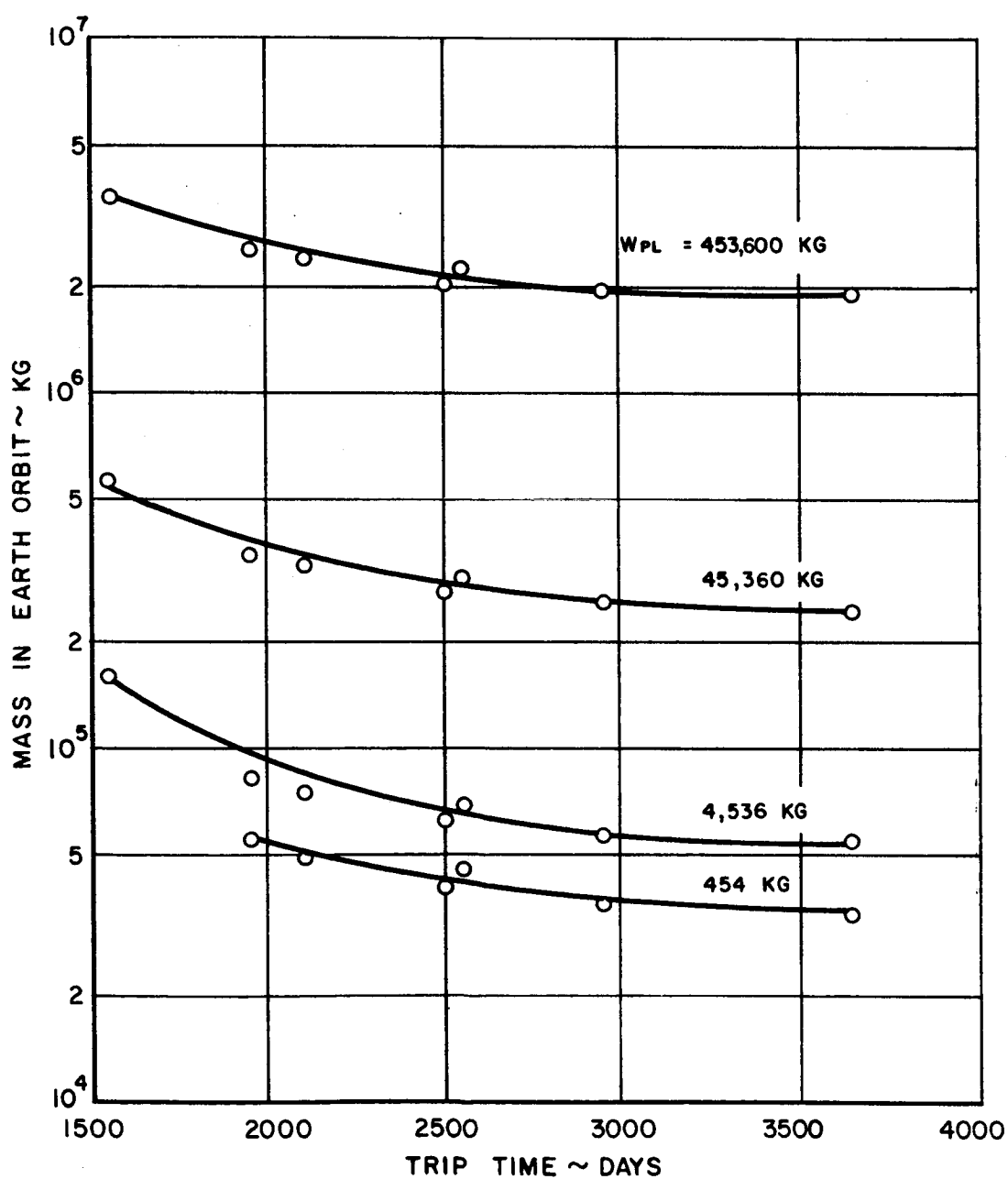
URANUS FLYBY MISSIONS WITH JUPITER SWINGBY

CHEMICAL

 $I_{SP} = 430 \text{ SEC}$ 

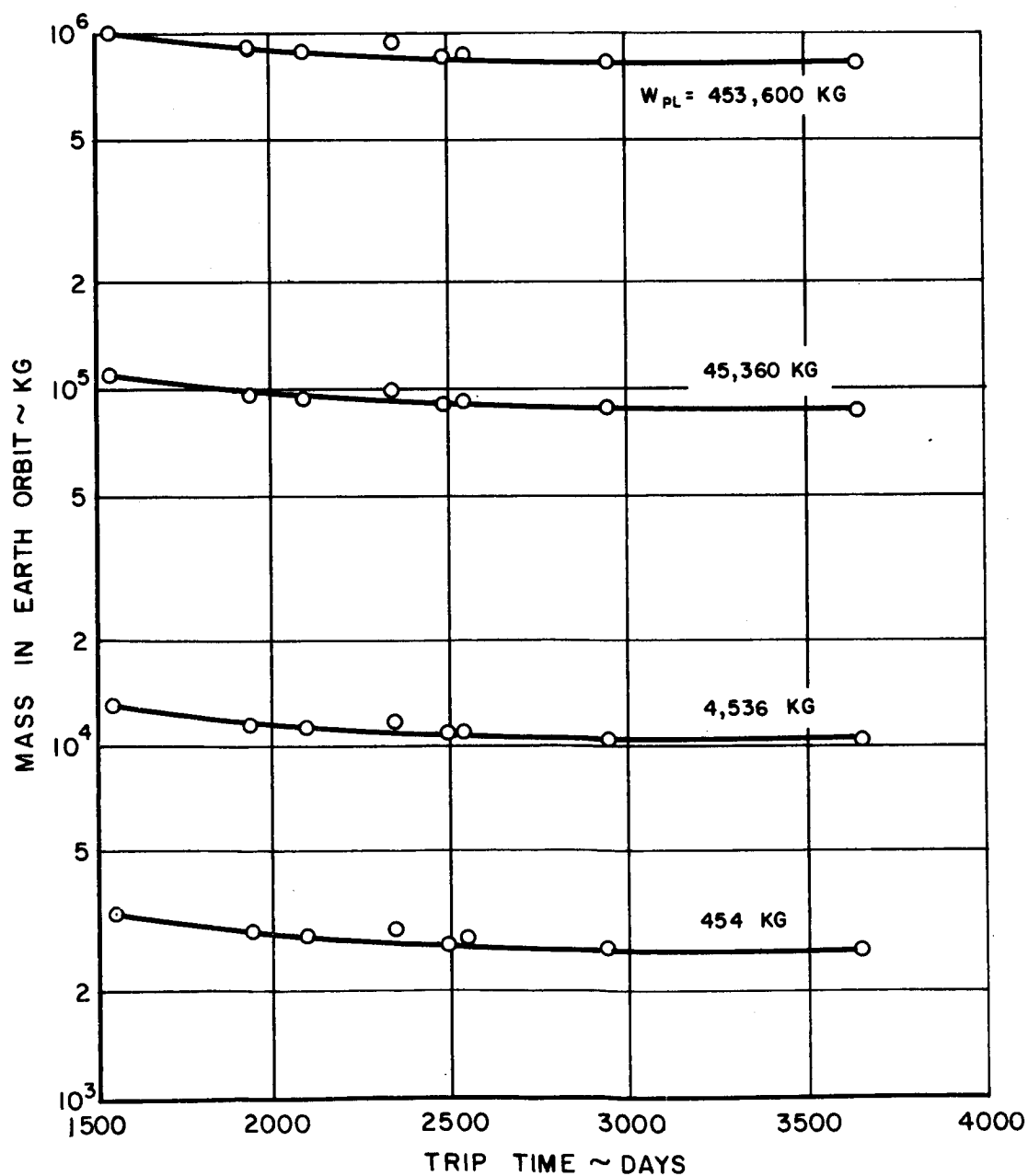
URANUS FLYBY MISSIONS WITH JUPITER SWINGBY

SOLID - CORE NUCLEAR

 $I_{SP} = 800 \text{ SEC}$ 

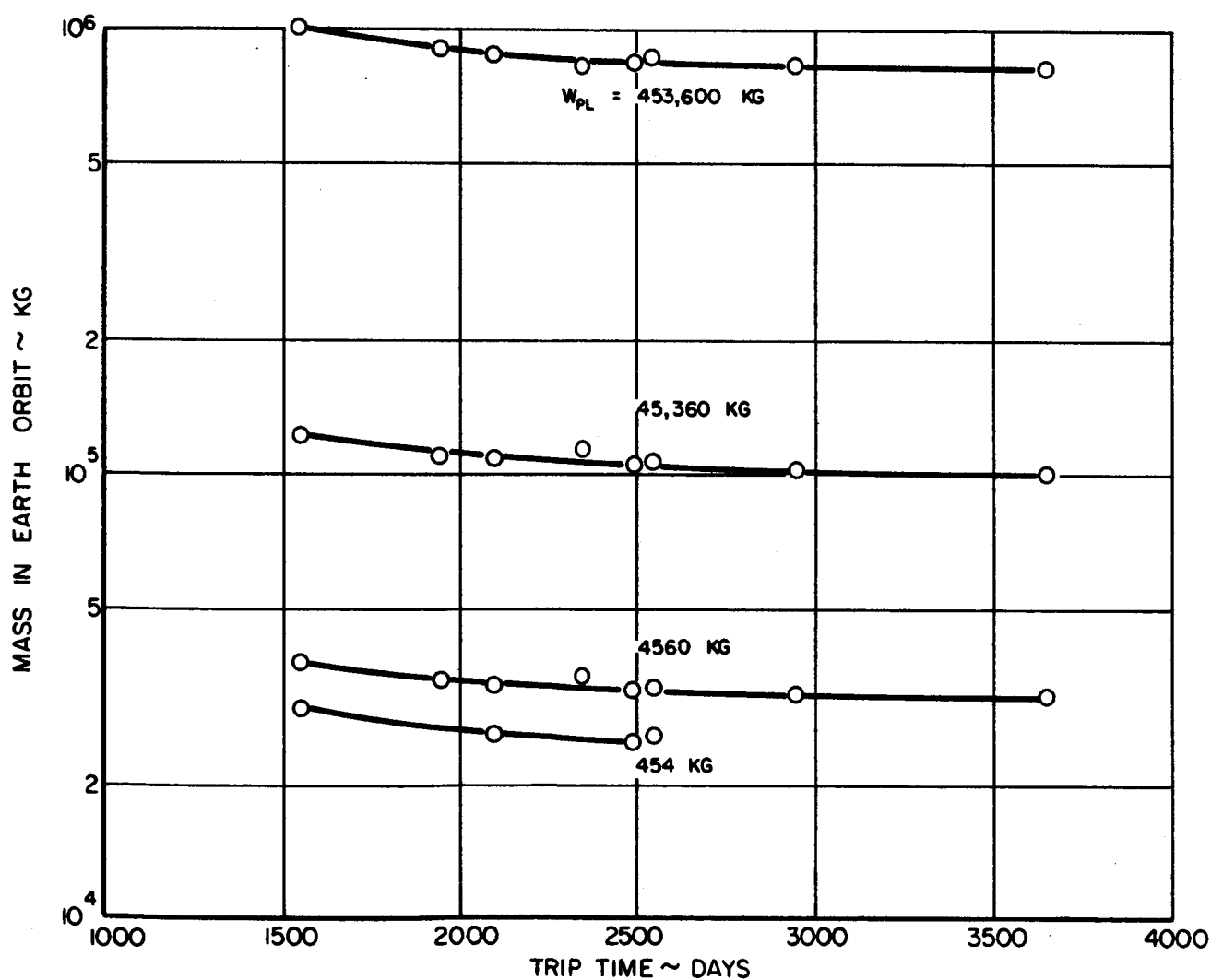
URANUS FLYBY MISSIONS WITH JUPITER SWINGBY

LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ 

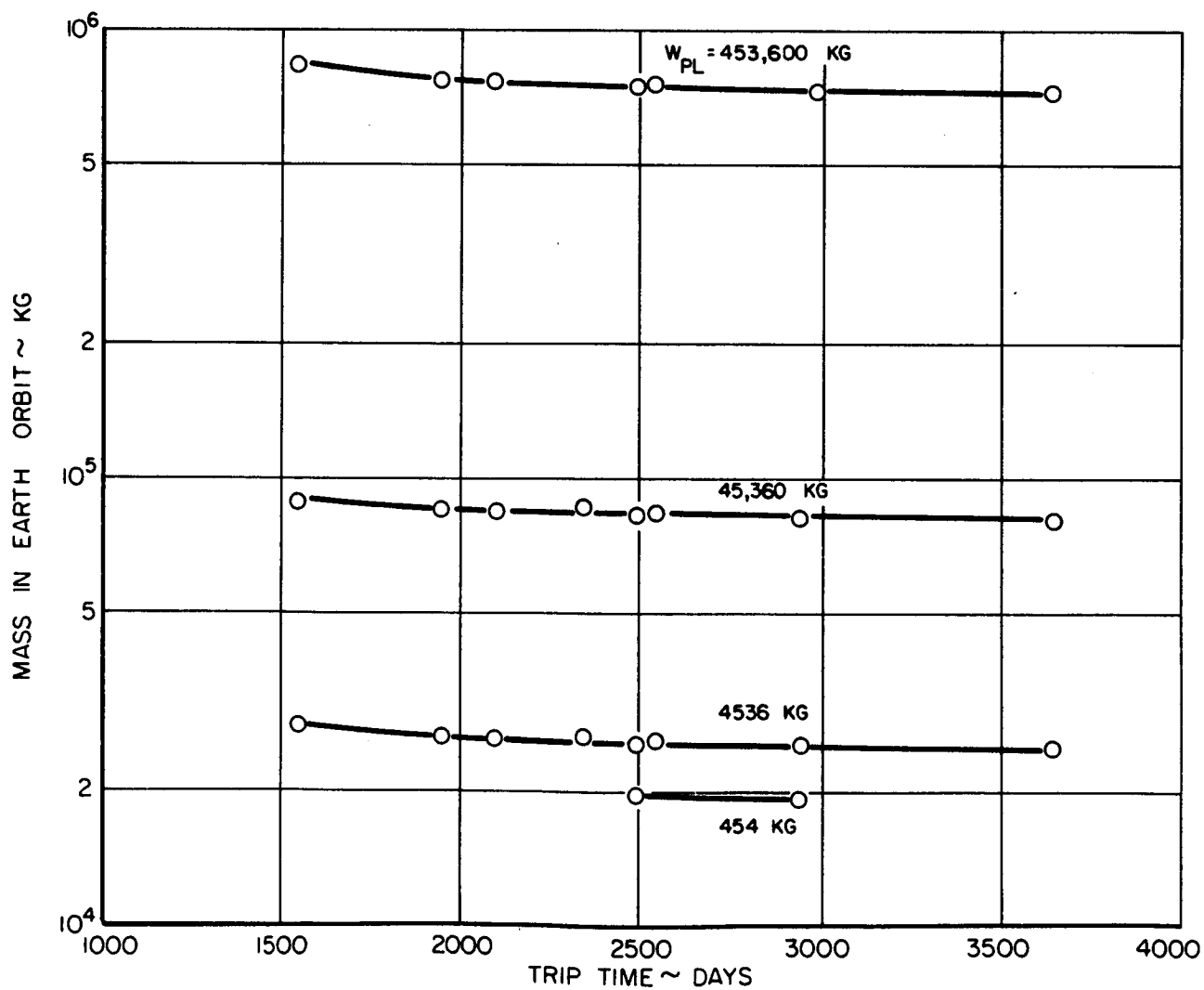
URANUS FLYBY MISSIONS WITH JUPITER SWINGBY

GASEOUS-CORE NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ 

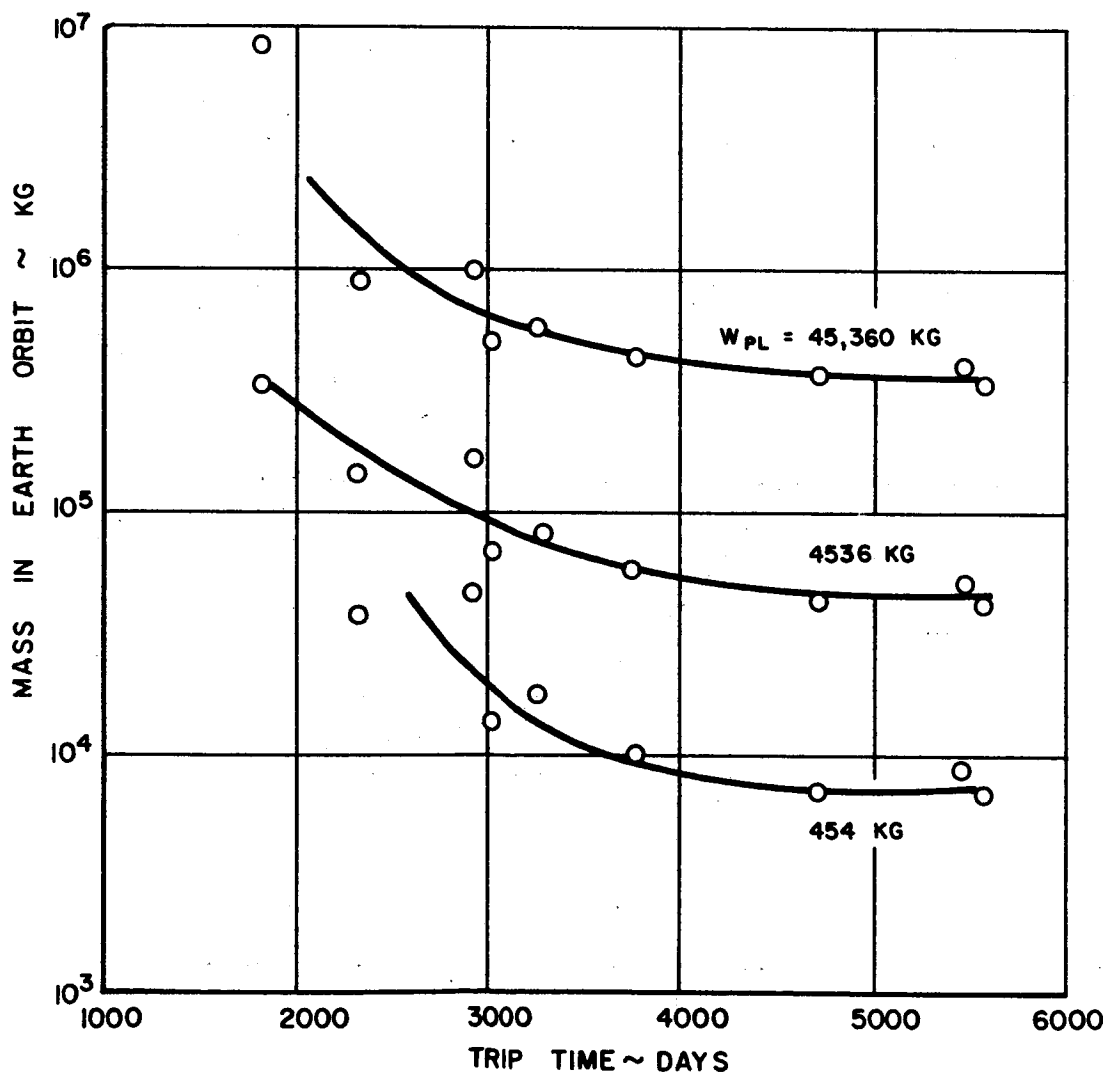
URANUS FLYBY MISSIONS WITH JUPITER SWINGBY

GASEOUS-CORE NUCLEAR

 $I_{SP} = 3500 \text{ SEC}$ 

NEPTUNE FLYBY MISSIONS WITH JUPITER SWINGBY

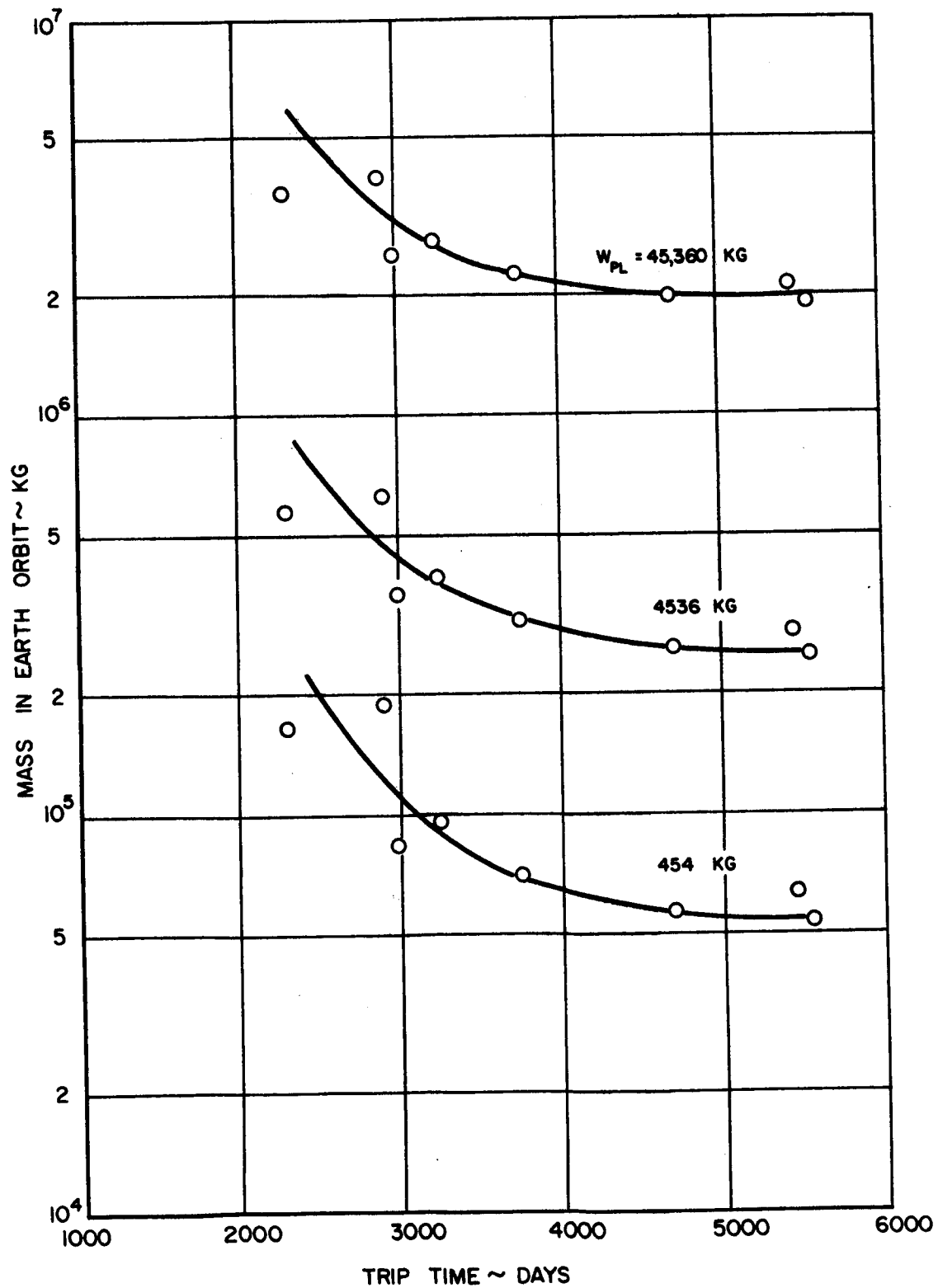
CHEMICAL

 $I_{SP} = 430 \text{ SEC}$ 

NEPTUNE FLYBY MISSIONS WITH JUPITER SWINGBY

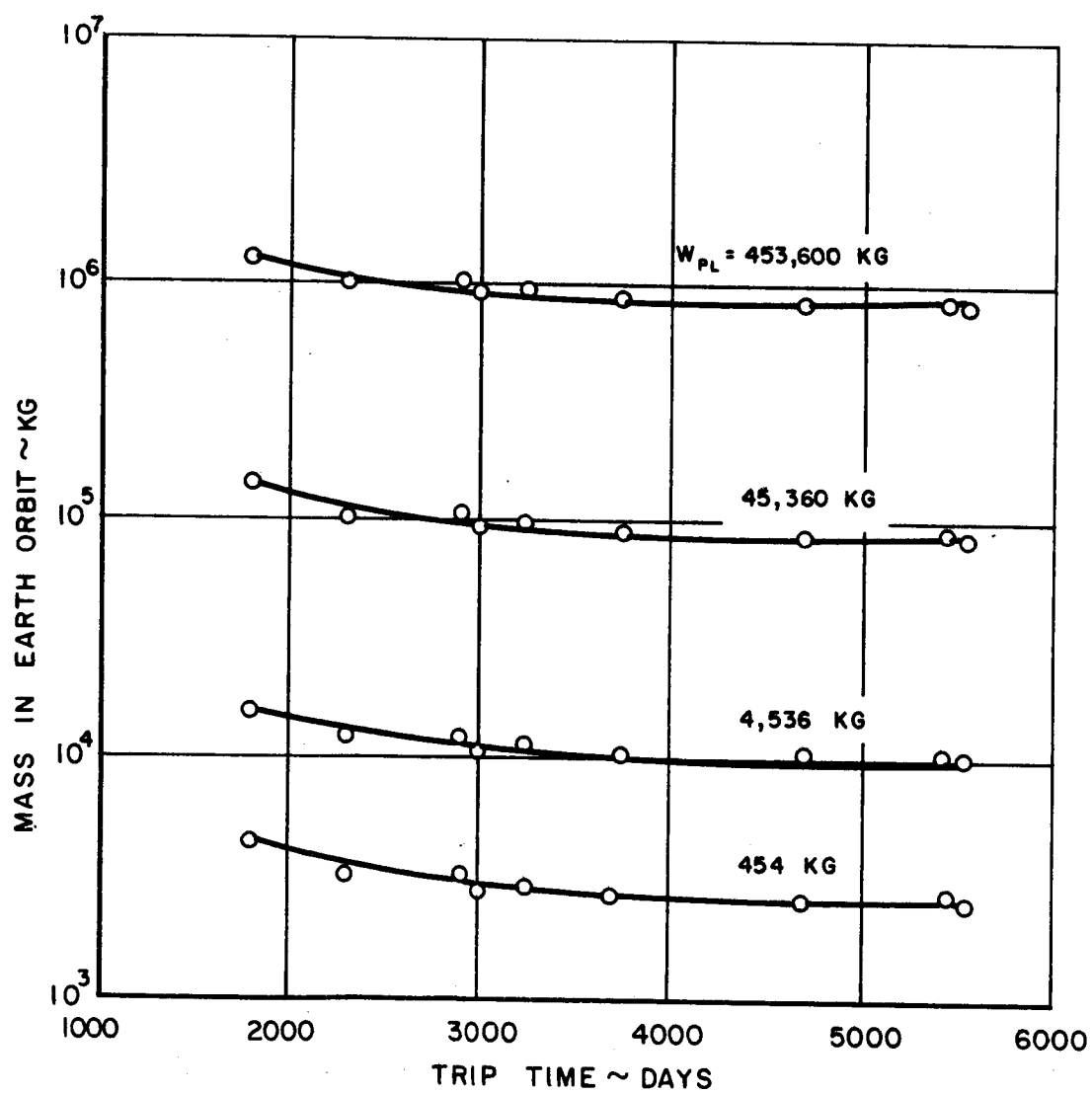
SOLID-CORE NUCLEAR

$$I_{SP} = 800$$



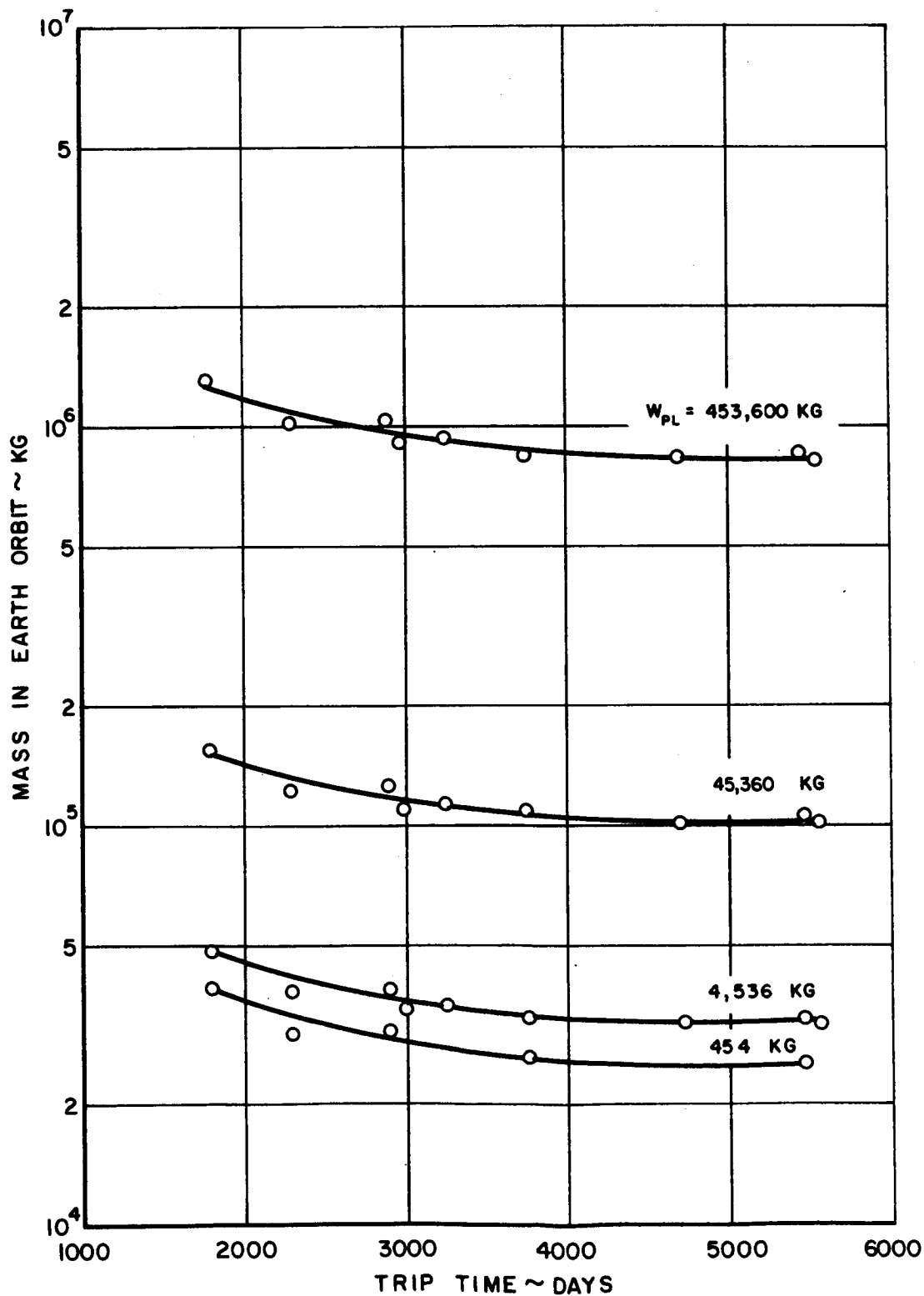
NEPTUNE FLYBY MISSIONS WITH JUPITER SWINGBY

LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ 

NEPTUNE FLYBY MISSIONS WITH JUPITER SWINGBY

GASEOUS - CORE NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ 

NEPTUNE FLYBY MISSIONS WITH JUPITER SWINGBYS

GASEOUS-CORE NUCLEAR

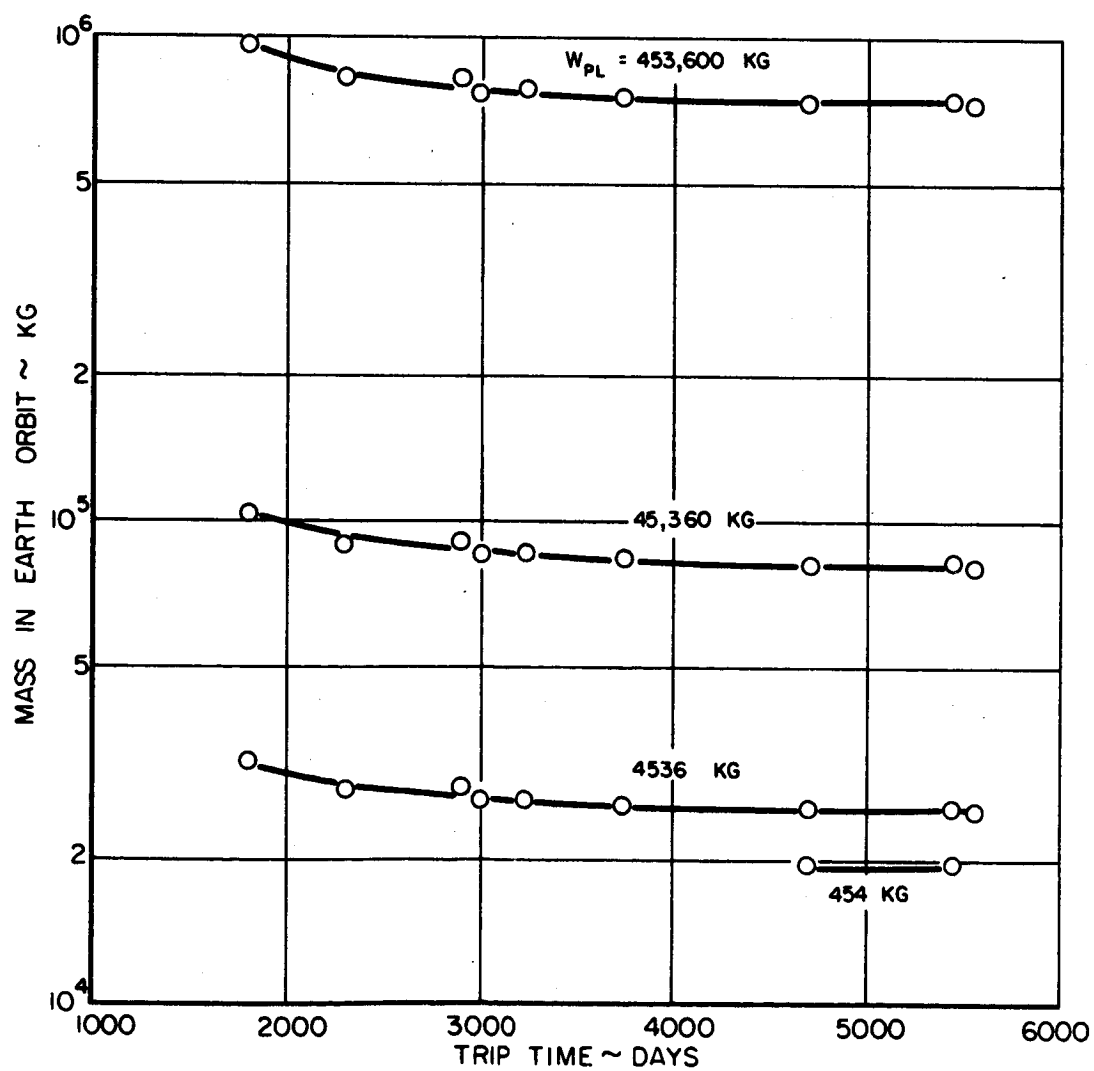
 $I_{SP} = 3500 \text{ SEC}$ 

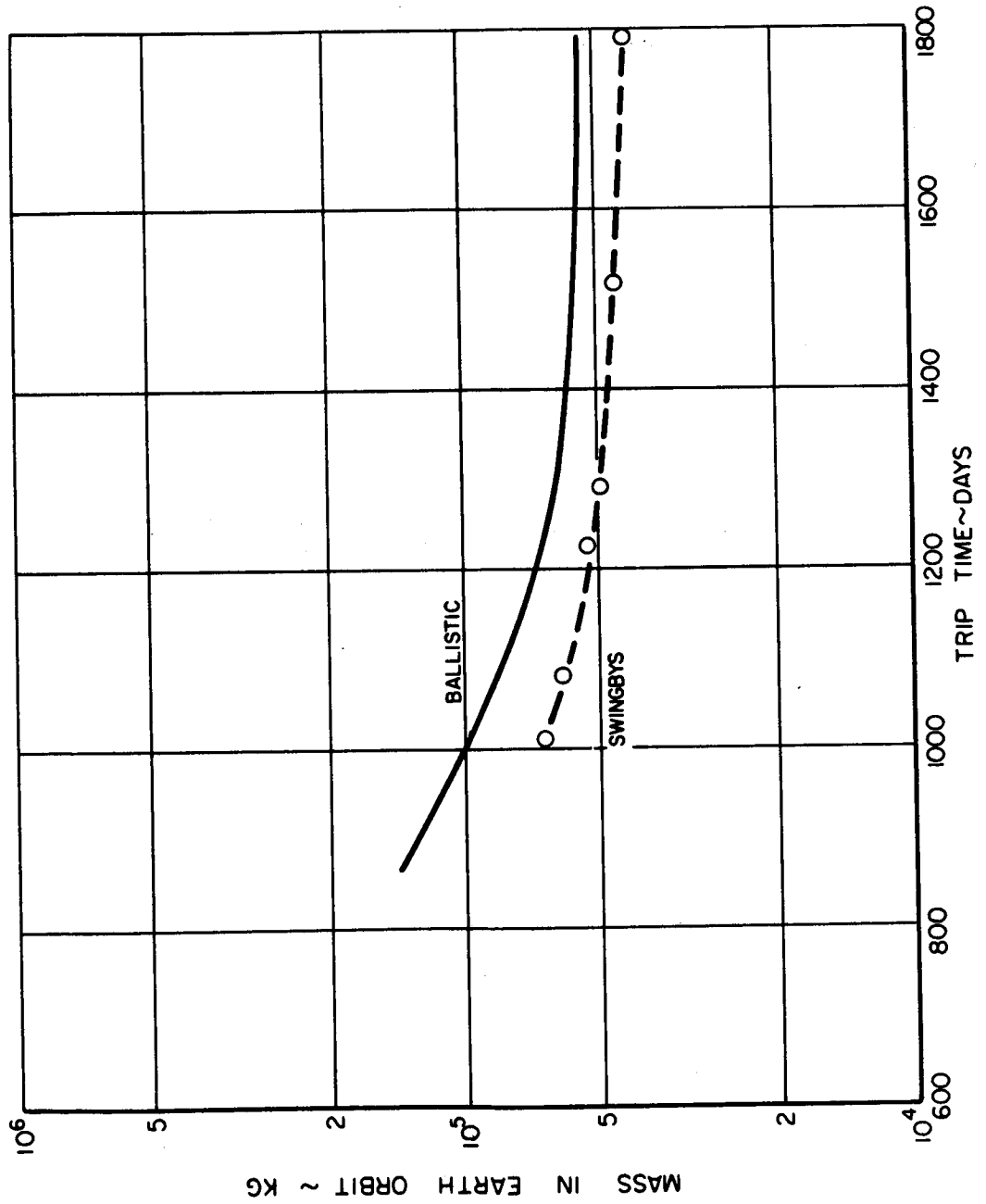
FIG. 146

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SATURN FLYBYS CHEMICAL

$I_{SP} = 430 \text{ SEC}$

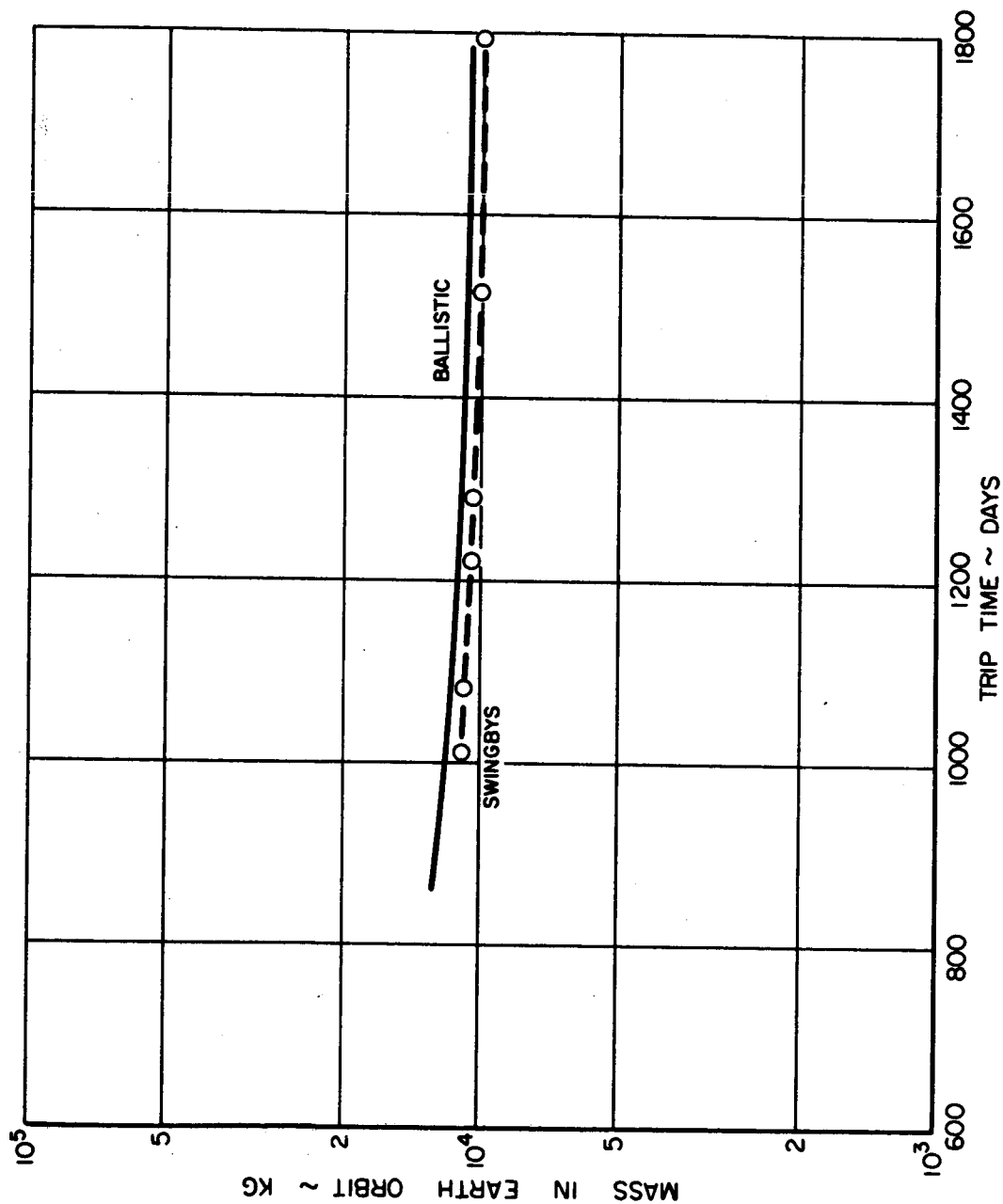
$W_{PL} = 4536 \text{ KG}$



SATURN FLYBYS
LIGHTWEIGHT ADVANCED NUCLEAR

$I_{sp} = 2000 \text{ SEC}$

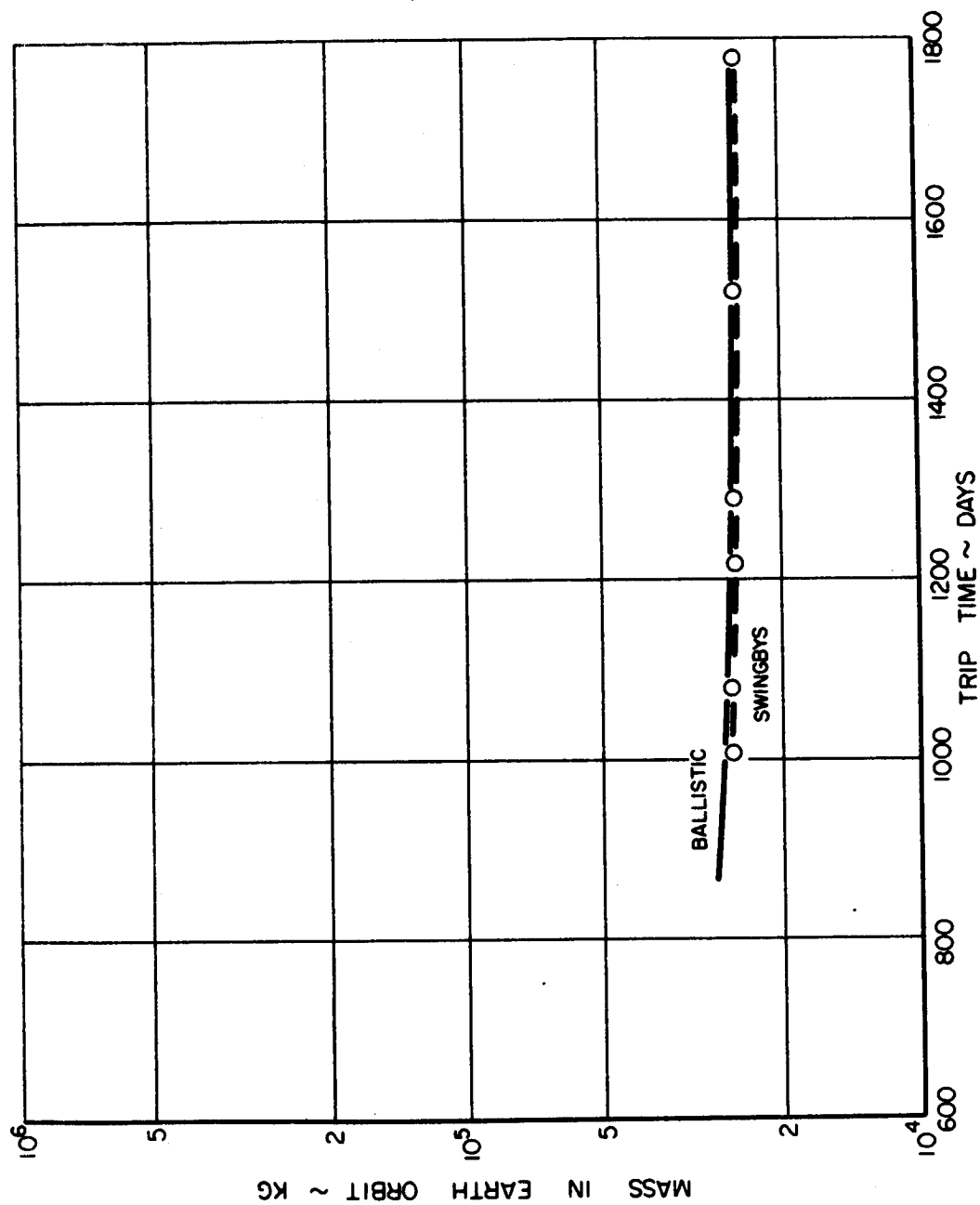
$W_{PL} = 4536 \text{ KG}$



SATURN FLYBYS GASEOUS-CORE NUCLEAR

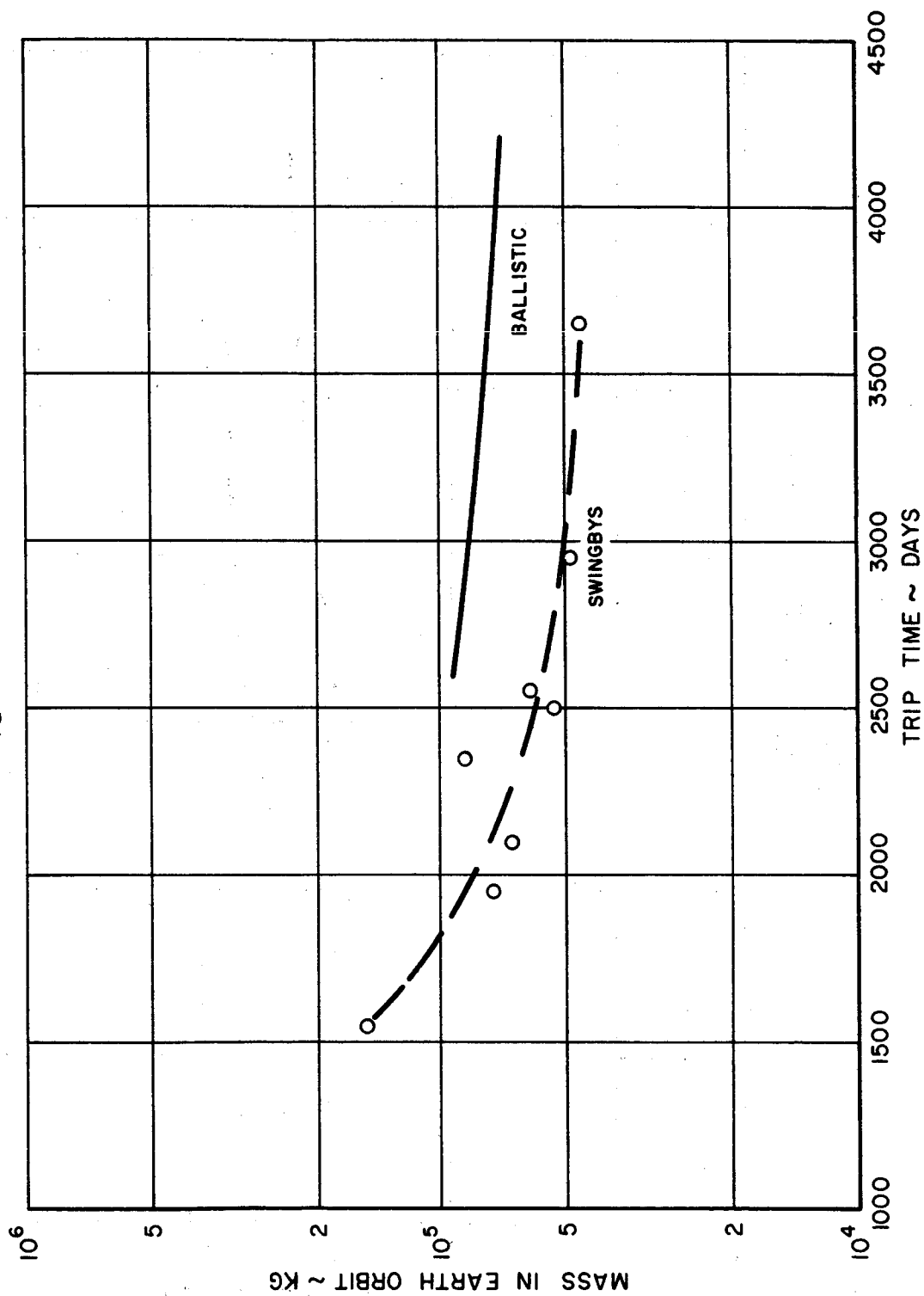
$I_{sp} = 3500 \text{ SEC}$

$W_{PL} = 4536 \text{ KG}$



URANUS FLYBYS

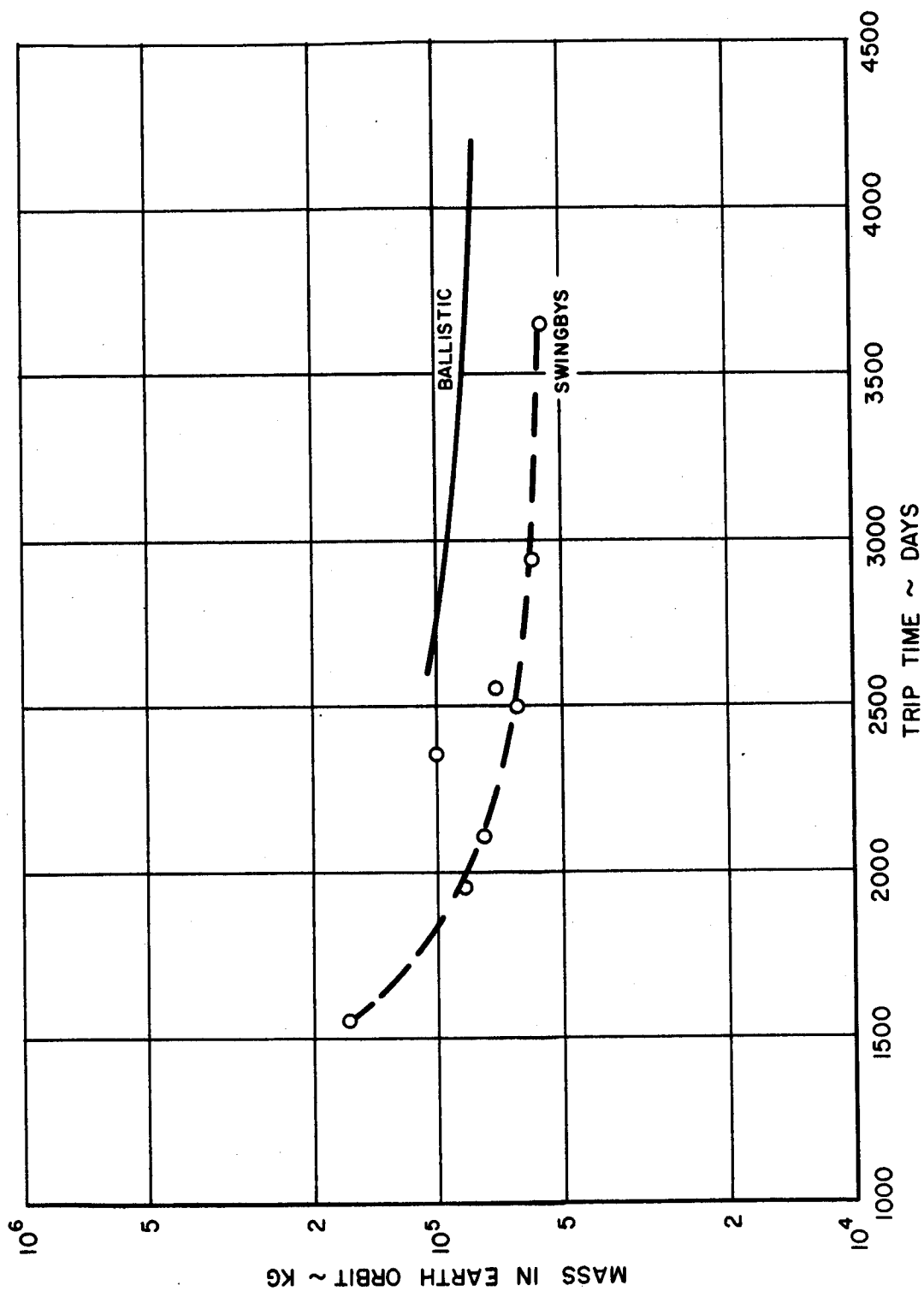
CHEMICAL

 $I_{SP} = 430 \text{ SEC}$ $W_{PL} = 4536 \text{ KG}$ 

URANUS FLYBYS
SOLID-CORE NUCLEAR

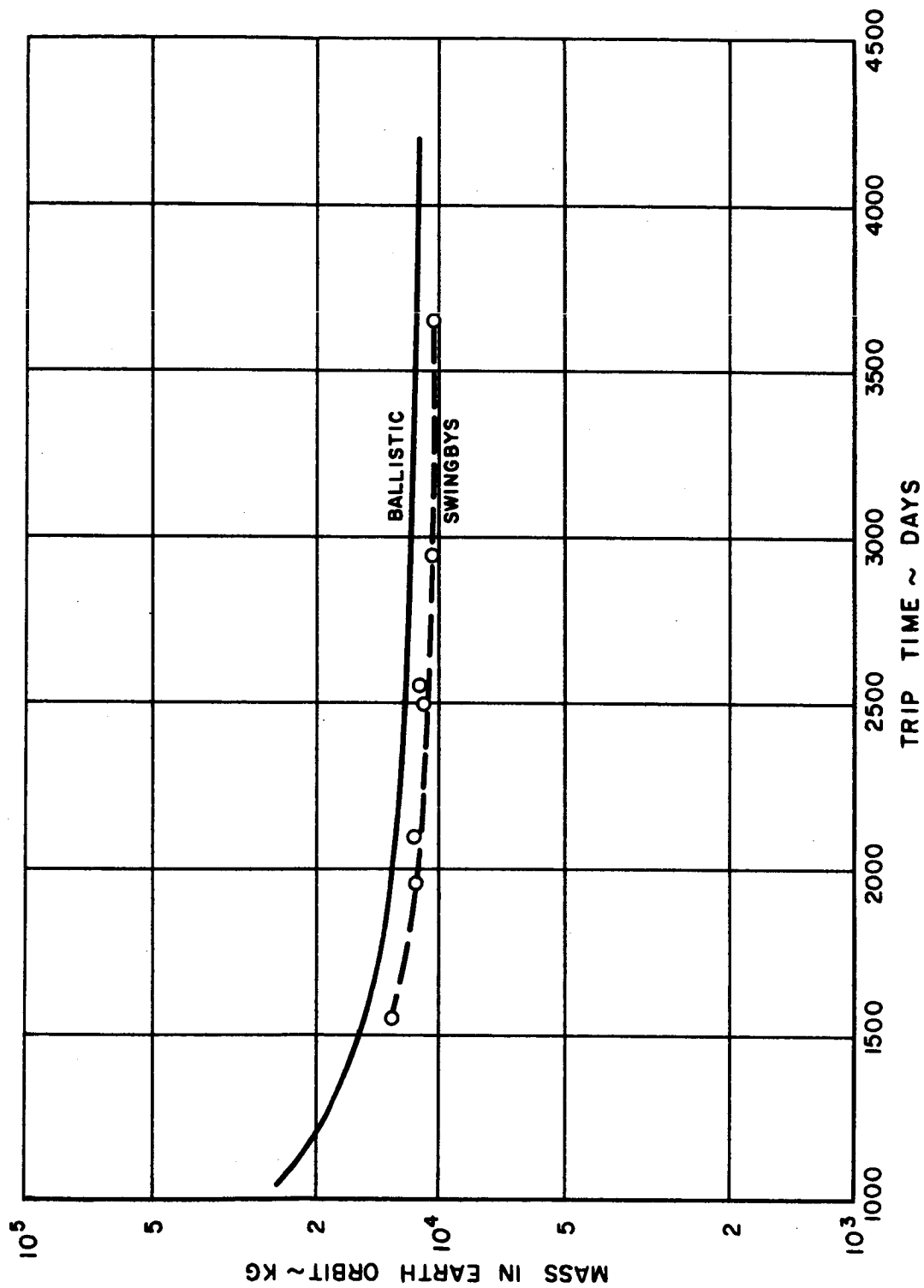
$I_{SP} = 800 \text{ SEC}$

$W_{PL} = 4536 \text{ KG}$



URANUS FLYBYS
LIGHTWEIGHT ADVANCED NUCLEAR

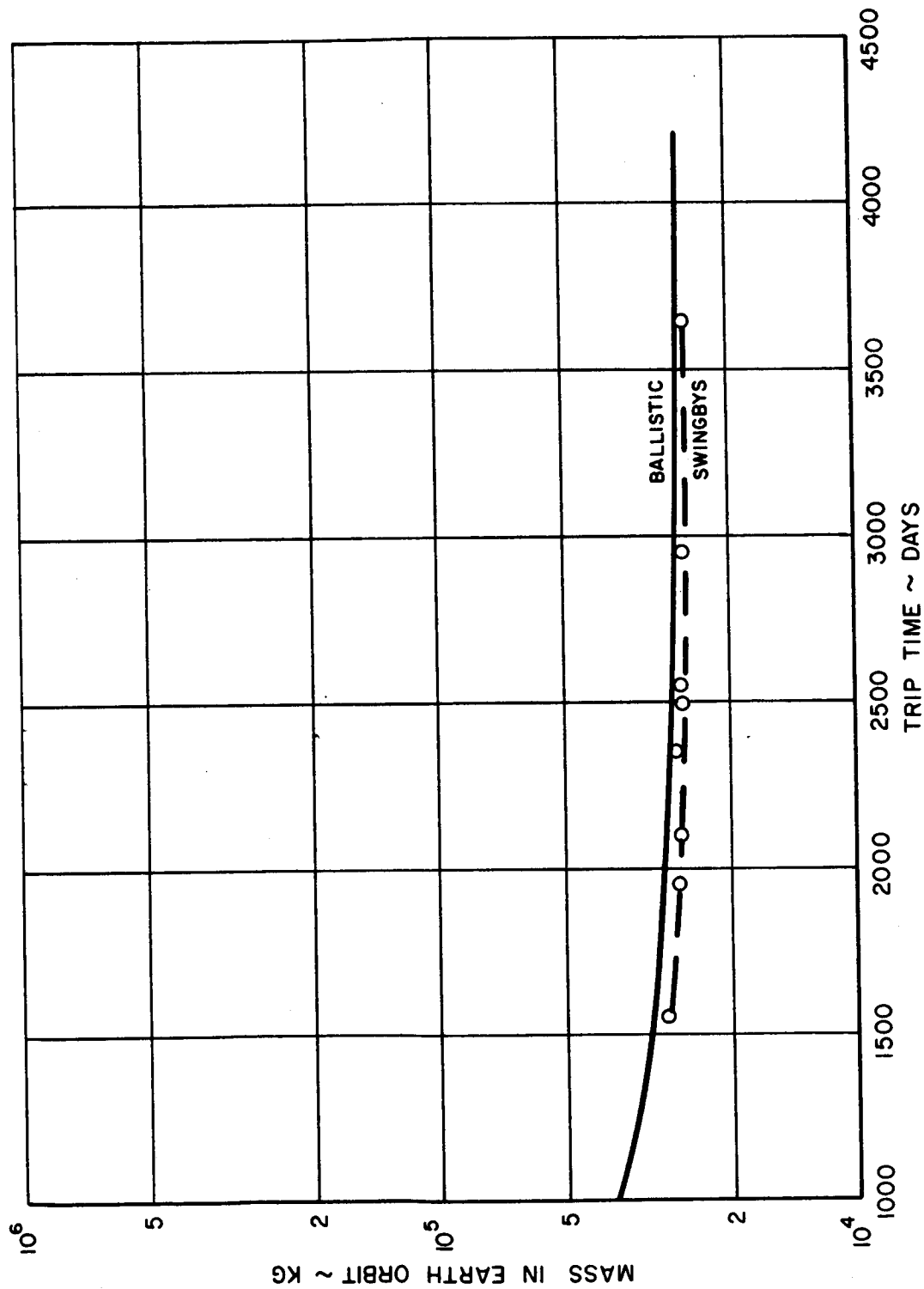
$I_{sp} = 2000 \text{ SEC}$
 $W_{PL} = 4536 \text{ KG}$



URANUS FLYBYS
GASEOUS-CORE NUCLEAR

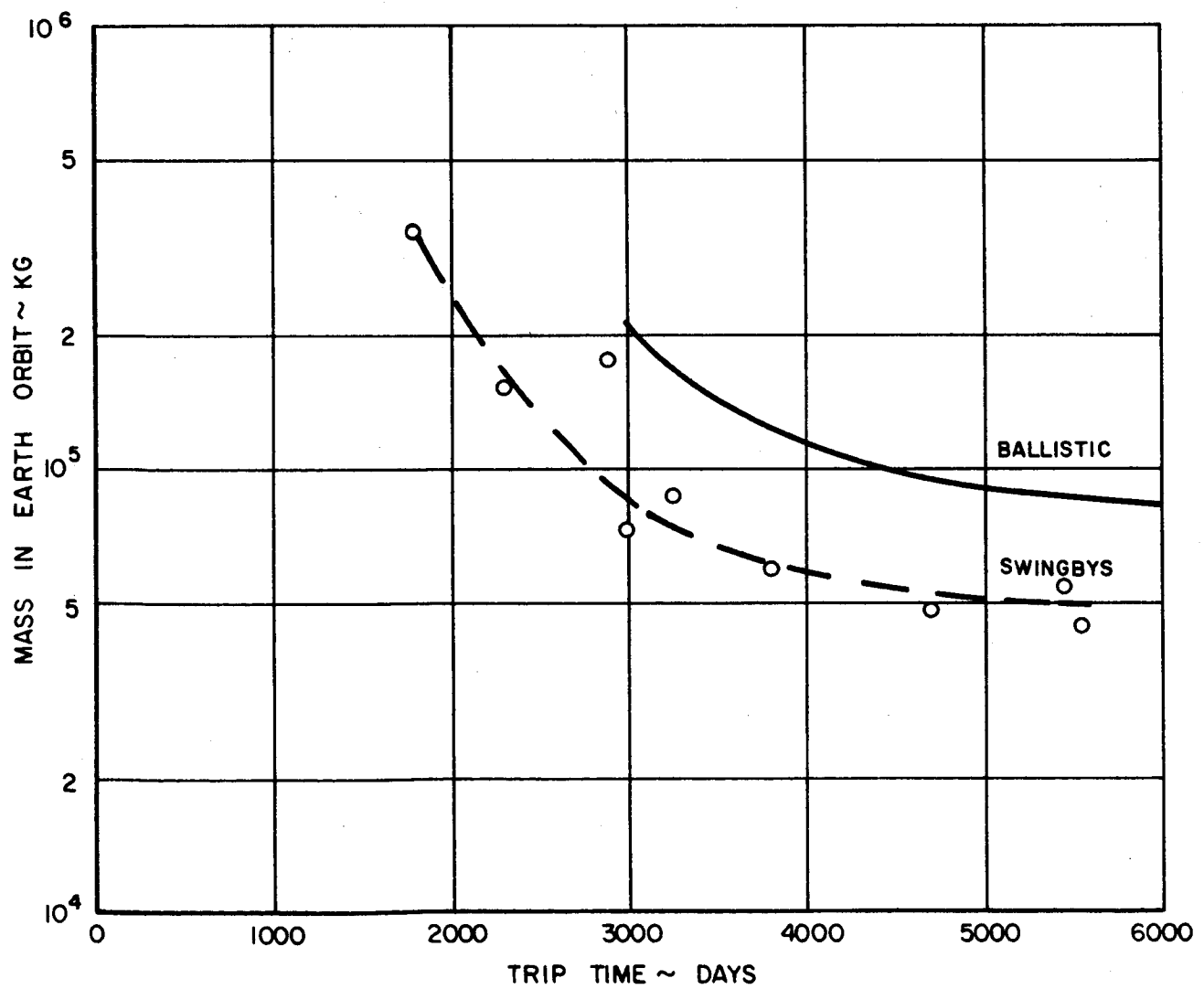
$I_{SP} = 3500 \text{ SEC}$

$W_{PL} = 4536 \text{ KG}$



NEPTUNE FLYBYS

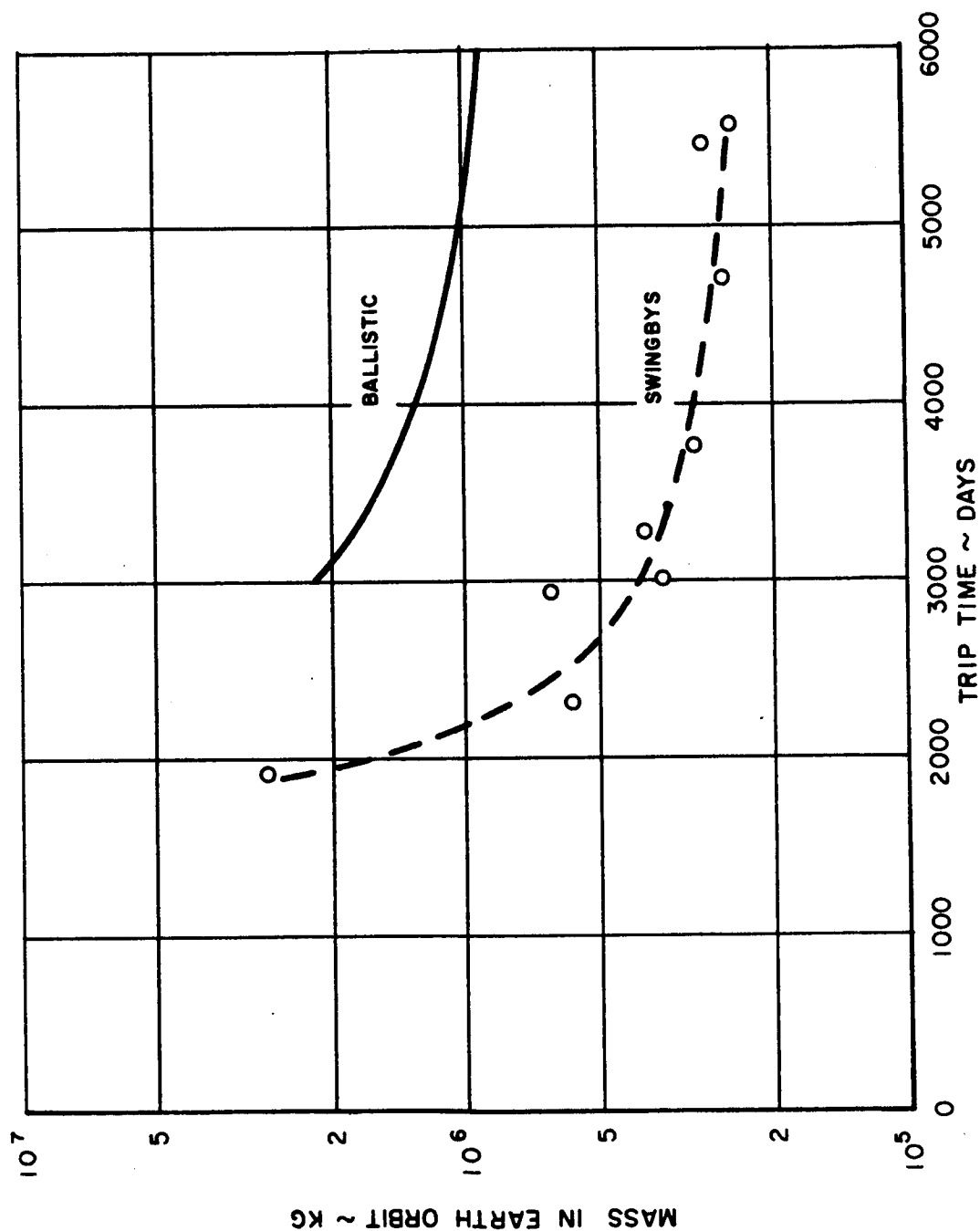
CHEMICAL

 $I_{SP} = 430 \text{ SEC}$ $W_{PL} = 4536 \text{ KG}$ 

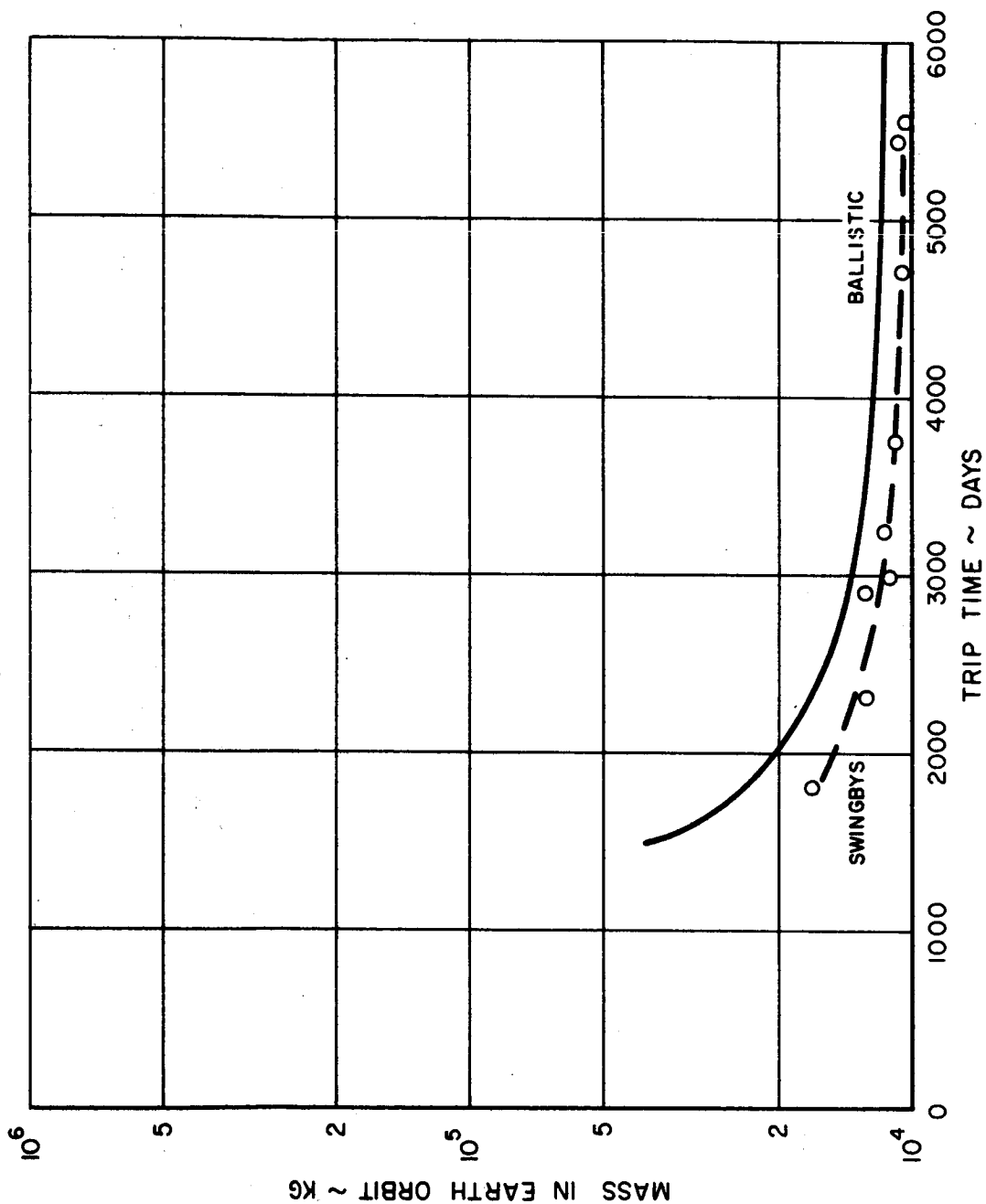
NEPTUNE FLYBYS
SOLID-CORE NUCLEAR

$I_{sp} = 800 \text{ SEC}$

$W_{PL} = 4536 \text{ KG}$



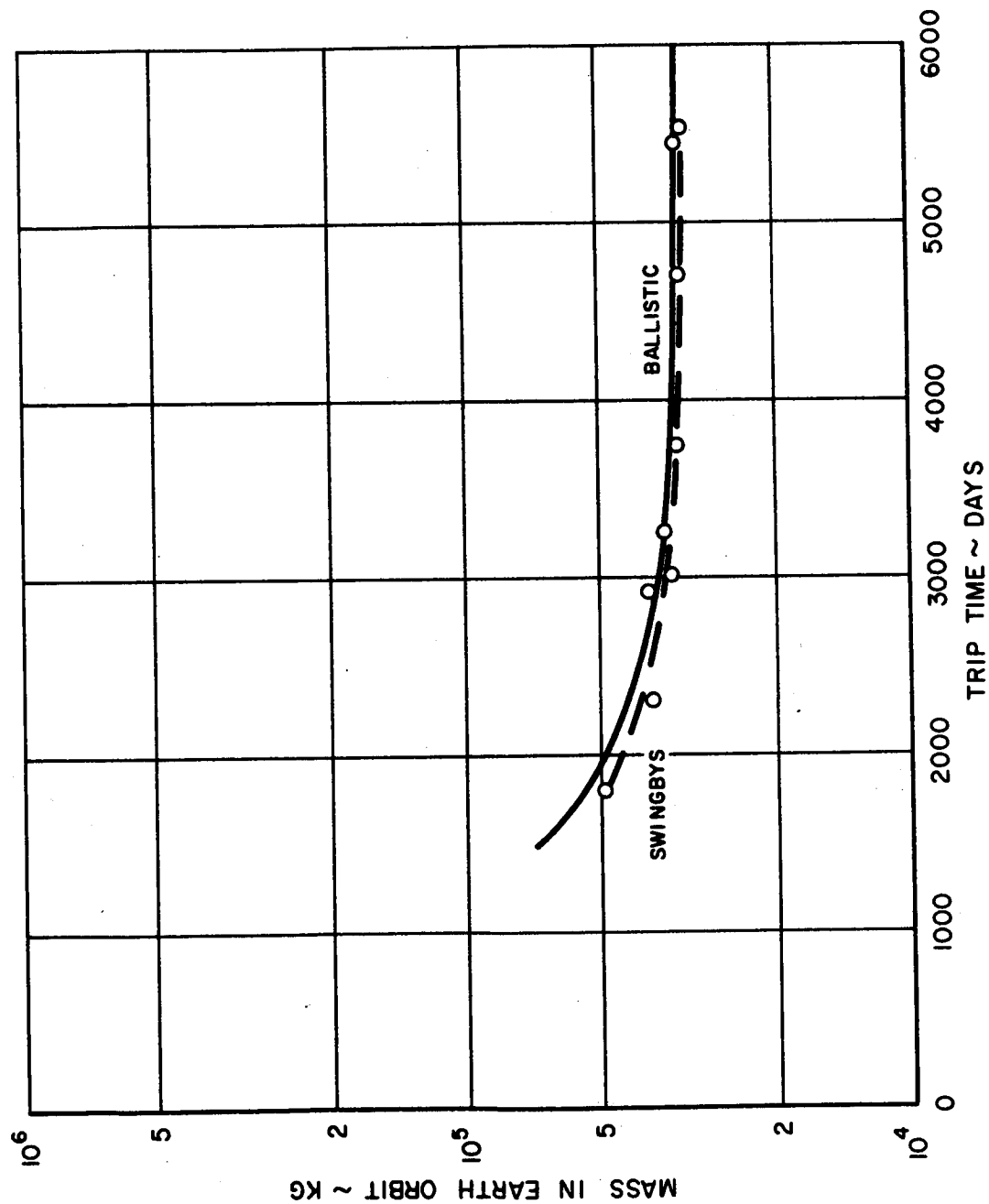
NEPTUNE FLYBYS
LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ $W_{PL} = 4536 \text{ KG}$ 

NEPTUNE FLYBYS GASEOUS-CORE NUCLEAR

$I_{SP} = 2000 \text{ SEC}$

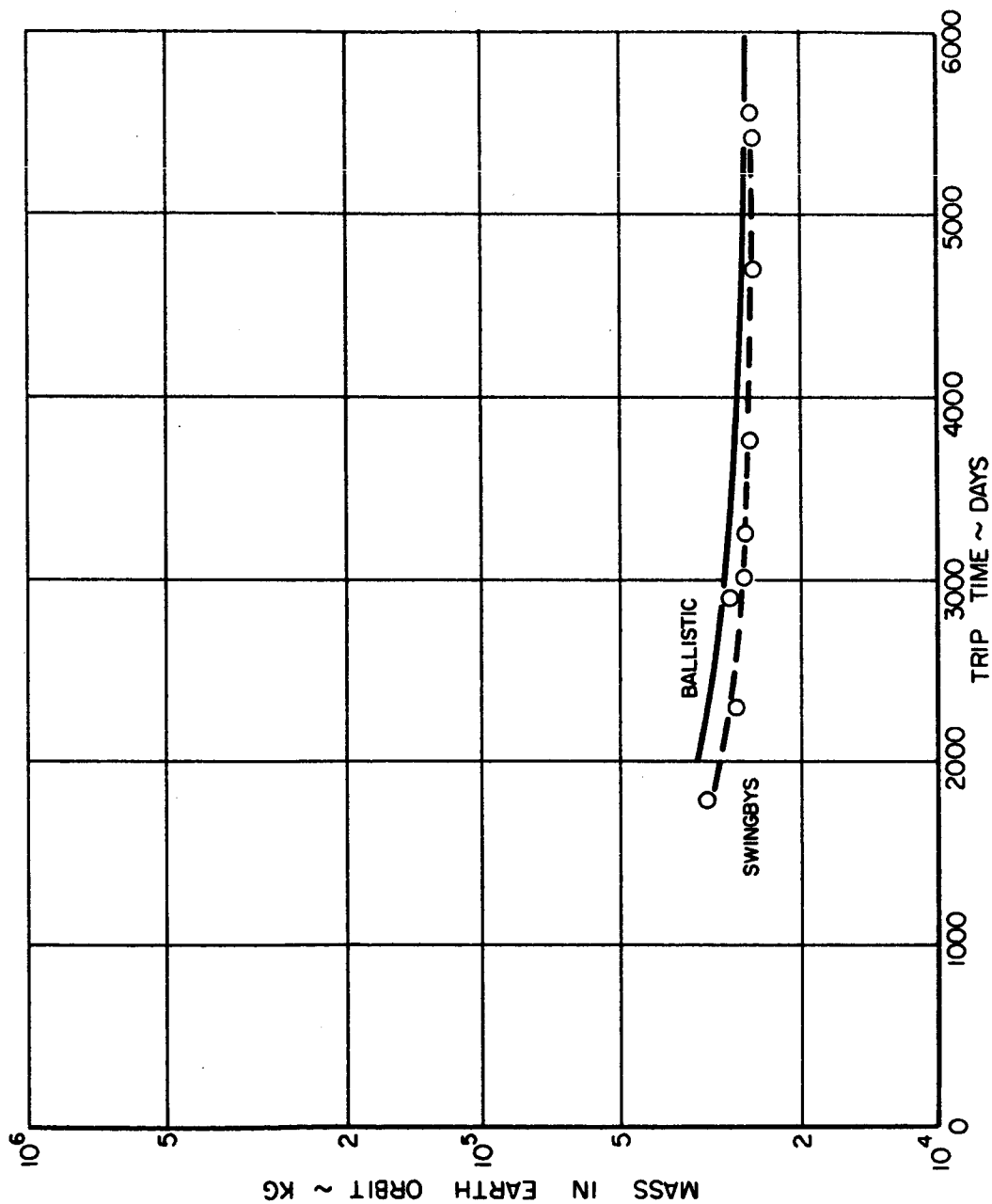
$W_{PL} = 4536 \text{ KG}$



NEPTUNE FLYBYS
GASEOUS-CORE NUCLEAR

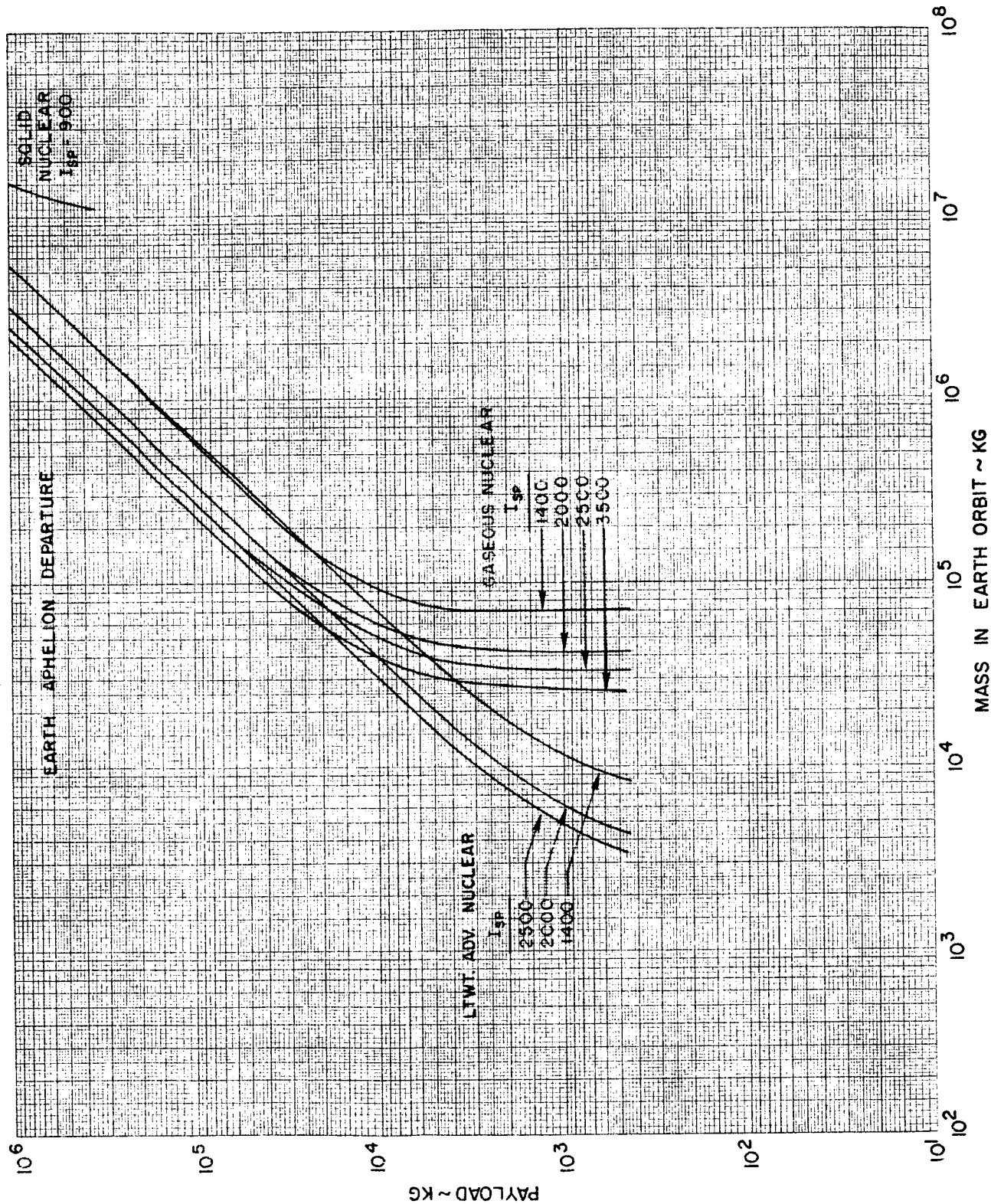
$I_{sp} = 3500 \text{ SEC}$

$W_{PL} = 4536 \text{ KG}$



SOLAR FLYBYS

$r_p = 0.05 \text{ A.U.}$



SOLAR FLYBYS

$$r_p = 0.05 \text{ A.U.}$$

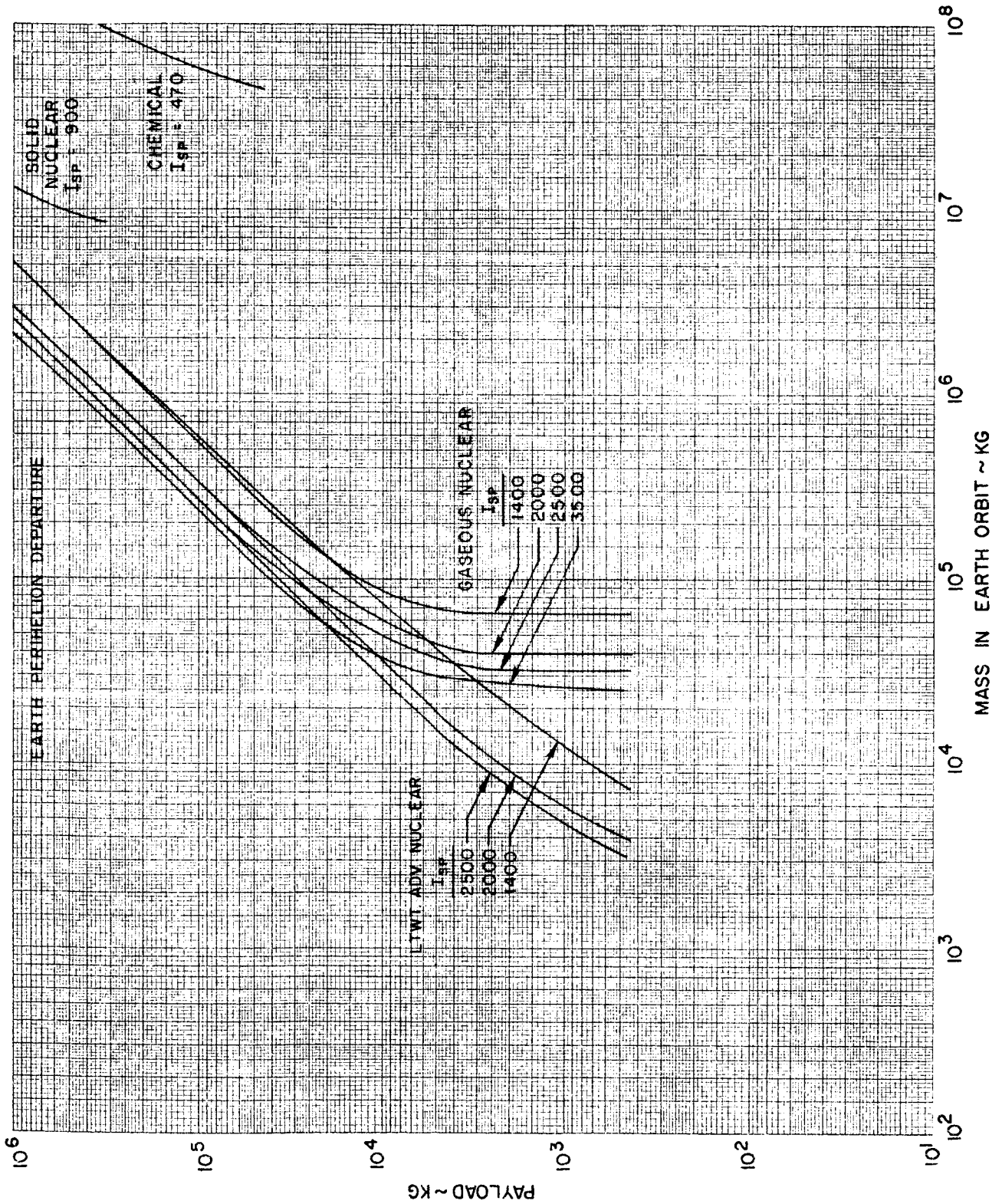
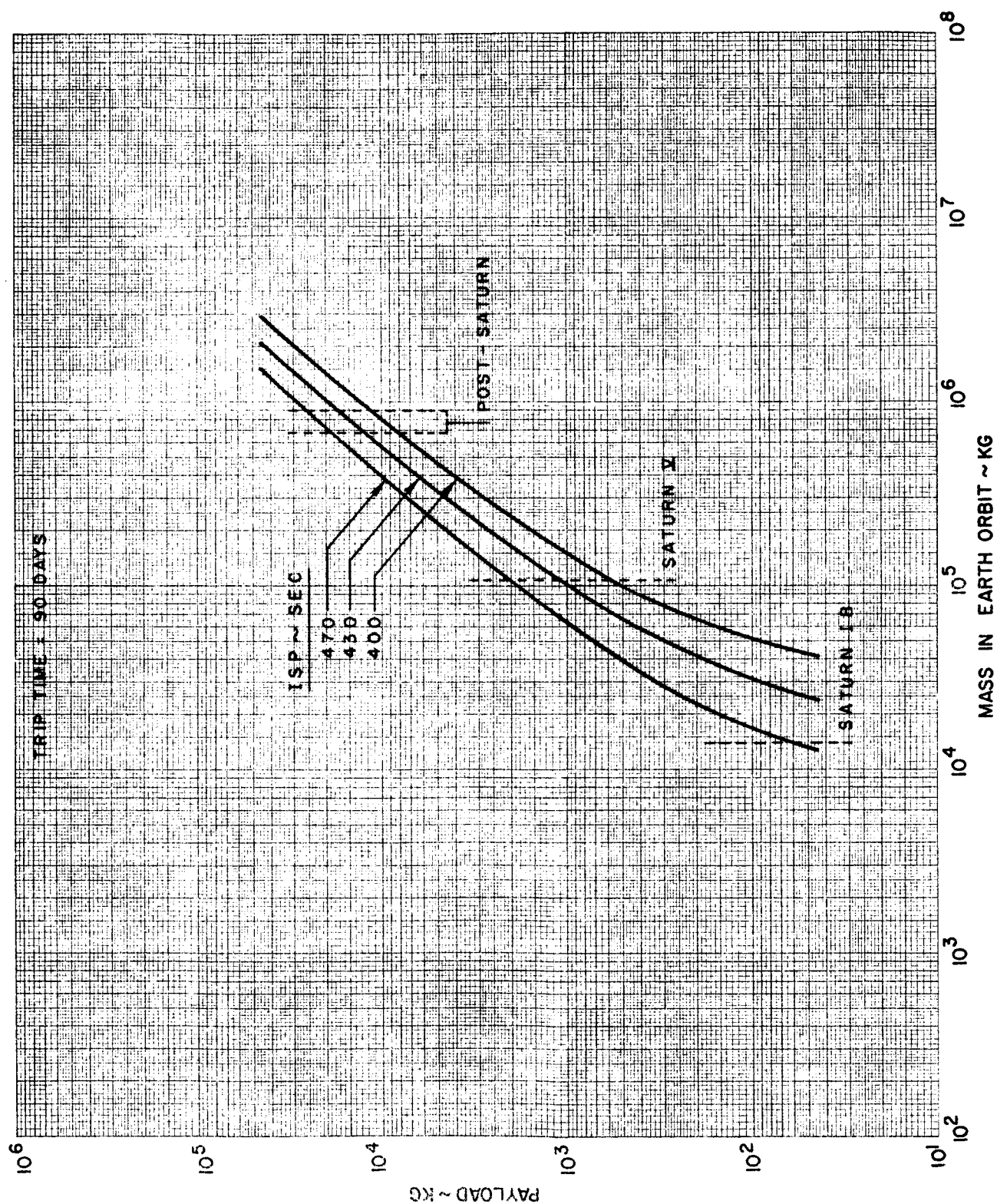


FIG. 160

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MERCURY ORBITAL STOPOVER

CHEMICAL



MERCURY ORBITAL STOPOVER

SOLID-CORE NUCLEAR

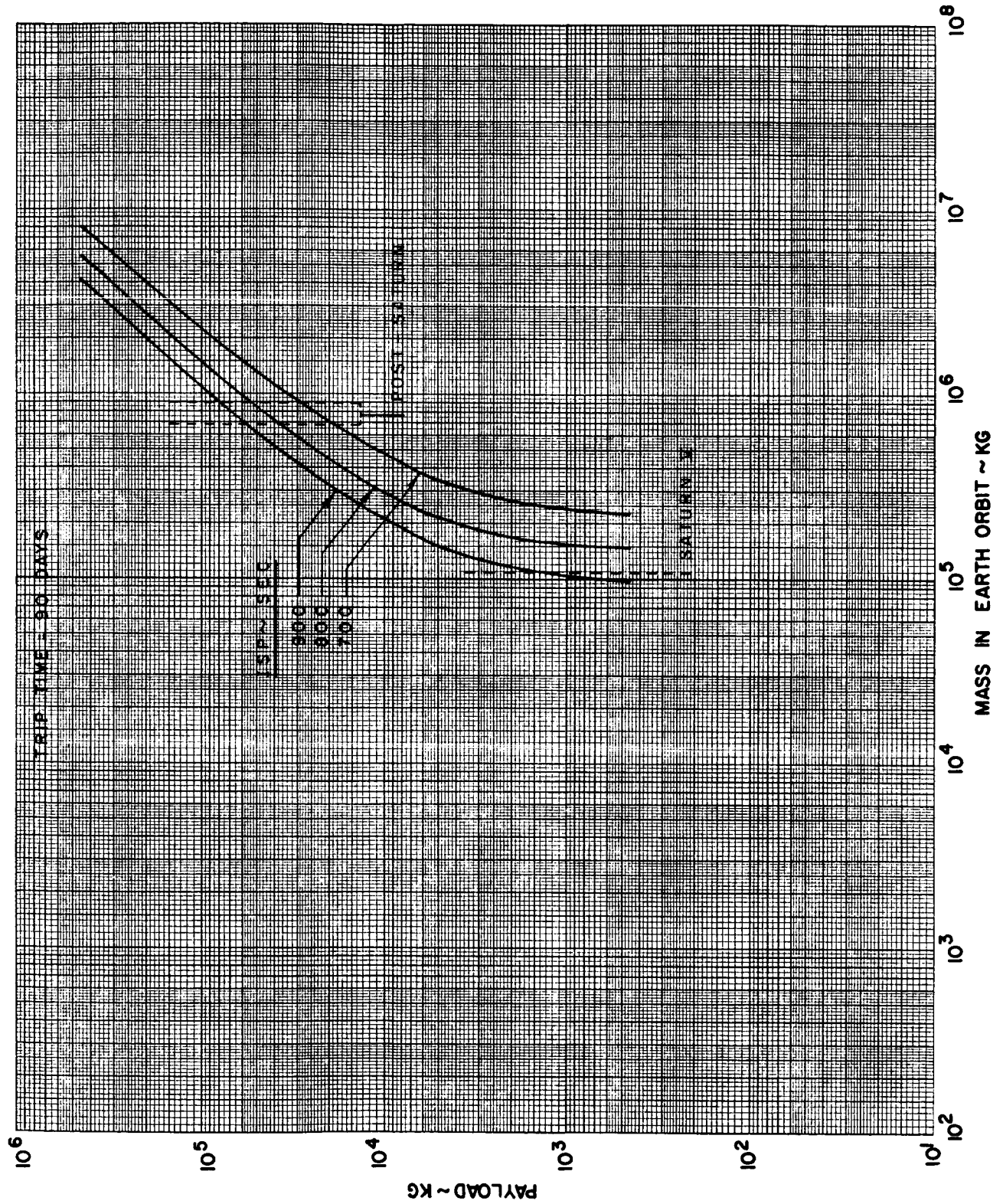
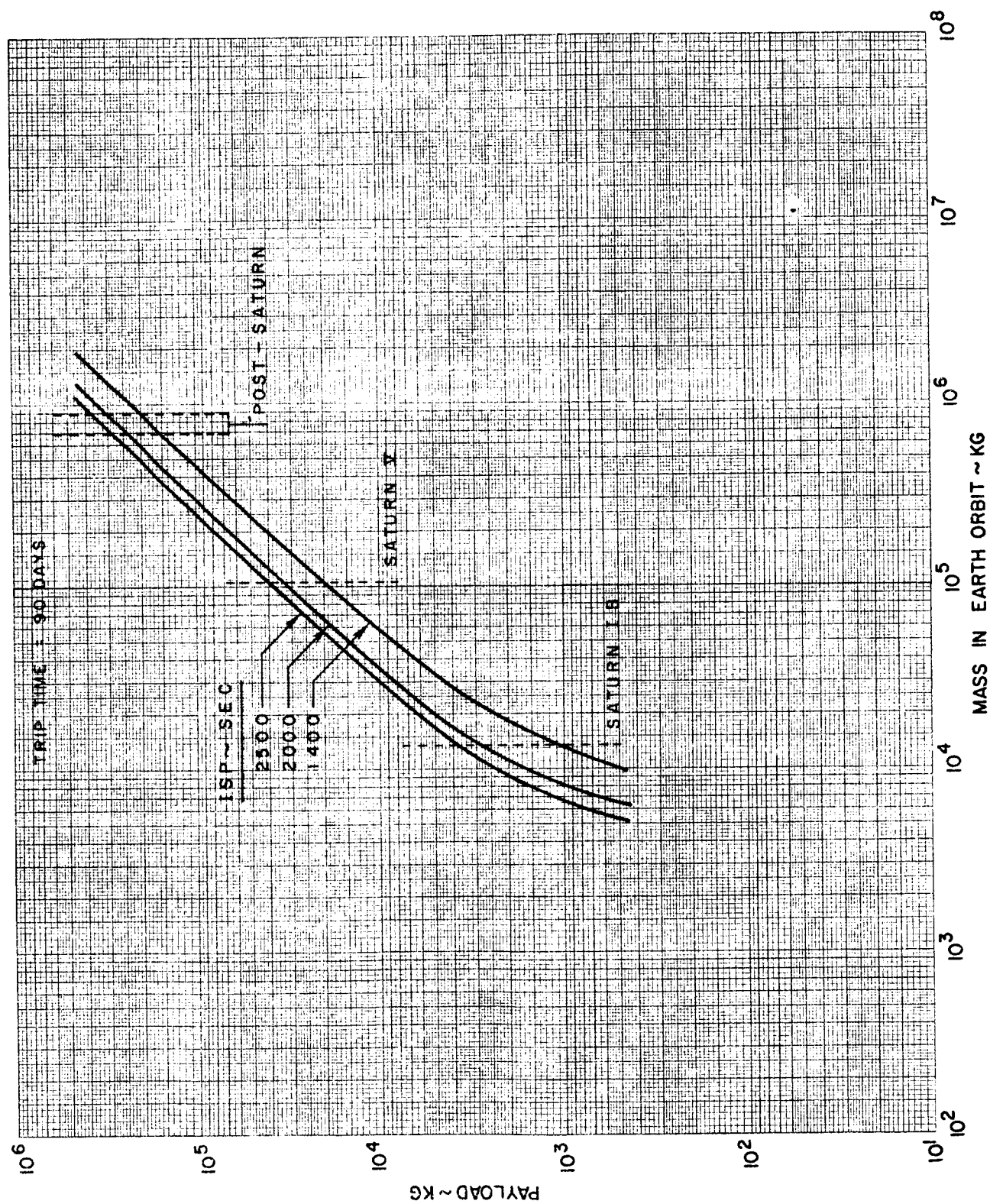


FIG. 162

E-910352-3

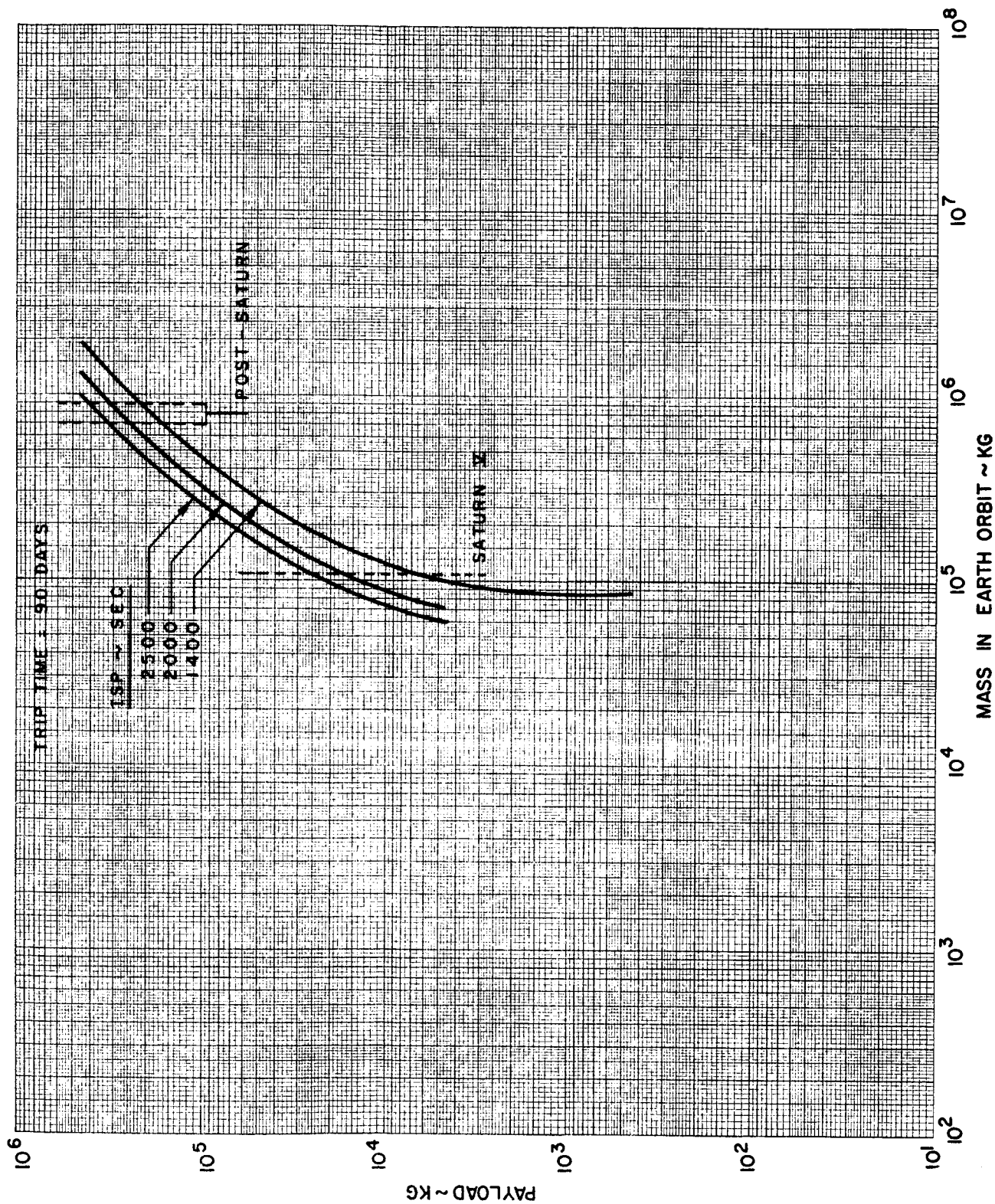
MERCURY ORBITAL STOPOVER

LIGHTWEIGHT ADVANCED NUCLEAR



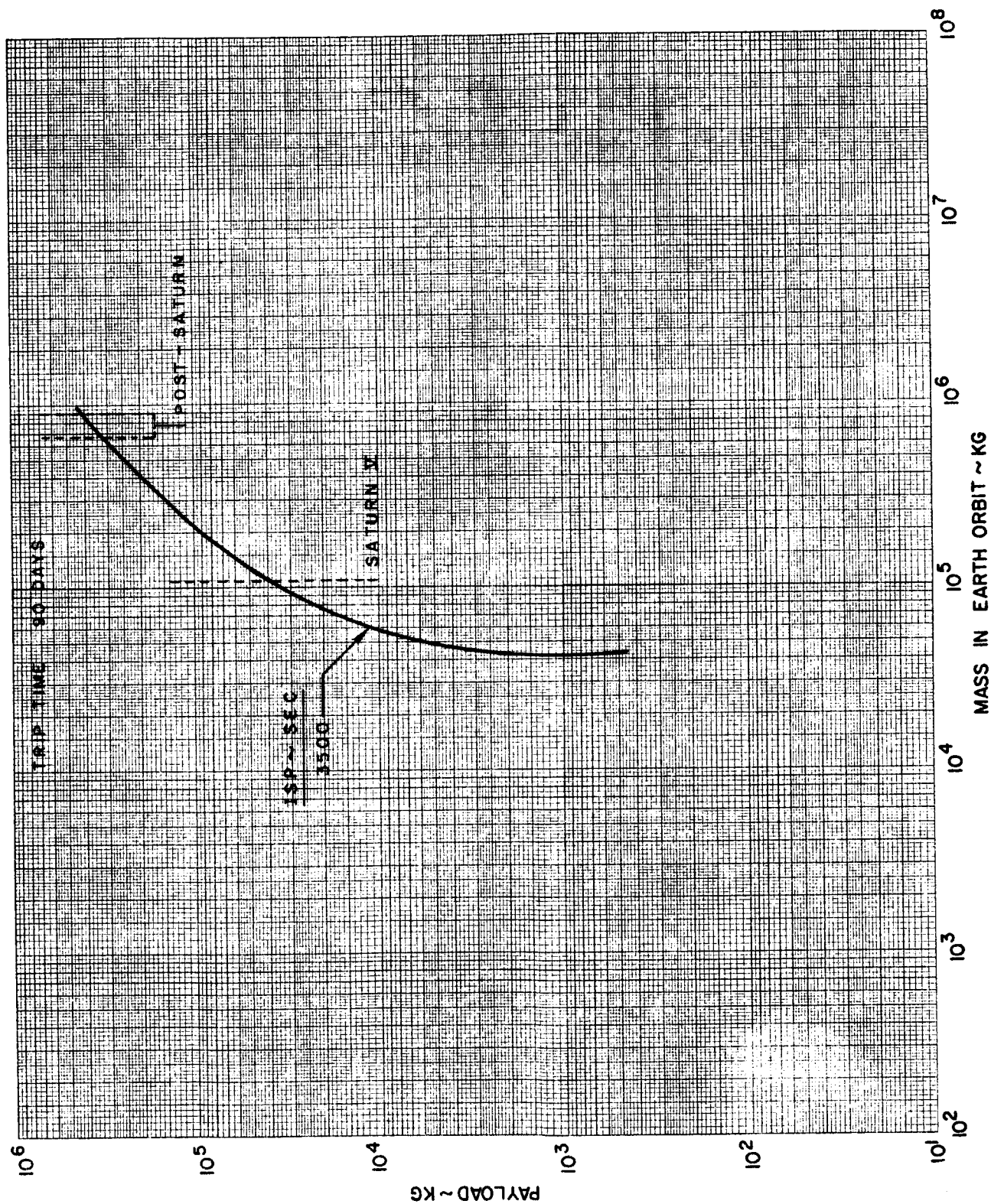
MERCURY ORBITAL STOPOVER

GASEOUS-CORE NUCLEAR

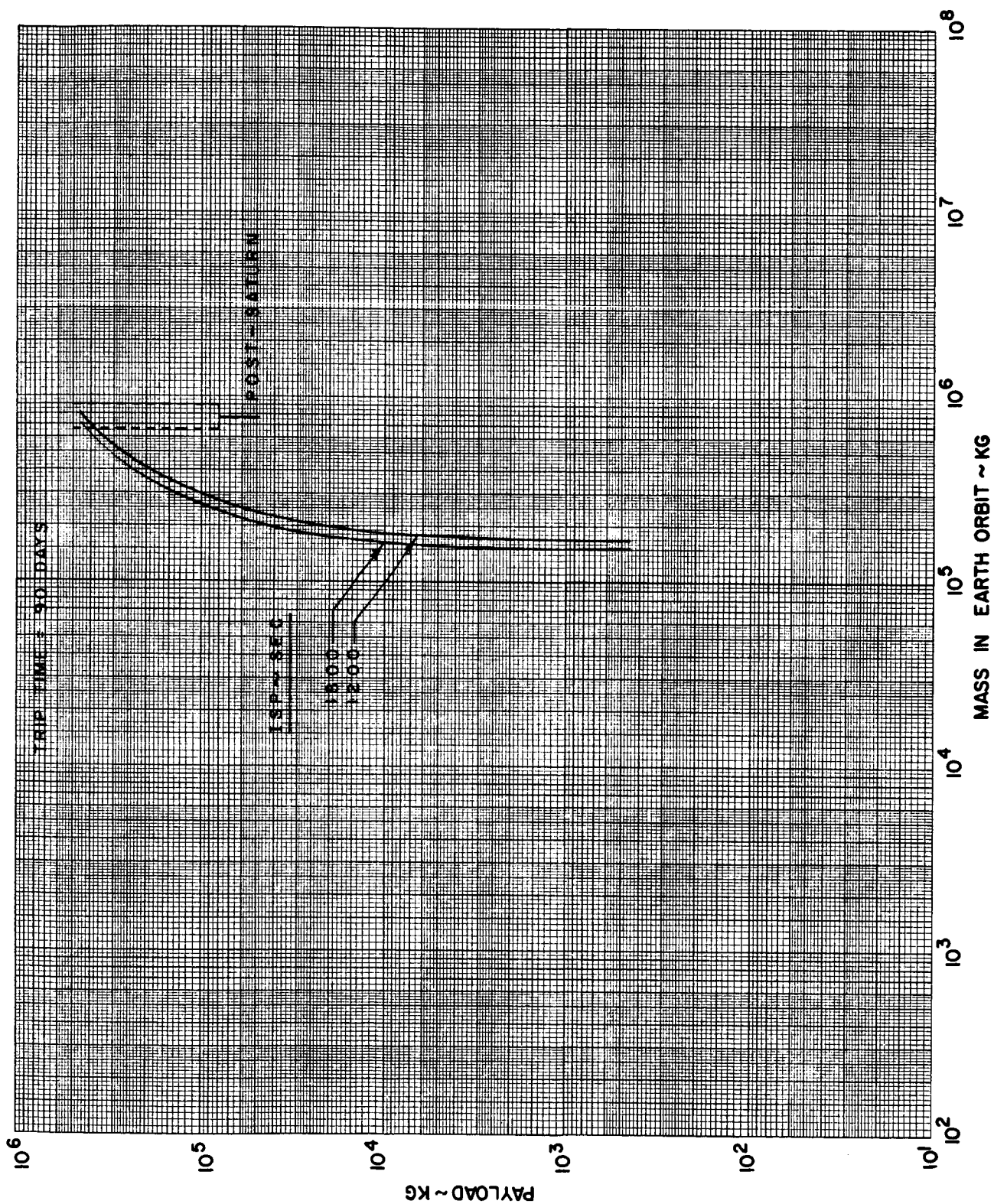


MERCURY ORBITAL STOPOVER

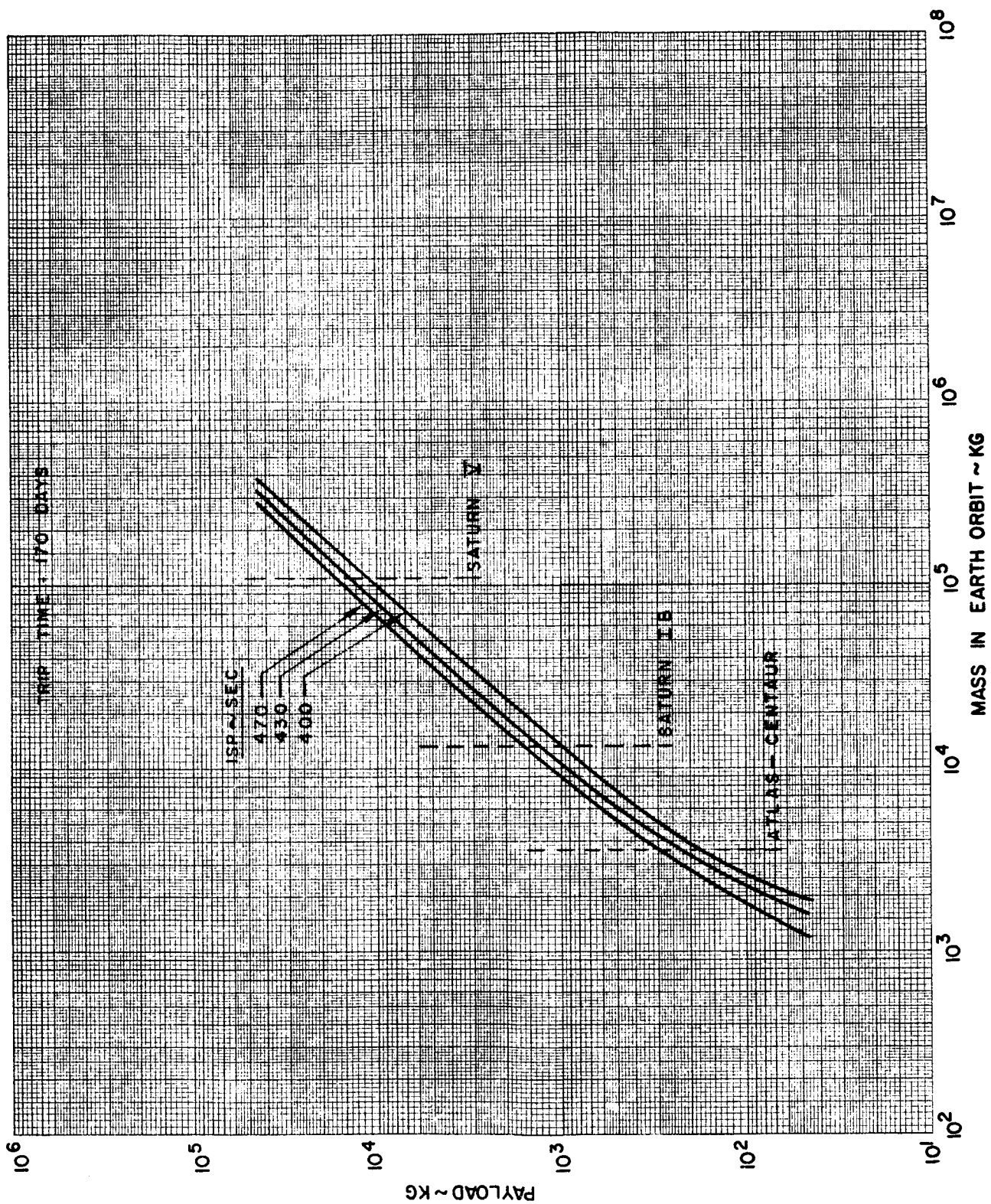
GASEOUS - CORE NUCLEAR



MERCURY ORBITAL STOPOVER PULSED NUCLEAR

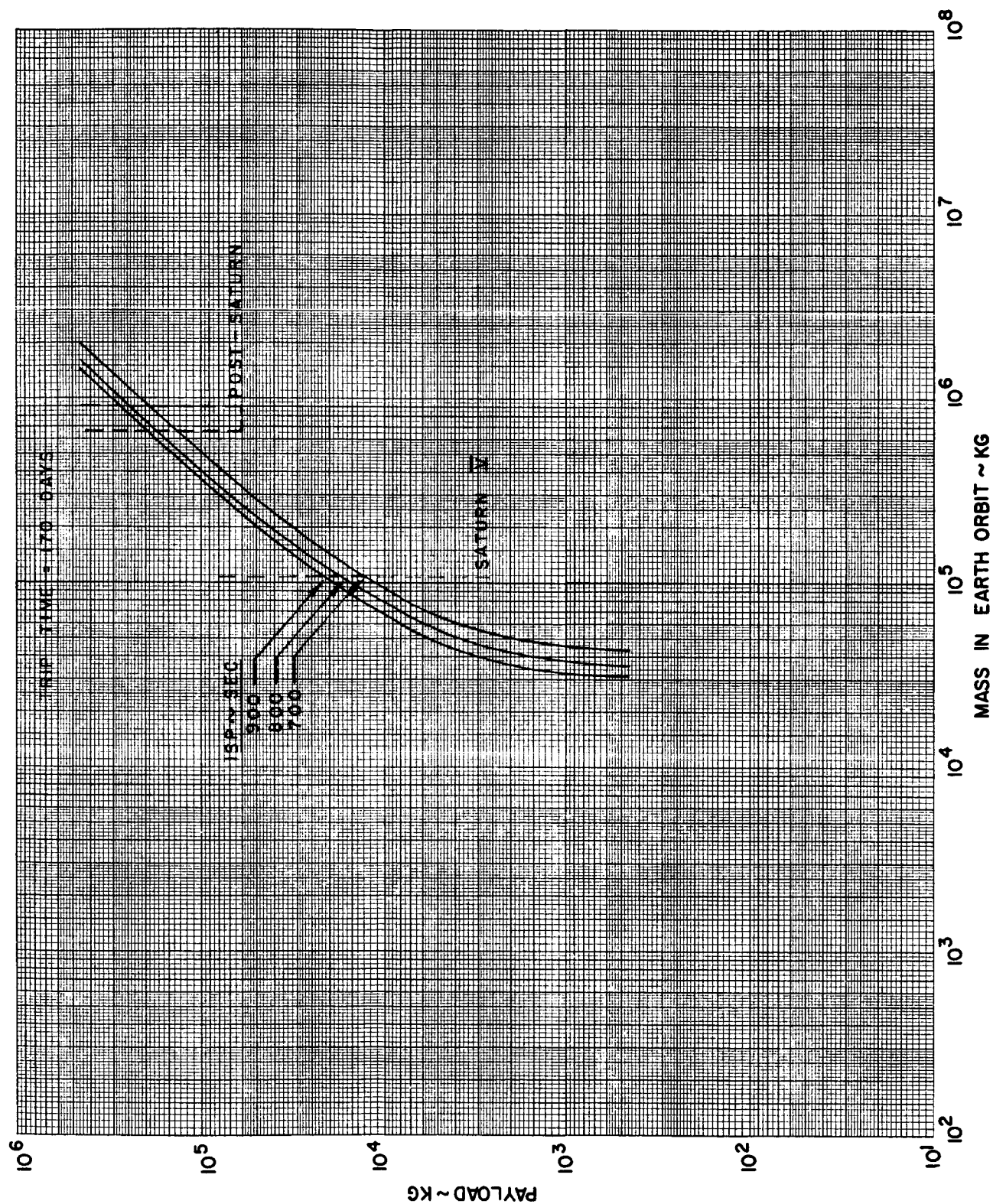


VENUS ORBITAL STOPOVER CHEMICAL



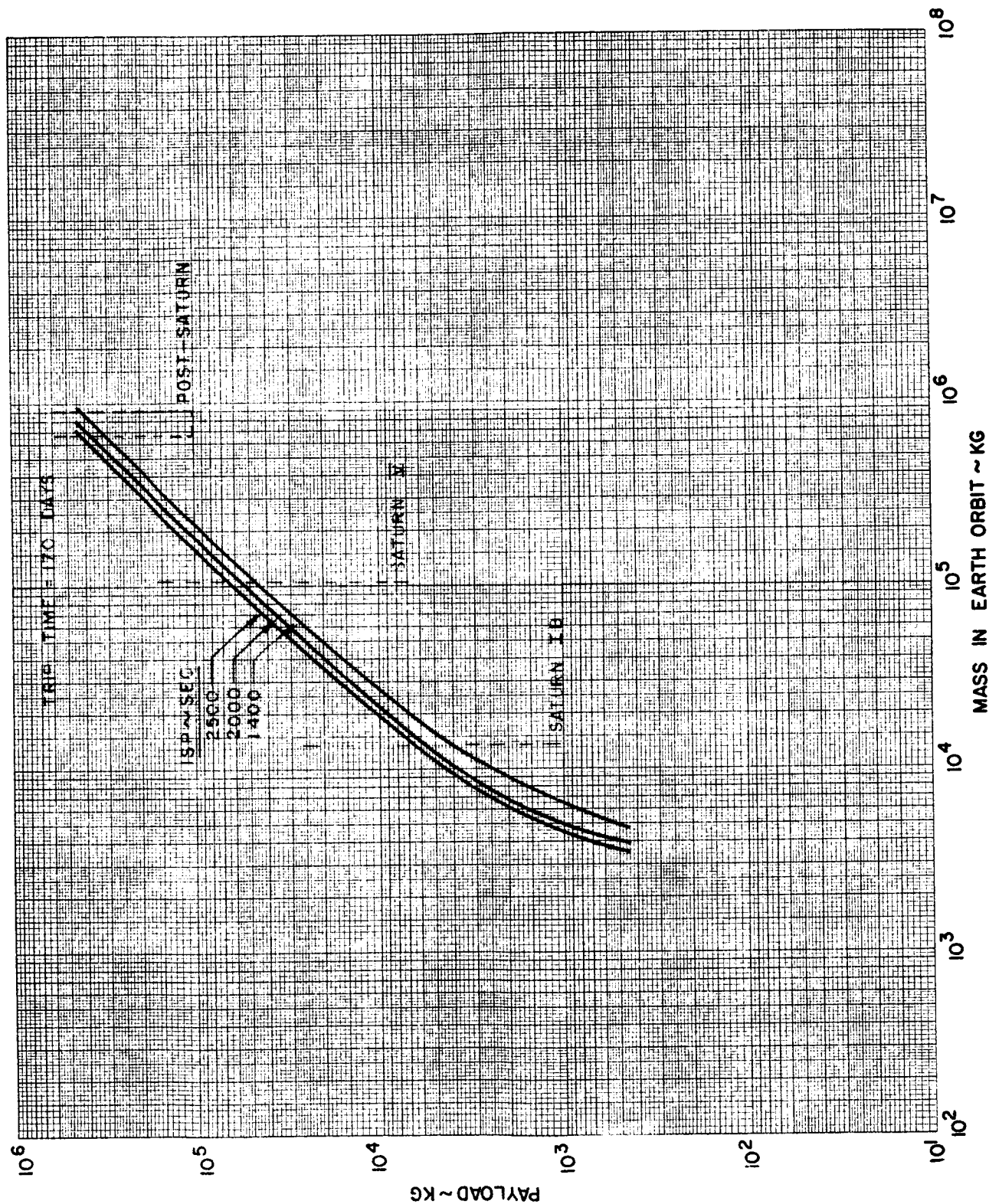
VENUS ORBITAL STOPOVER

SOLID - CORE NUCLEAR



VENUS ORBITAL STOPOVER

LIGHTWEIGHT ADVANCED NUCLEAR



VENUS ORBITAL STOPOVER
GASEOUS - CORE NUCLEAR

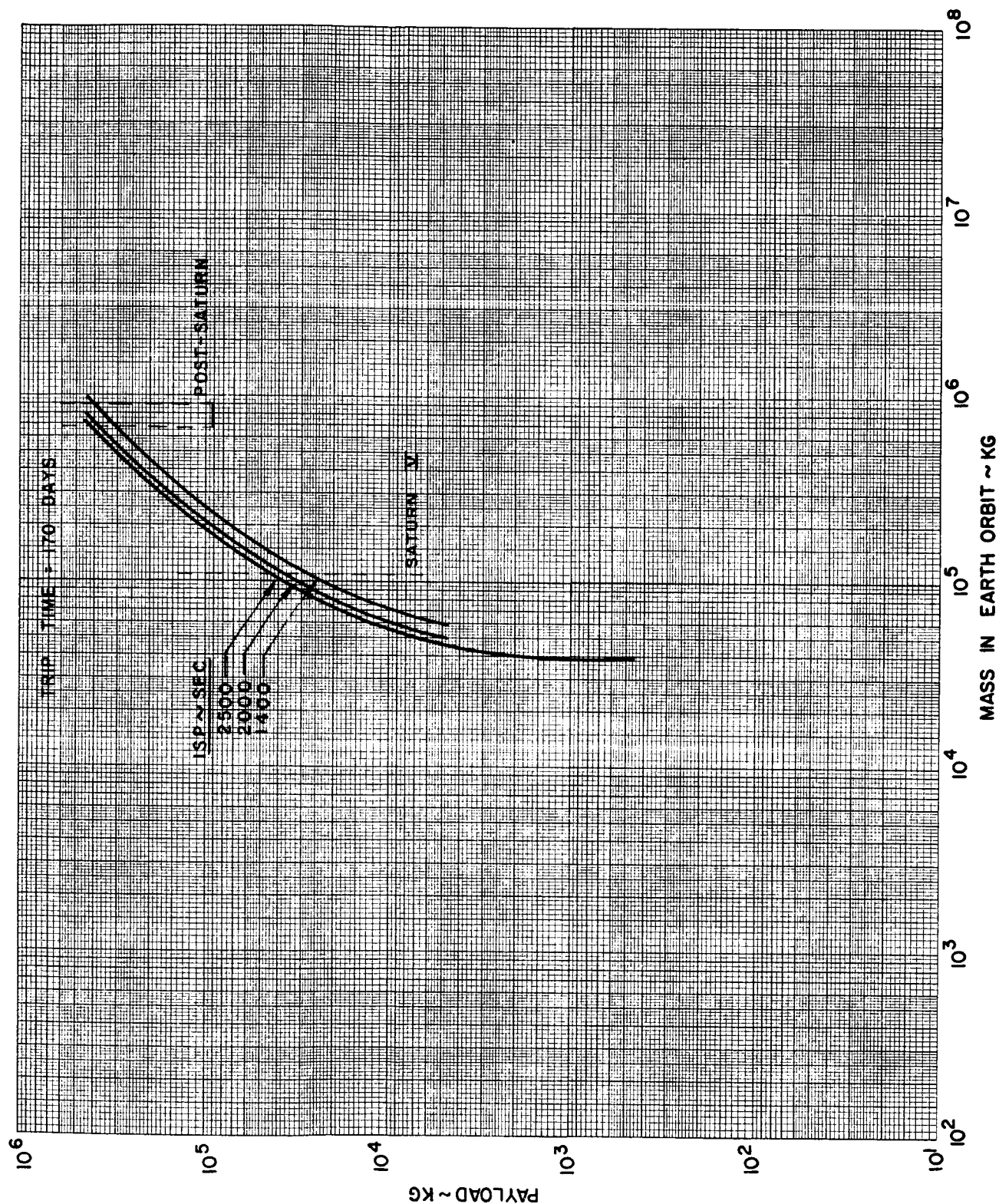
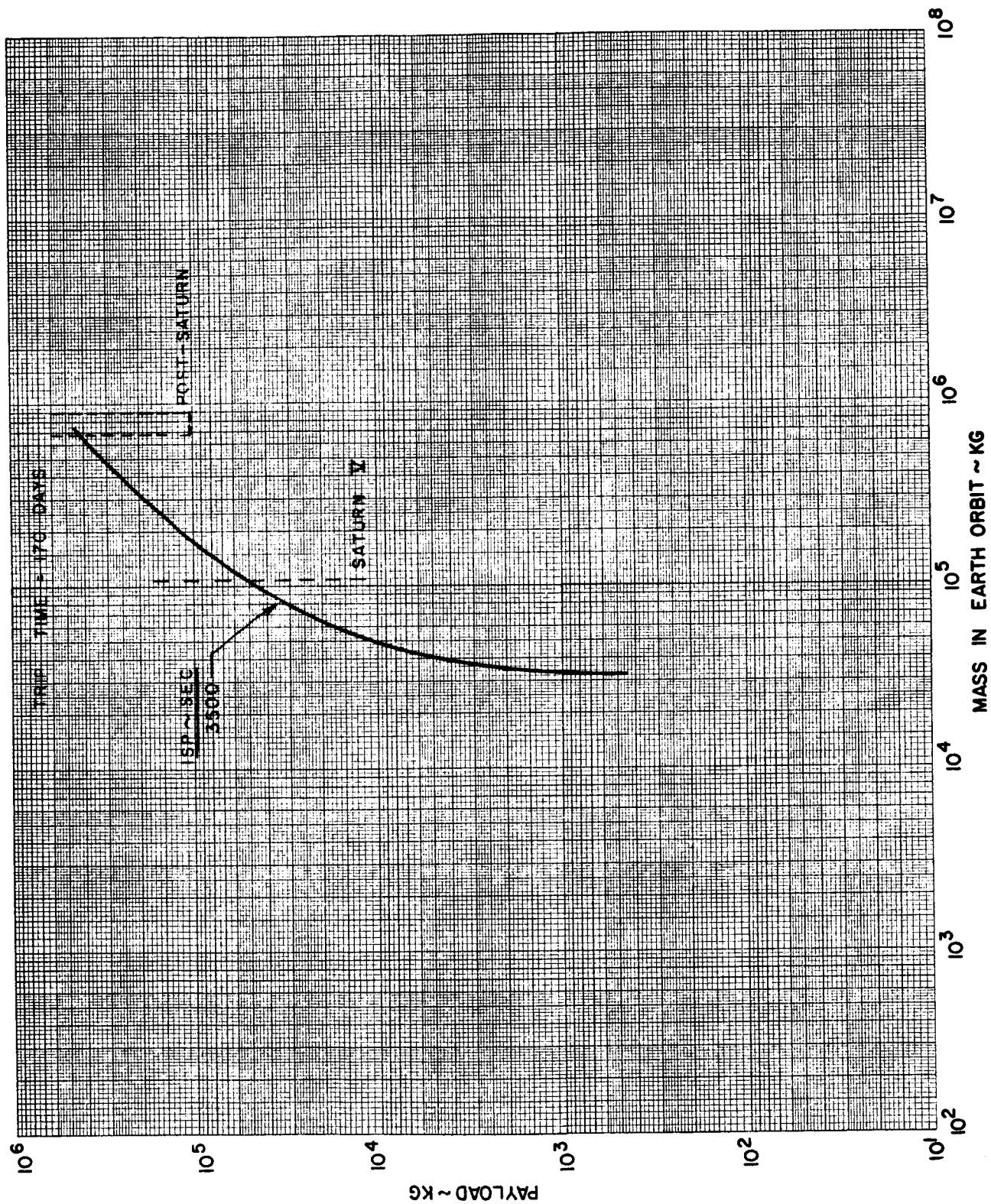


FIG. 170

E-910352-9

VENUS ORBITAL STOPOVER GASEOUS - CORE NUCLEAR



VENUS ORBITAL STOPOVER PULSED NUCLEAR

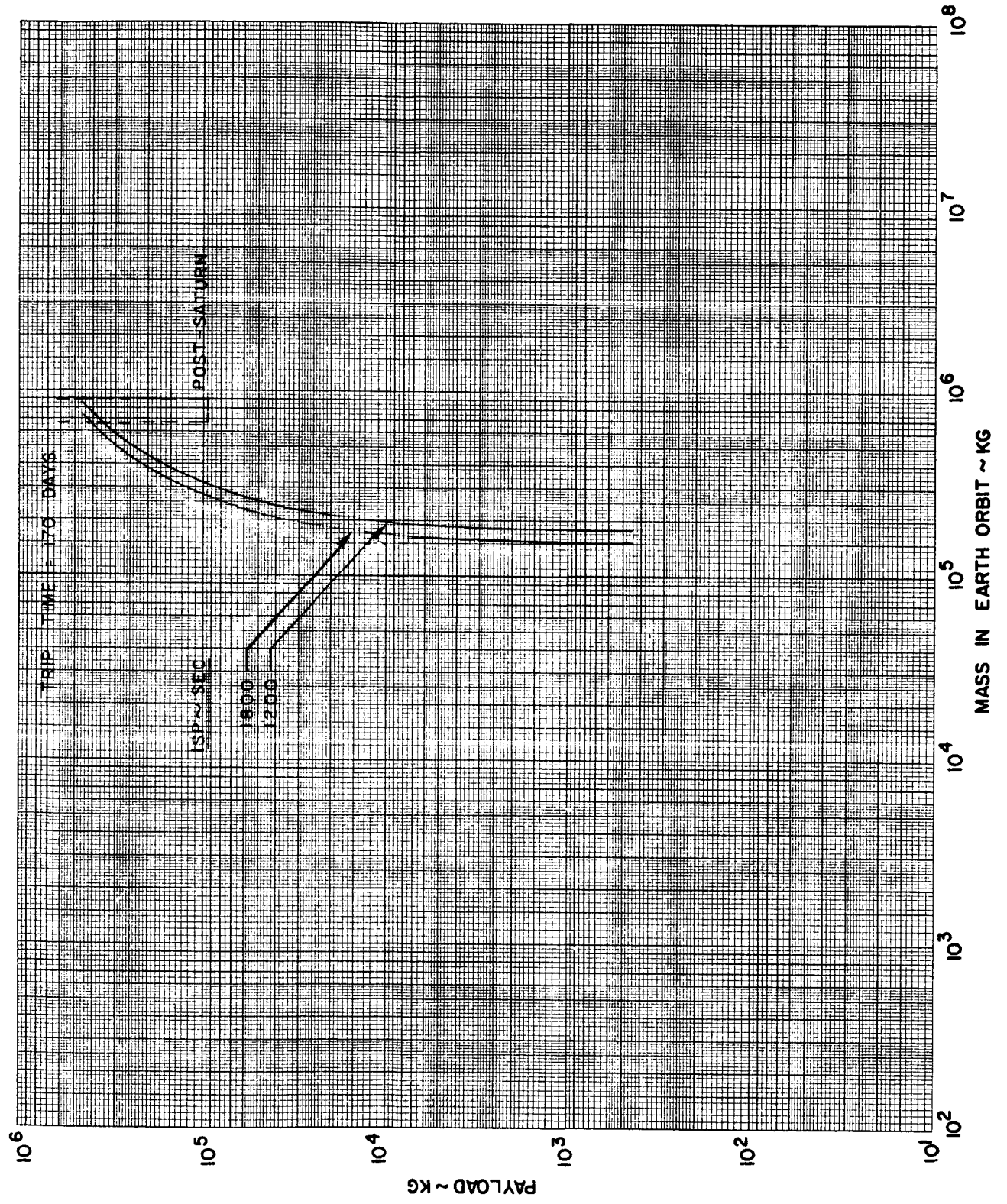
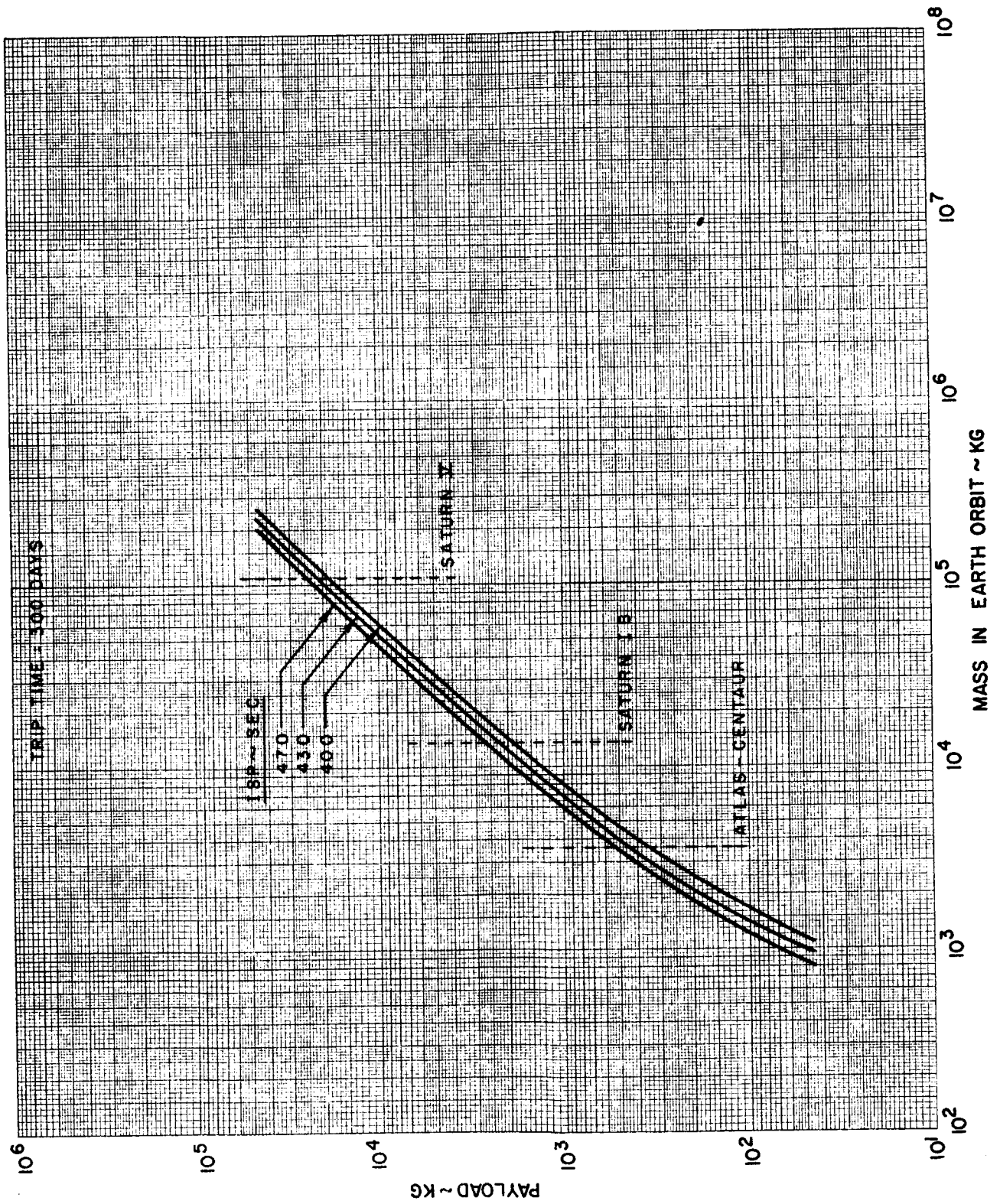


FIG. 172

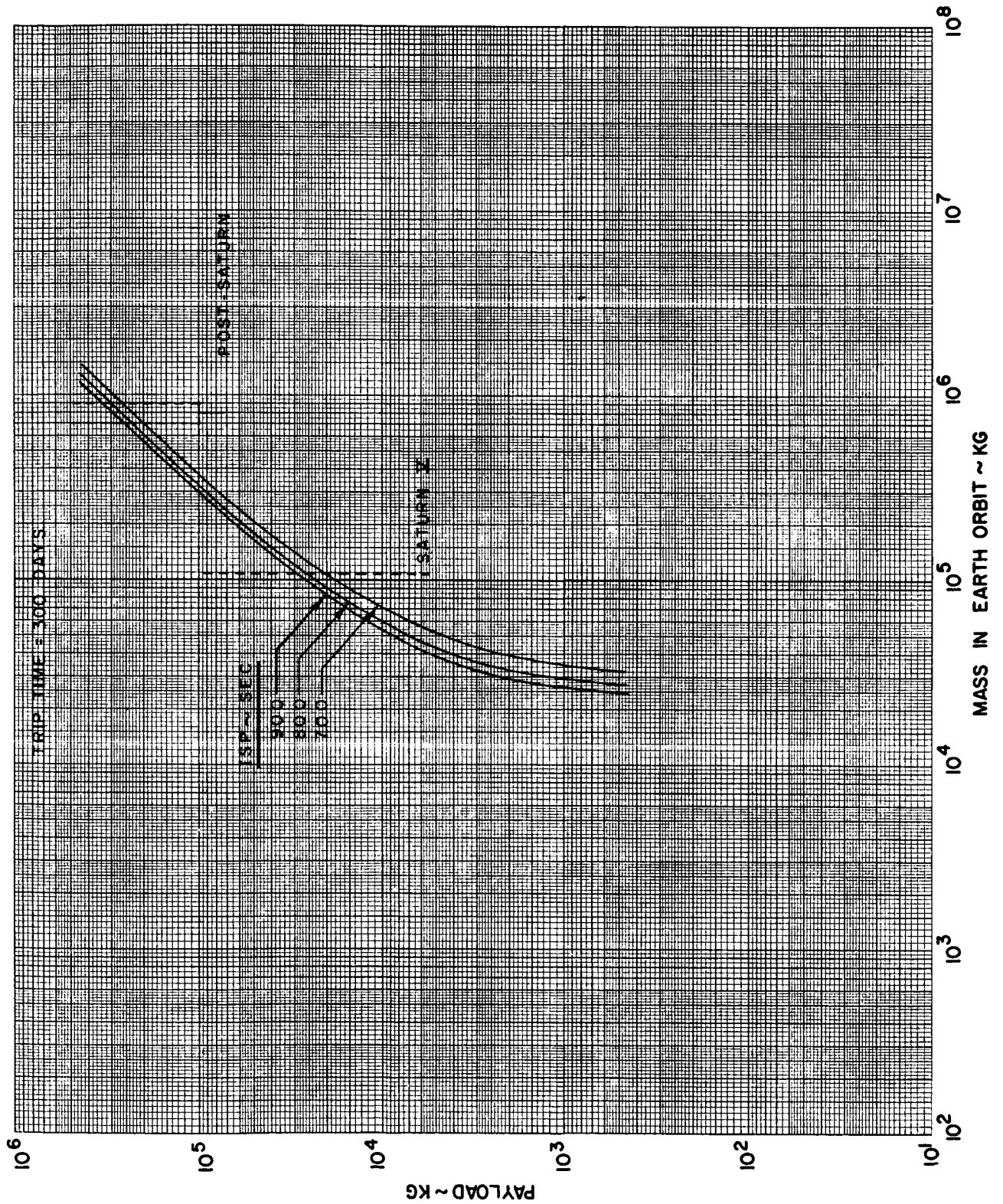
E-910352-9

MARS ORBITAL STOPOVER CHEMICAL

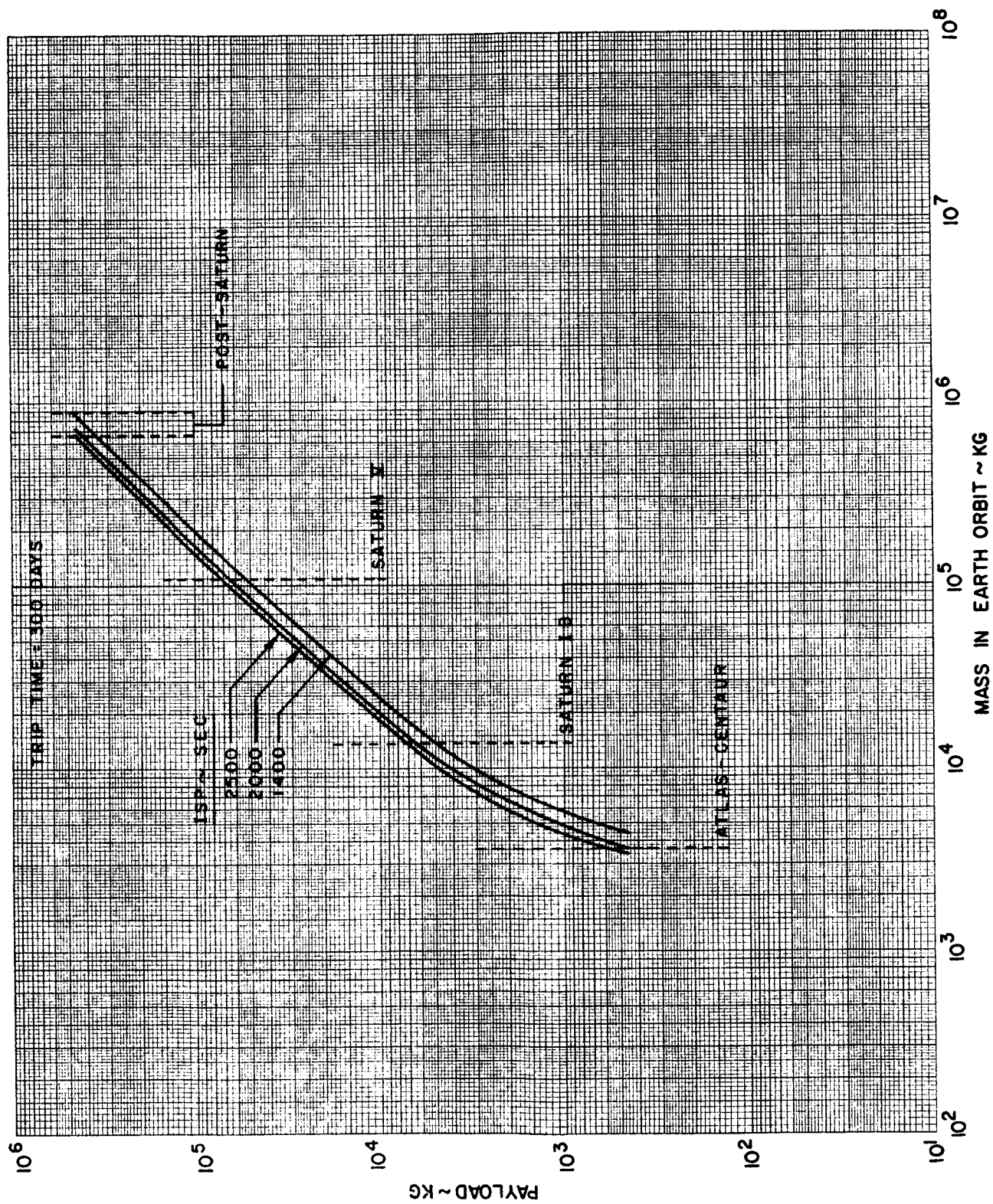


MARS ORBITAL STOPOVER

SOLID-CORE NUCLEAR



MARS ORBITAL STOPOVER LIGHTWEIGHT ADVANCED NUCLEAR



MARS ORBITAL STOPOVER

GASEOUS-CORE NUCLEAR

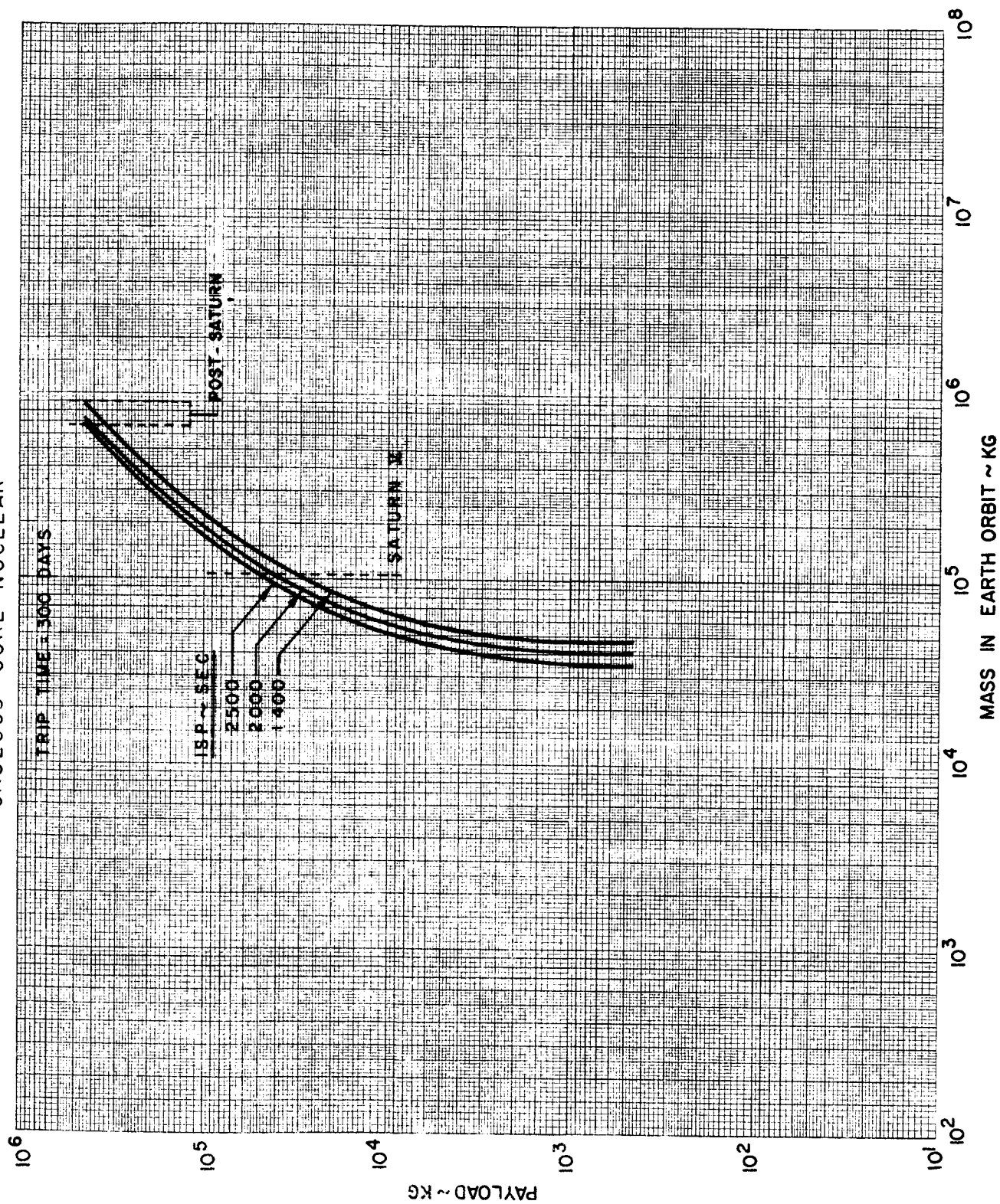
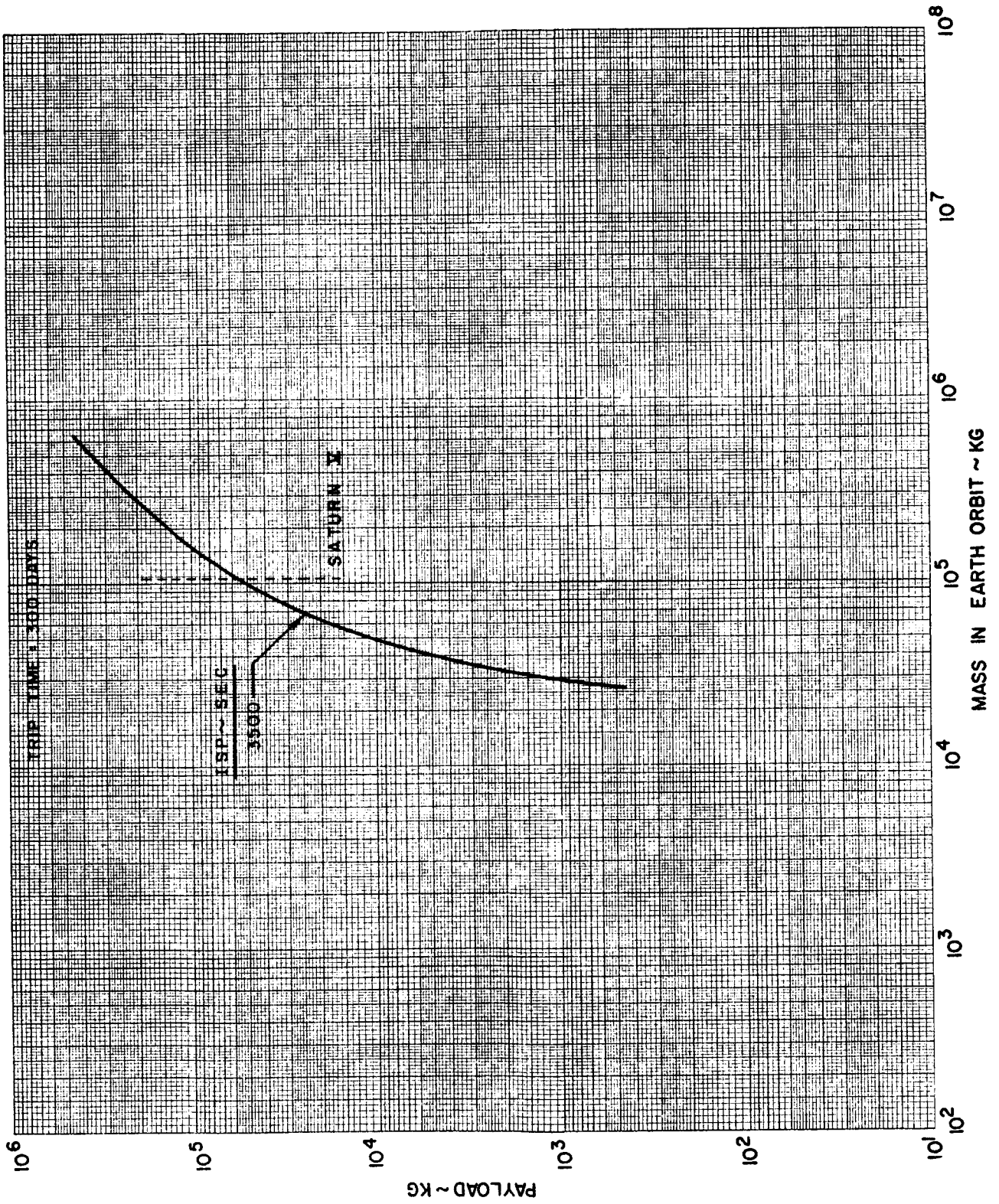


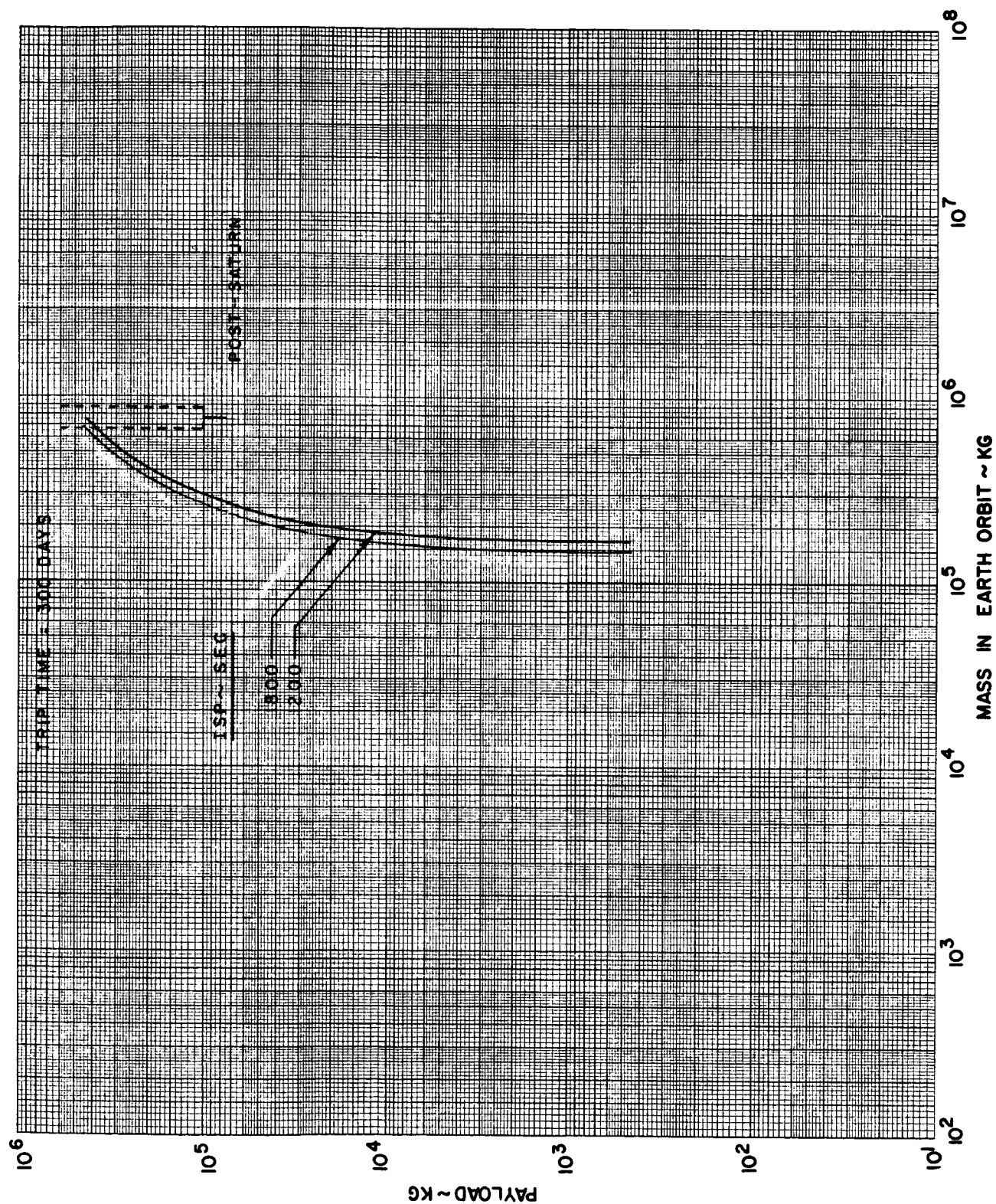
FIG. 176

E-910352-9

MARS ORBITAL STOPOVER GASEOUS-CORE NUCLEAR

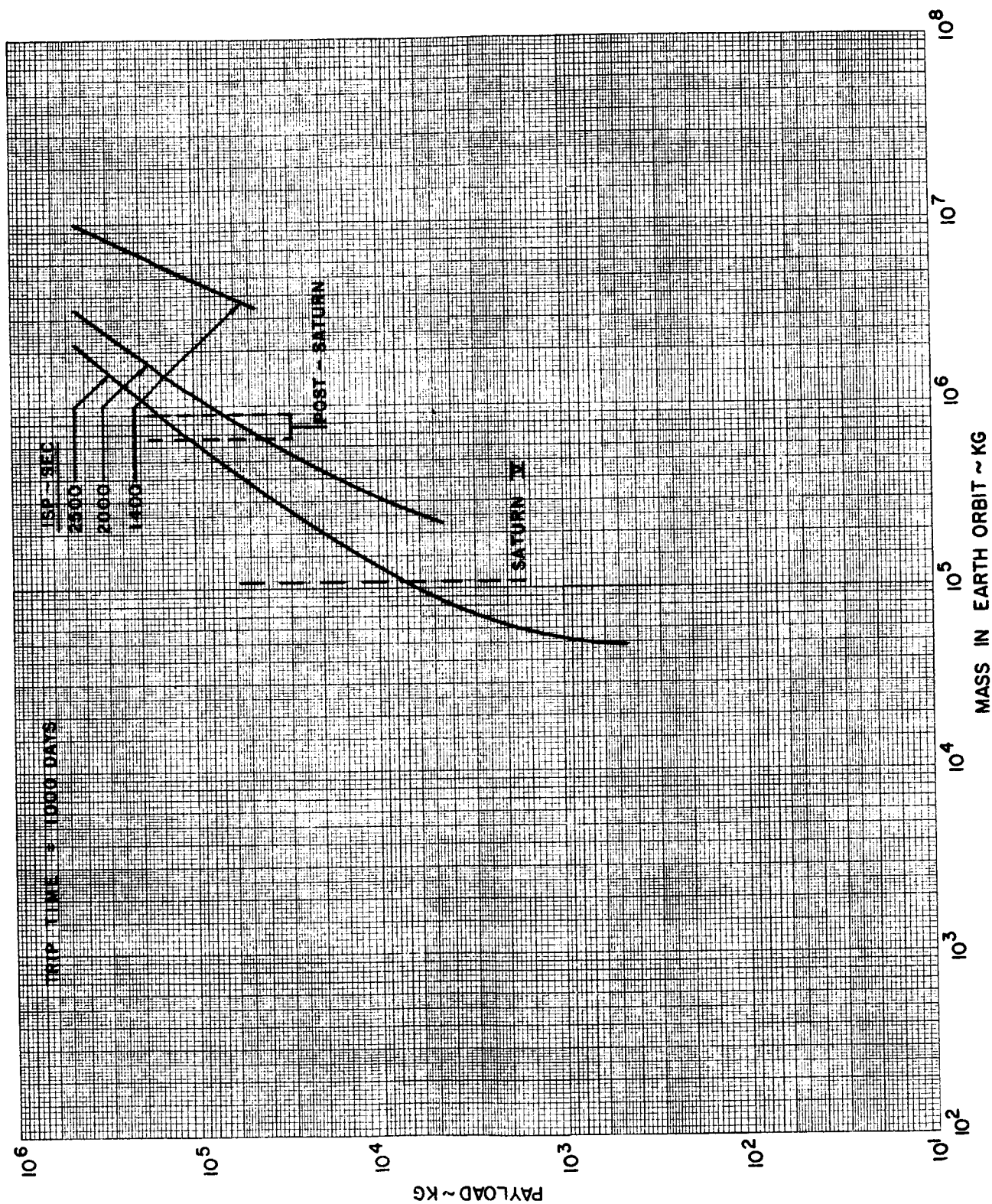


MARS ORBITAL STOPOVER PULSED NUCLEAR



JUPITER ORBITAL STOPOVER

LIGHTWEIGHT ADVANCED NUCLEAR



JUPITER ORBITAL STOPOVER

GASEOUS-CORE NUCLEAR

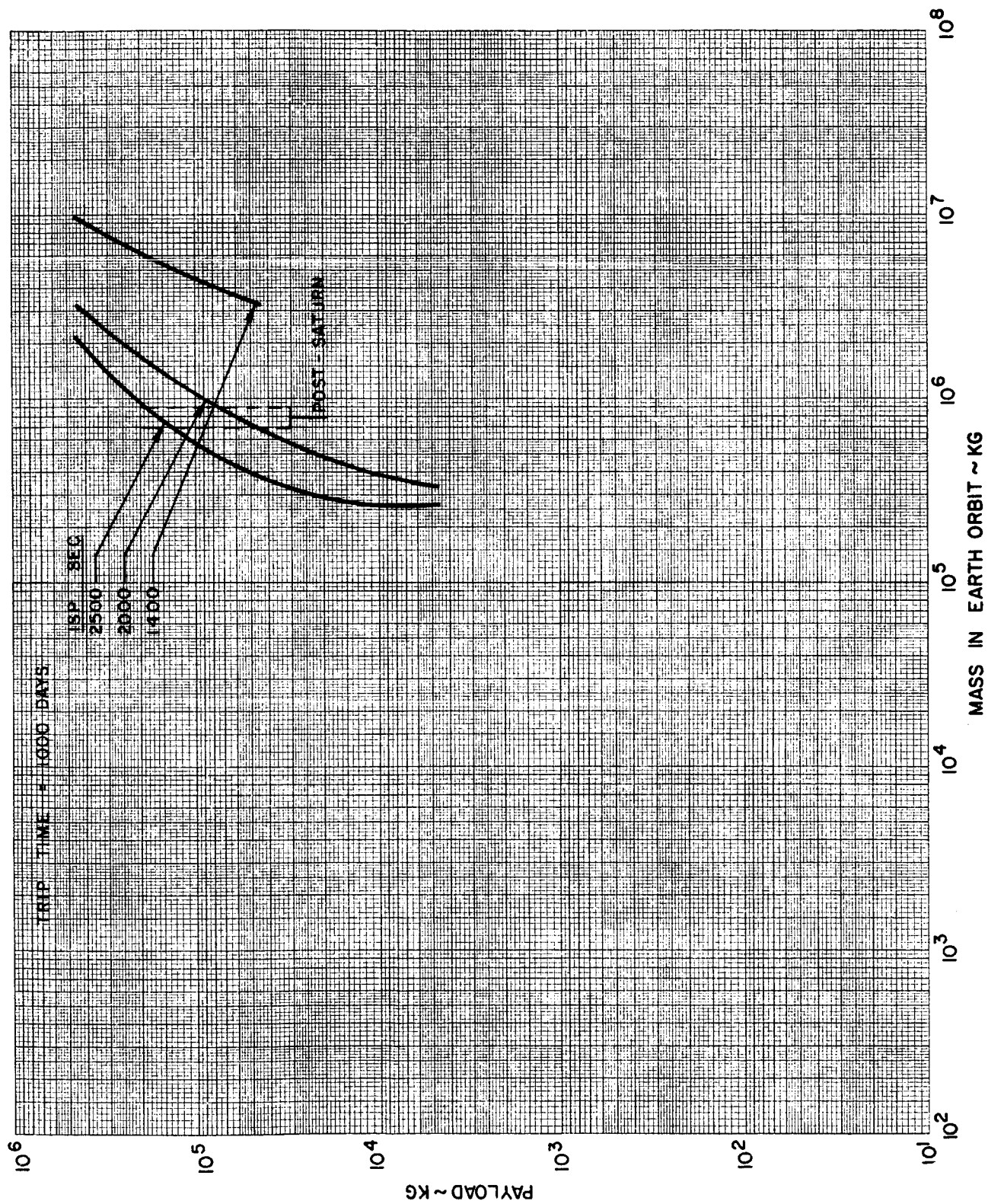
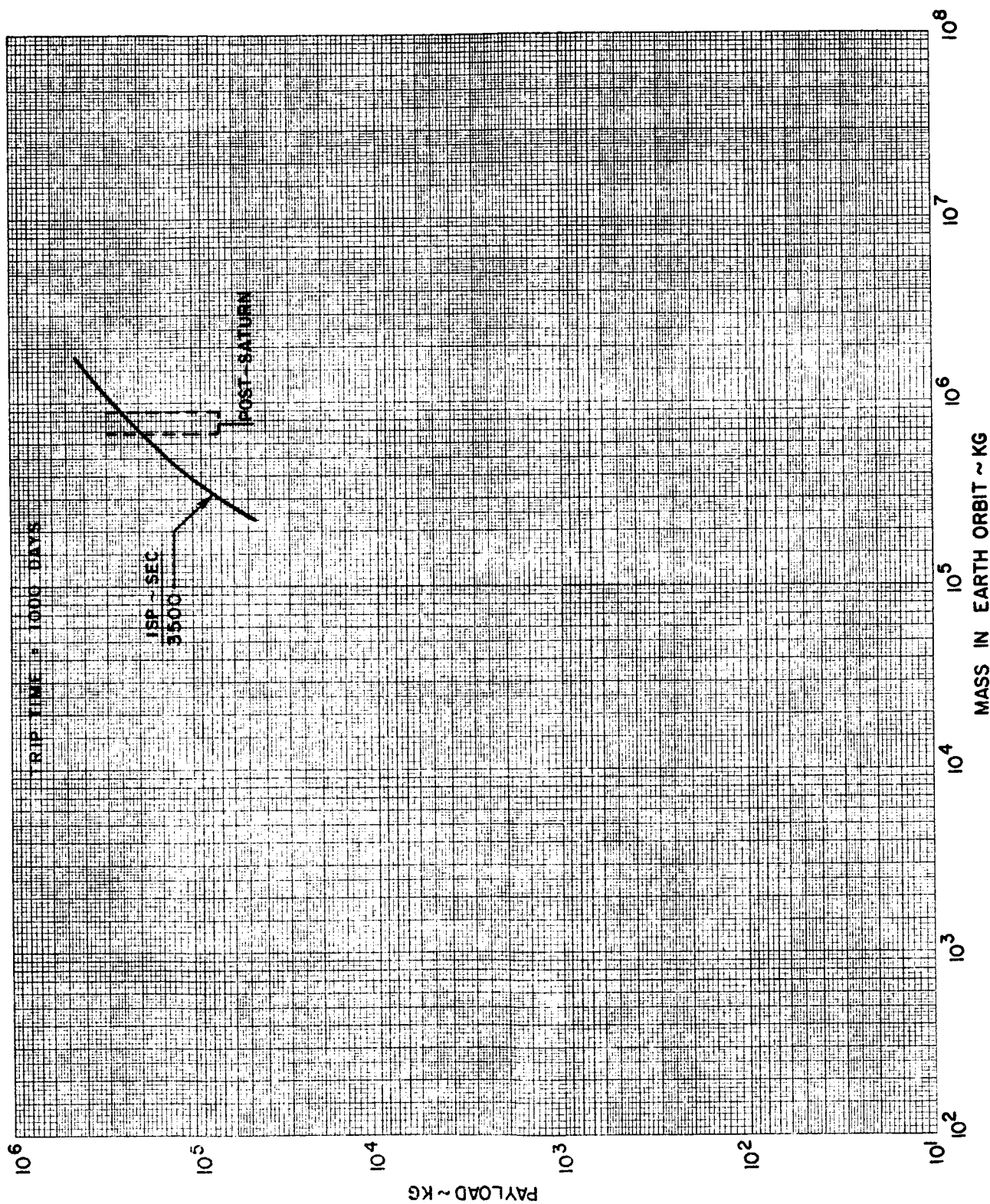


FIG. 180

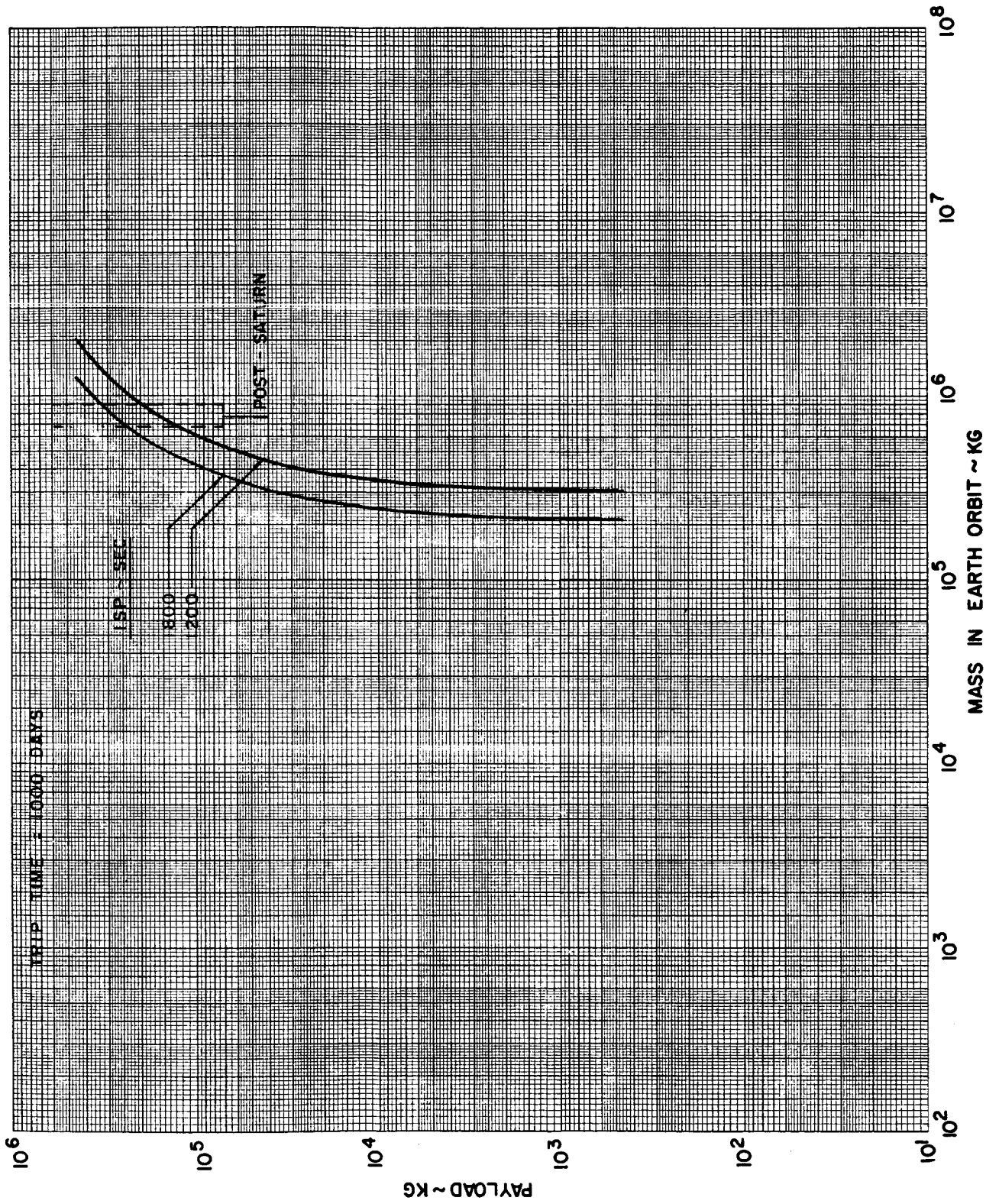
E-910352-9

JUPITER ORBITAL STOPOVER

GASEOUS-CORE NUCLEAR

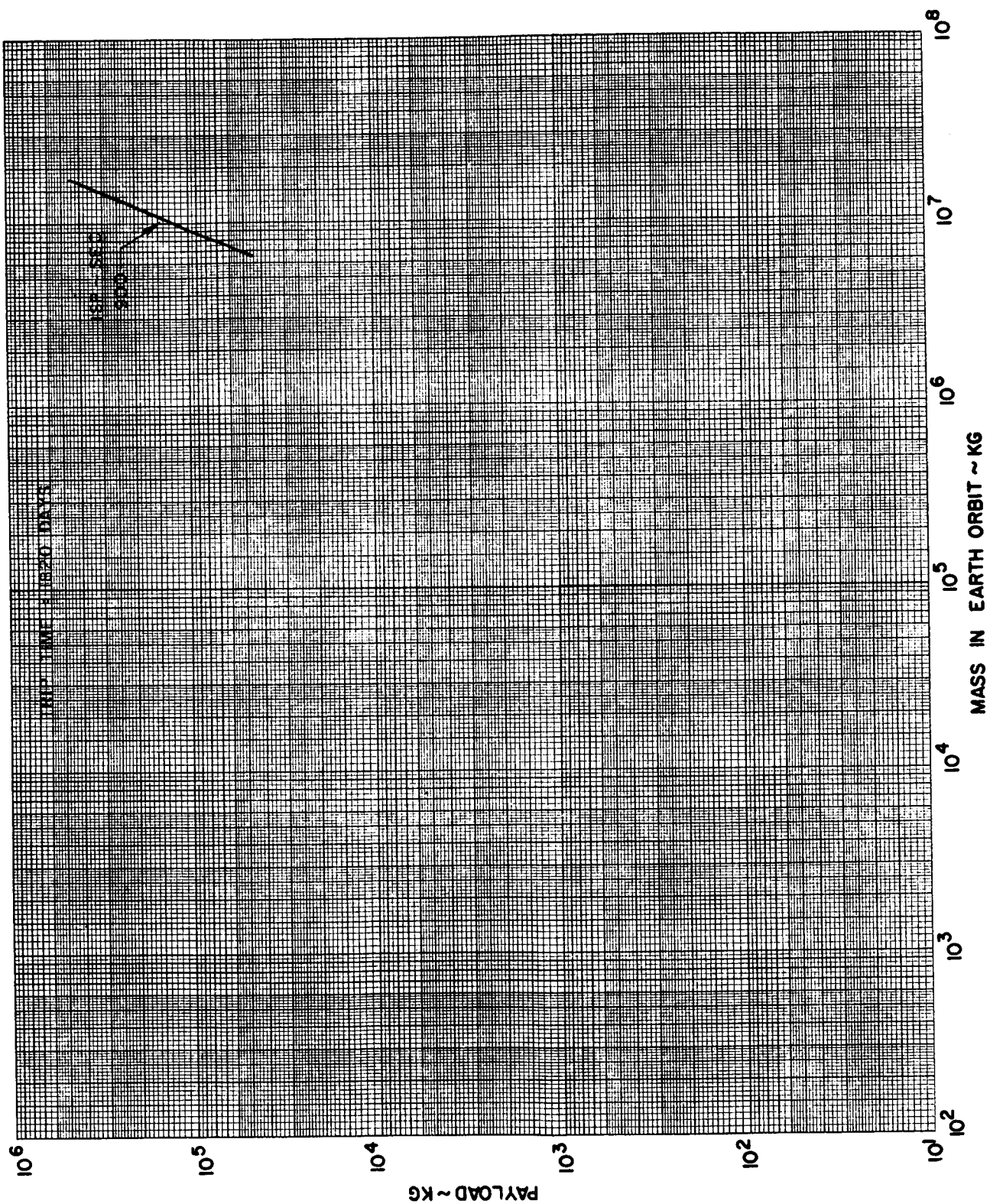


JUPITER ORBITAL STOPOVER PULSED NUCLEAR



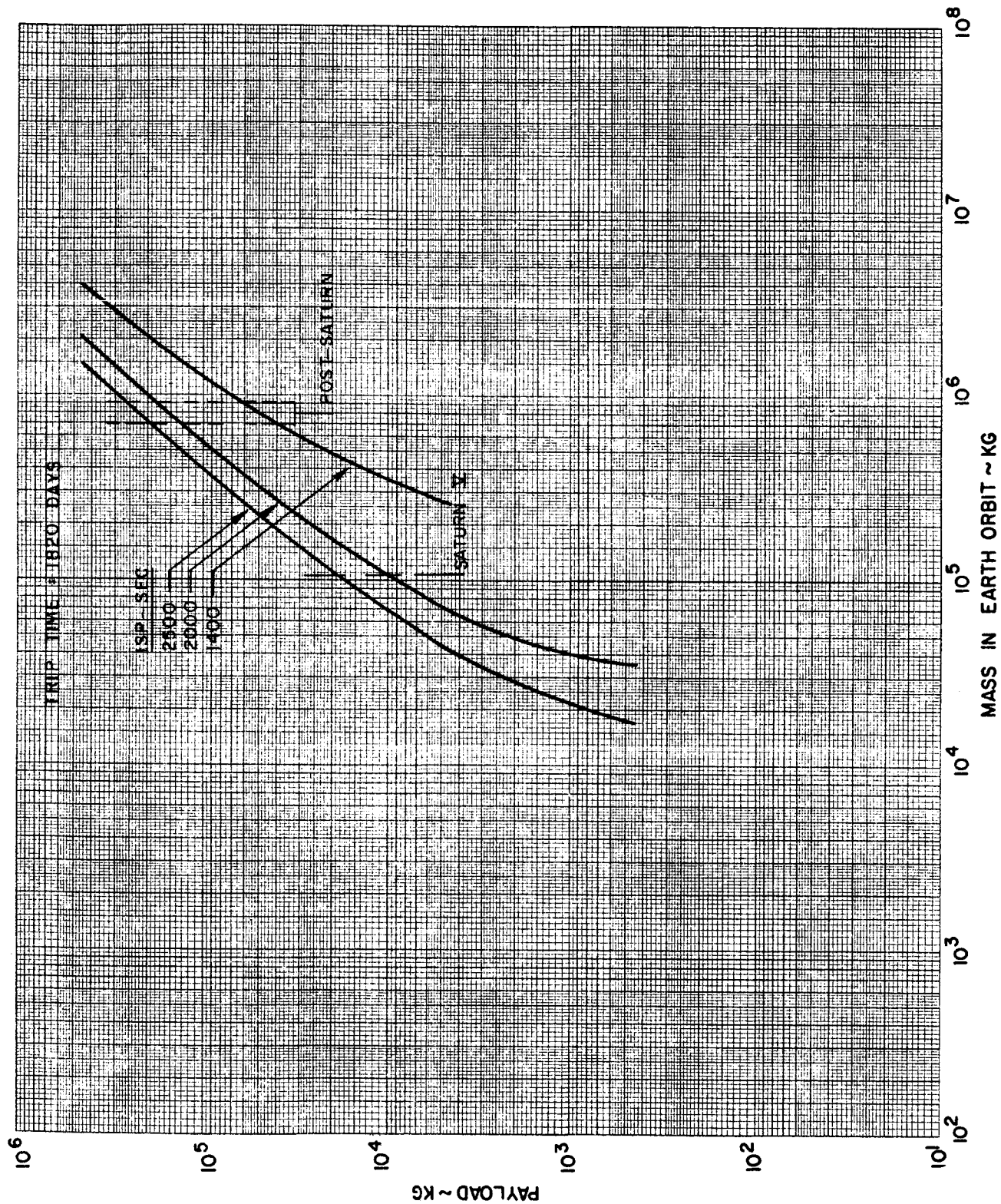
SATURN ORBITAL STOPOVER

SOLID - CORE NUCLEAR



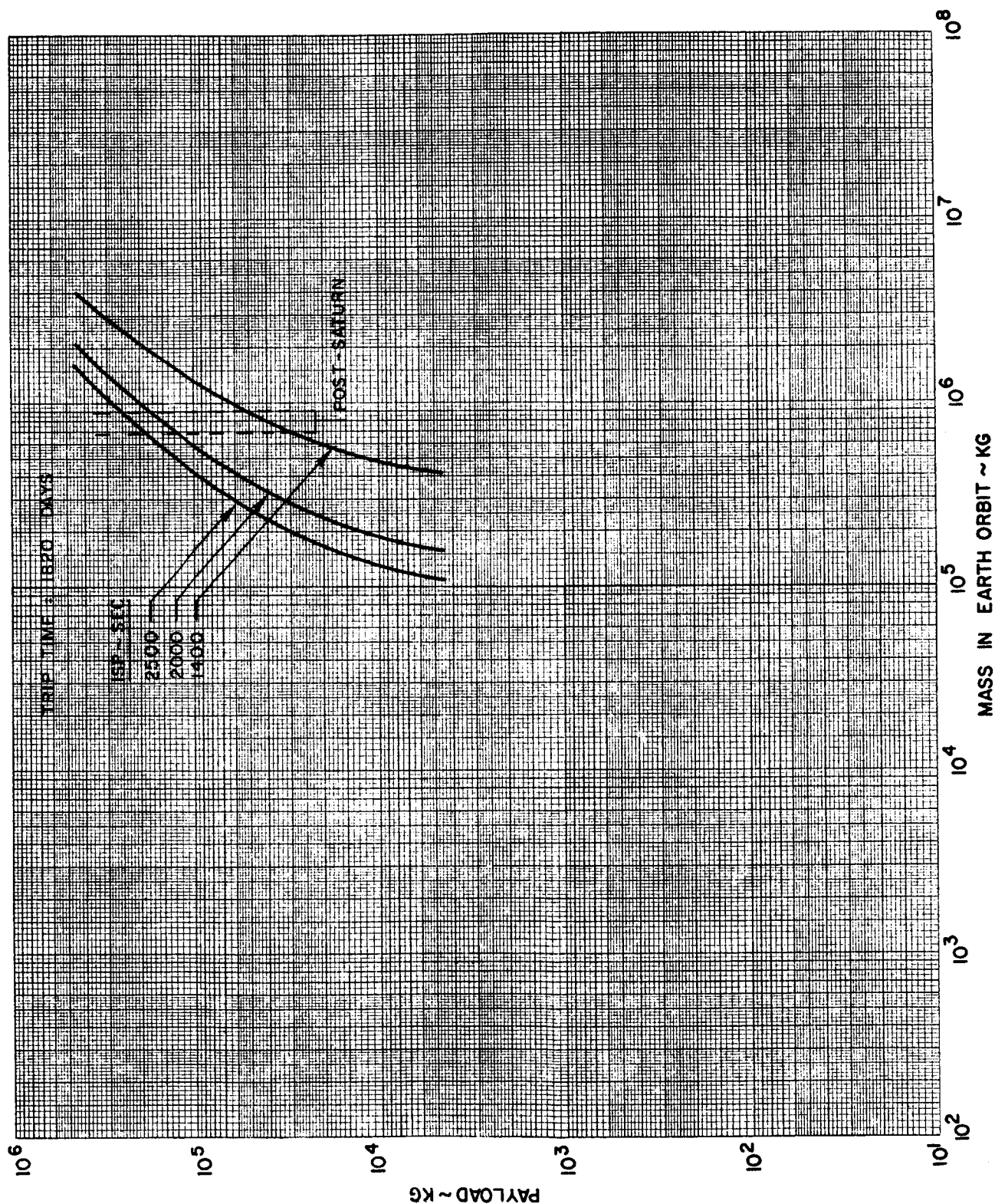
SATURN ORBITAL STOPOVER

LIGHTWEIGHT ADVANCED NUCLEAR



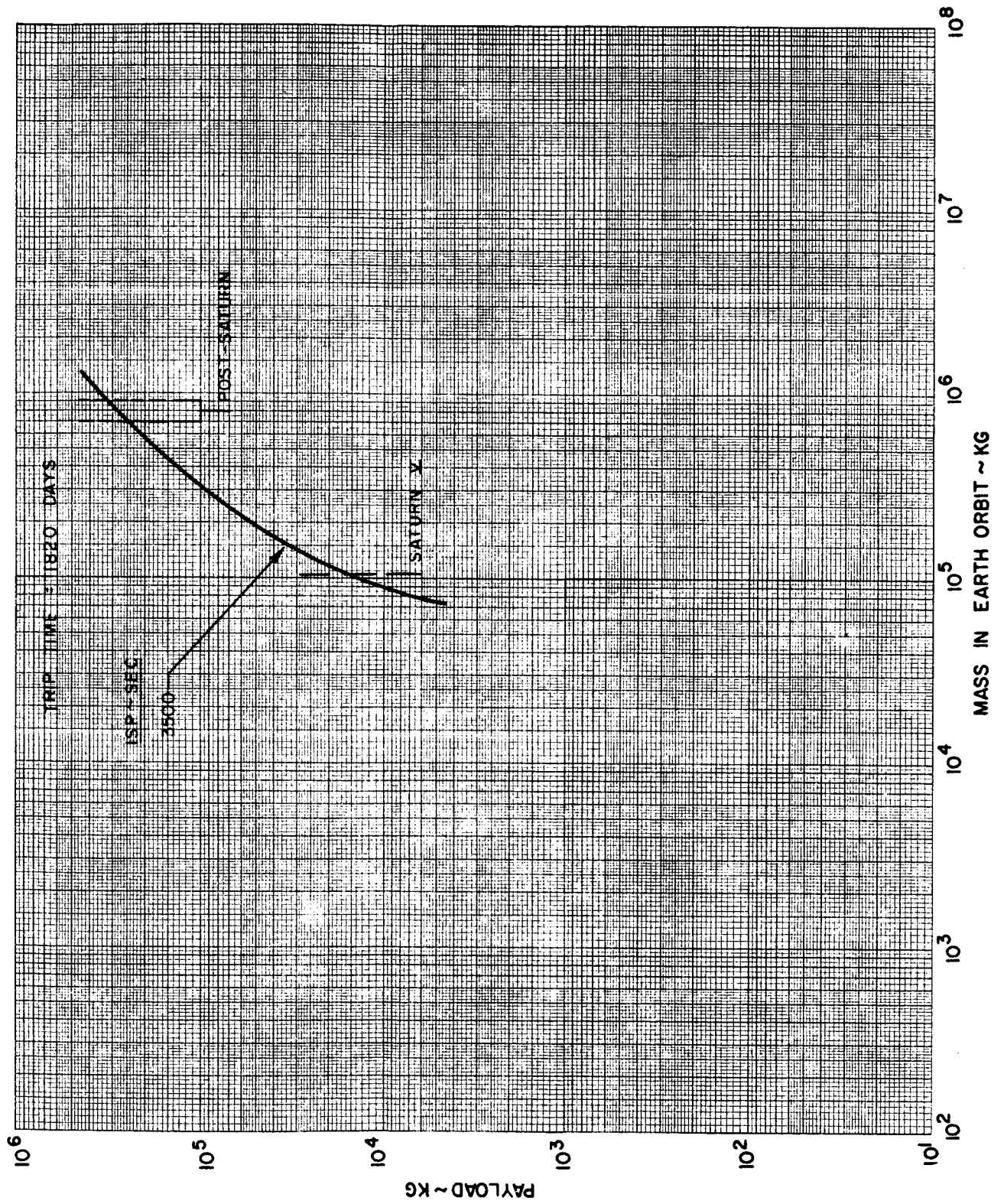
SATURN ORBITAL STOPOVER

GASEOUS - CORE NUCLEAR



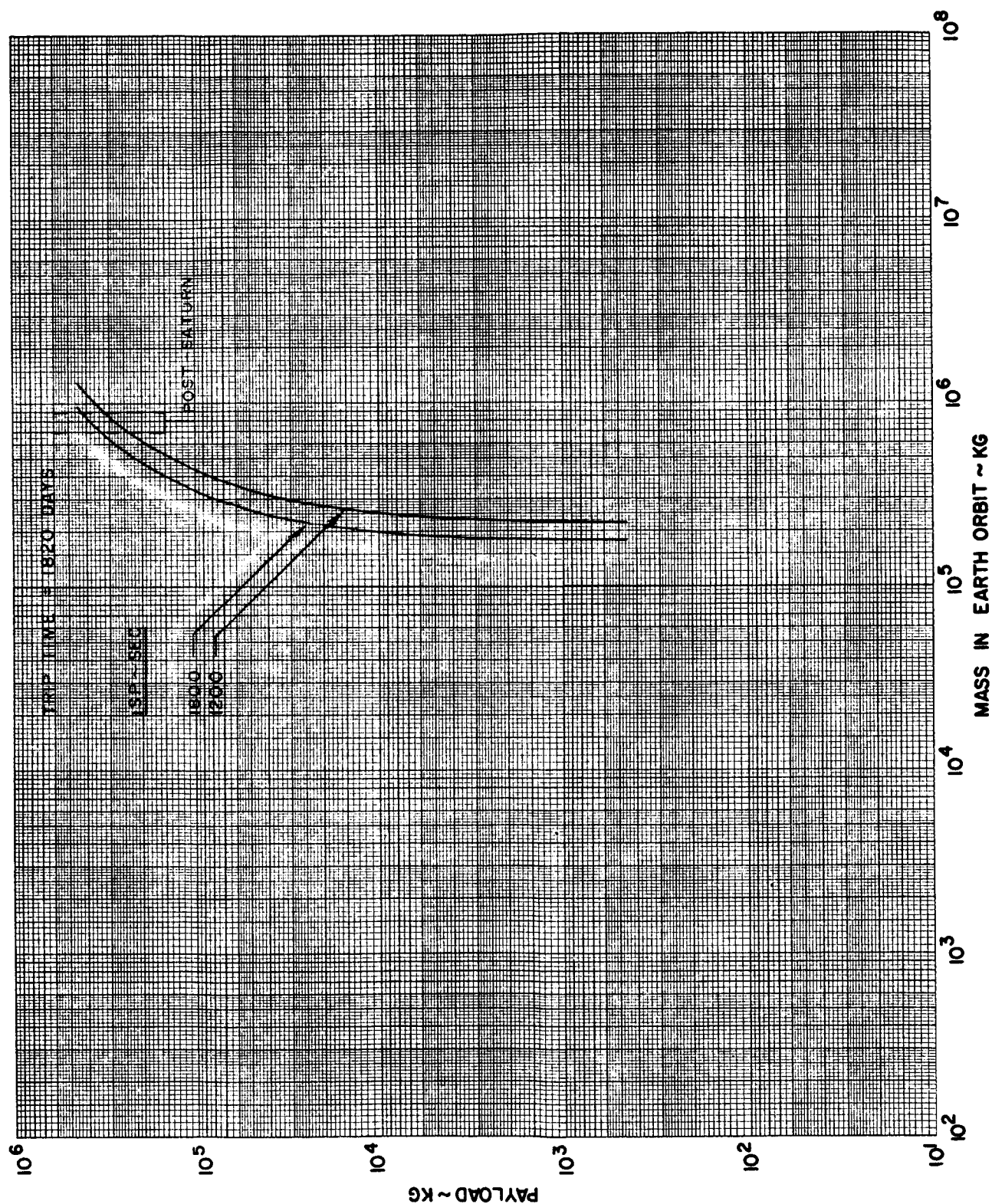
SATURN ORBITAL STOPOVER

GASEOUS - CORE NUCLEAR



SATURN ORBITAL STOPOVER

PULSED NUCLEAR



URANUS ORBITAL STOPOVER
SOLID-CORE NUCLEAR

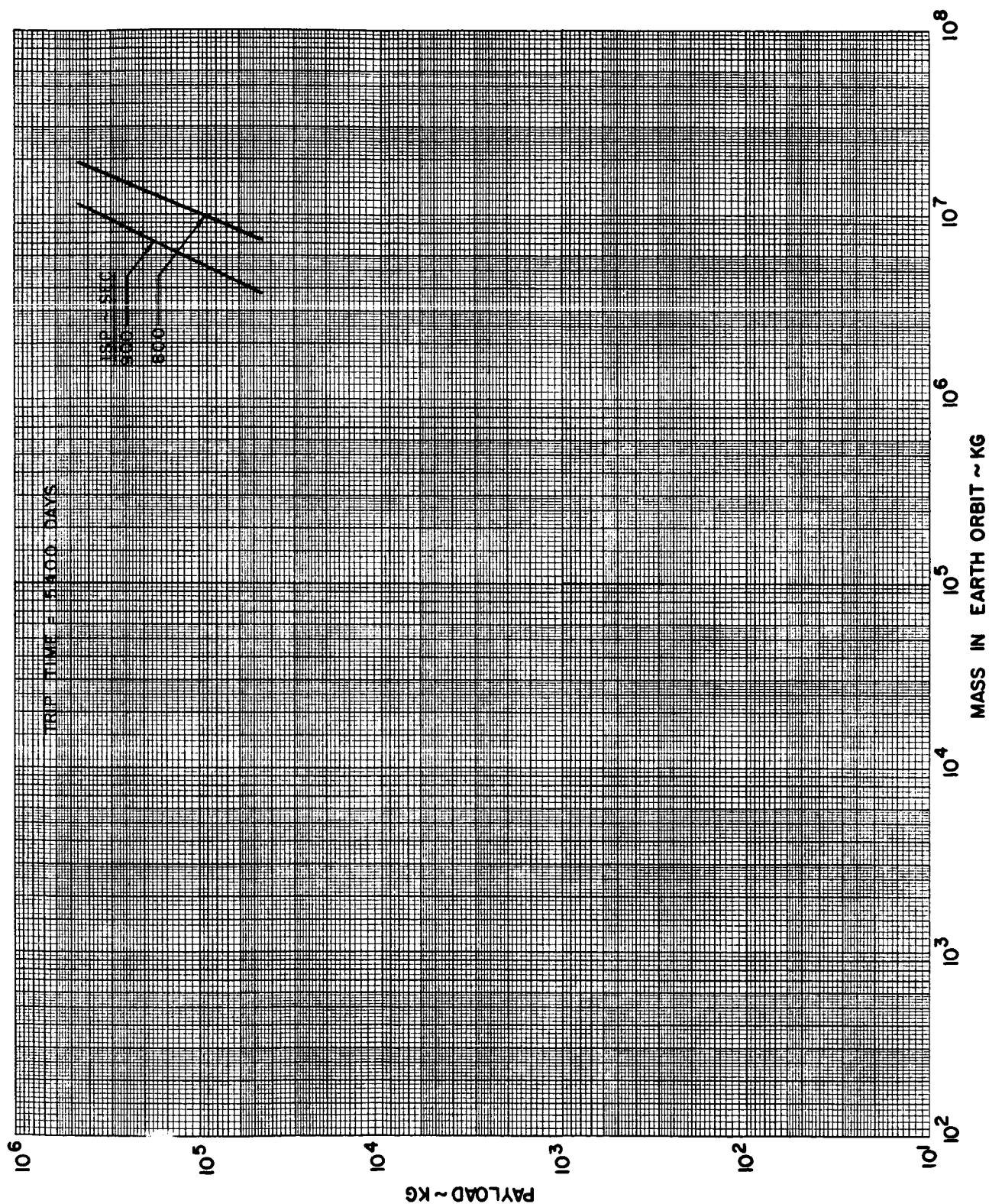
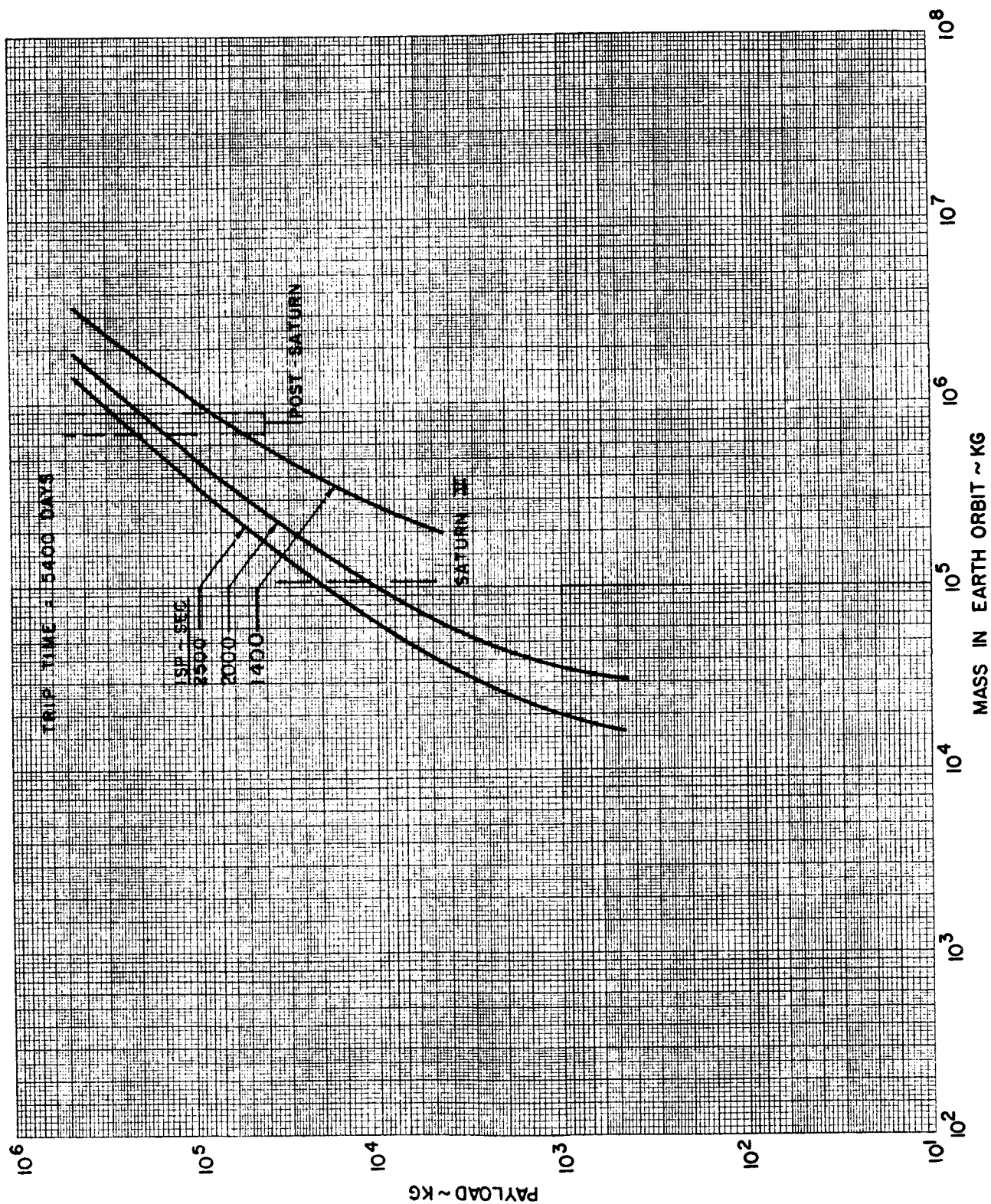


FIG. 188

E-910352-9

URANUS ORBITAL STOPOVER

LIGHTWEIGHT ADVANCED NUCLEAR



URANUS ORBITAL STOPOVER GASEOUS-CORE NUCLEAR

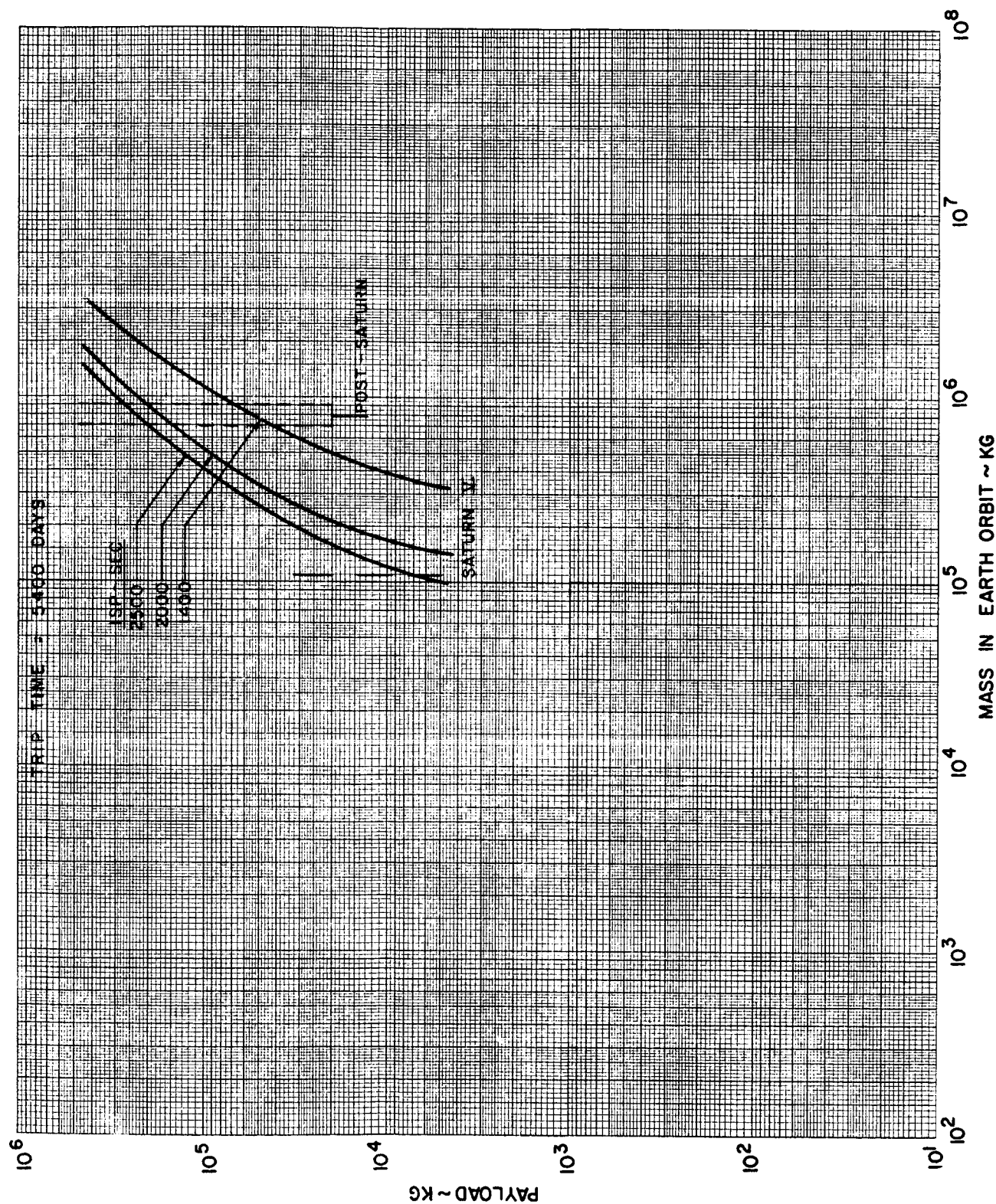
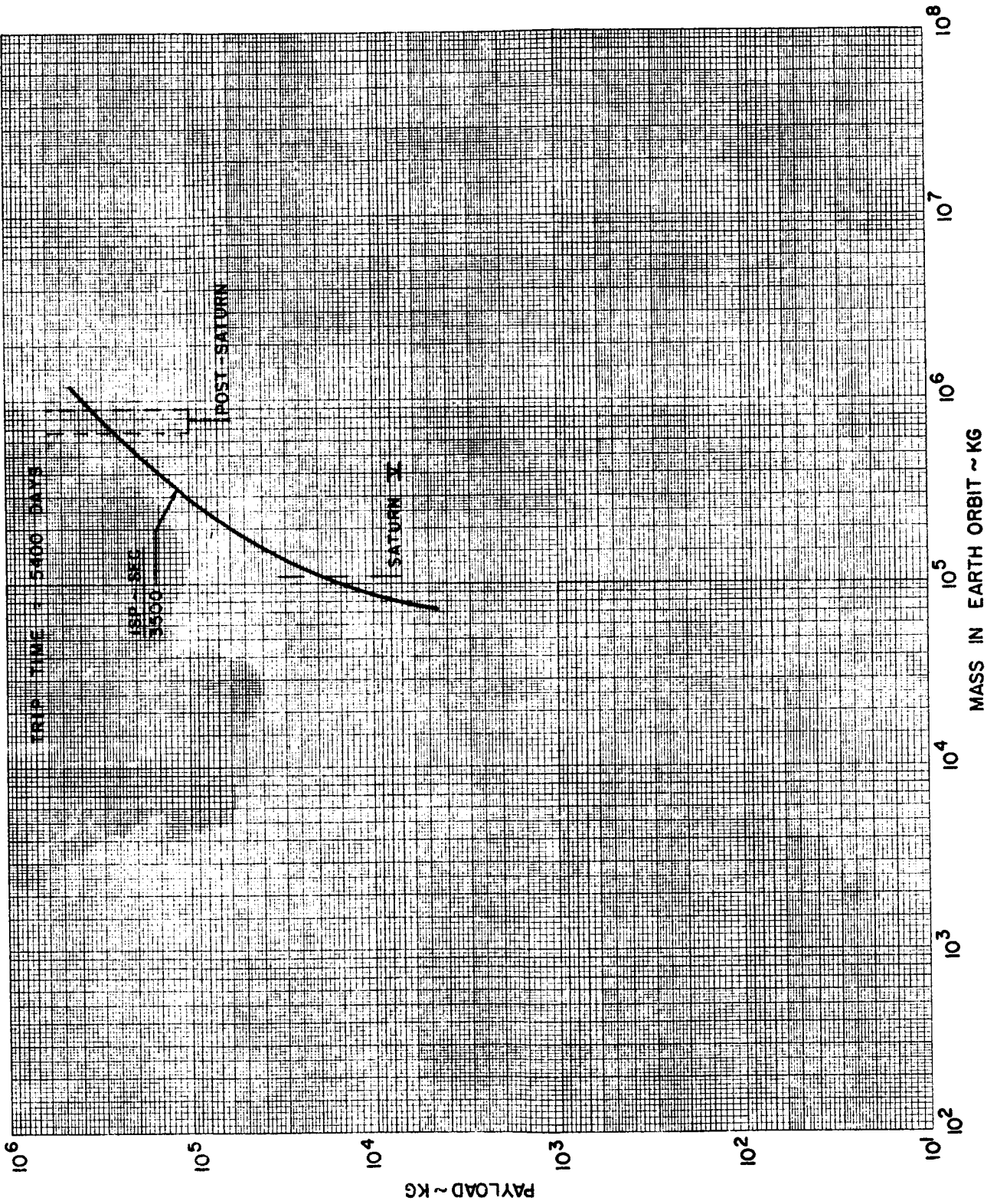


FIG. 190

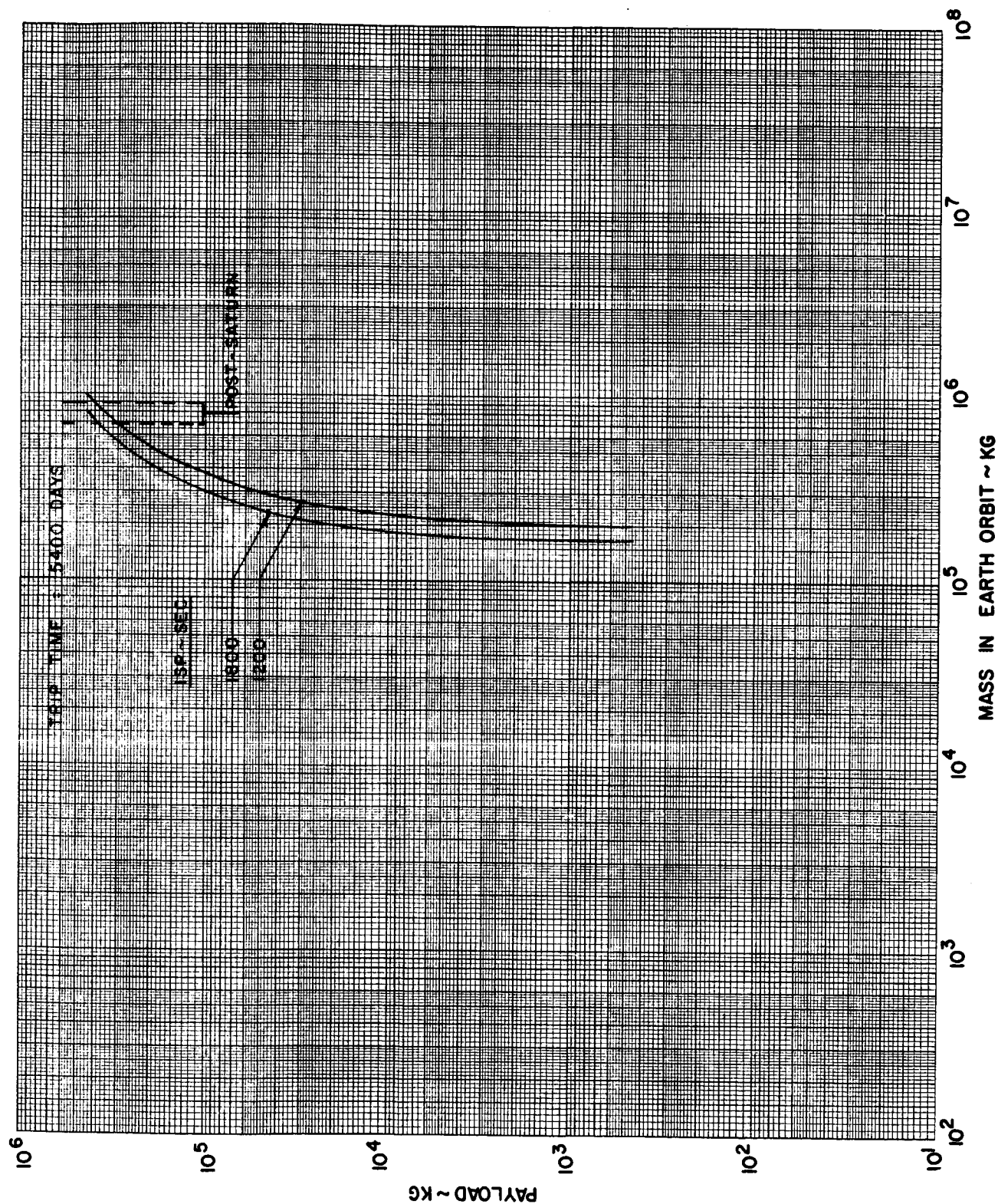
E-910352-9

URANUS ORBITAL STOPOVER GASEOUS-CORE NUCLEAR

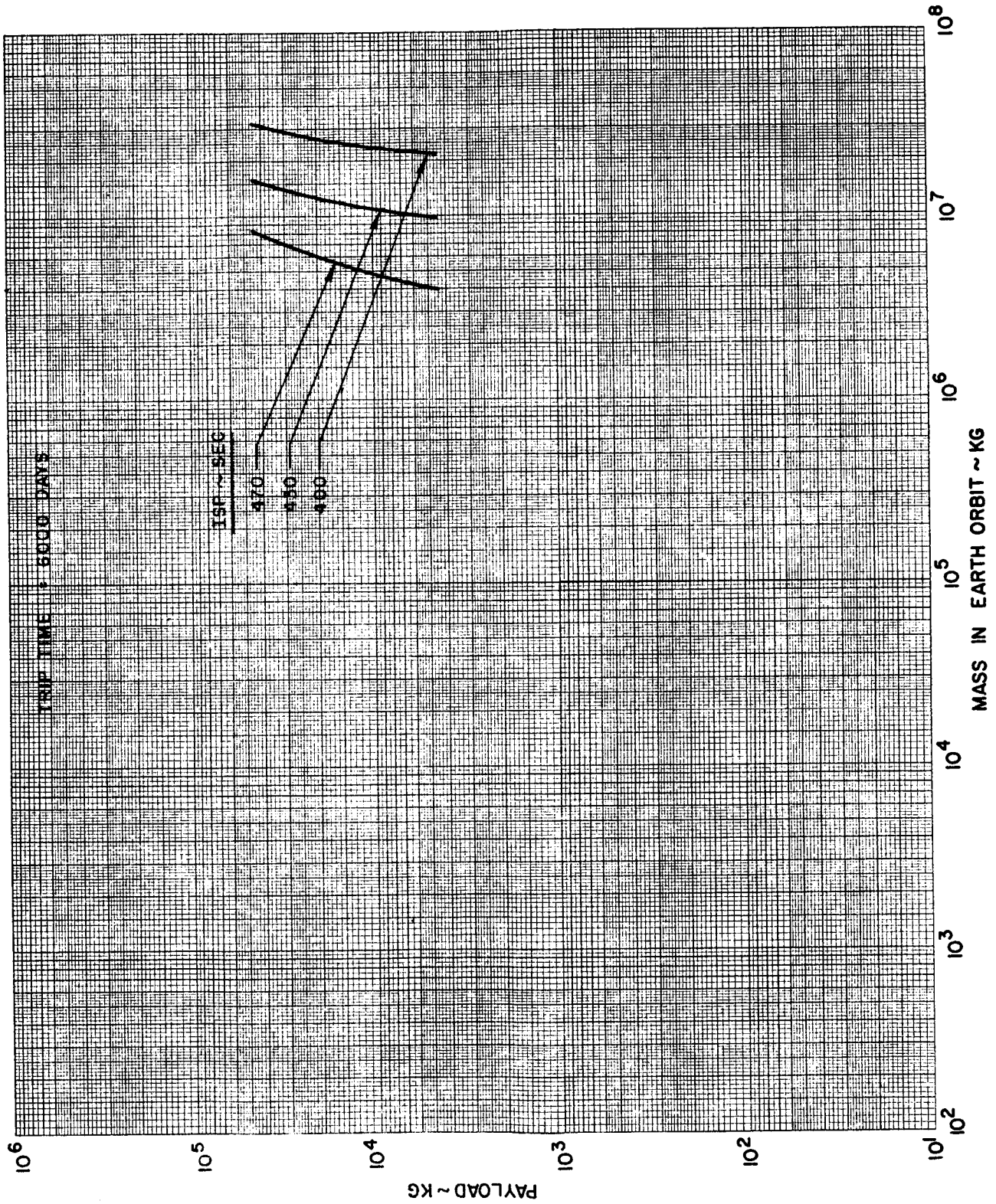


URANUS ORBITAL STOPOVER

PULSED NUCLEAR

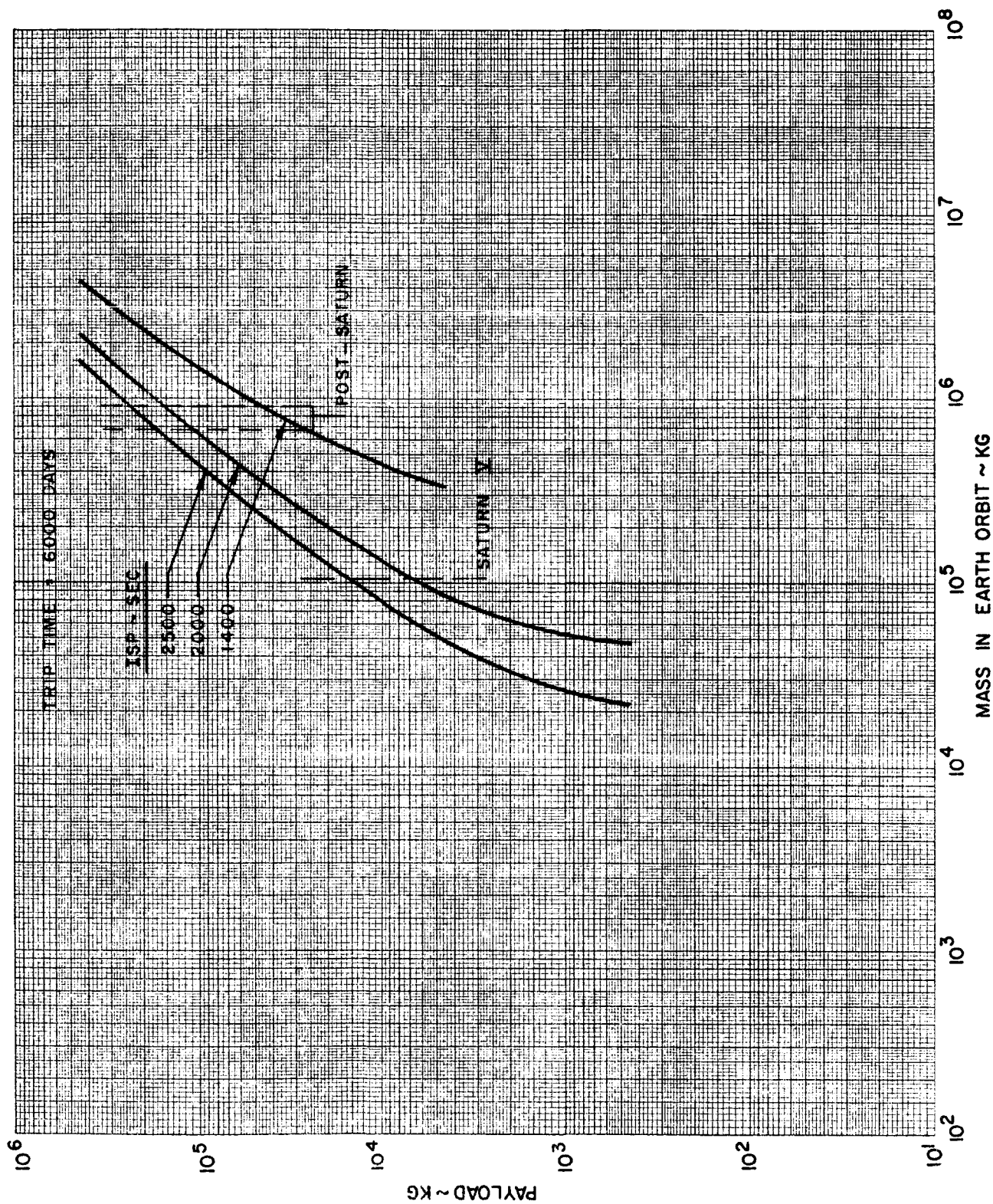


NEPTUNE ORBITAL STOPOVER
CHEMICAL



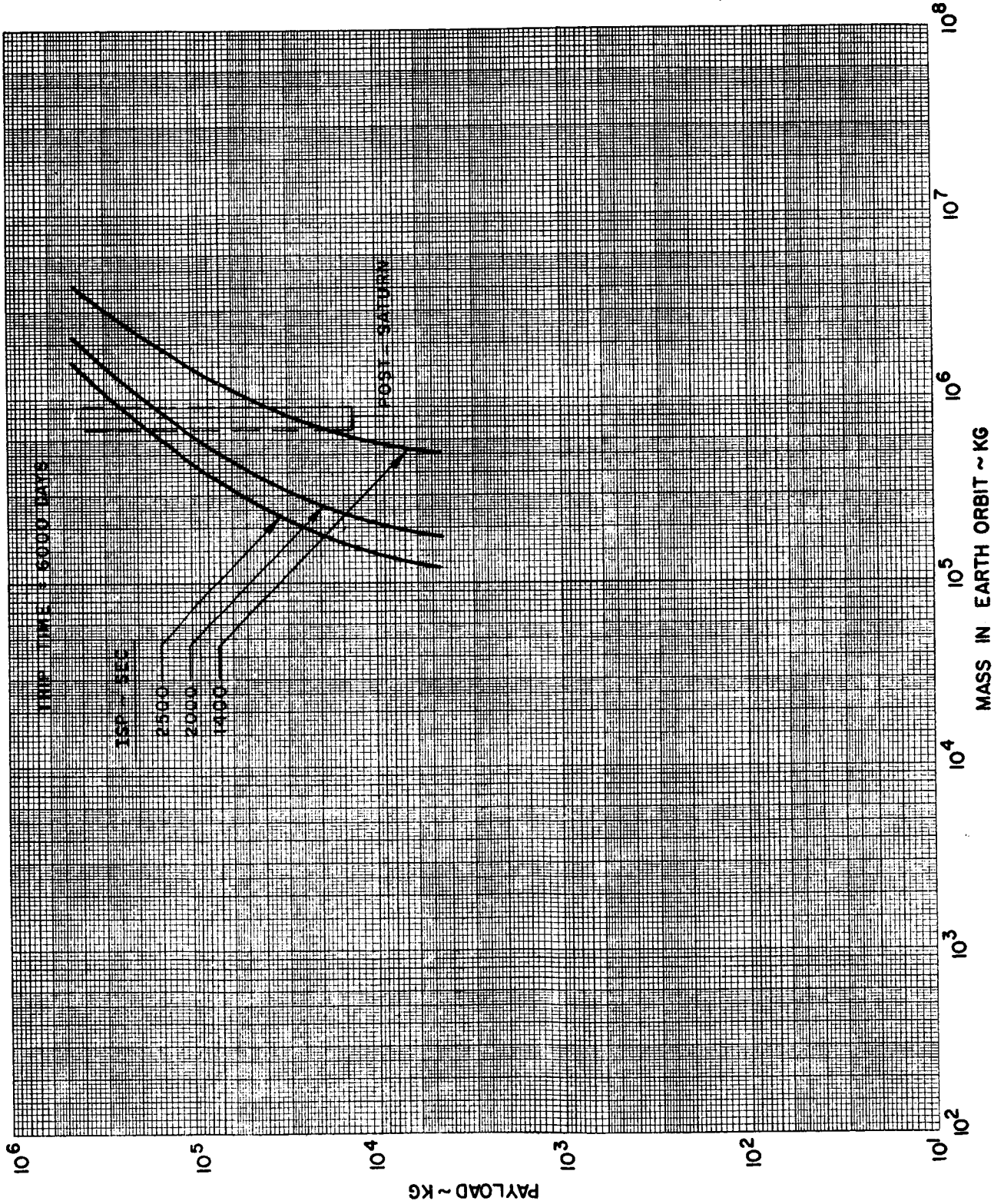
NEPTUNE ORBITAL STOPOVER

LIGHTWEIGHT ADVANCED NUCLEAR



NEPTUNE ORBITAL STOPOVER

GASEOUS-CORE NUCLEAR



NEPTUNE ORBITAL STOPOVER

GASEOUS - CORE NUCLEAR

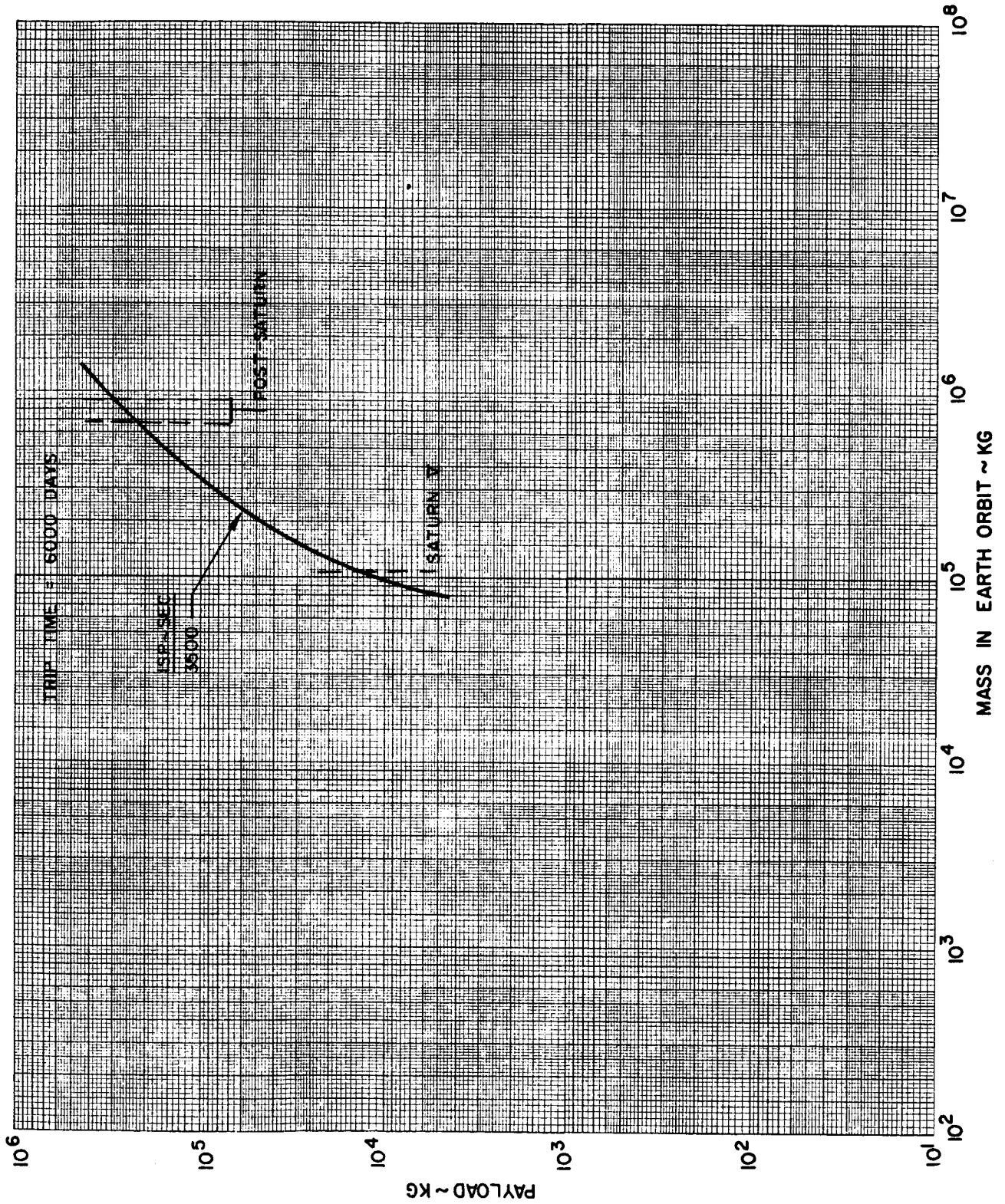
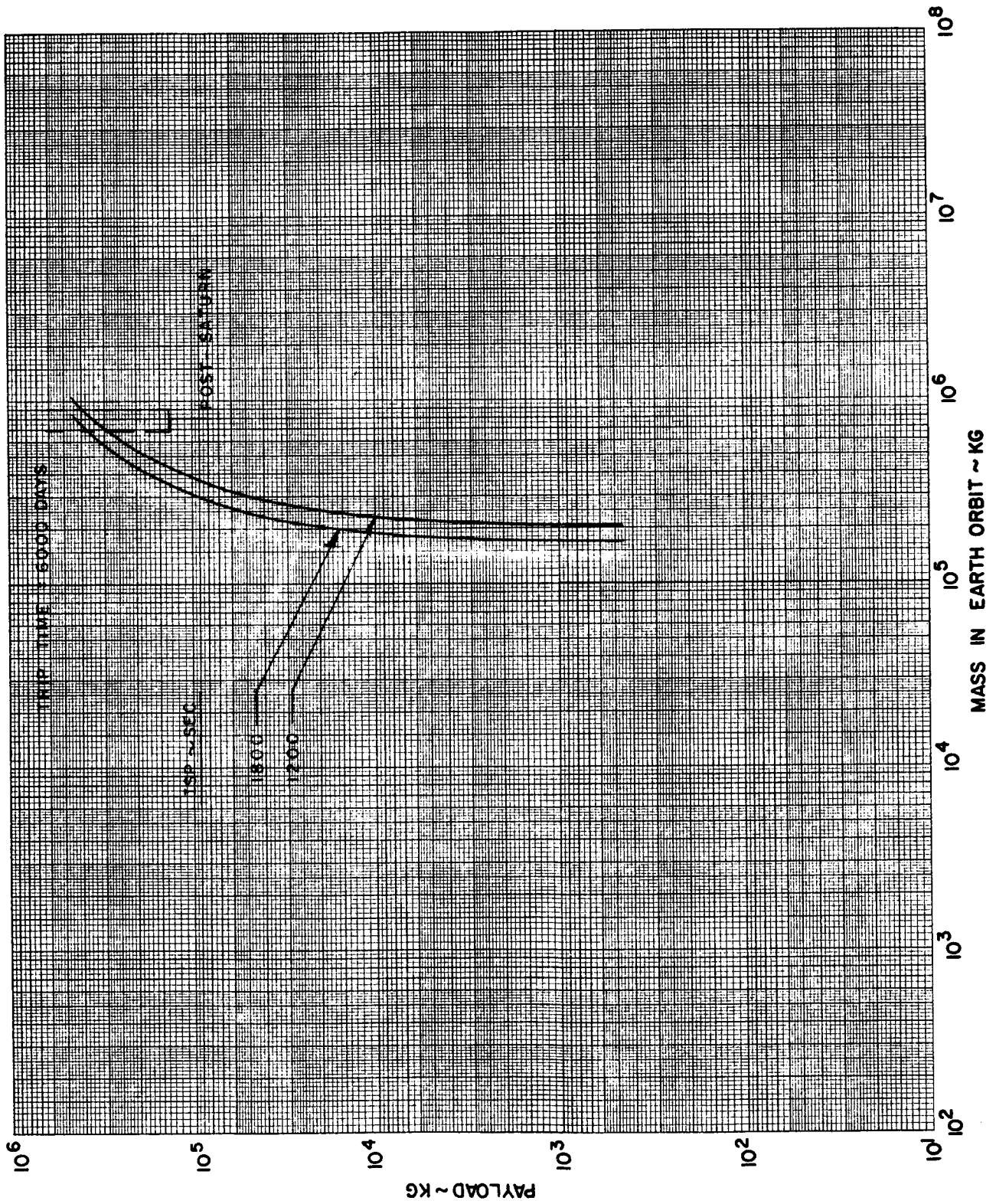


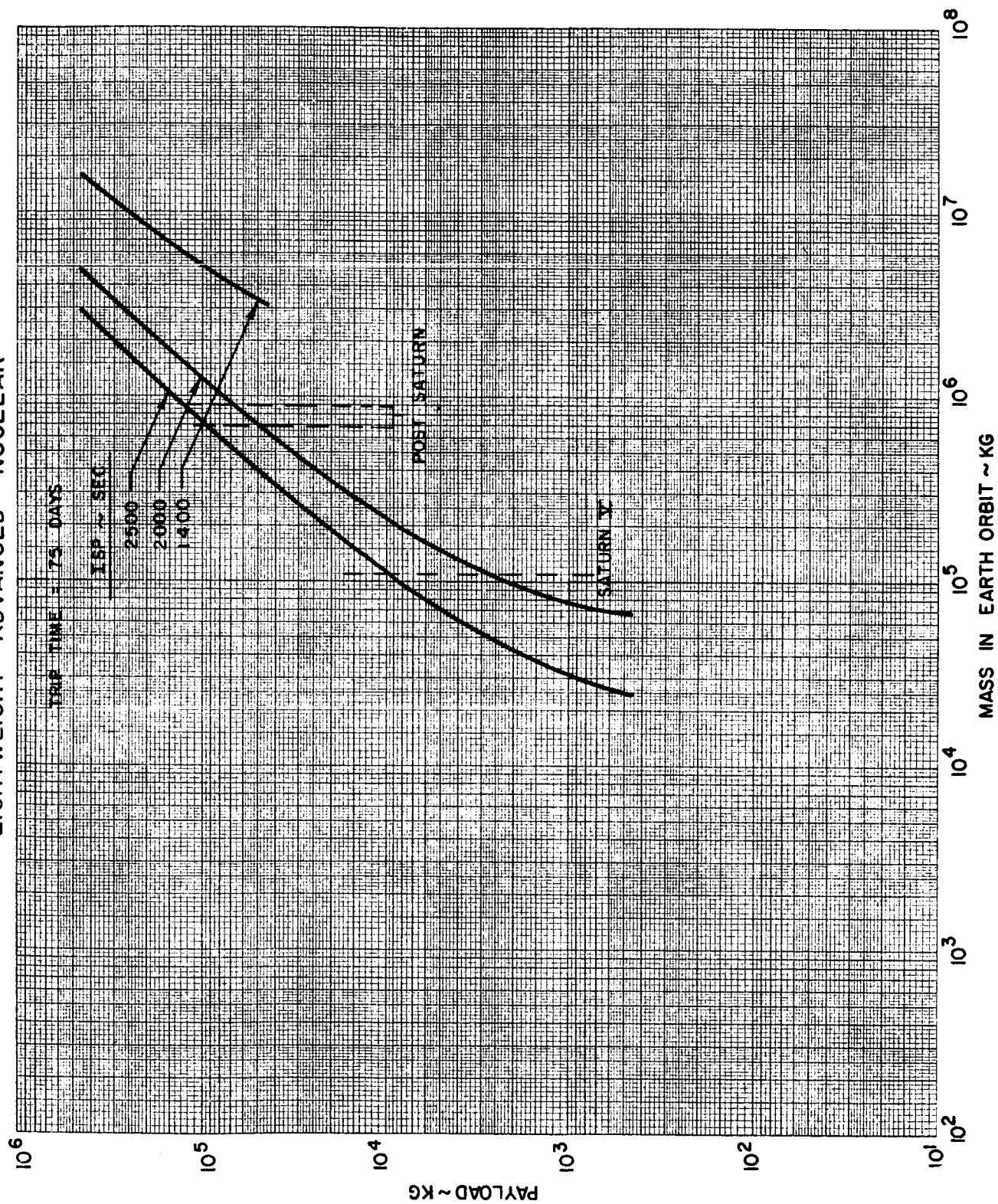
FIG. 196

E-910352-9

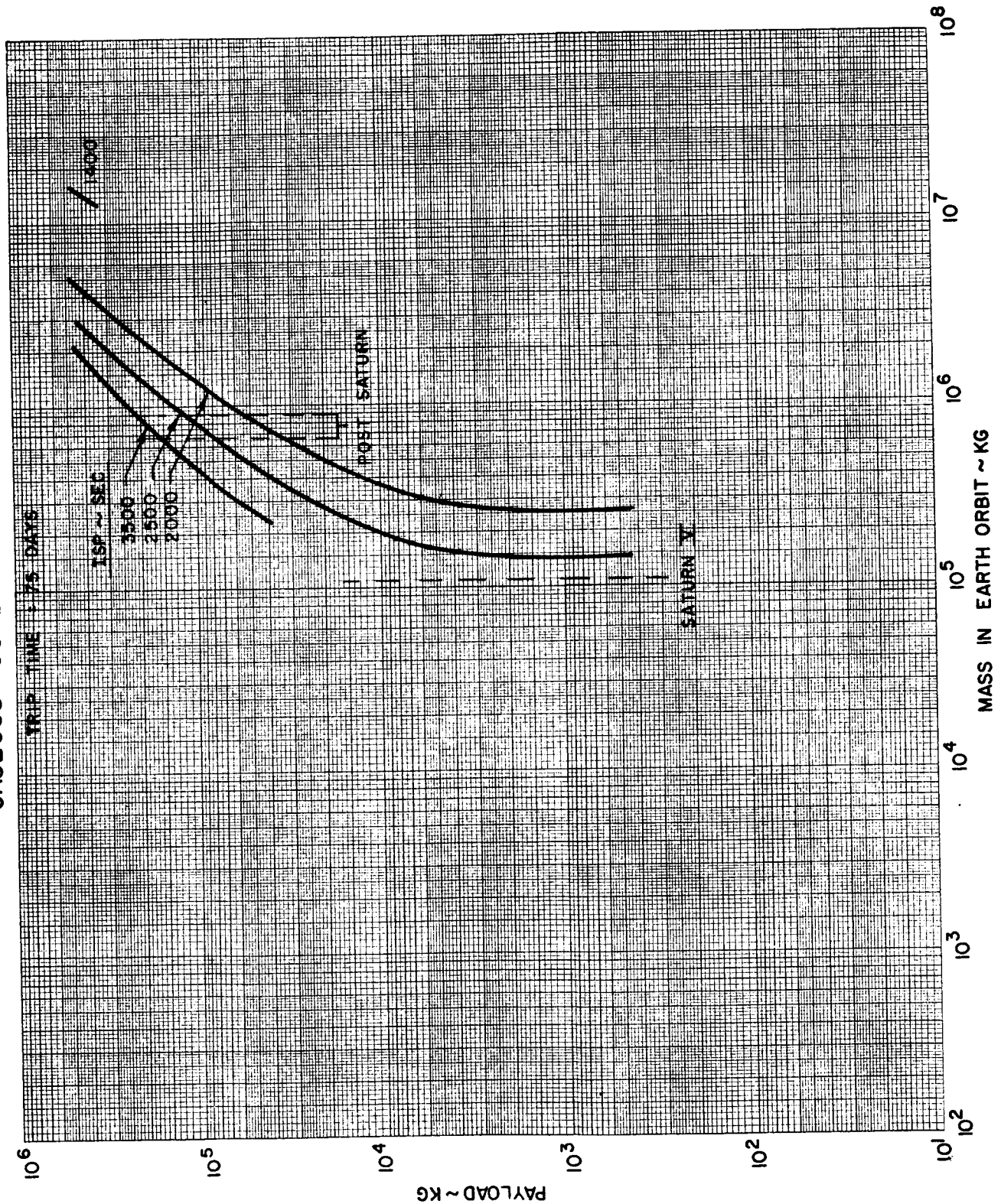
NEPTUNE ORBITAL STOPOVER PULSED NUCLEAR



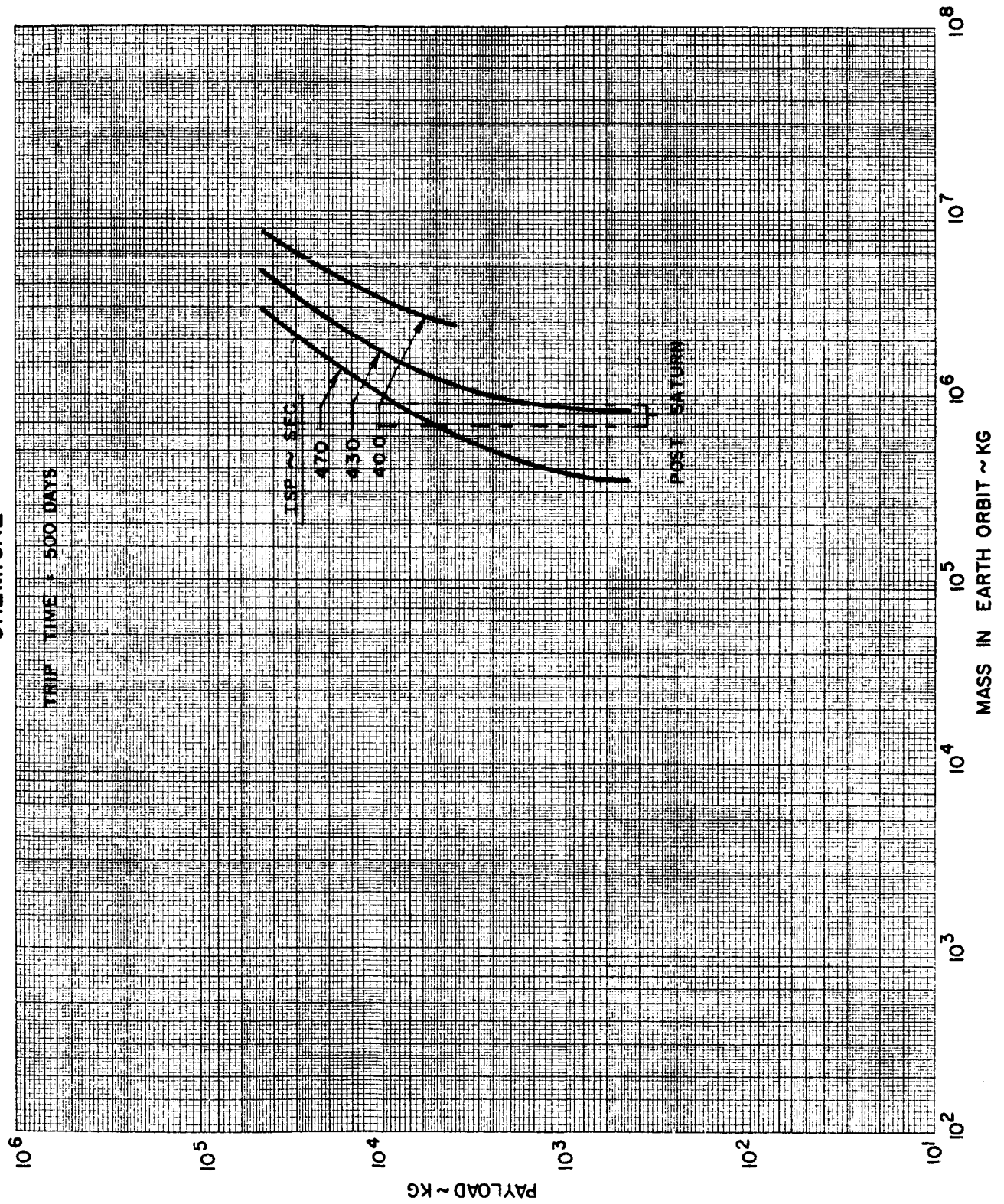
SYNCHRONOUS SOLAR ORBITER LIGHTWEIGHT ADVANCED NUCLEAR



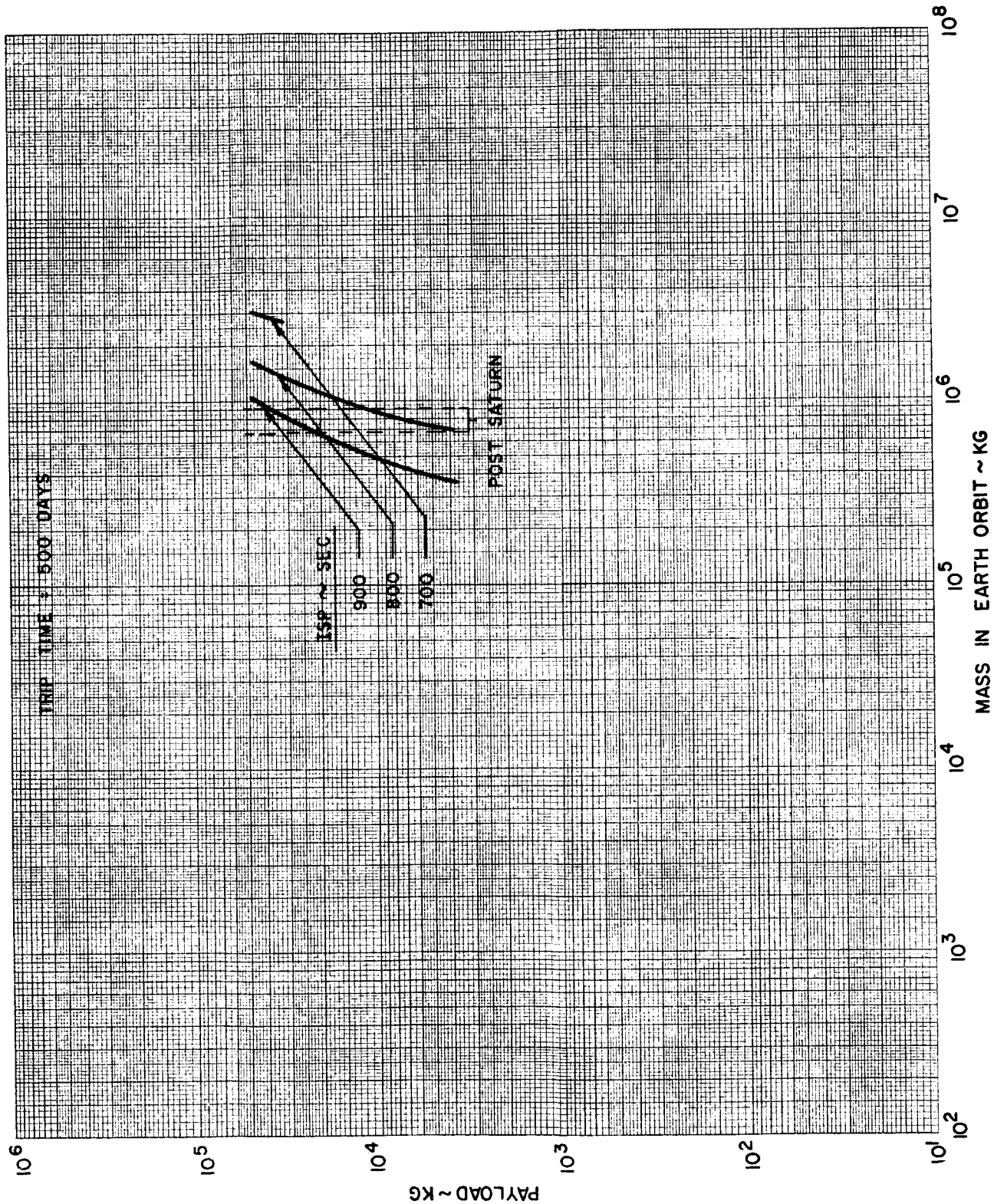
SYNCHRONOUS SOLAR ORBITER GASEOUS - CORE NUCLEAR



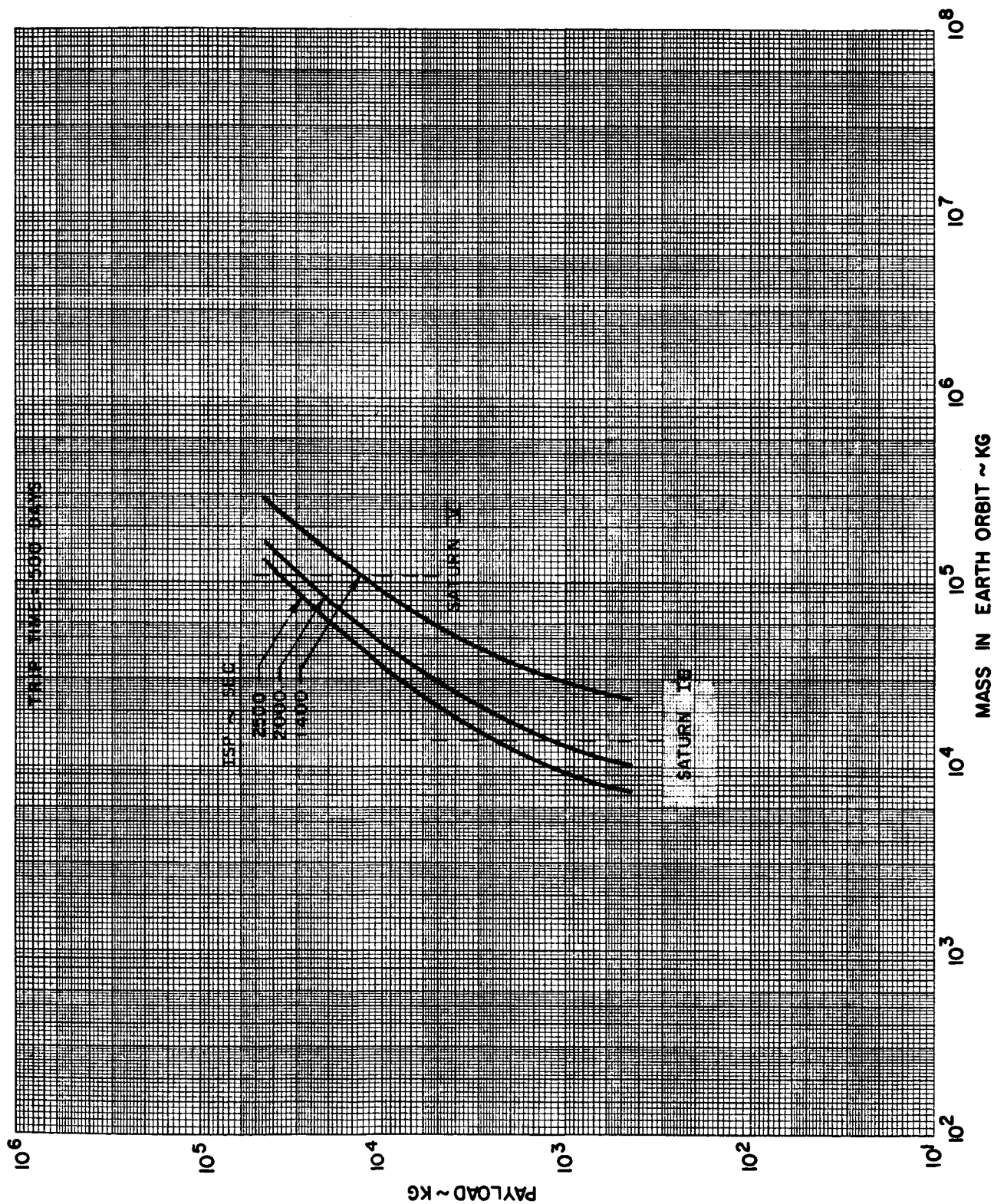
SCHWASSMANN - WACHMANN II RENDEZVOUS
CHEMICAL



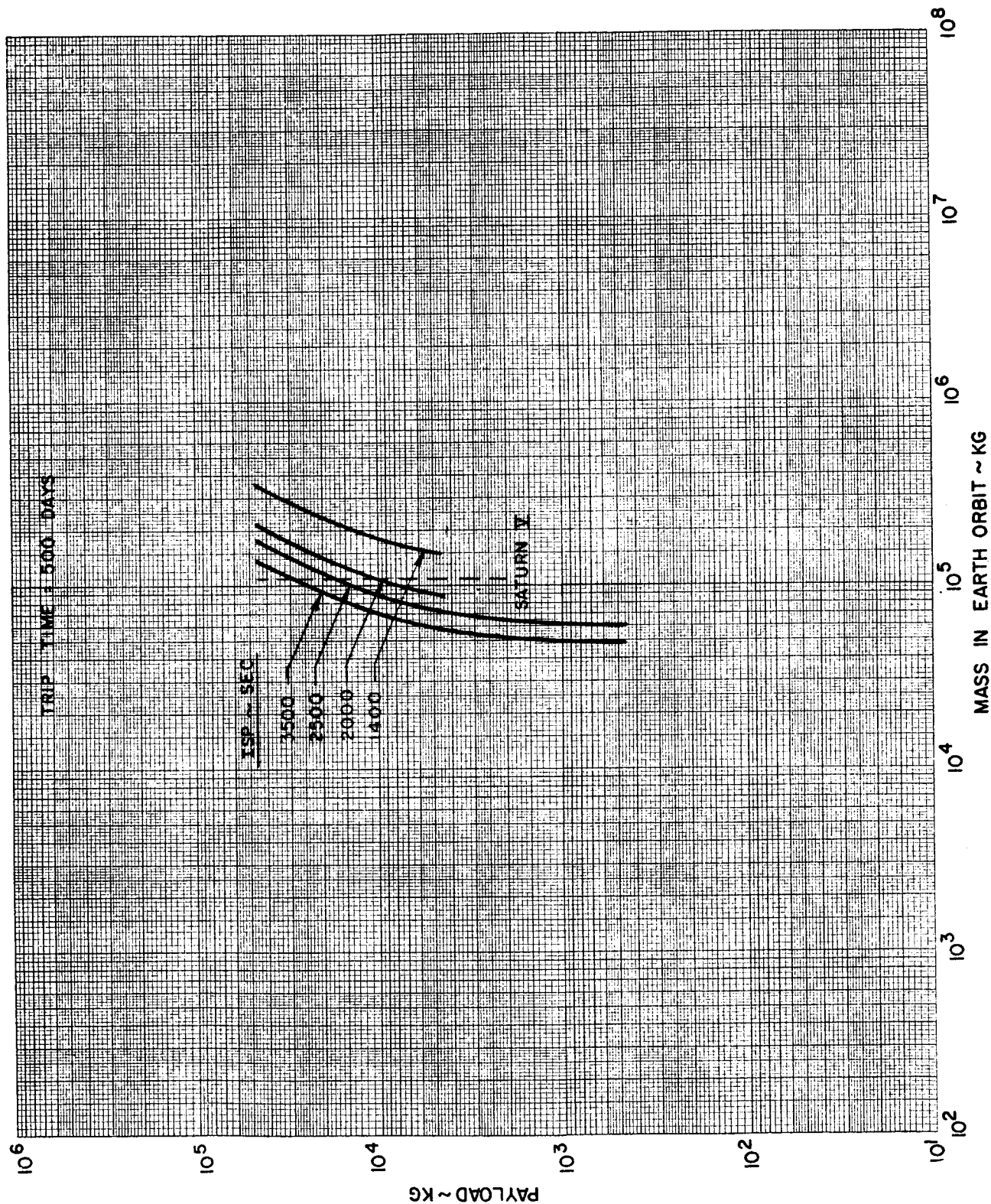
SCHWASSMANN - WACHMANN II RENDEZVOUS SOLID-CORE NUCLEAR



SCHWASSMANN - WACHMANN II RENDEZVOUS
LIGHTWEIGHT ADVANCED NUCLEAR

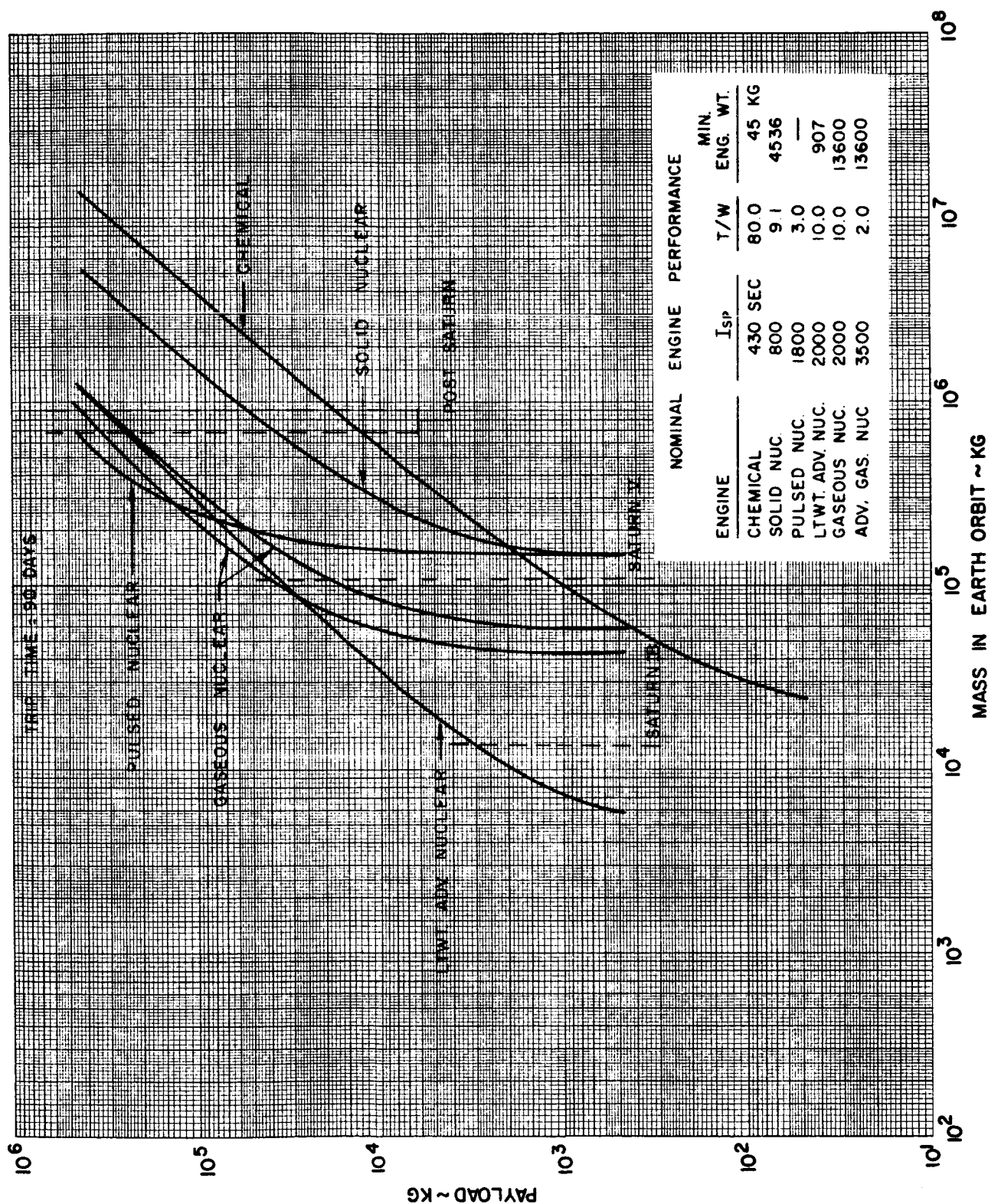


SCHWASSMANN - WACHMANN II RENDEZVOUS GASEOUS - CORE NUCLEAR



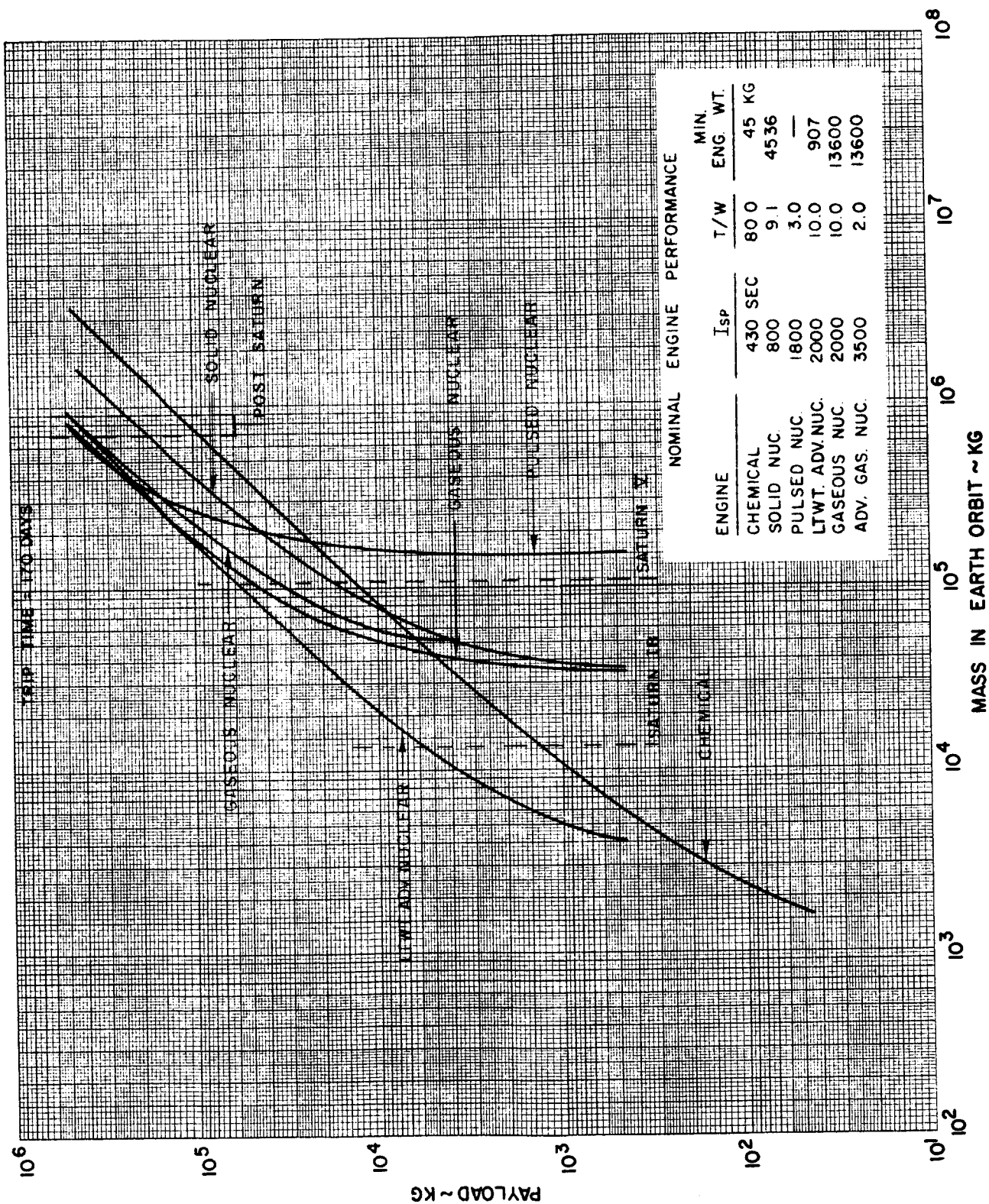
MERCURY ORBITAL STOPOVERS

PROPULSION SYSTEM COMPARISON



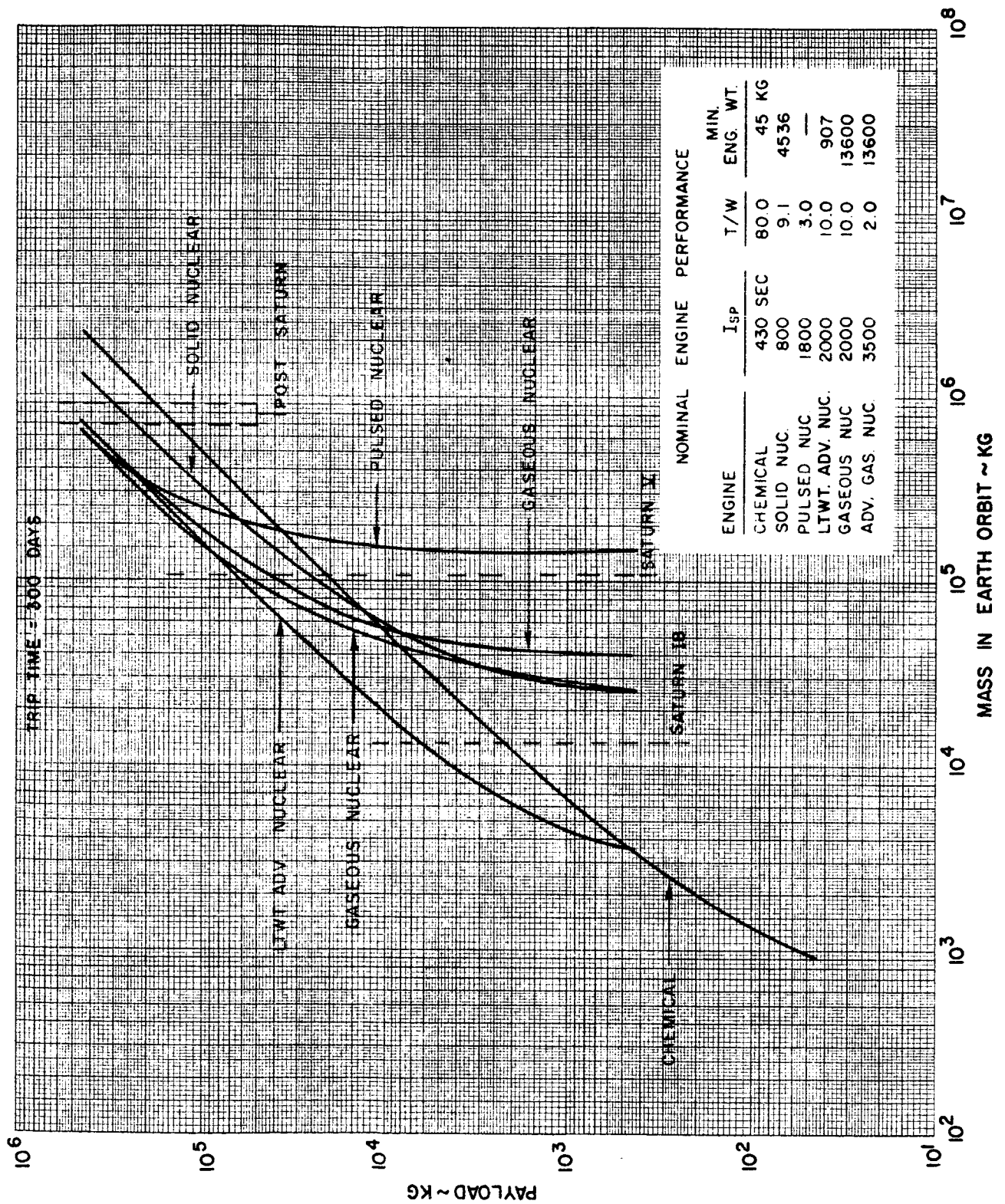
VENUS ORBITAL STOPOVERS

PROPULSION SYSTEM COMPARISON

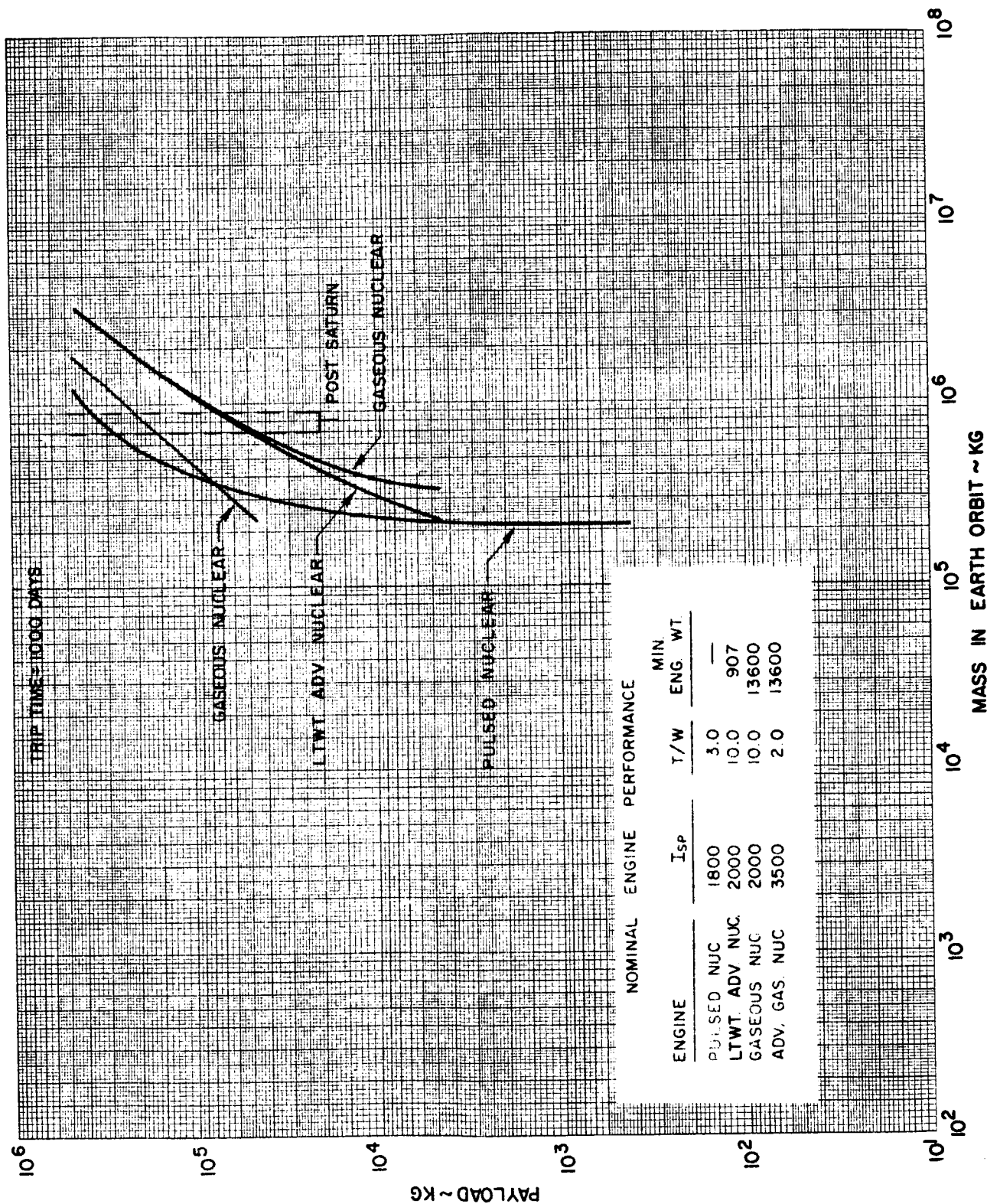


MARS ORBITAL STOPOVERS

PROPULSION SYSTEM COMPARISON

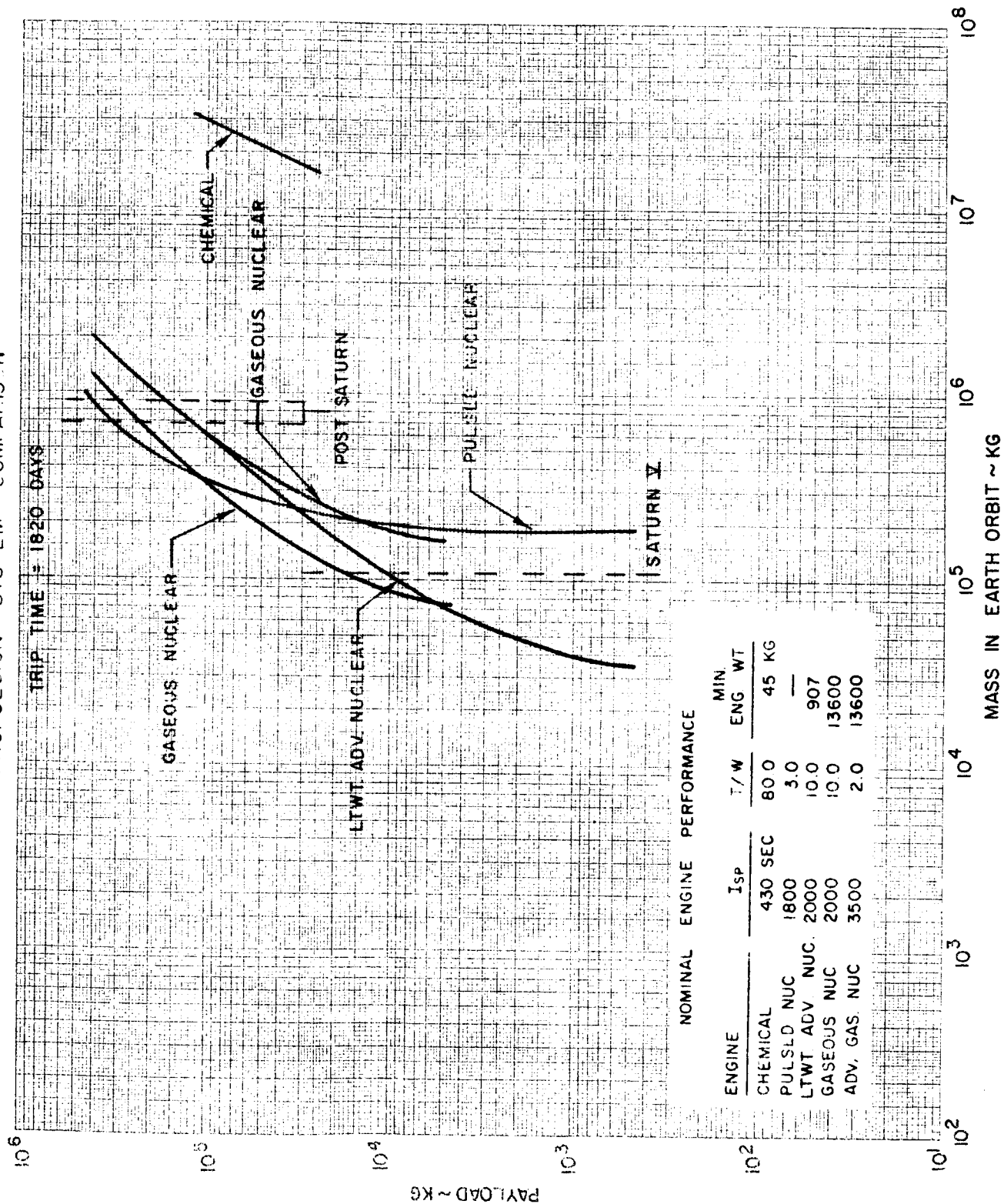


JUPITER ORBITAL STOPOVERS PROPULSION SYSTEM COMPARISON

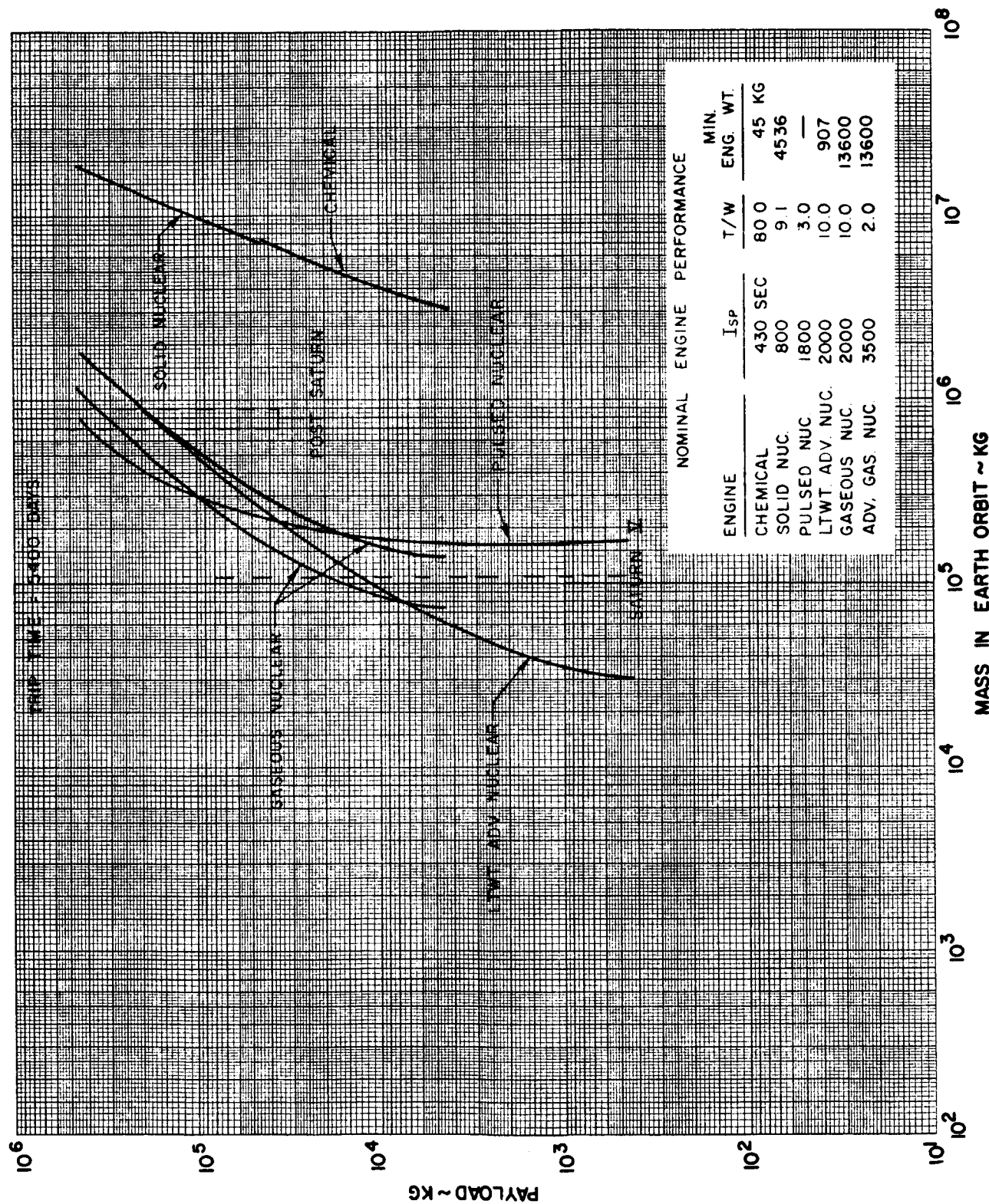


SATURN ORBITAL STOPOVERS

PROPULSION SYSTEM COMPARISON

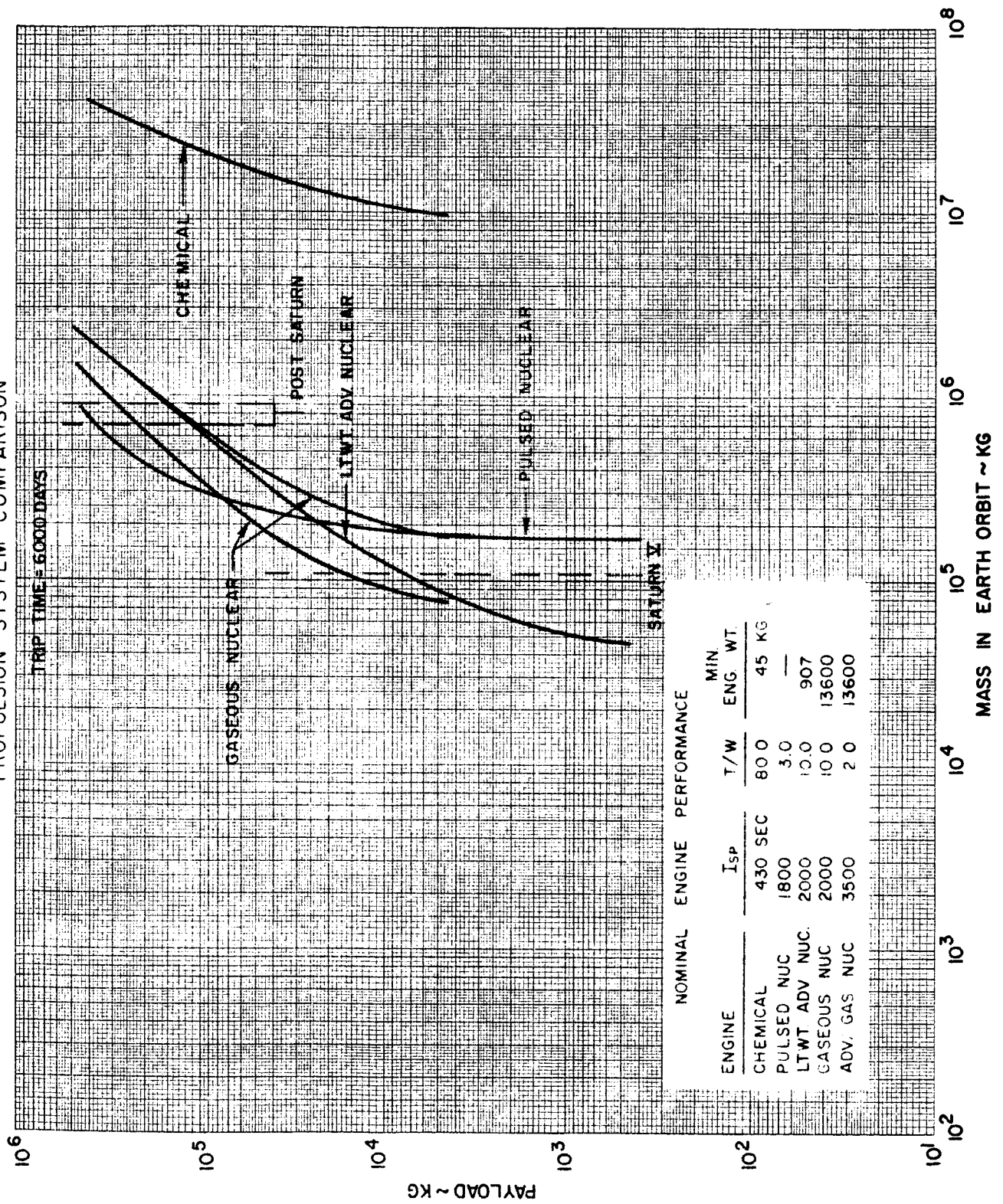


URANUS ORBITAL STOPOVERS
PROPULSION SYSTEM COMPARISON

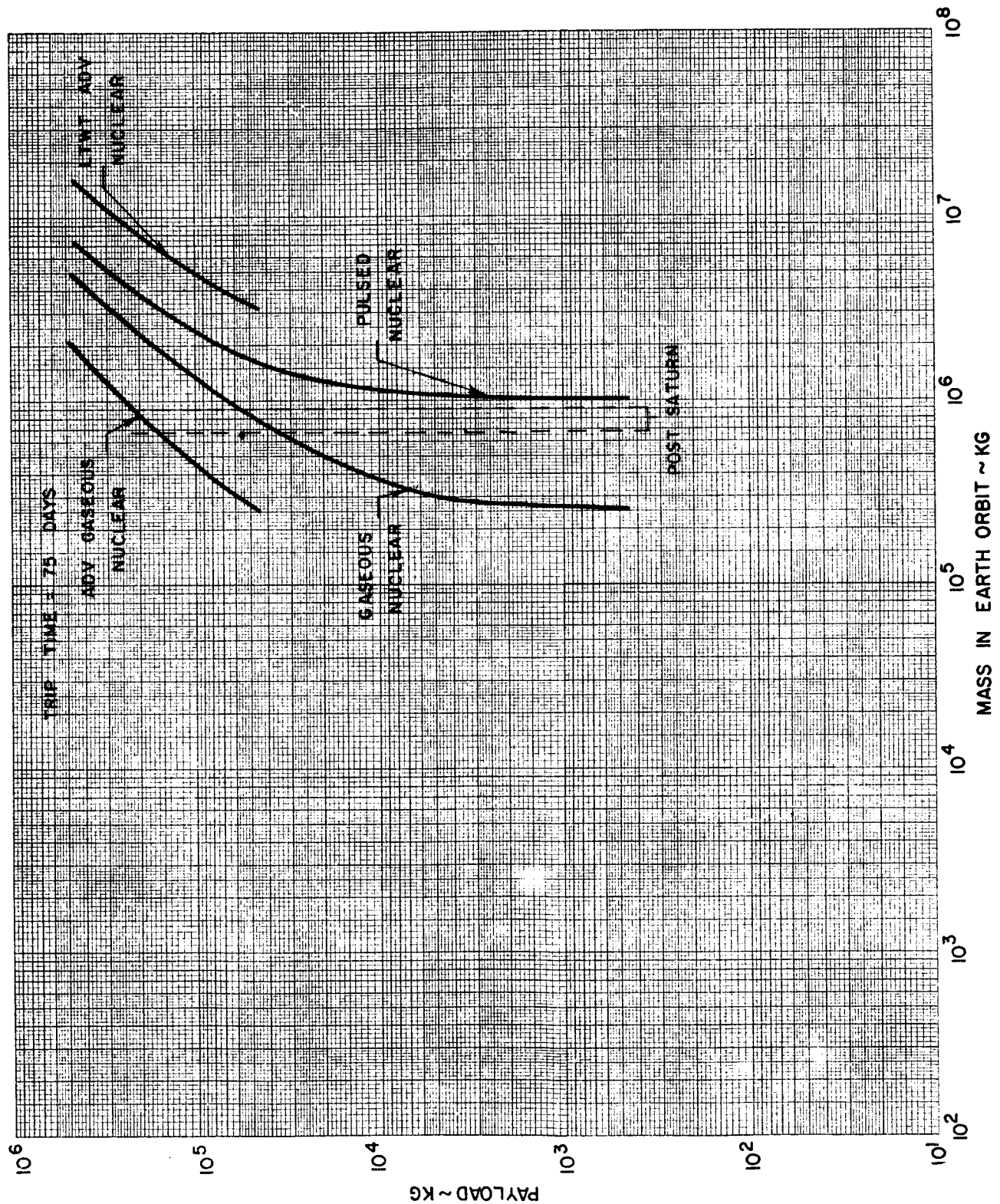


NEPTUNE ORBITAL STOPOVERS

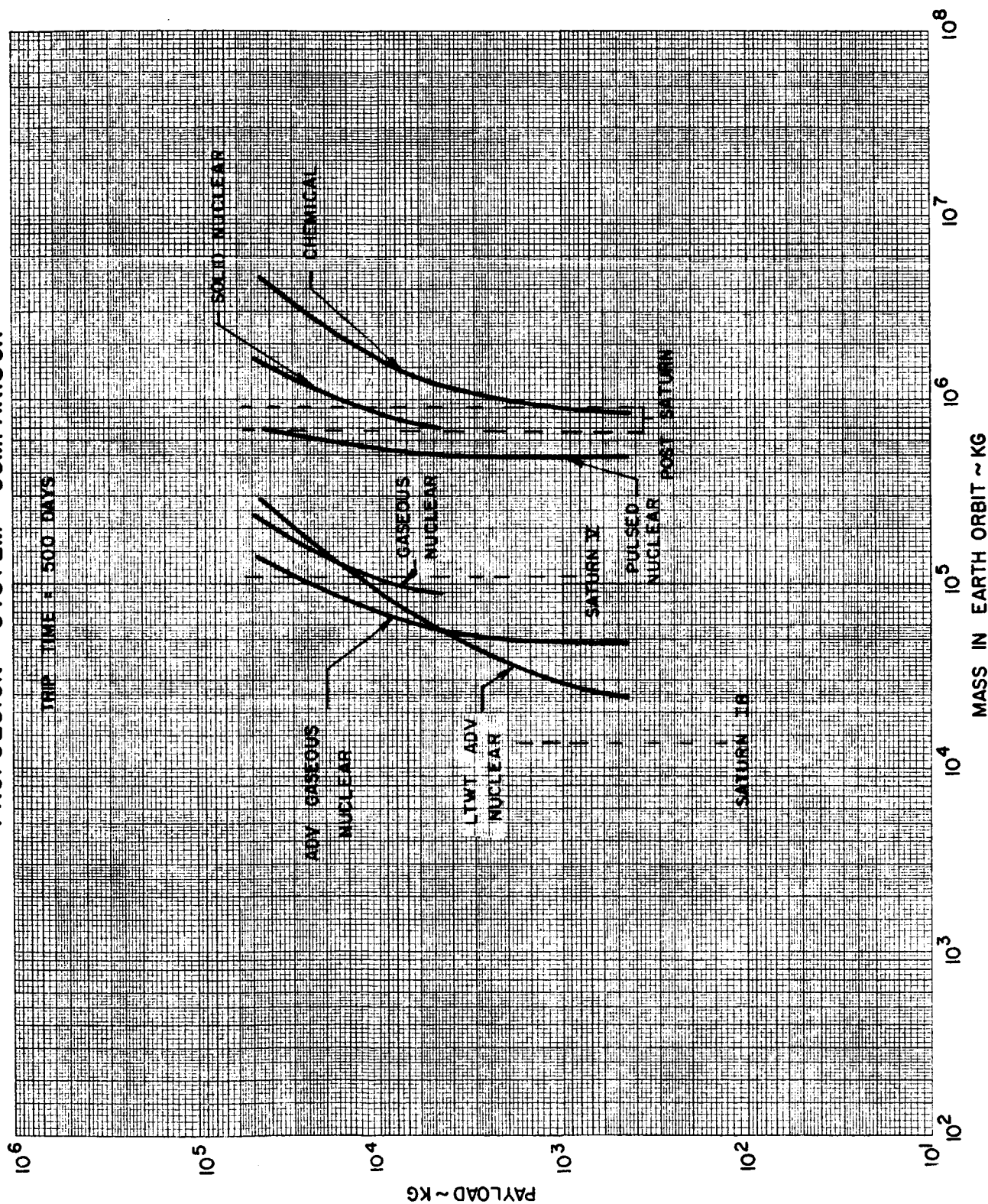
PROPULSION SYSTEM COMPARISON



SYNCHRONOUS SOLAR ORBITER PROPULSION SYSTEM COMPARISON



SCHWASSMANN - WACHMANN II RENDEZVOUS PROPULSION SYSTEM COMPARISON

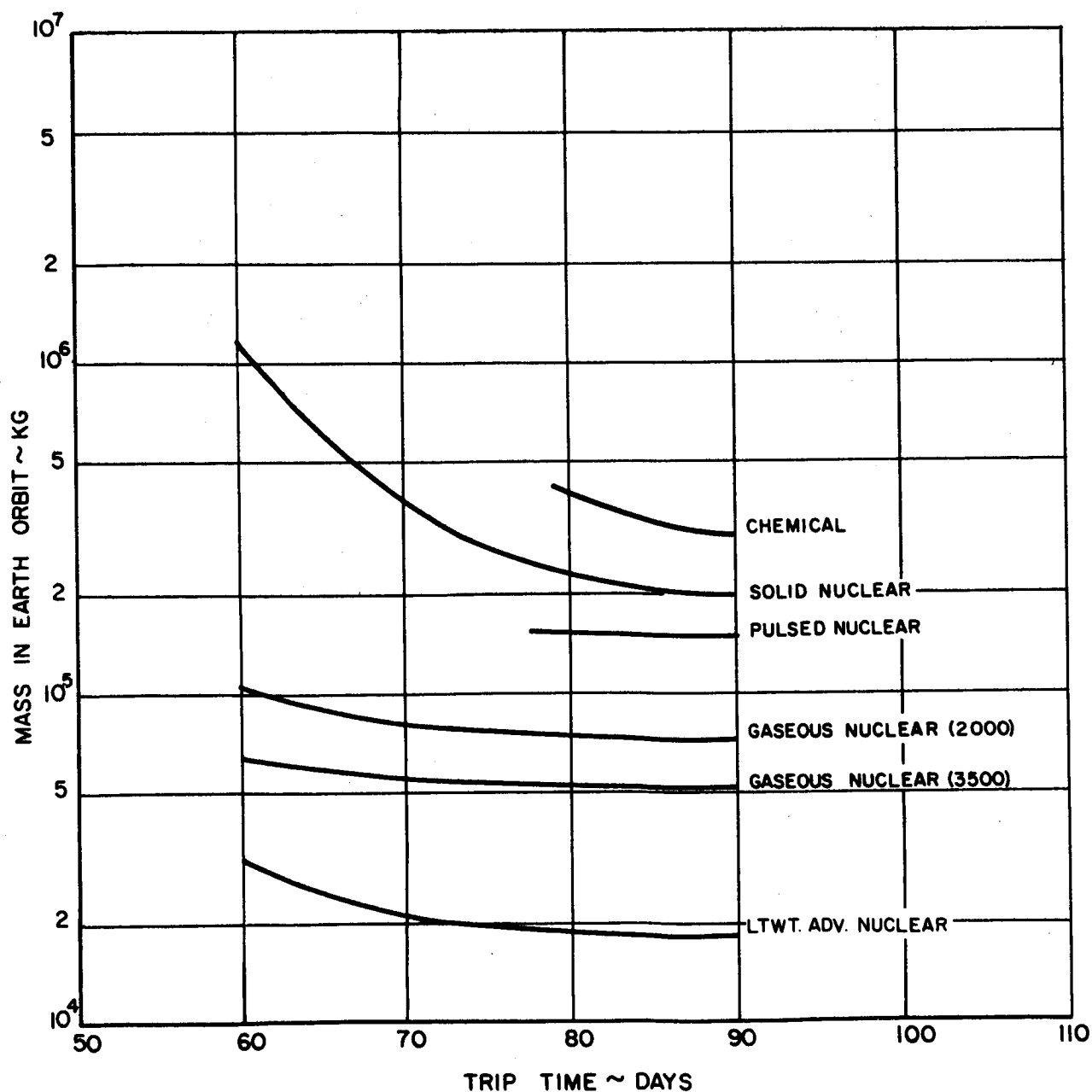


MERCURY ORBITAL STOPOVERS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

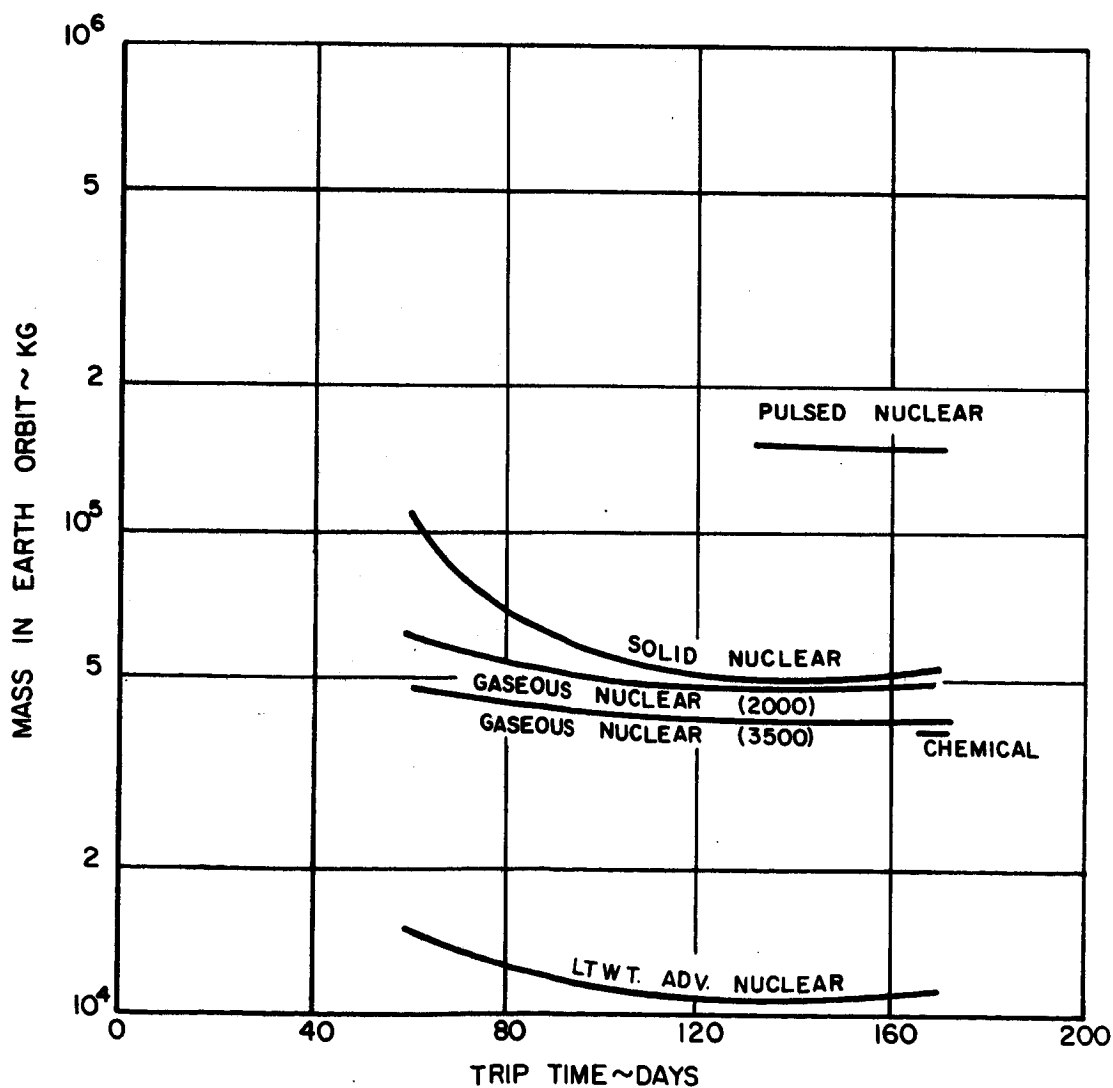


VENUS ORBITAL STOPOVERS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I _{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

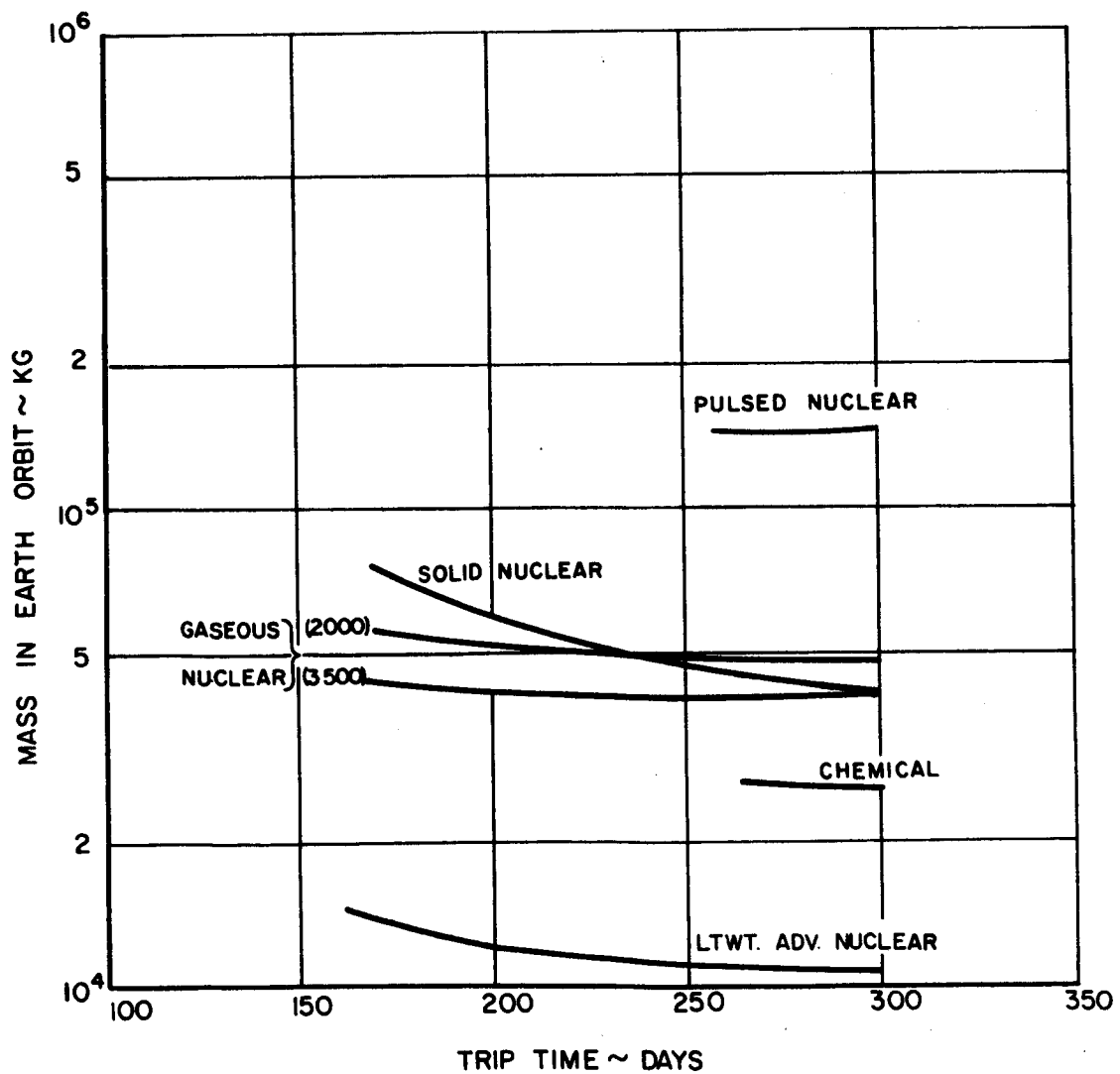


MARS ORBITAL STOPOVERS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

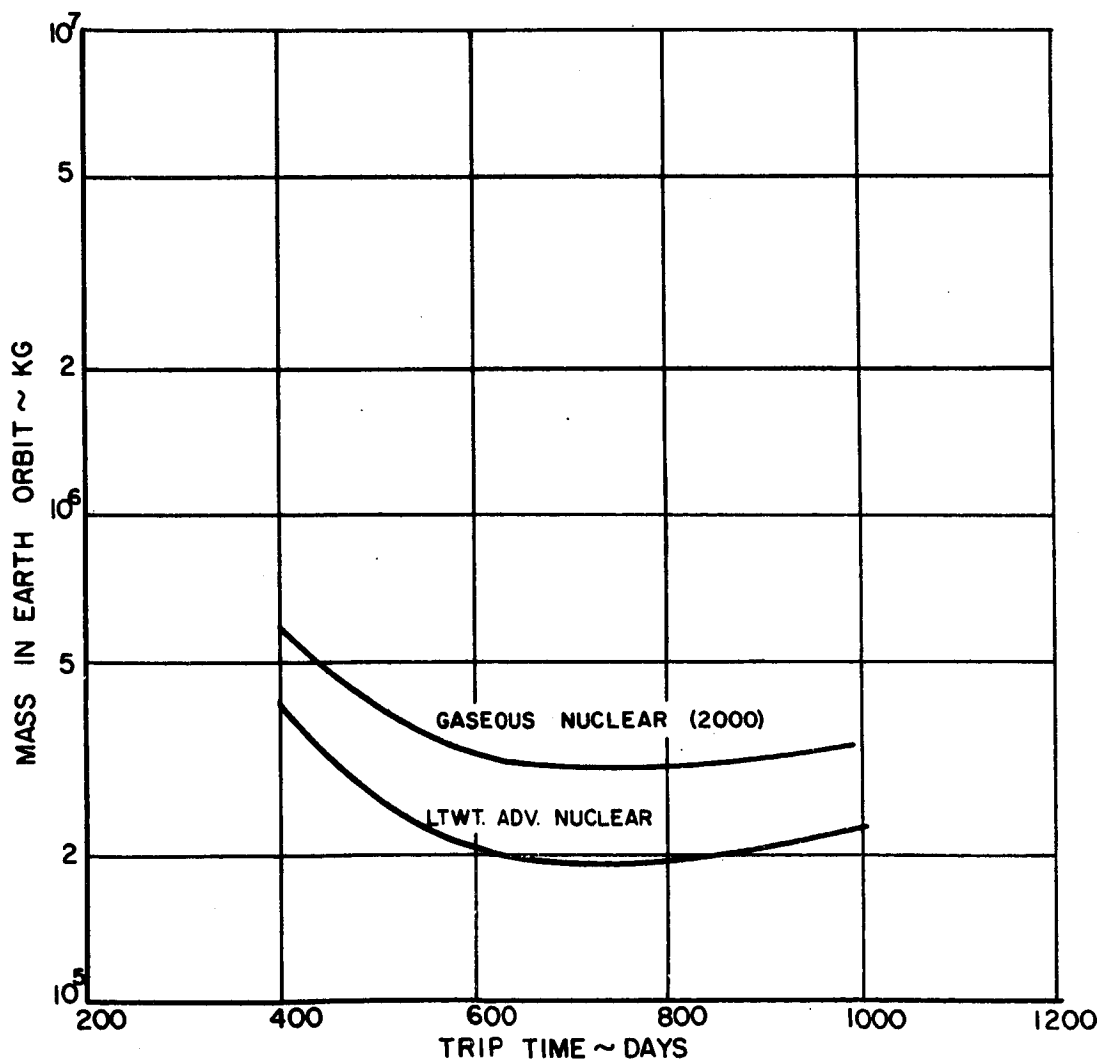


JUPITER ORBITAL STOPOVERS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600

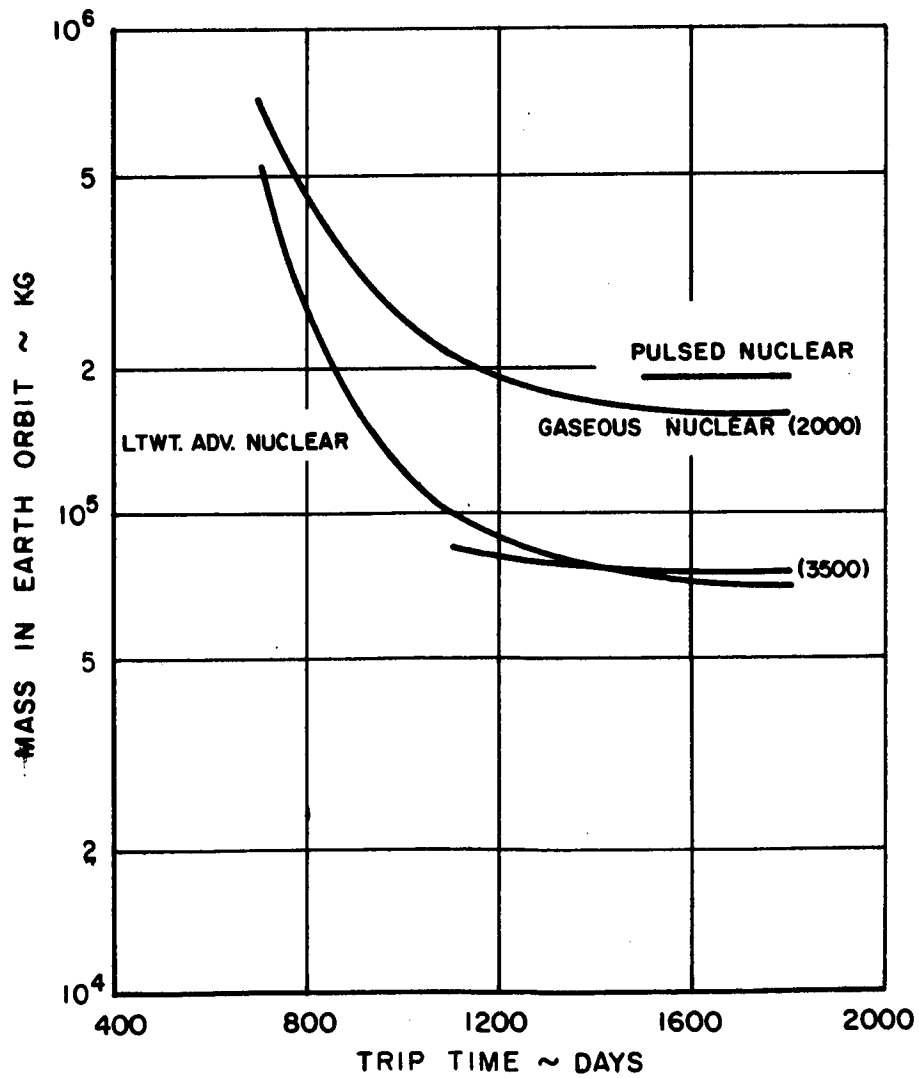


SATURN ORBITAL STOPOVERS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{SP}	T/W	MIN. ENG. WT.
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

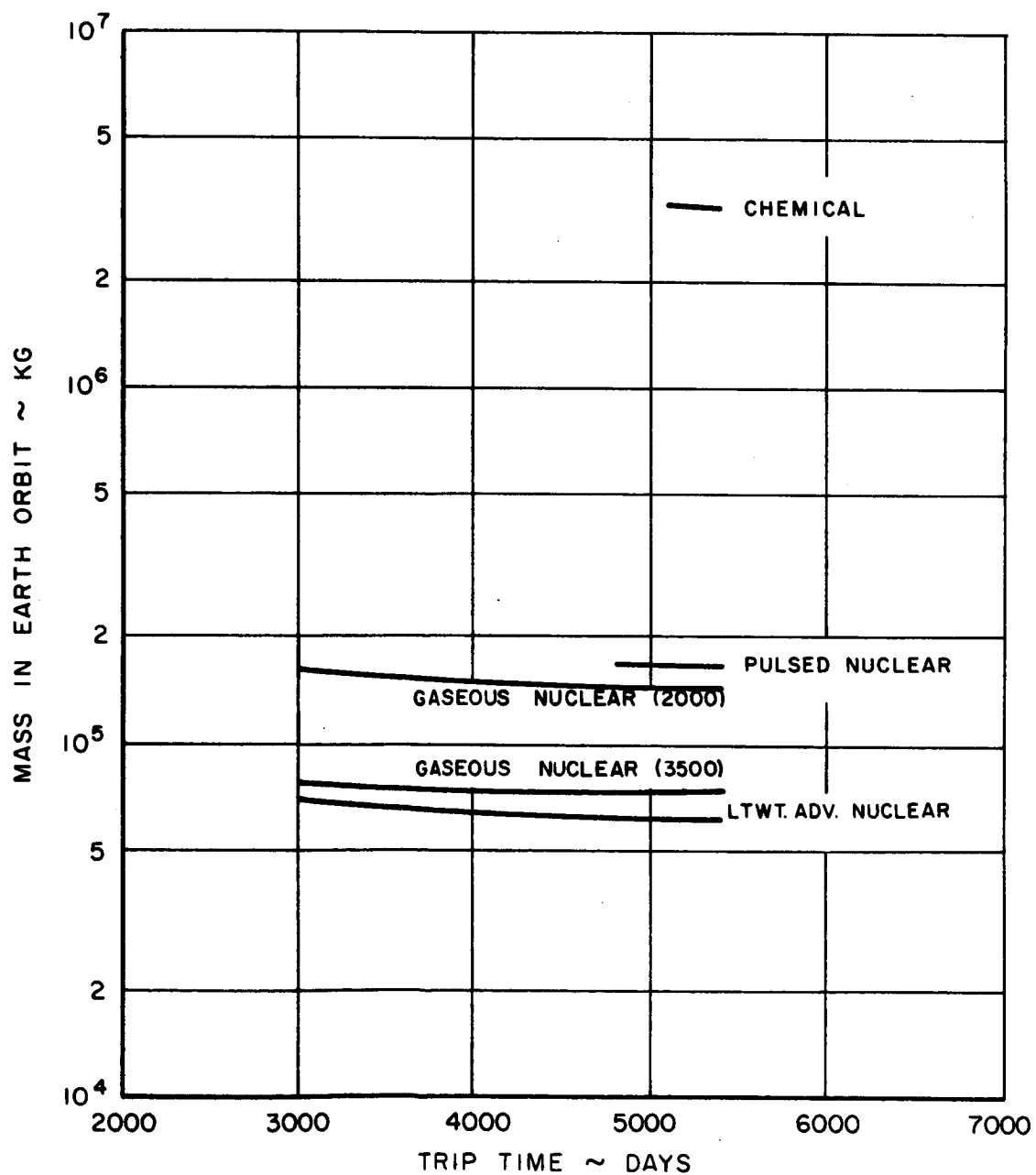


URANUS ORBITAL STOPOVERS

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{SP}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

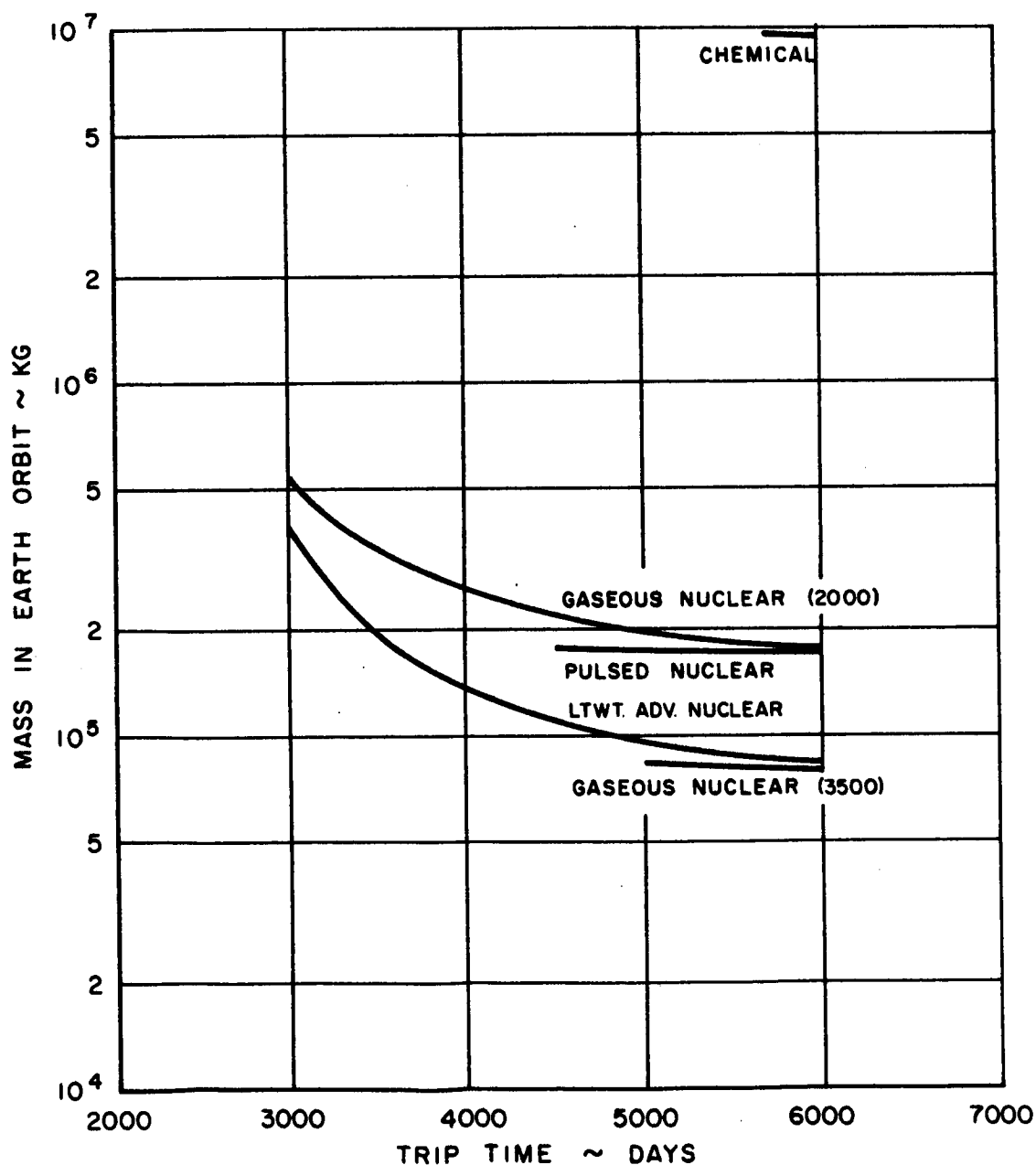


NEPTUNE ORBITAL STOPOVER

$$W_{PL} = 4536 \text{ KG}$$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

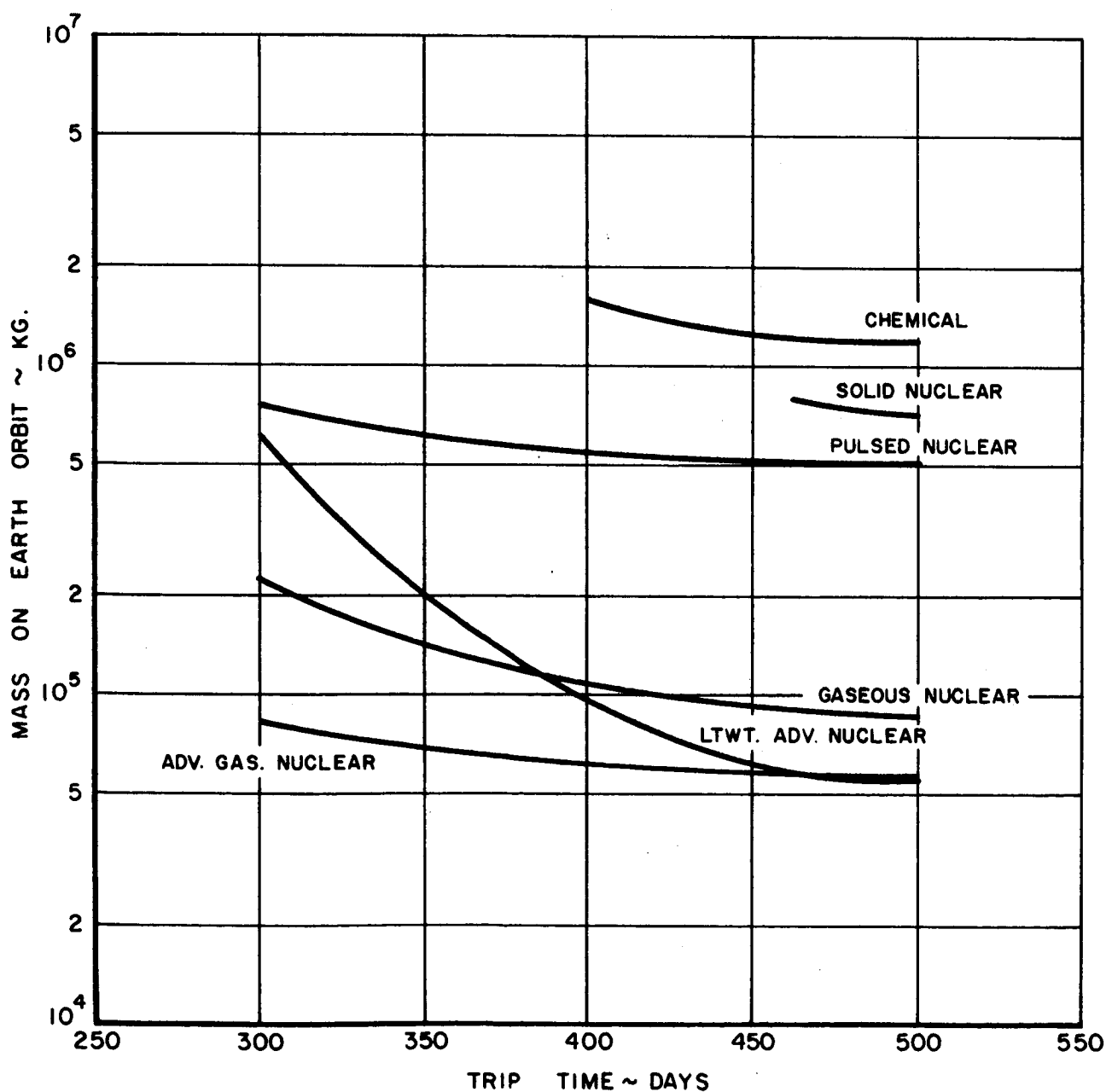


SCHWASSMANN - WACHMANN II RENDEZVOUS

$$W_{PL} = 4536 \text{ KG}$$

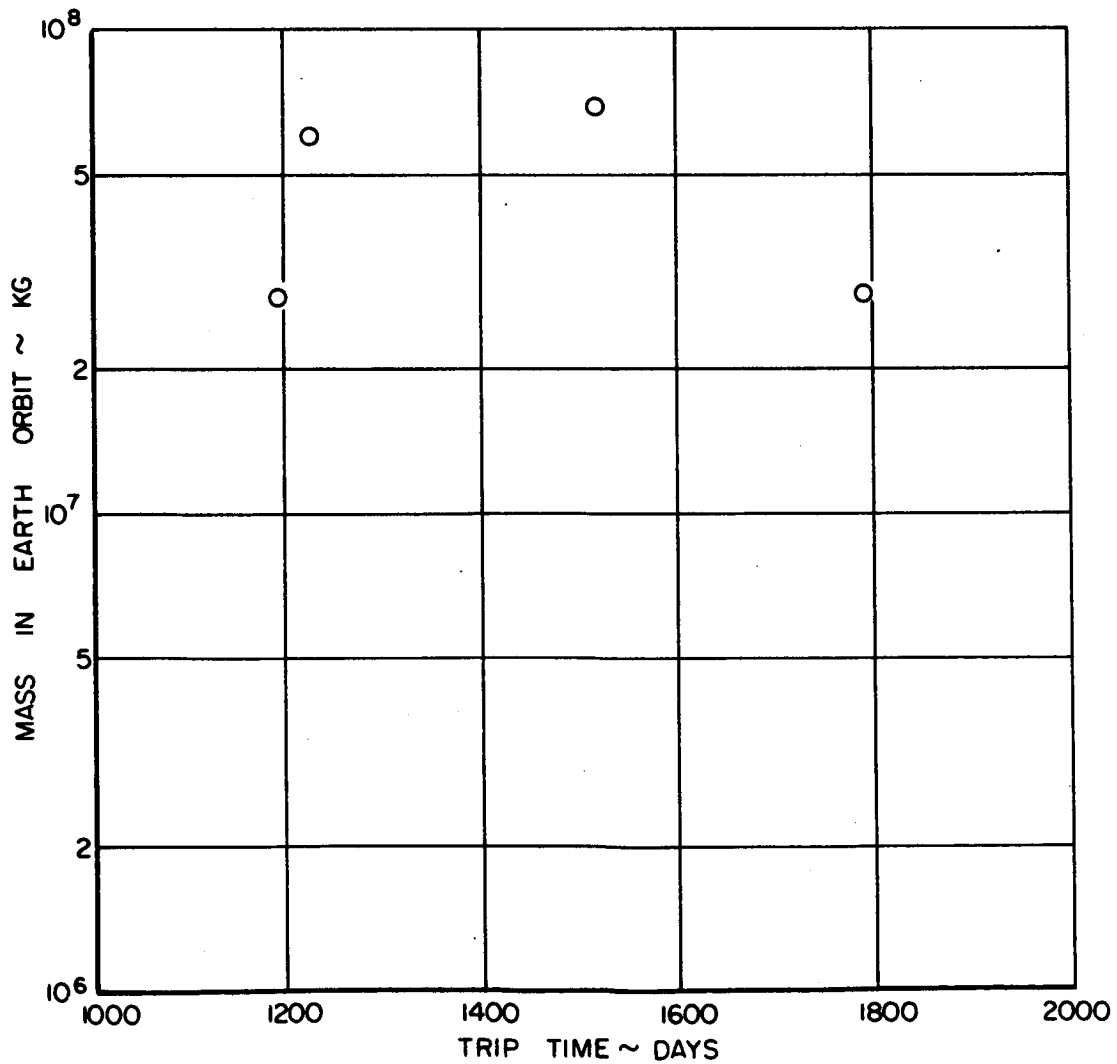
NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600



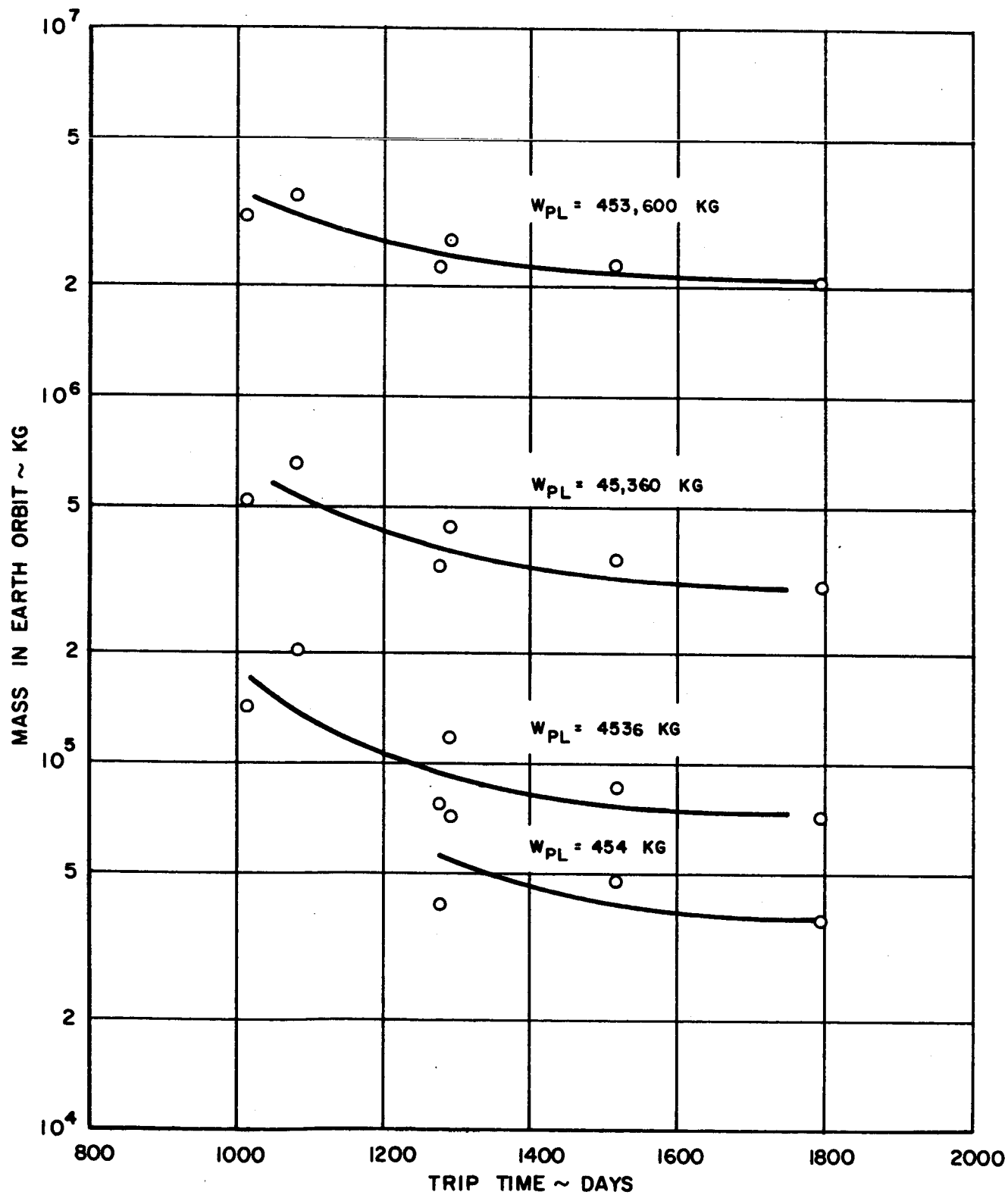
SATURN STOPOVERS WITH JUPITER SWINGBY

CHEMICAL

 $I_{SP} = 430 \text{ SEC}$ $W_{PL} = 45,360 \text{ KG}$ 

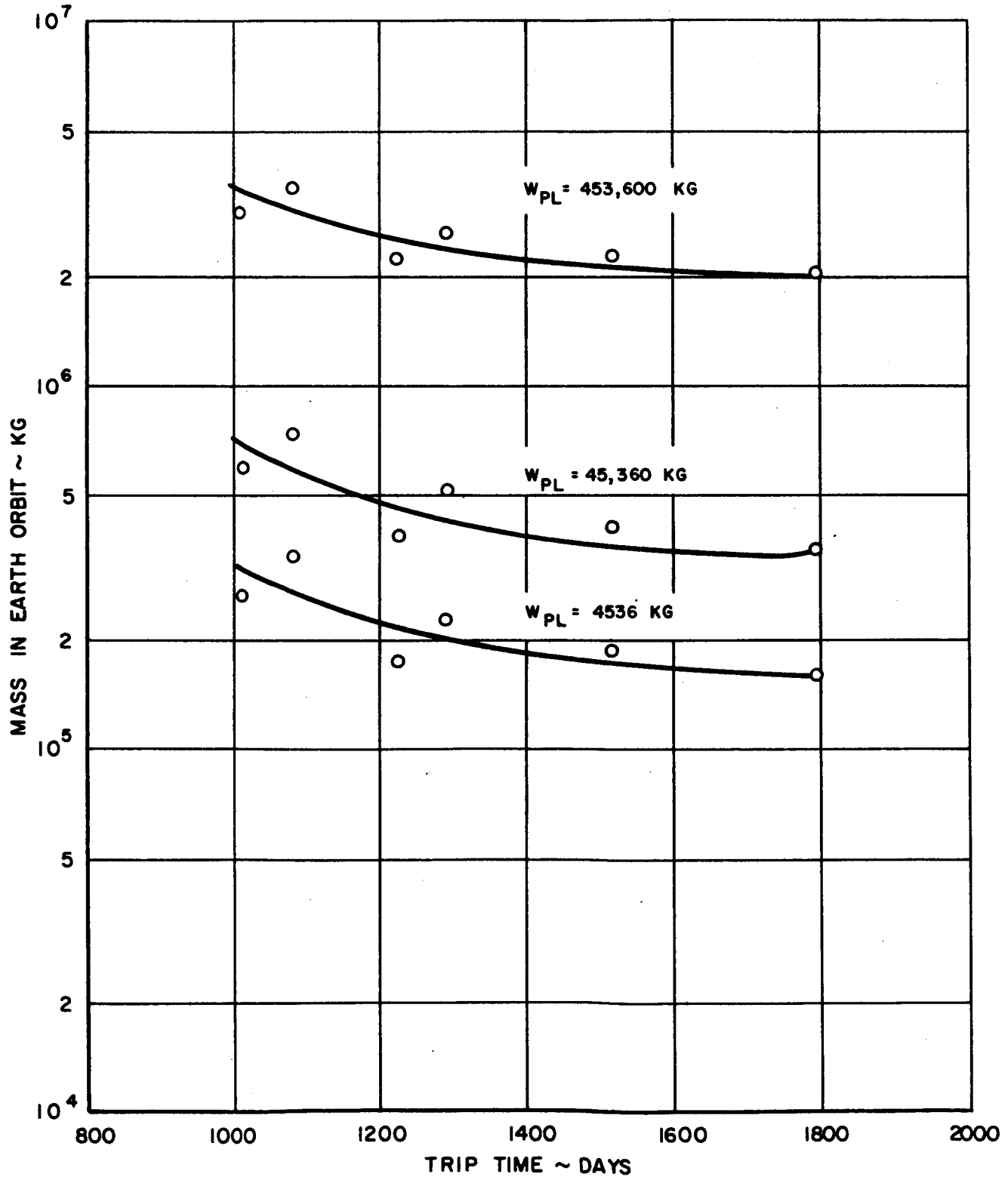
SATURN STOPOVERS WITH JUPITER SWINGBY

LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{sp} = 2000 \text{ SEC}$ 

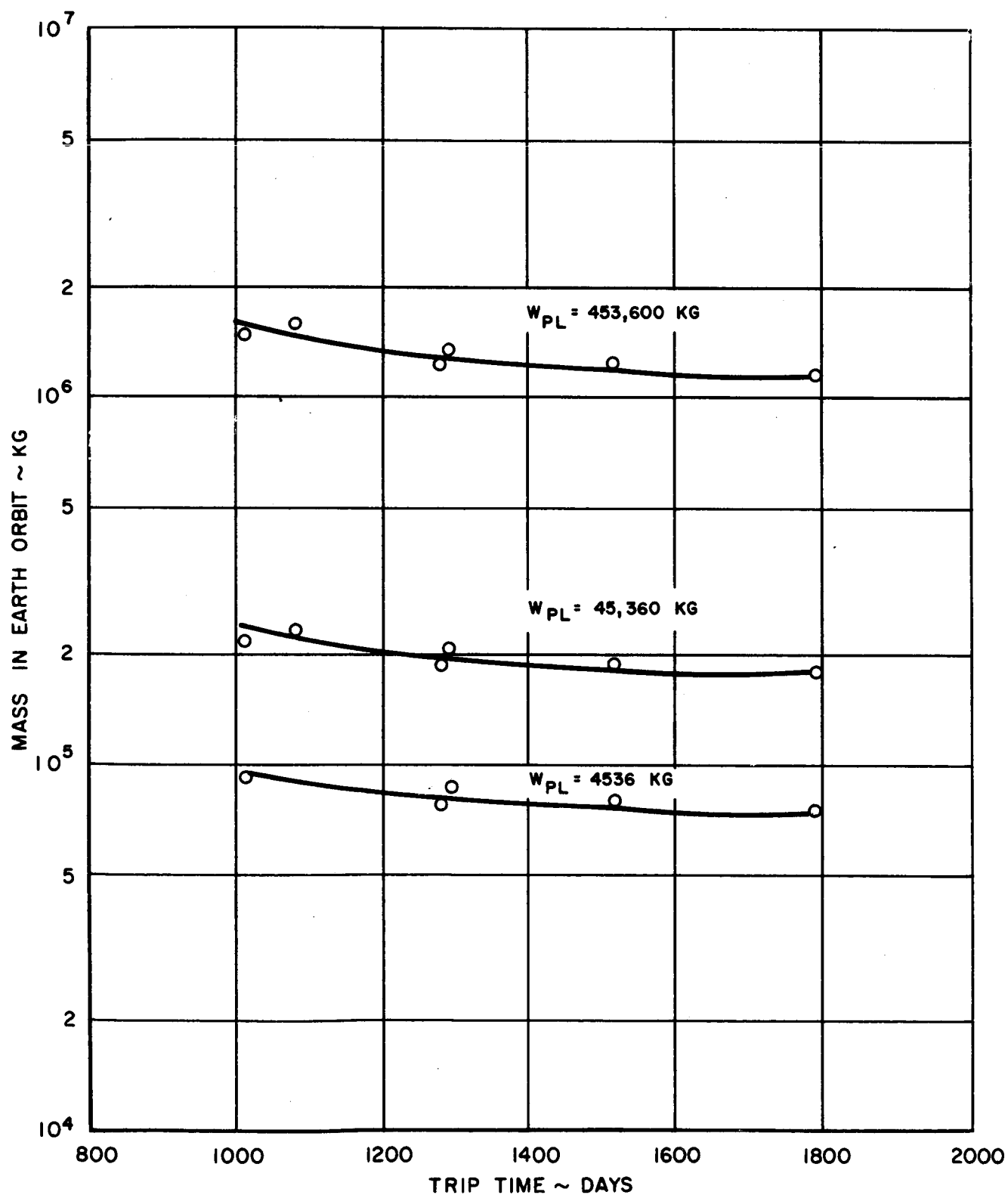
SATURN STOPOVERS WITH JUPITER SWINGBY

GASEOUS-CORE NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ 

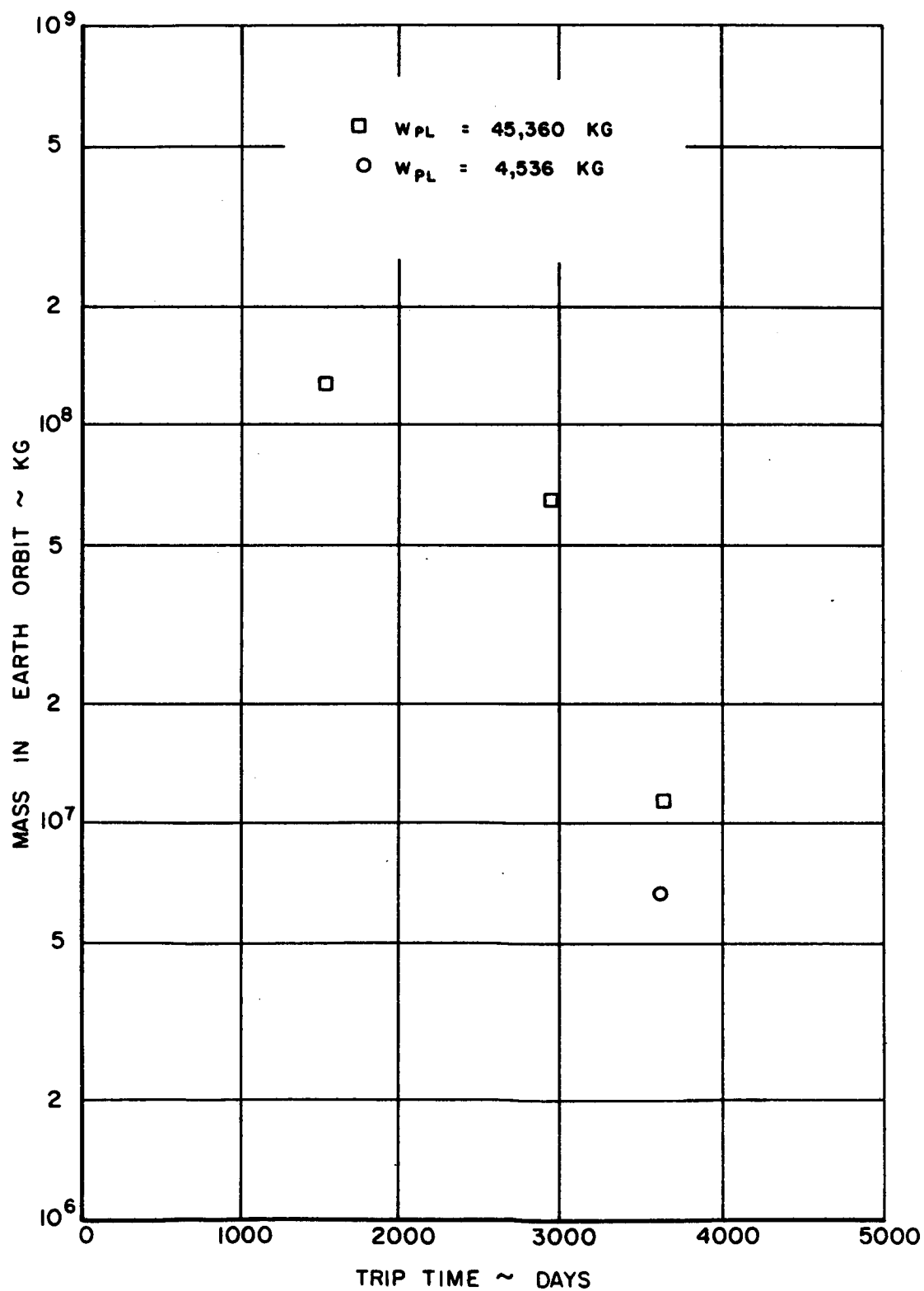
SATURN STOPOVERS WITH JUPITER SWINGBY

GASEOUS-CORE NUCLEAR

 $I_{SP} = 3500 \text{ SEC}$ 

URANUS STOPOVERS WITH JUPITER SWINGBY

CHEMICAL

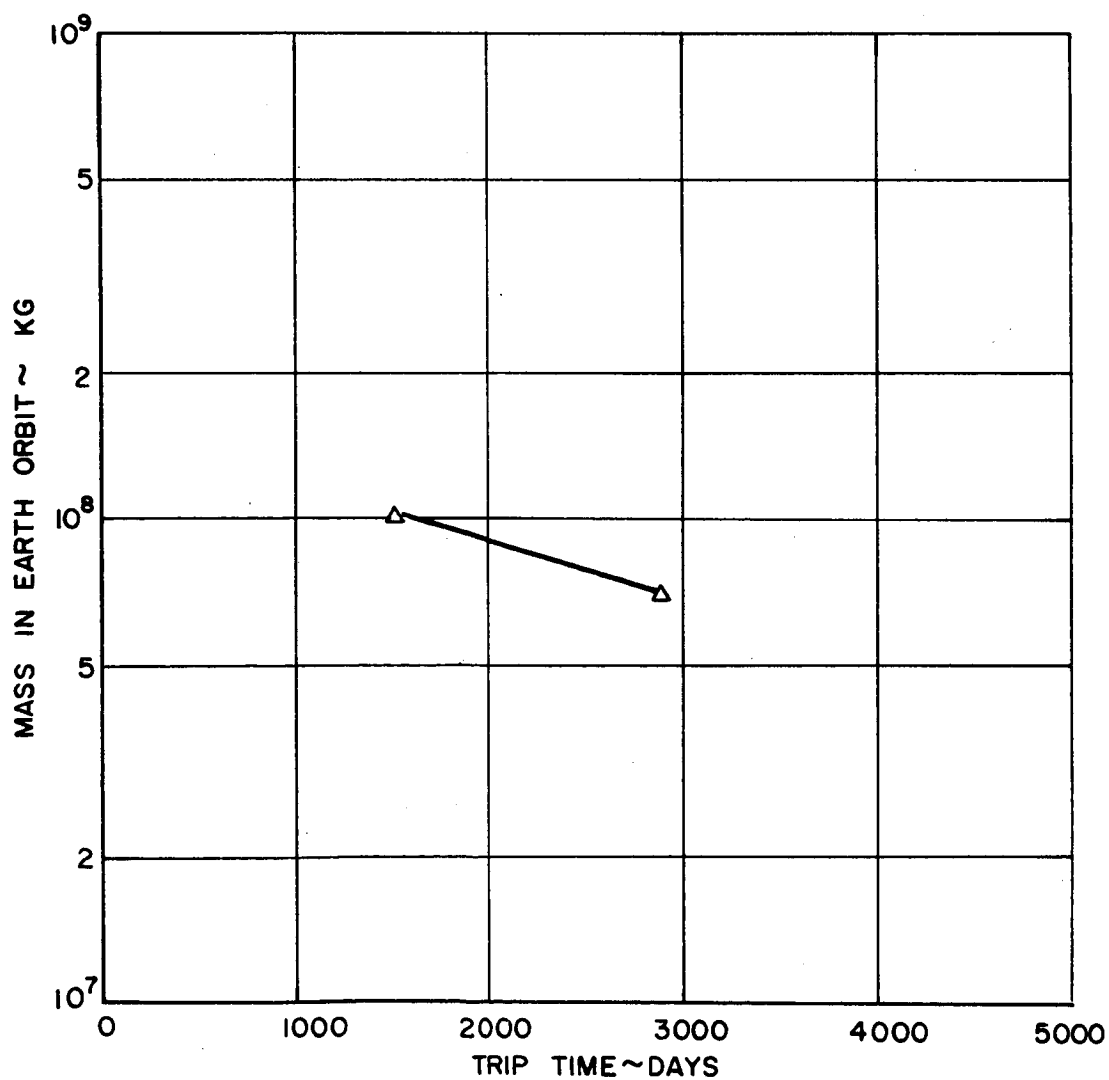
 $I_{SP} = 430 \text{ SEC}$ 

URANUS STOPOVERS WITH JUPITER SWINGBY

SOLID-CORE NUCLEAR

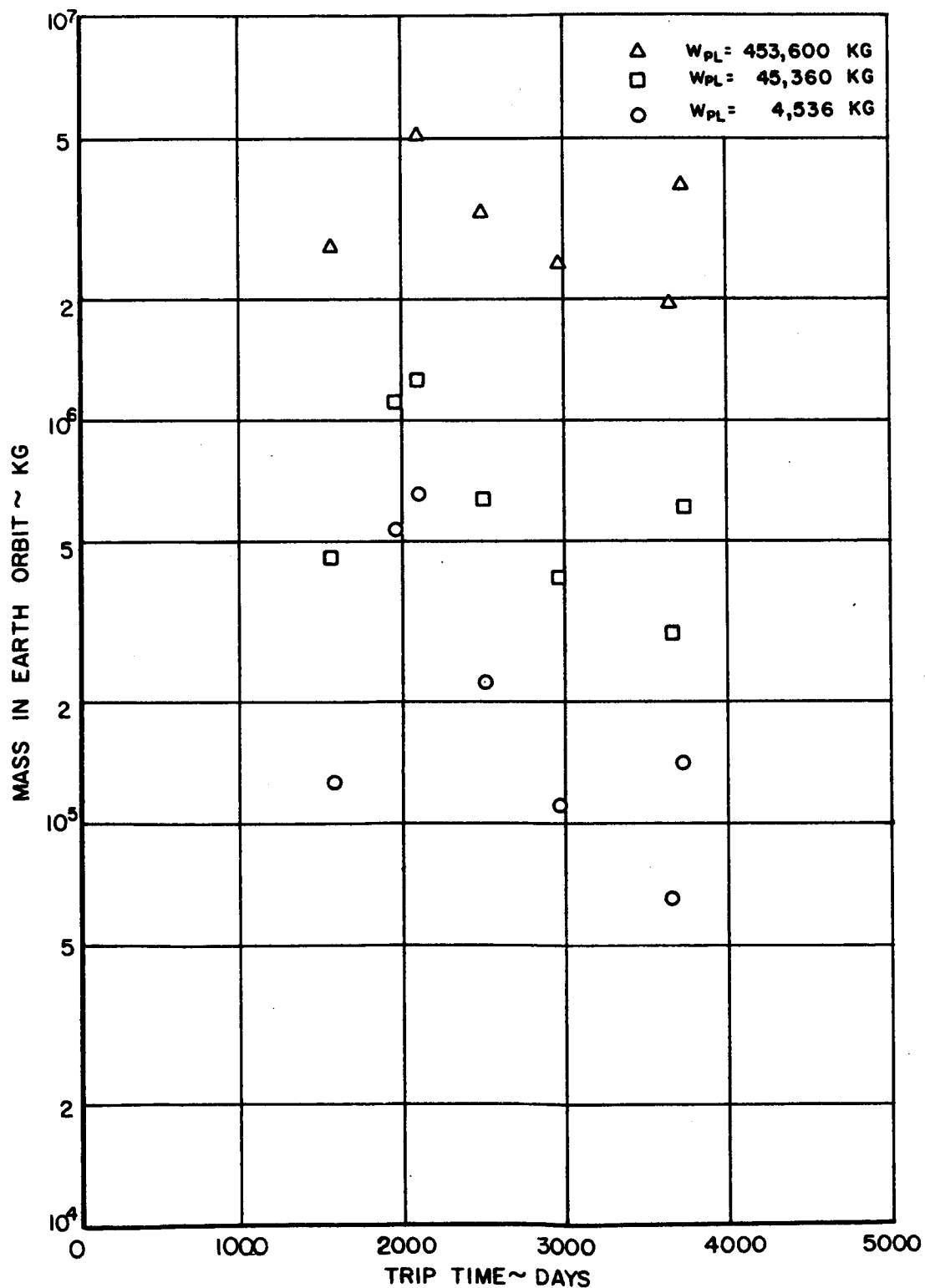
$$I_{SP} = 800 \text{ SEC}$$

$$\Delta W_{PL} = 453,600 \text{ KG}$$



URANUS STOPOVERS WITH JUPITER SWINGBY

LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{sp} = 2000 \text{ SEC}$ 

URANUS STOPOVERS WITH JUPITER SWINGBY

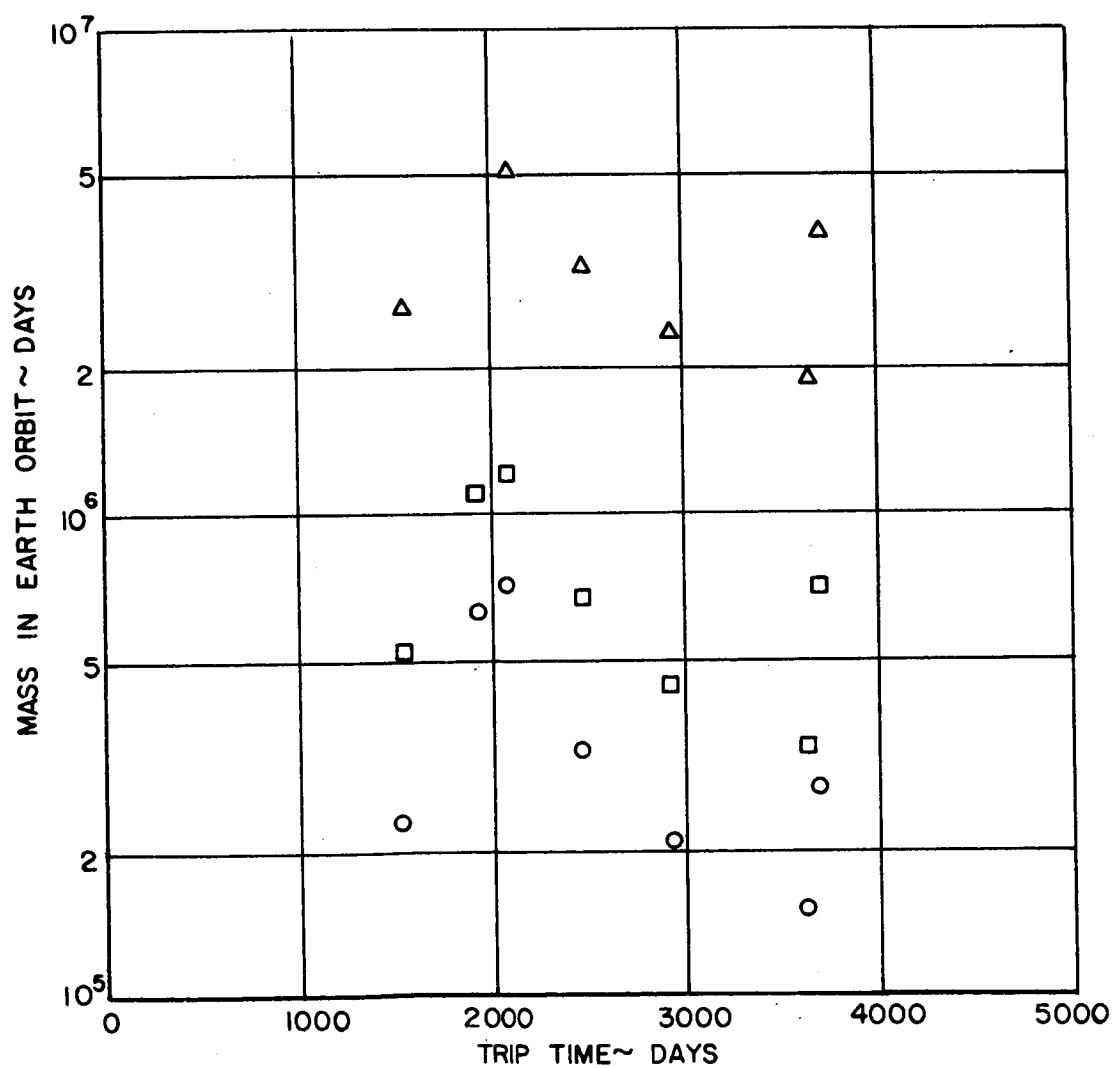
GASEOUS-CORE NUCLEAR

$$I_{SP} = 2000 \text{ SEC}$$

$$\Delta W_{PL} = 453,600 \text{ KG}$$

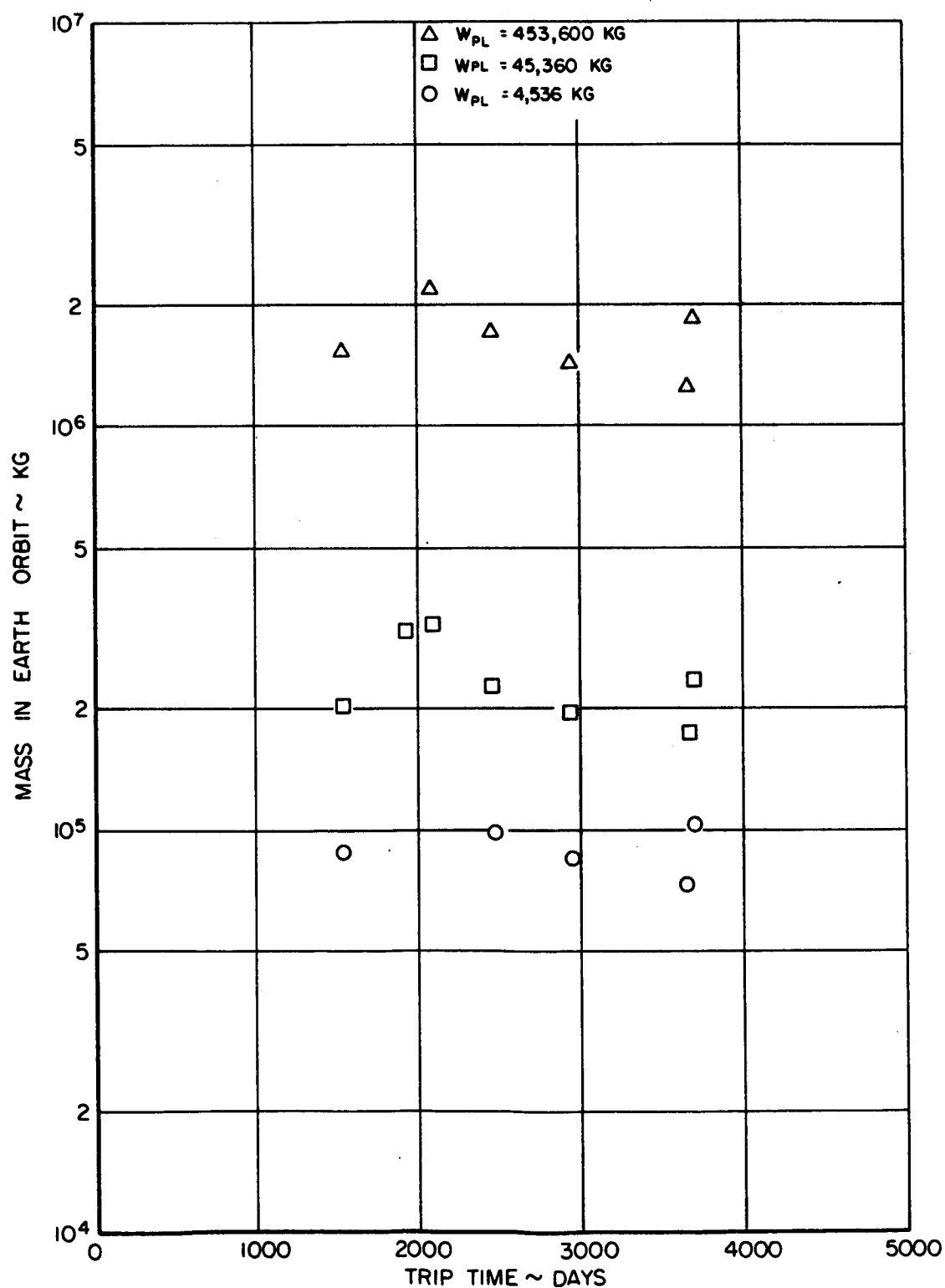
$$\square W_{PL} = 45,360 \text{ KG}$$

$$\circ W_{PL} = 4,536 \text{ KG}$$



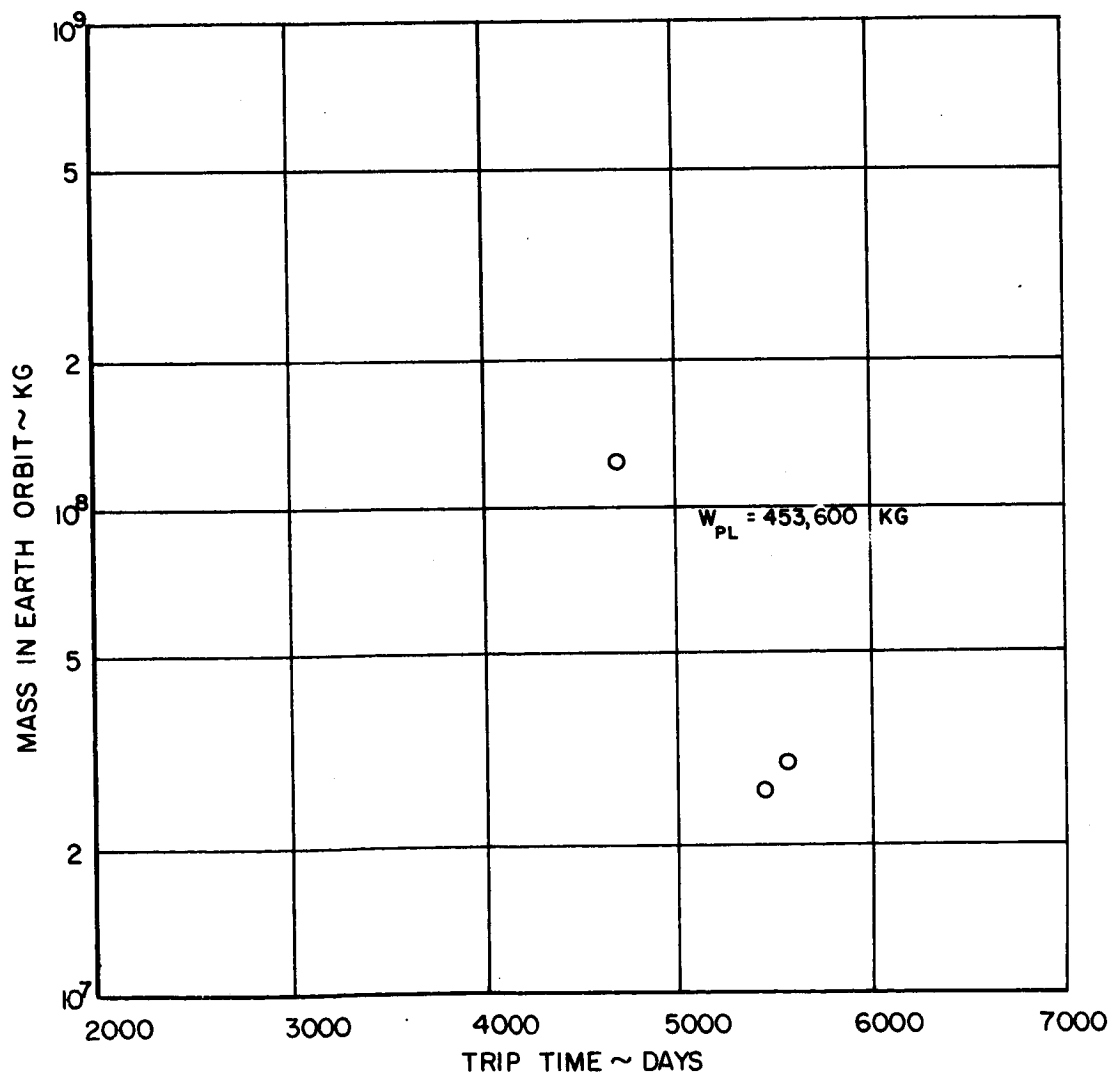
URANUS STOPOVERS WITH JUPITER SWINGBY

GASEOUS-CORE NUCLEAR

 $I_{sp} = 3500 \text{ SEC}$ 

NEPTUNE STOPOVERS WITH JUPITER SWINGBY

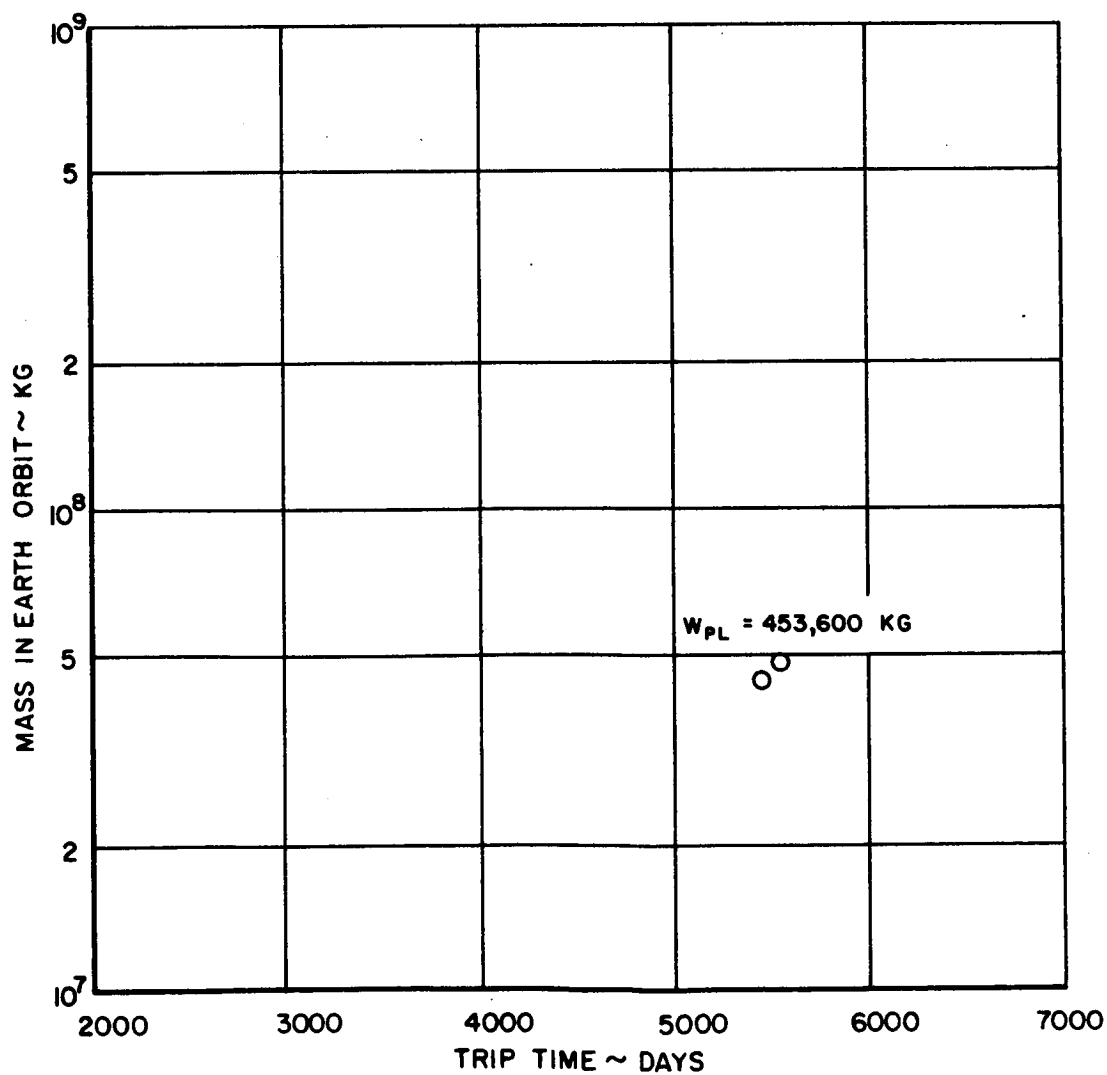
CHEMICAL

 $I_{SP} = 430 \text{ SEC}$ 

NEPTUNE STOPOVERS WITH JUPITER SWINGBY

SOLID-CORE NUCLEAR

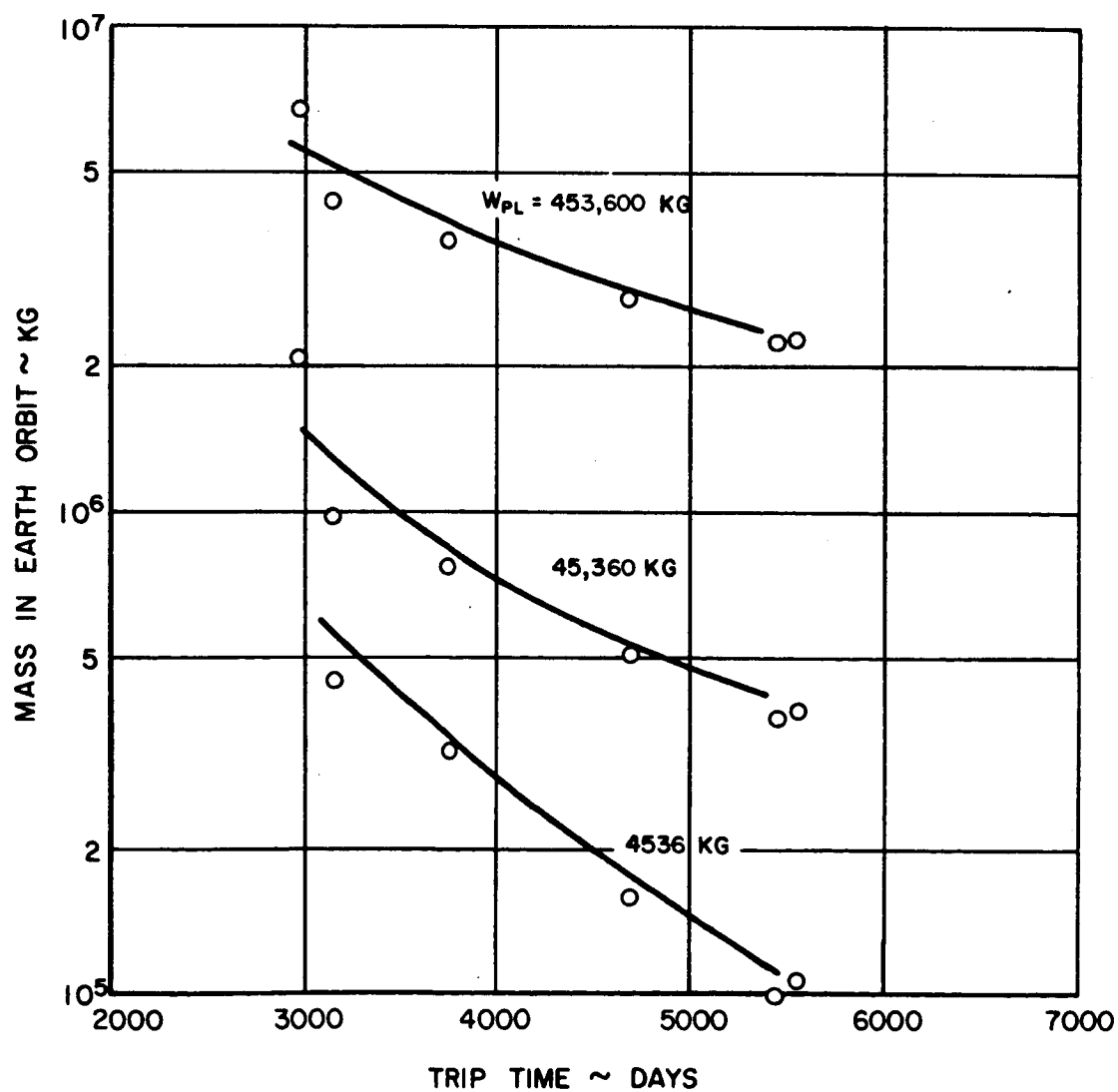
$$I_{sp} = 800 \text{ SEC}$$



NEPTUNE STOPOVERS WITH JUPITER SWINGBY

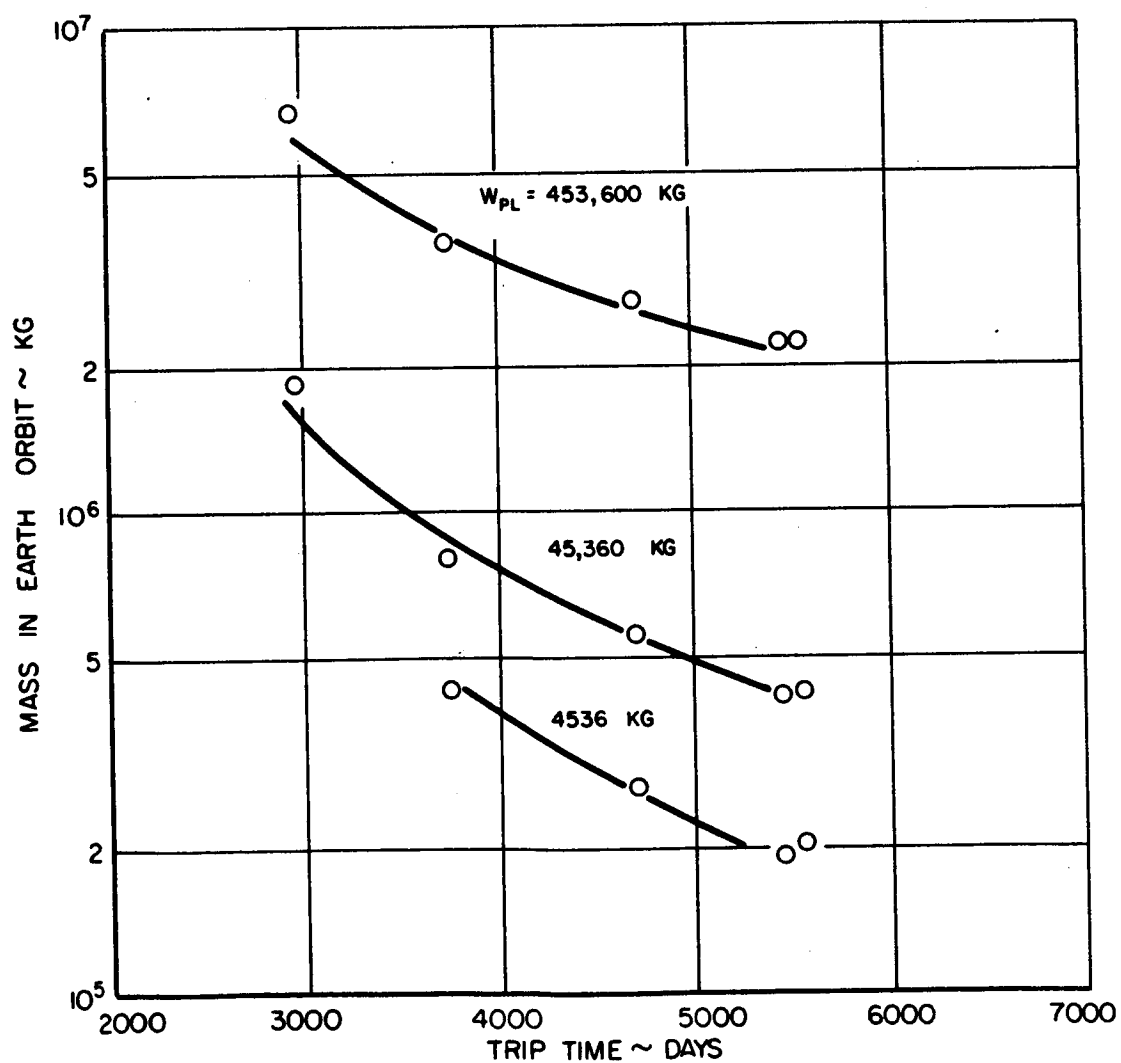
LIGHTWEIGHT ADVANCED NUCLEAR

$$I_{SP} = 2000 \text{ SEC}$$



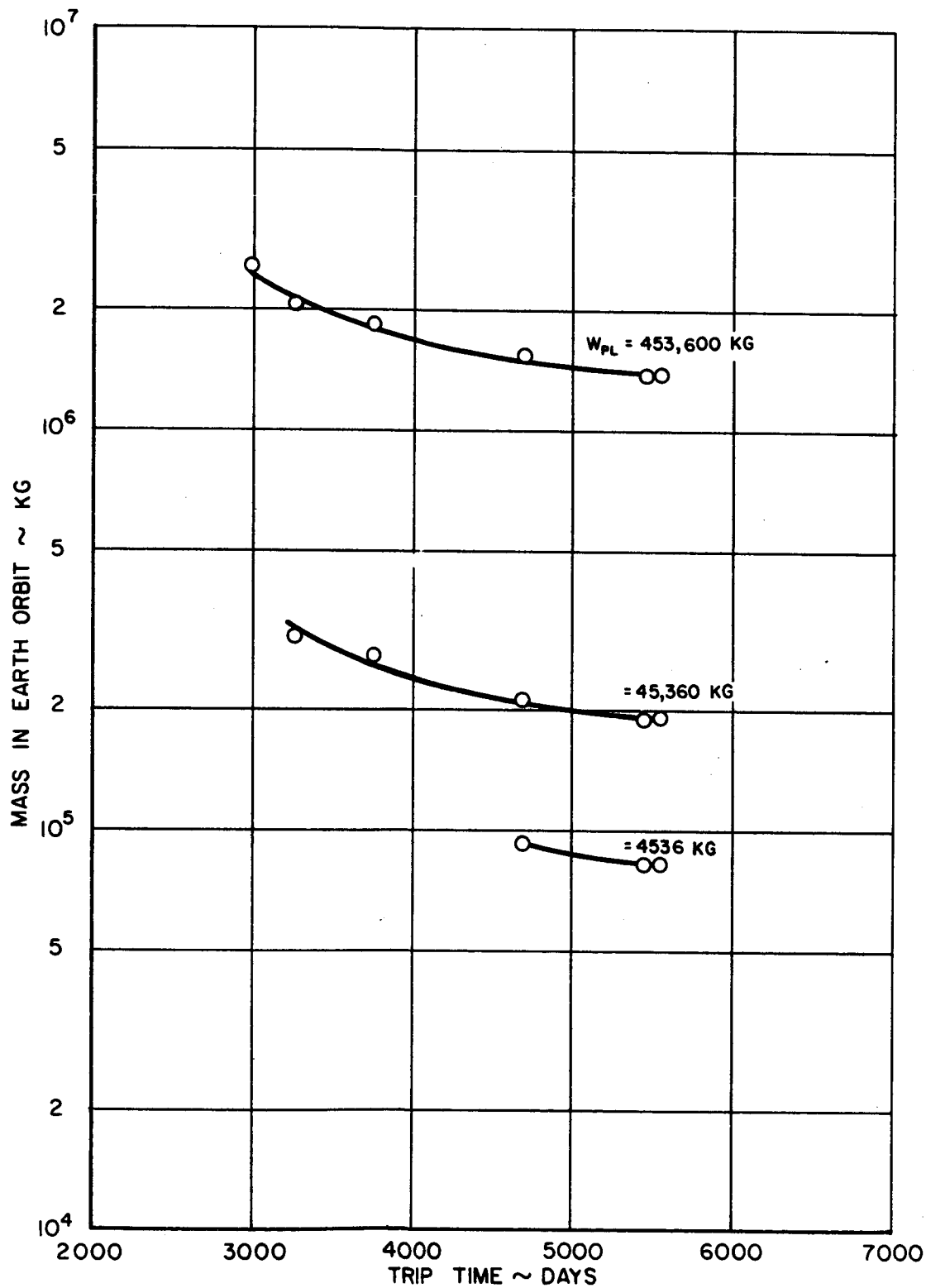
NEPTUNE STOPOVERS WITH JUPITER SWINGBY

GASEOUS-CORE NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ 

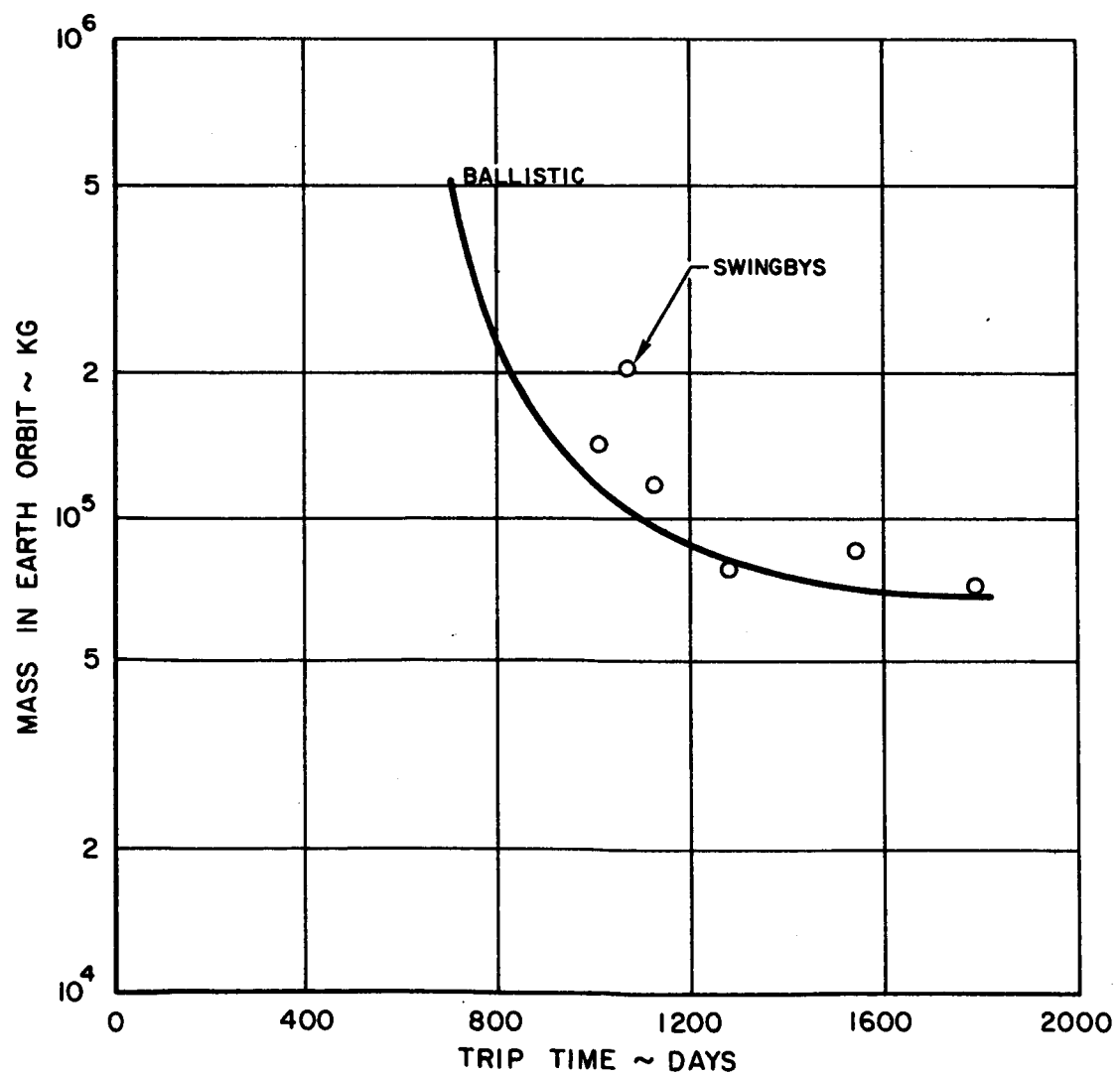
NEPTUNE STOPOVERS WITH JUPITER SWINGBY

GASEOUS-CORE NUCLEAR

 $I_{SP} = 3500 \text{ SEC}$ 

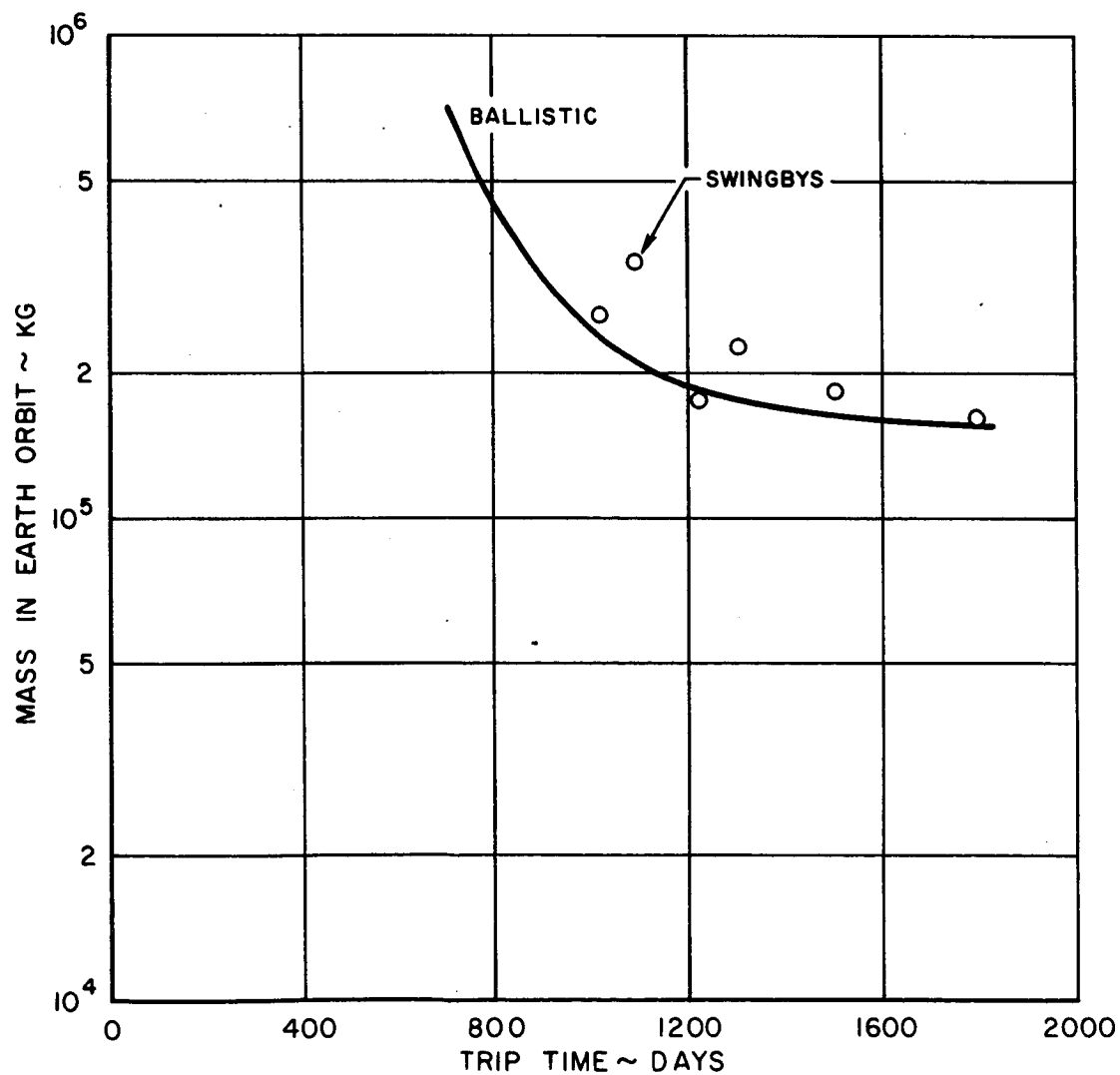
SATURN ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ $W_{PL} = 4536 \text{ KG}$ 

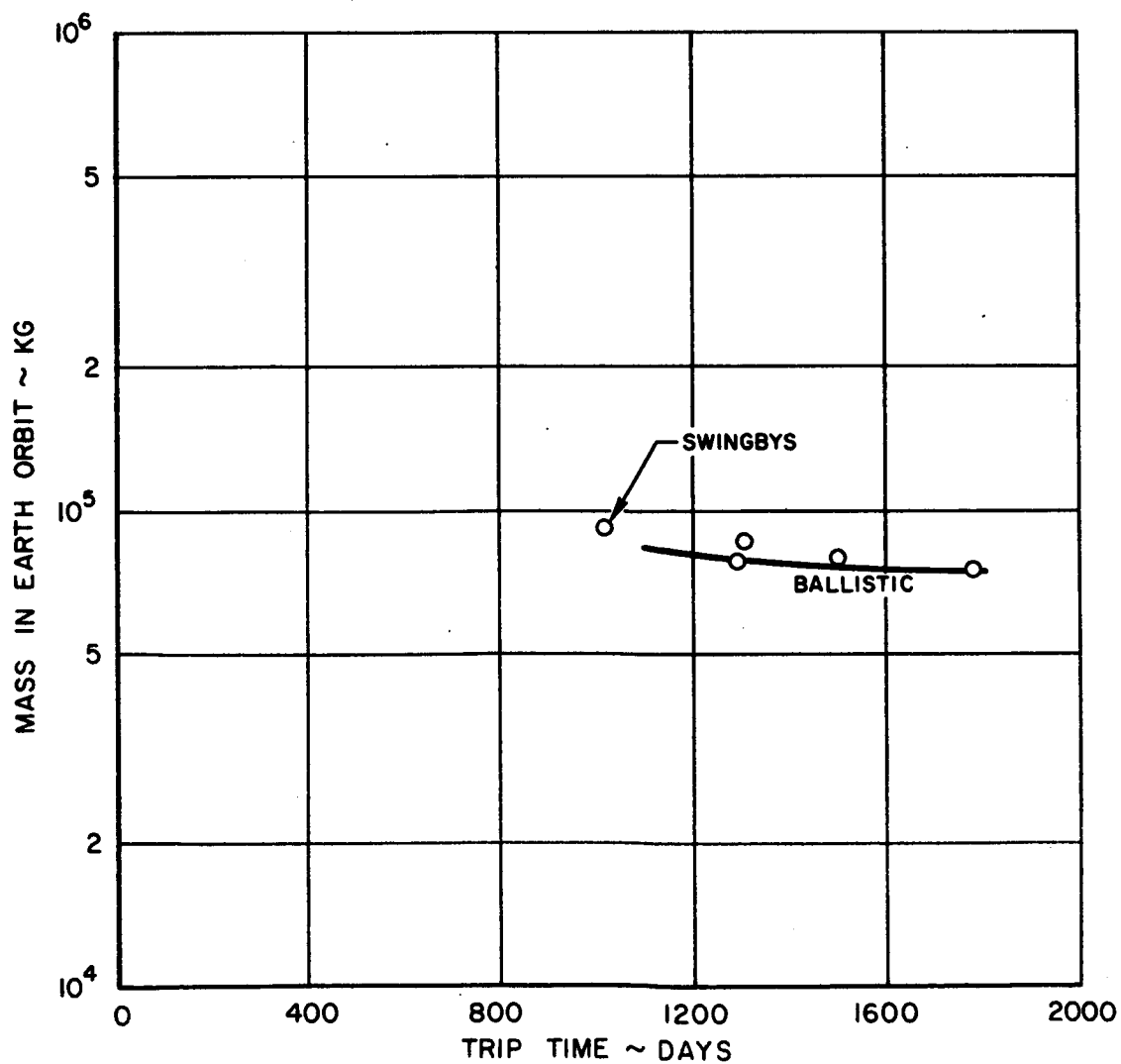
SATURN ORBITAL STOPOVERS

GASEOUS - CORE NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ $W_{PL} = 4536 \text{ KG}$ 

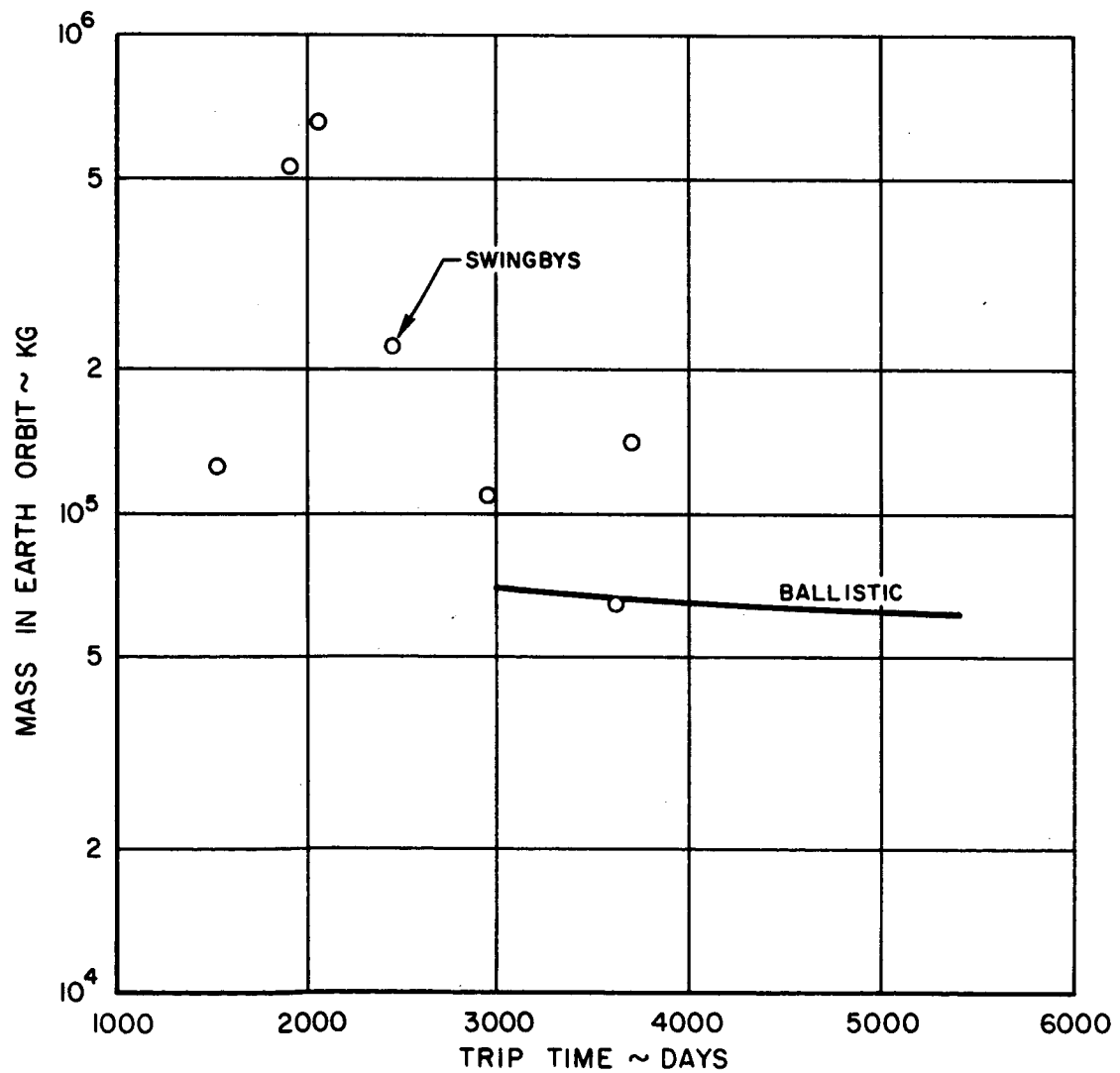
SATURN ORBITAL STOPOVERS

GASEOUS - CORE NUCLEAR

 $I_{SP} = 3500 \text{ SEC}$ $W_{PL} = 4536 \text{ KG}$ 

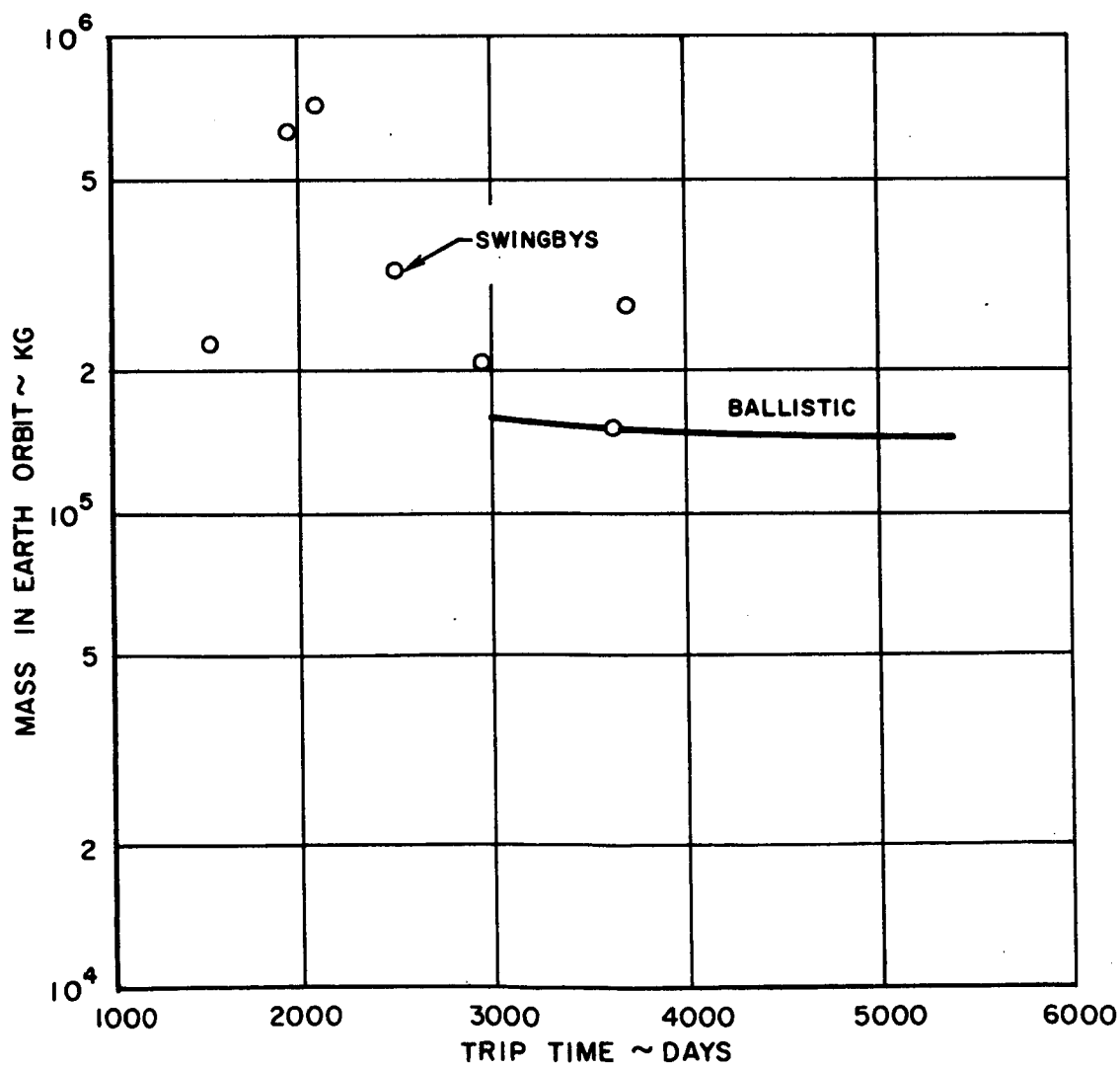
URANUS ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ $W_{PL} = 4536 \text{ KG}$ 

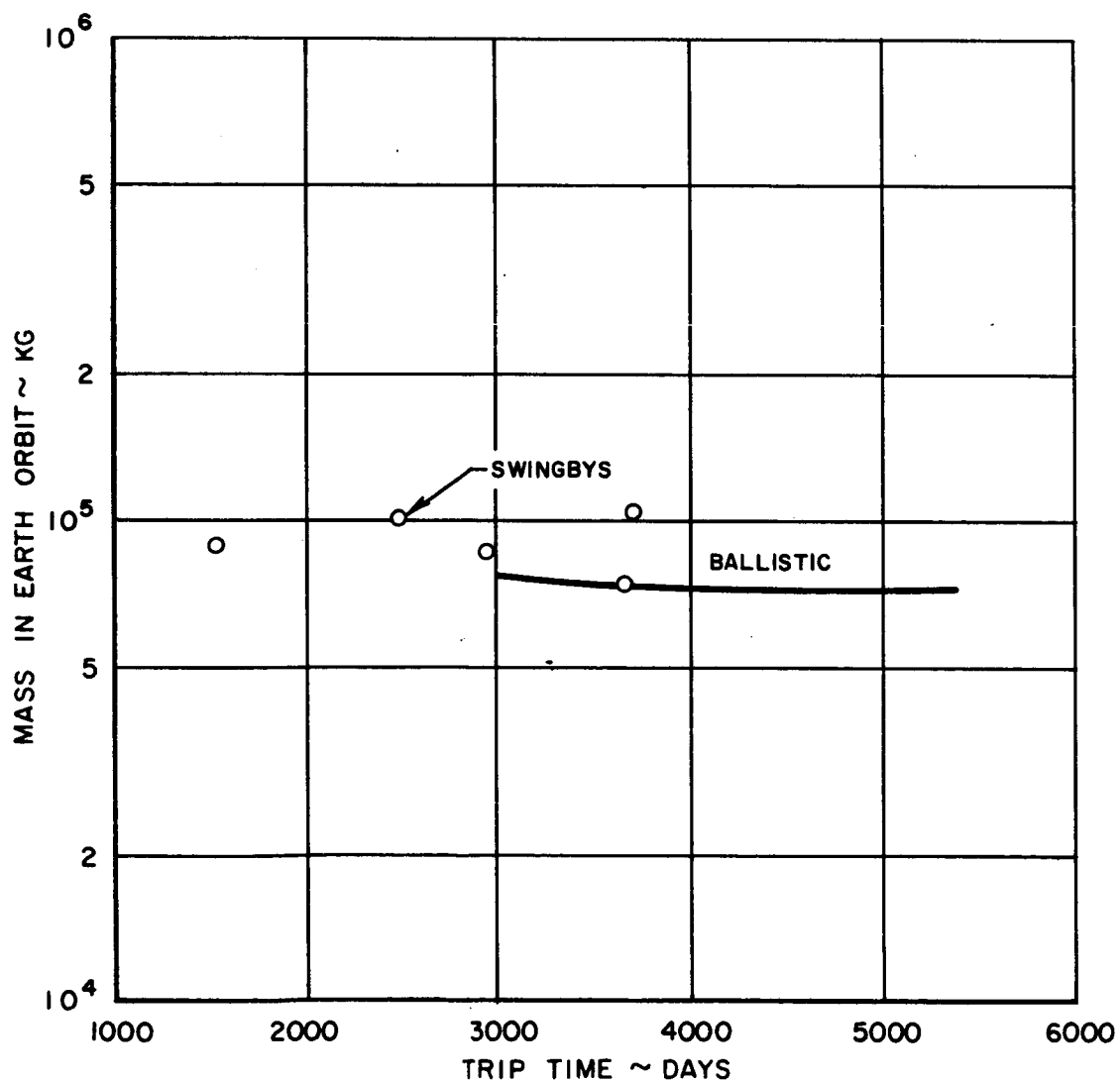
URANUS ORBITAL STOPOVERS

GASEOUS - CORE NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ $W_{PL} = 4536 \text{ KG}$ 

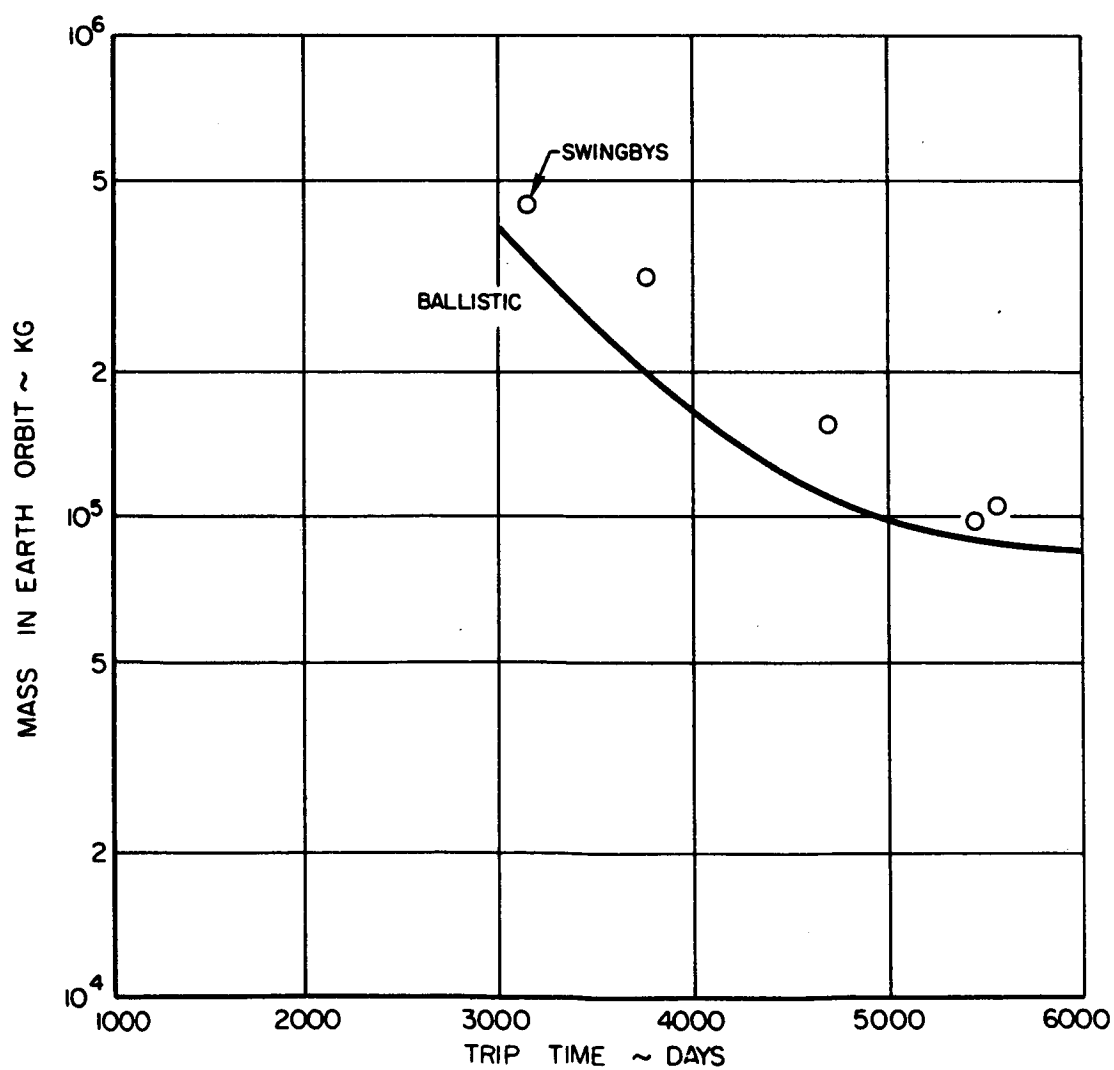
URANUS ORBITAL STOPOVERS

GASEOUS - CORE NUCLEAR

 $I_{sp} = 3500 \text{ SEC}$ $W_{PL} = 4536 \text{ KG}$ 

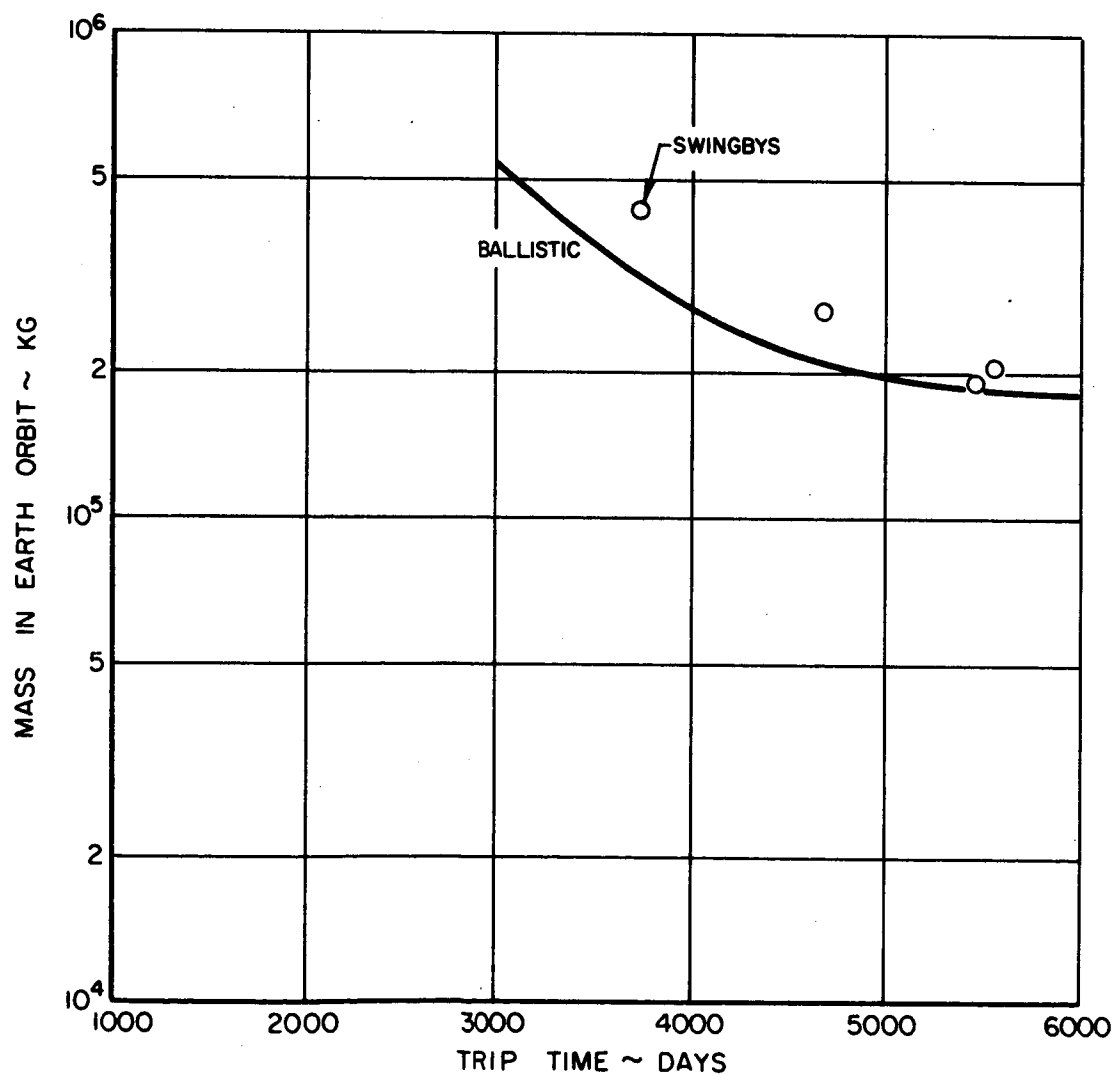
NEPTUNE ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ $W_{PL} = 4536 \text{ KG}$ 

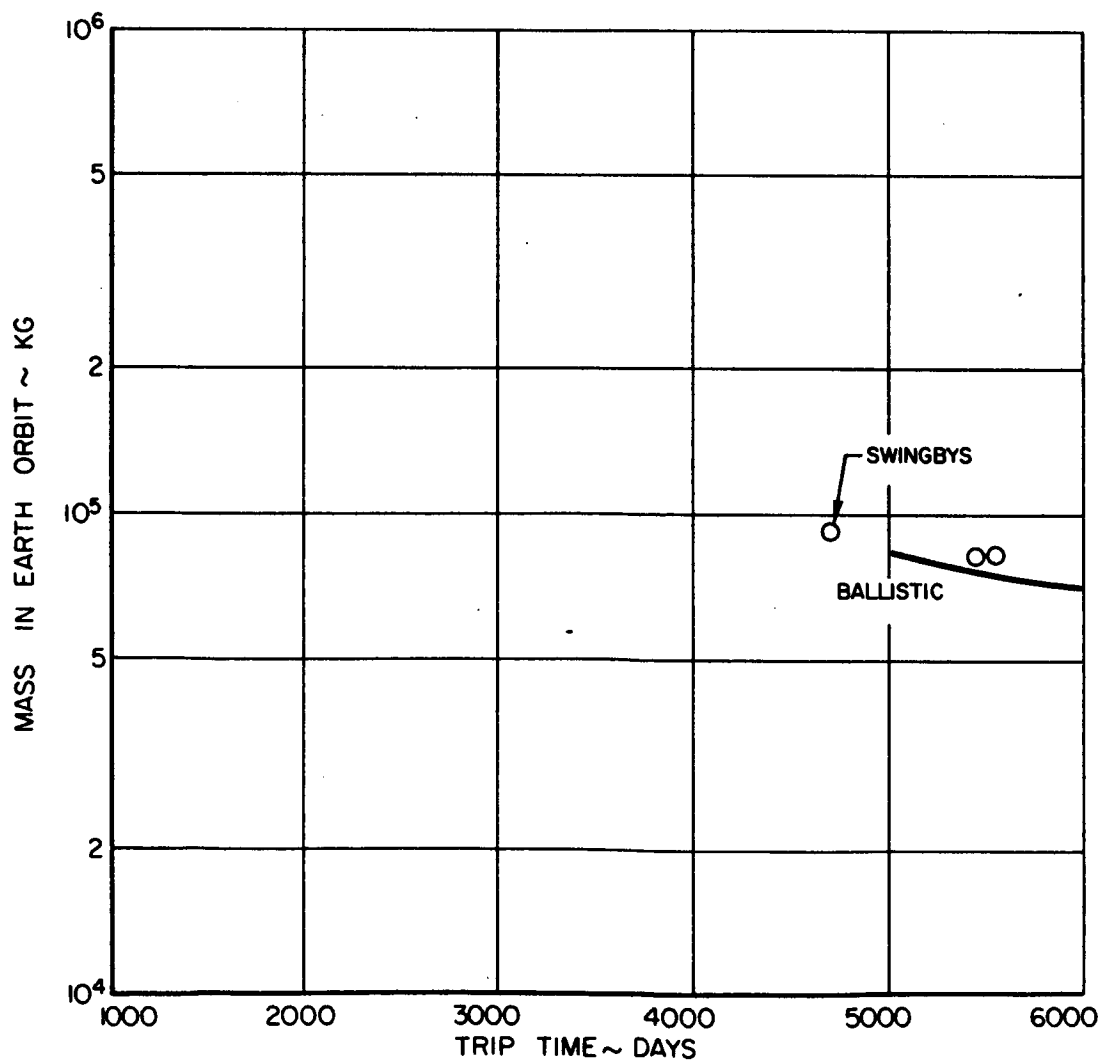
NEPTUNE ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ $W_{PL} = 4536 \text{ KG}$ 

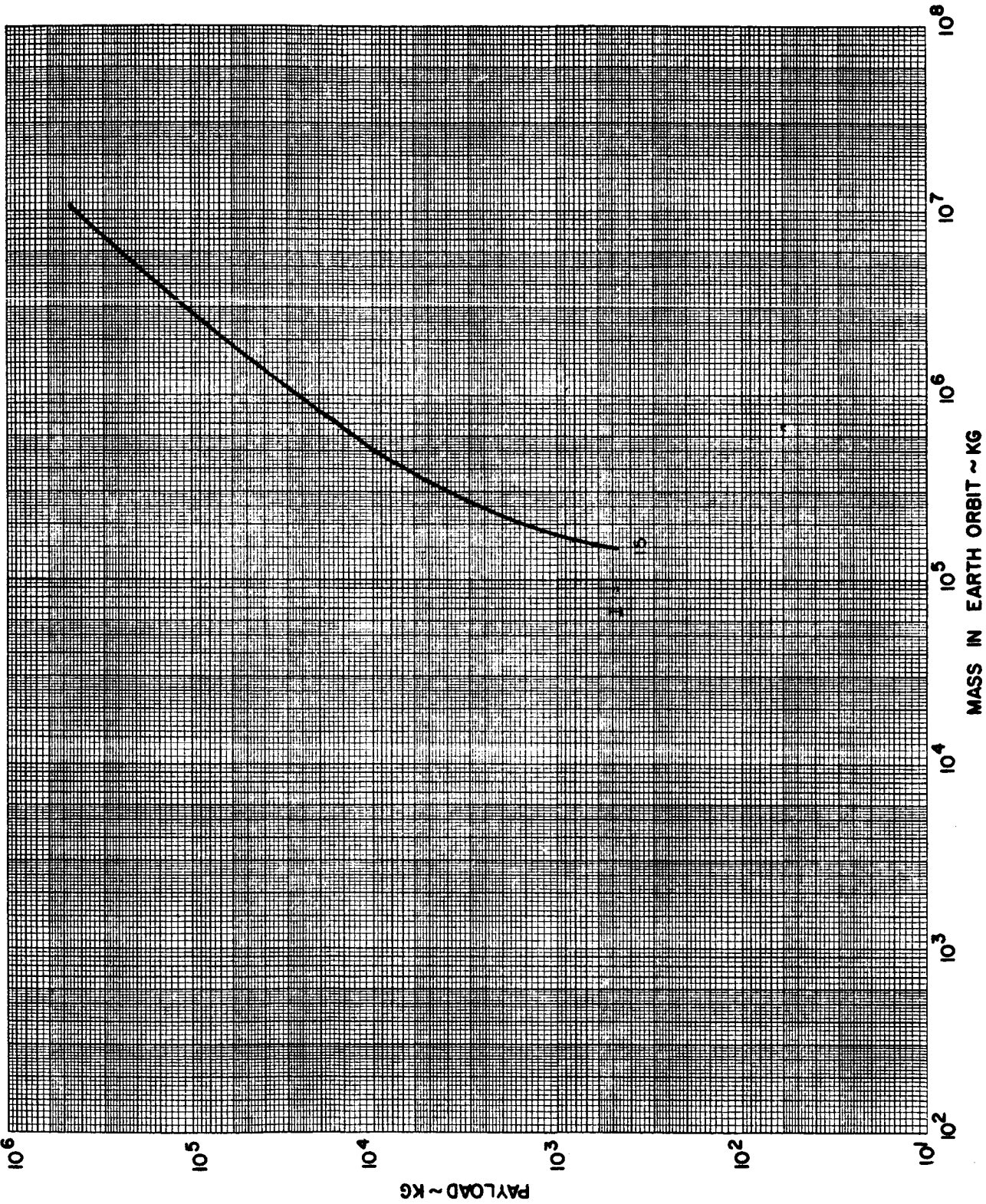
NEPTUNE ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR

 $I_{SP} = 3500 \text{ SEC}$ $W_{PL} = 4536 \text{ KG}$ 

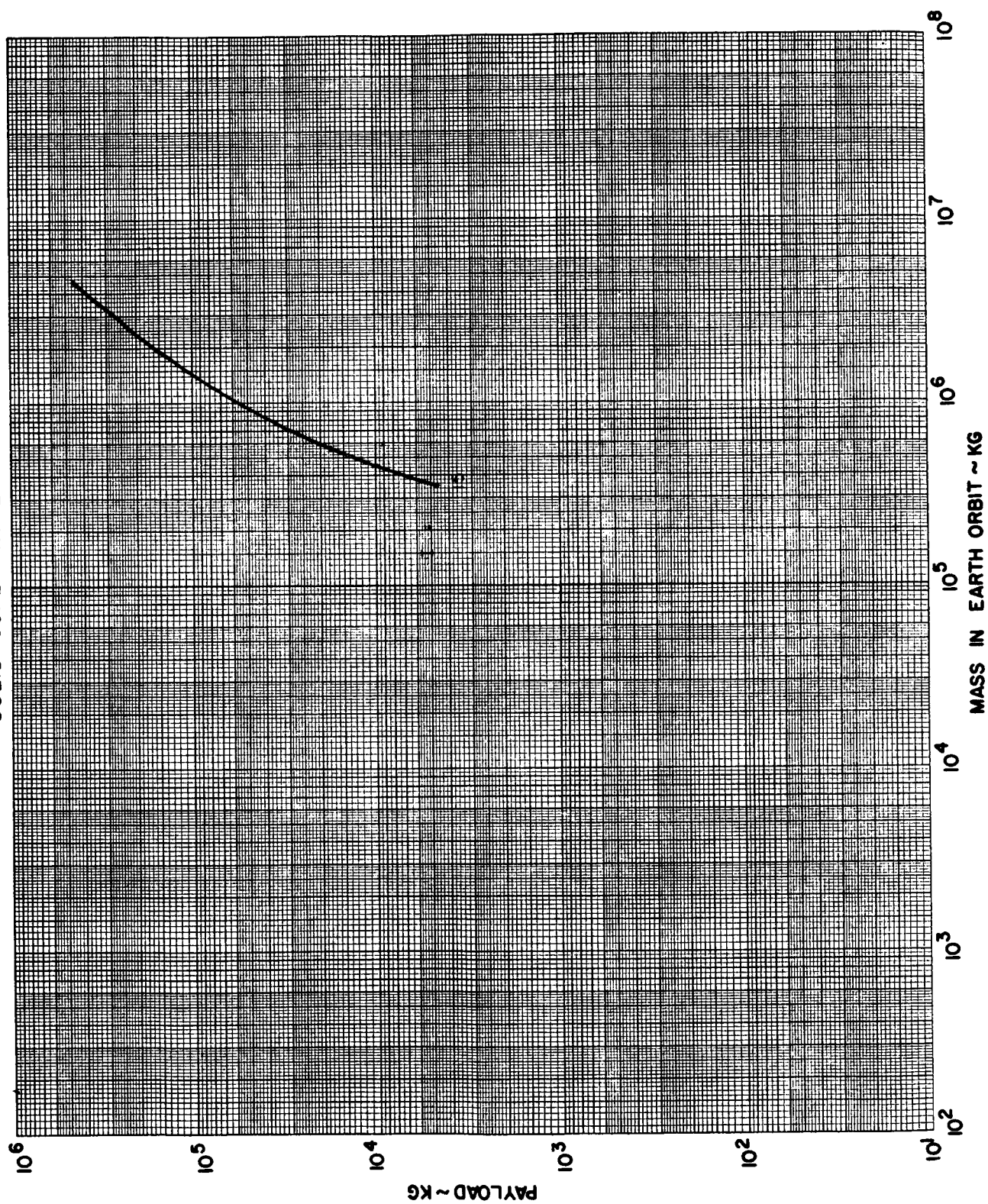
1 A.U. INCLINED ORBITERS

CHEMICAL

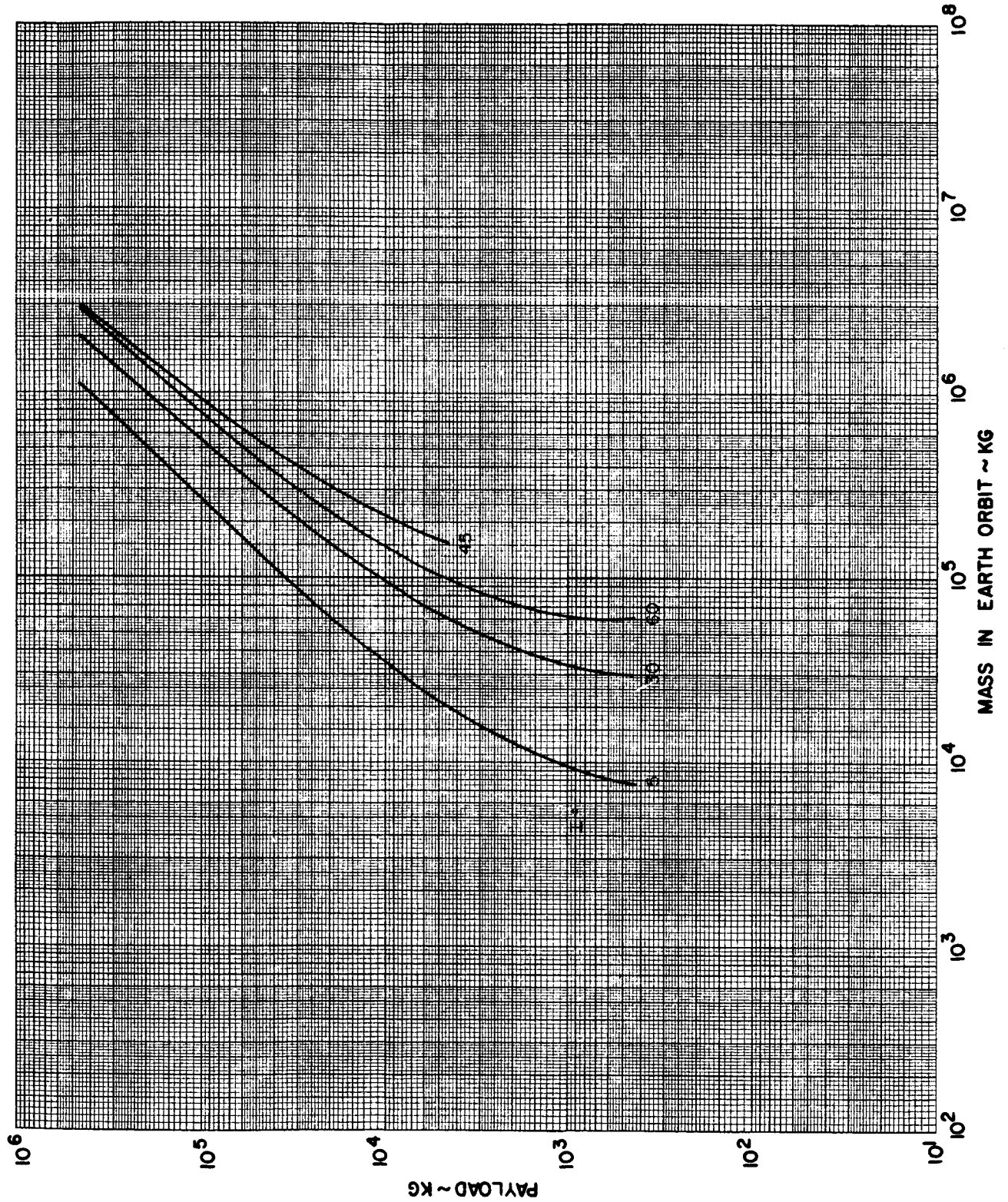


1 A.U. INCLINED ORBITERS

SOLID-CORE NUCLEAR

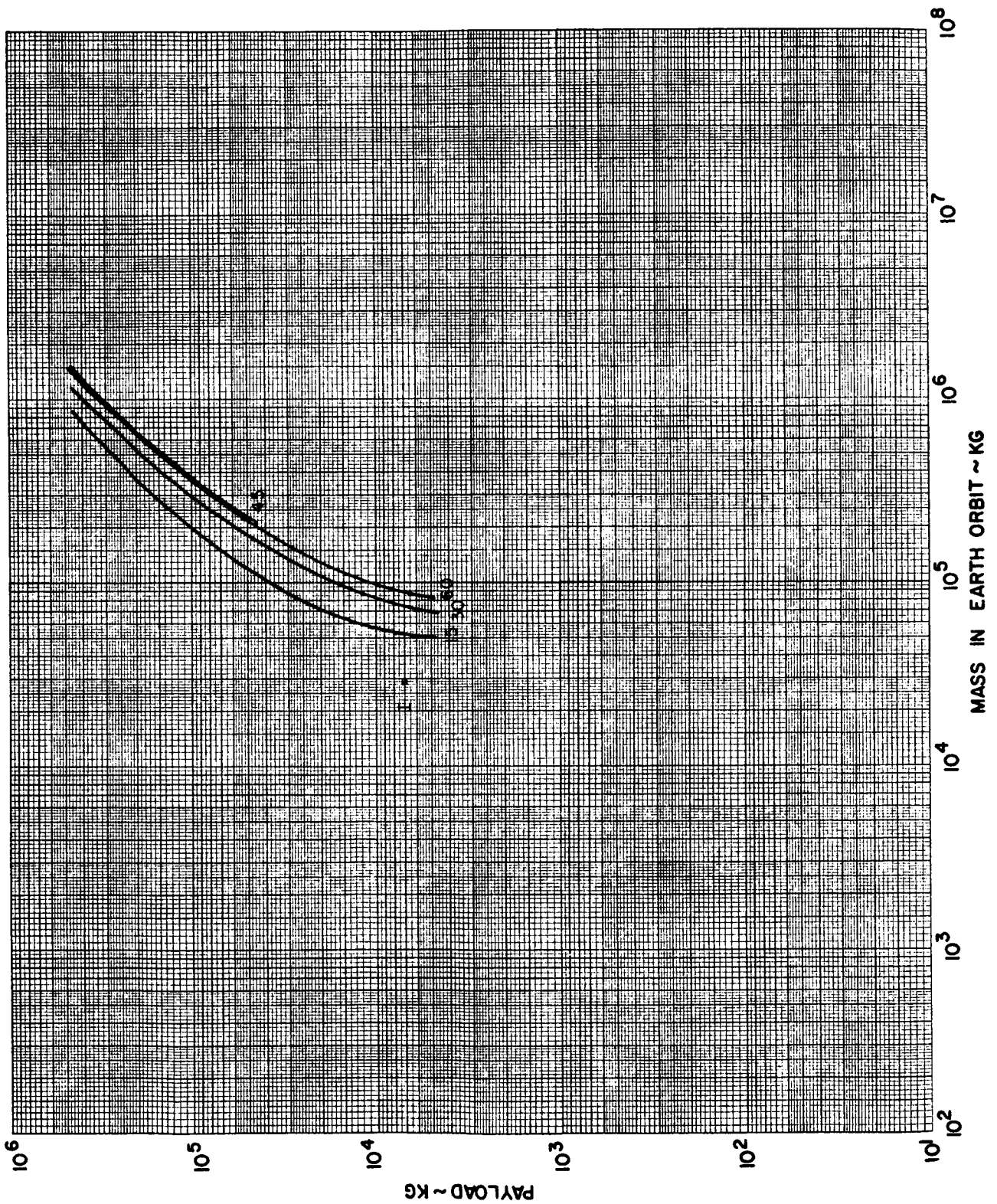


1 A.U. INCLINED ORBITERS
LIGHTWEIGHT ADVANCED NUCLEAR

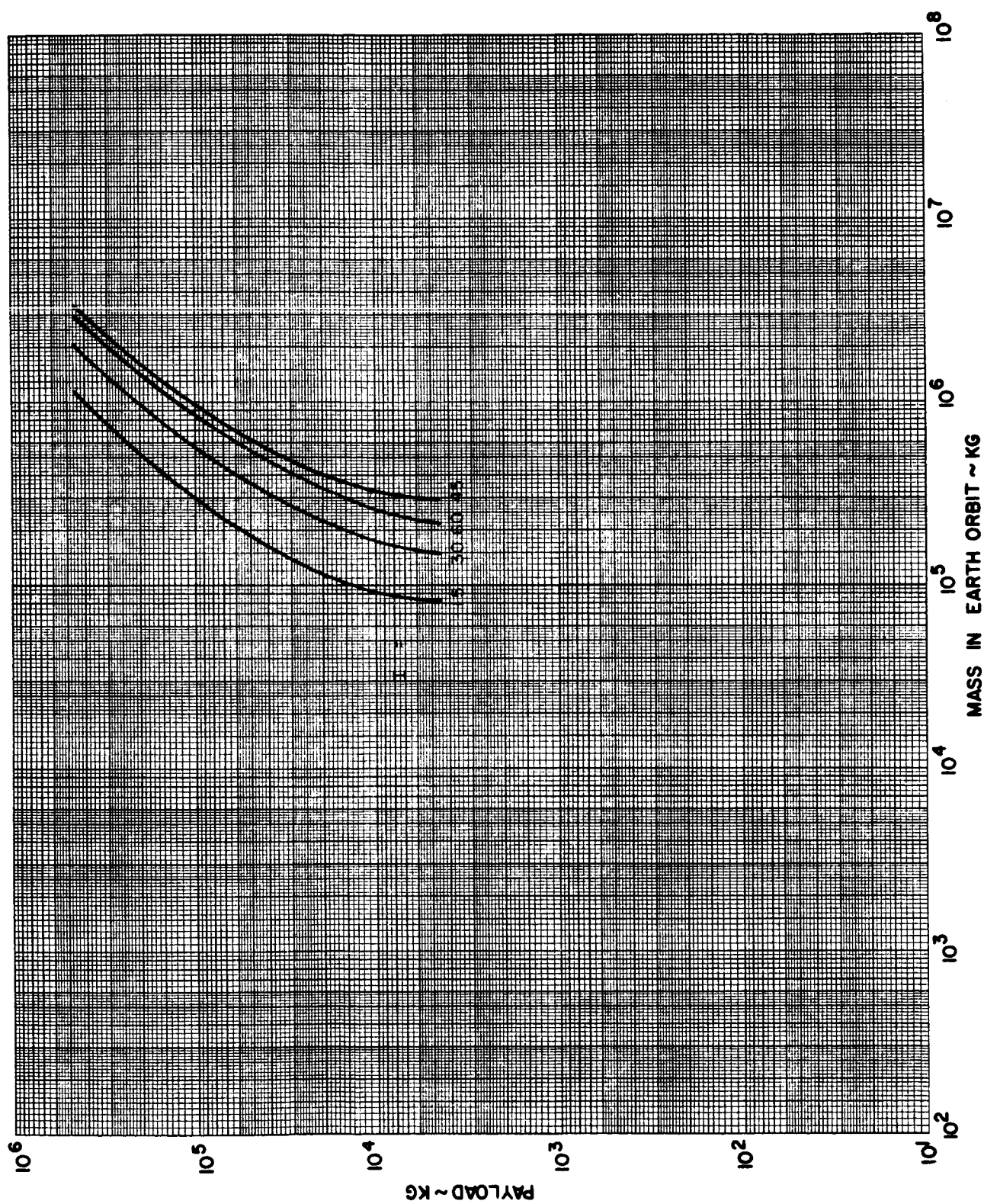


1 A.U. INCLINED ORBITERS

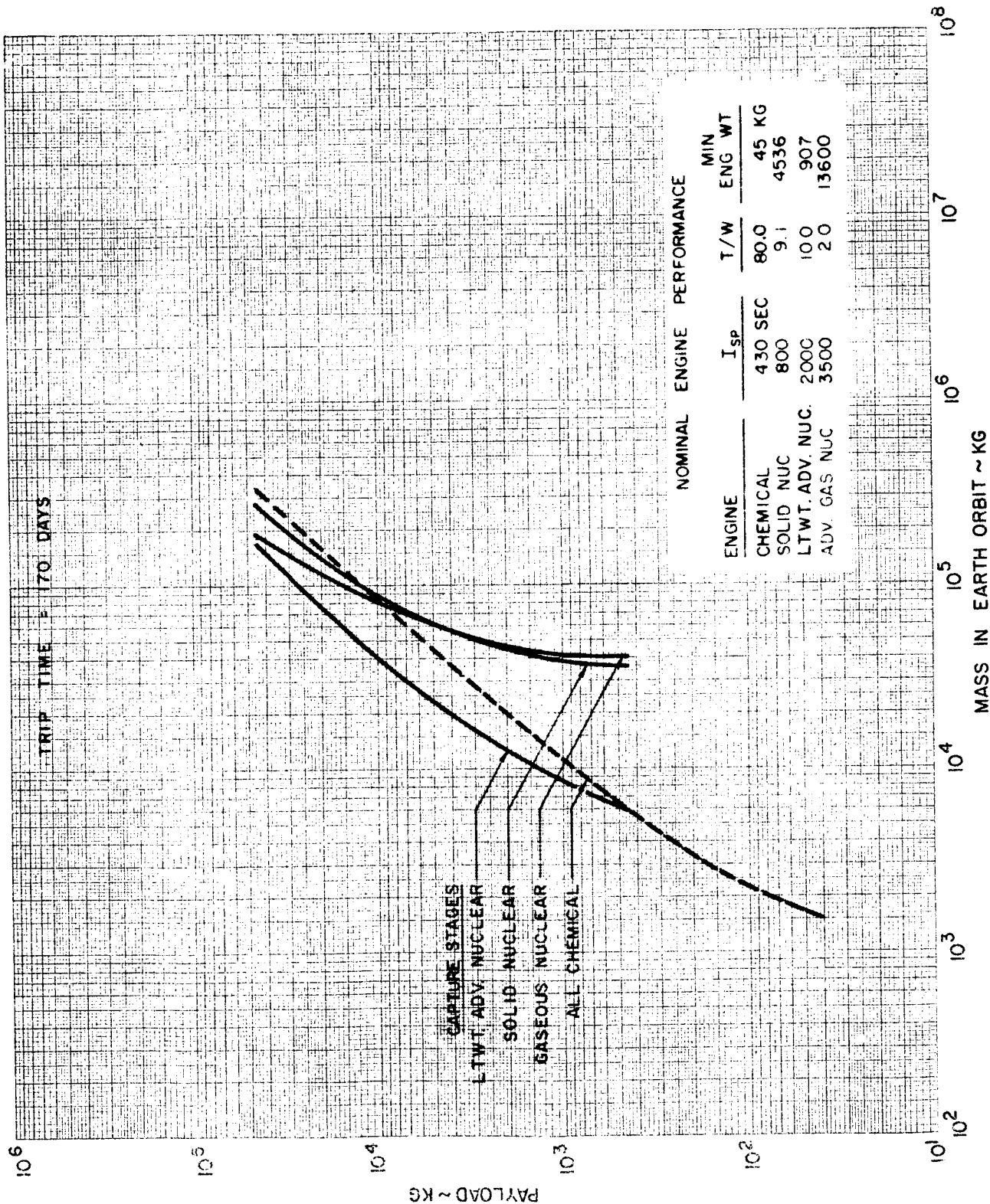
GASEOUS-CORE NUCLEAR



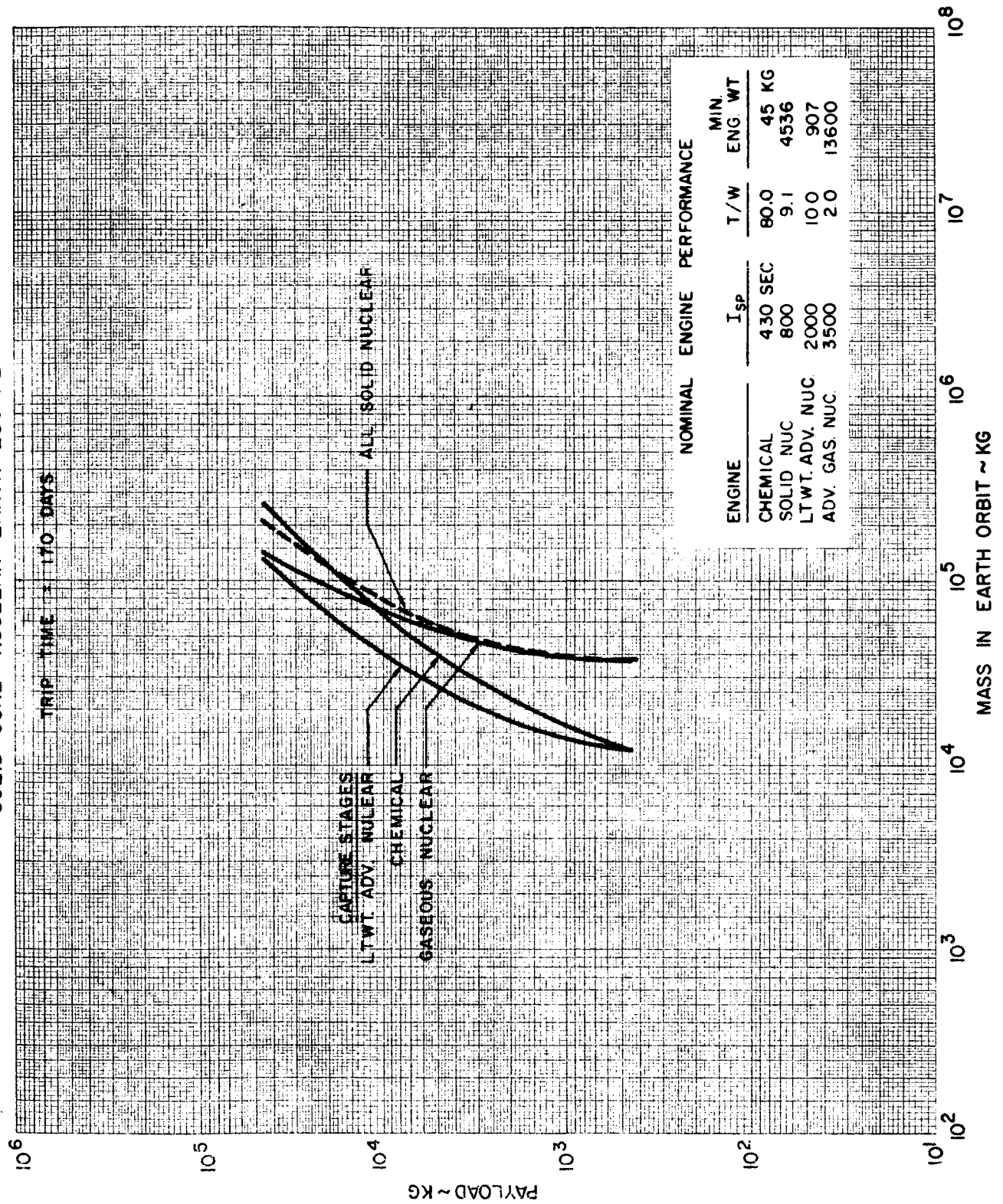
1 A.U. INCLINED ORBITERS
GASEOUS - CORE NUCLEAR



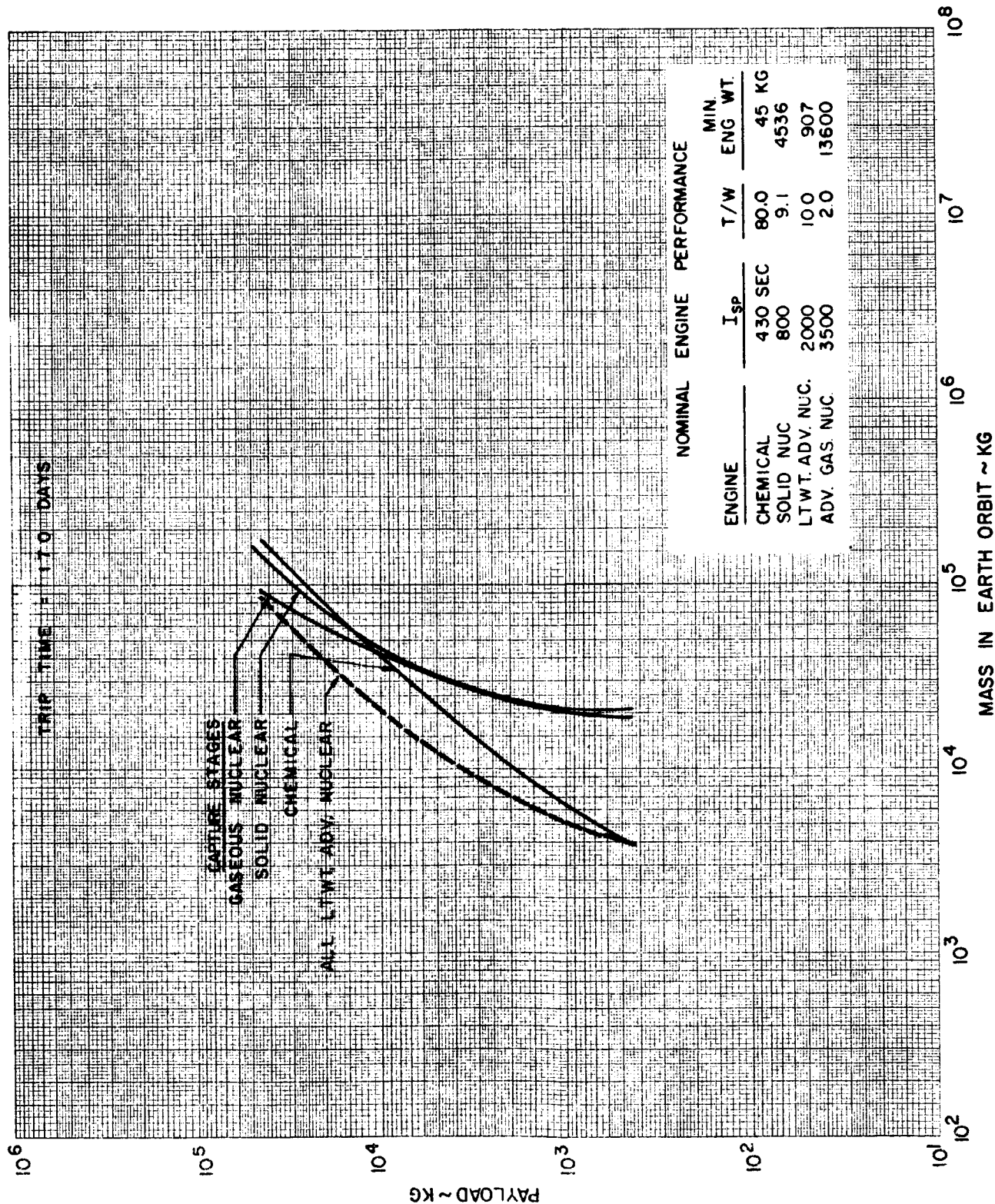
VENUS ORBITAL STOPOVER CHEMICAL EARTH ESCAPE



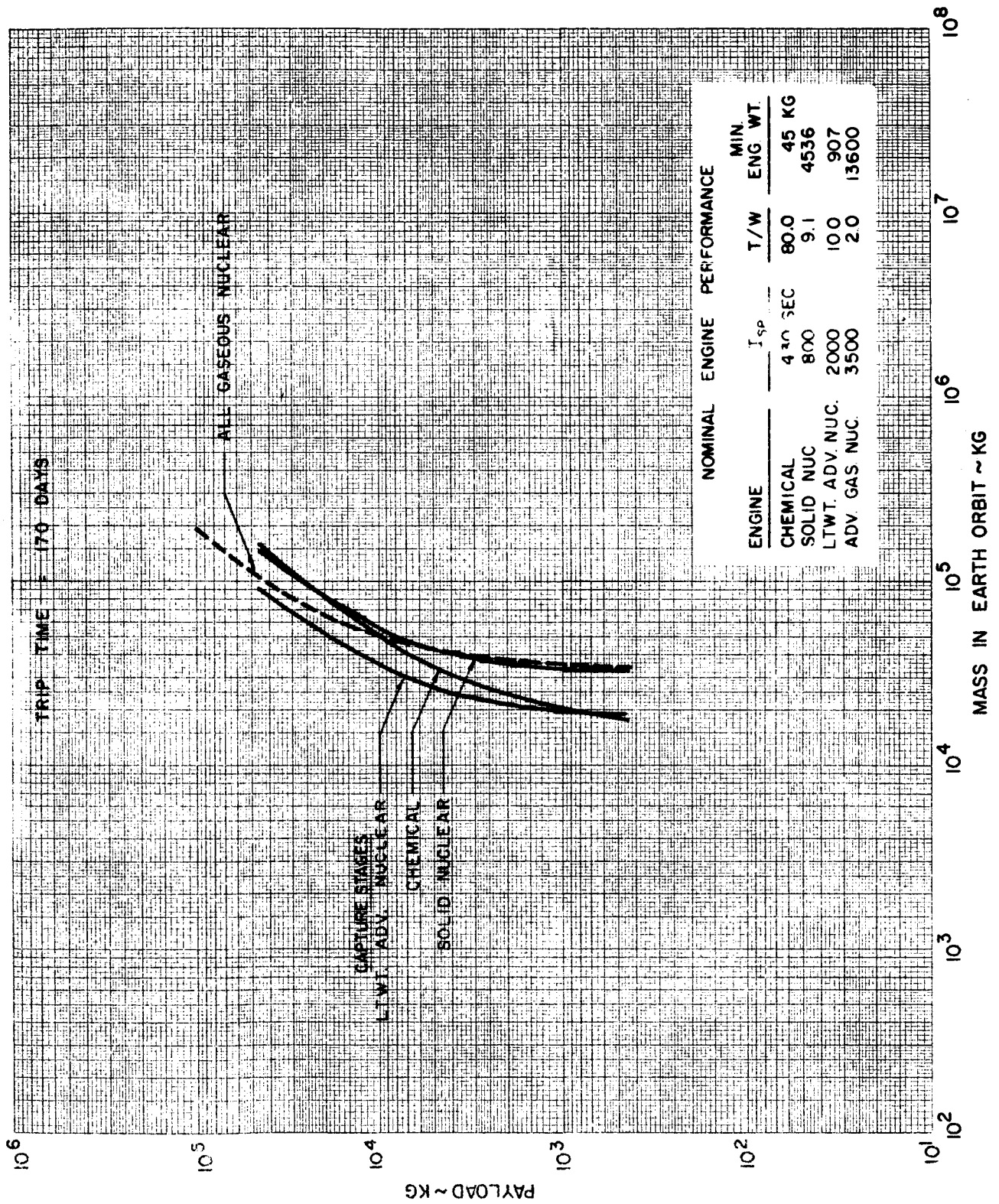
VENUS ORBITAL STOPOVER SOLID-CORE NUCLEAR EARTH ESCAPE



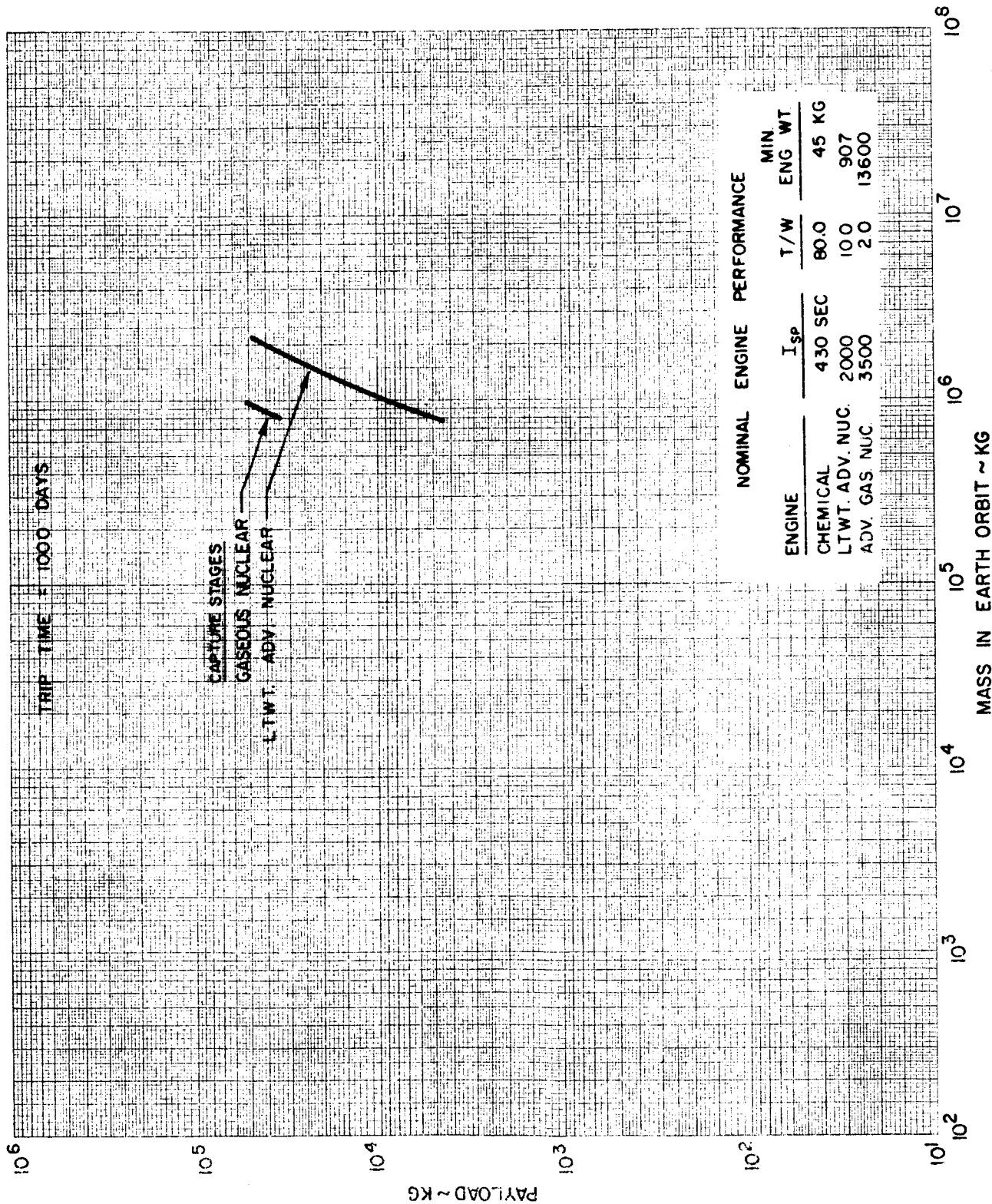
VENUS ORBITAL STOPOVER LIGHTWEIGHT ADVANCED NUCLEAR EARTH ESCAPE



VENUS ORBITAL STOPOVER GASEOUS-CORE NUCLEAR EARTH ESCAPE

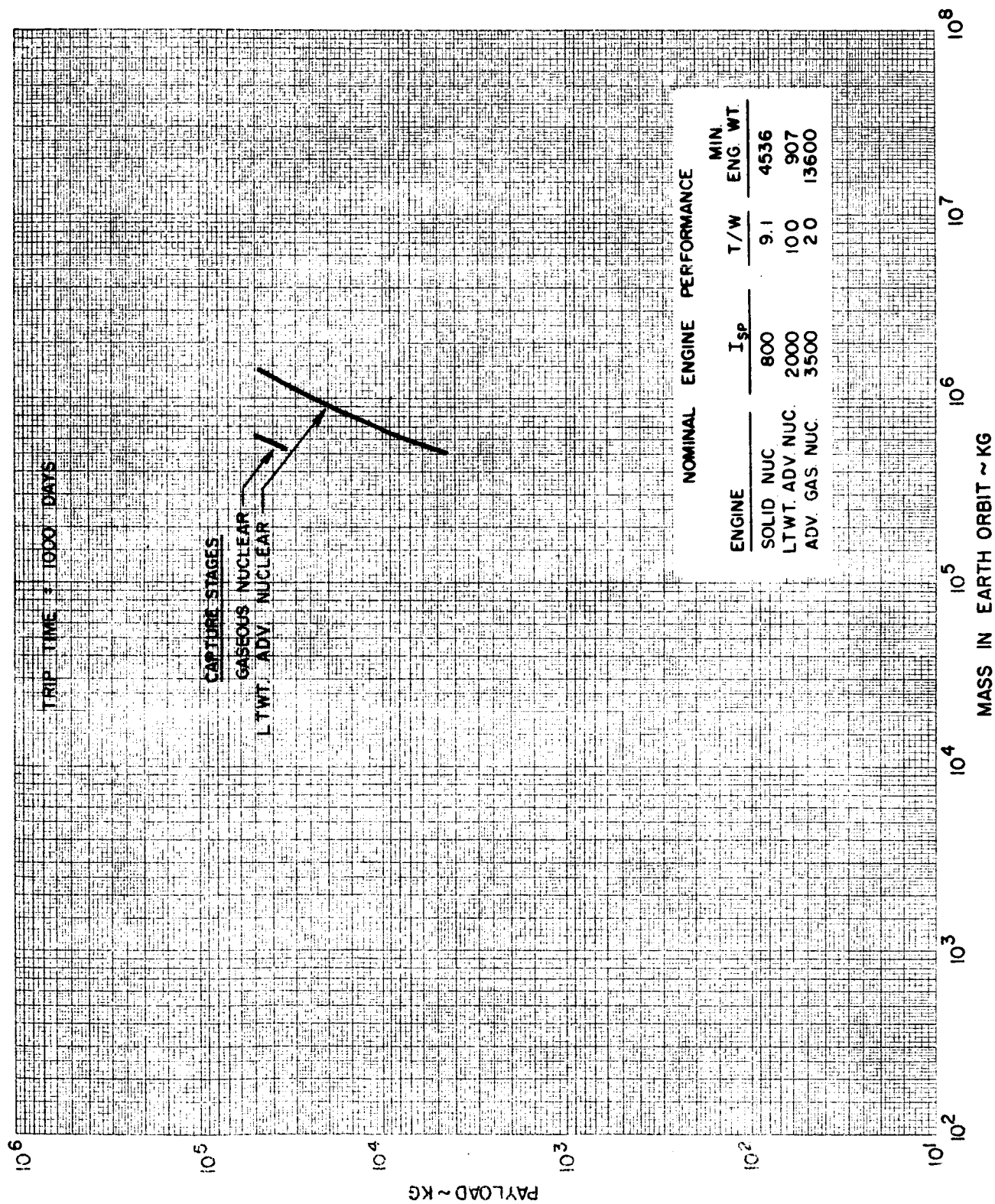


JUPITER ORBITAL STOPOVER CHEMICAL EARTH ESCAPE



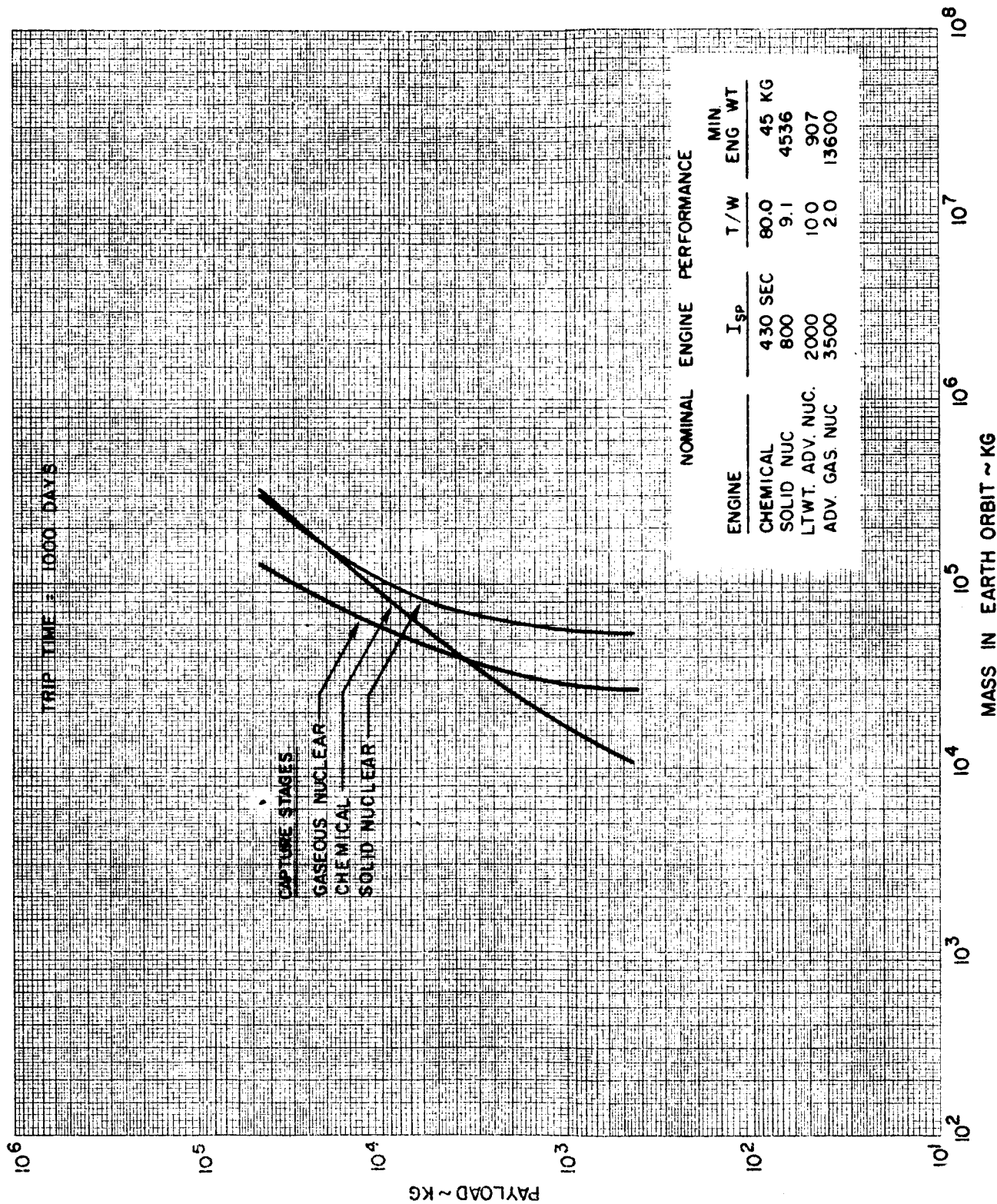
JUPITER ORBITAL STOPOVER

SOLID-CORE NUCLEAR EARTH ESCAPE

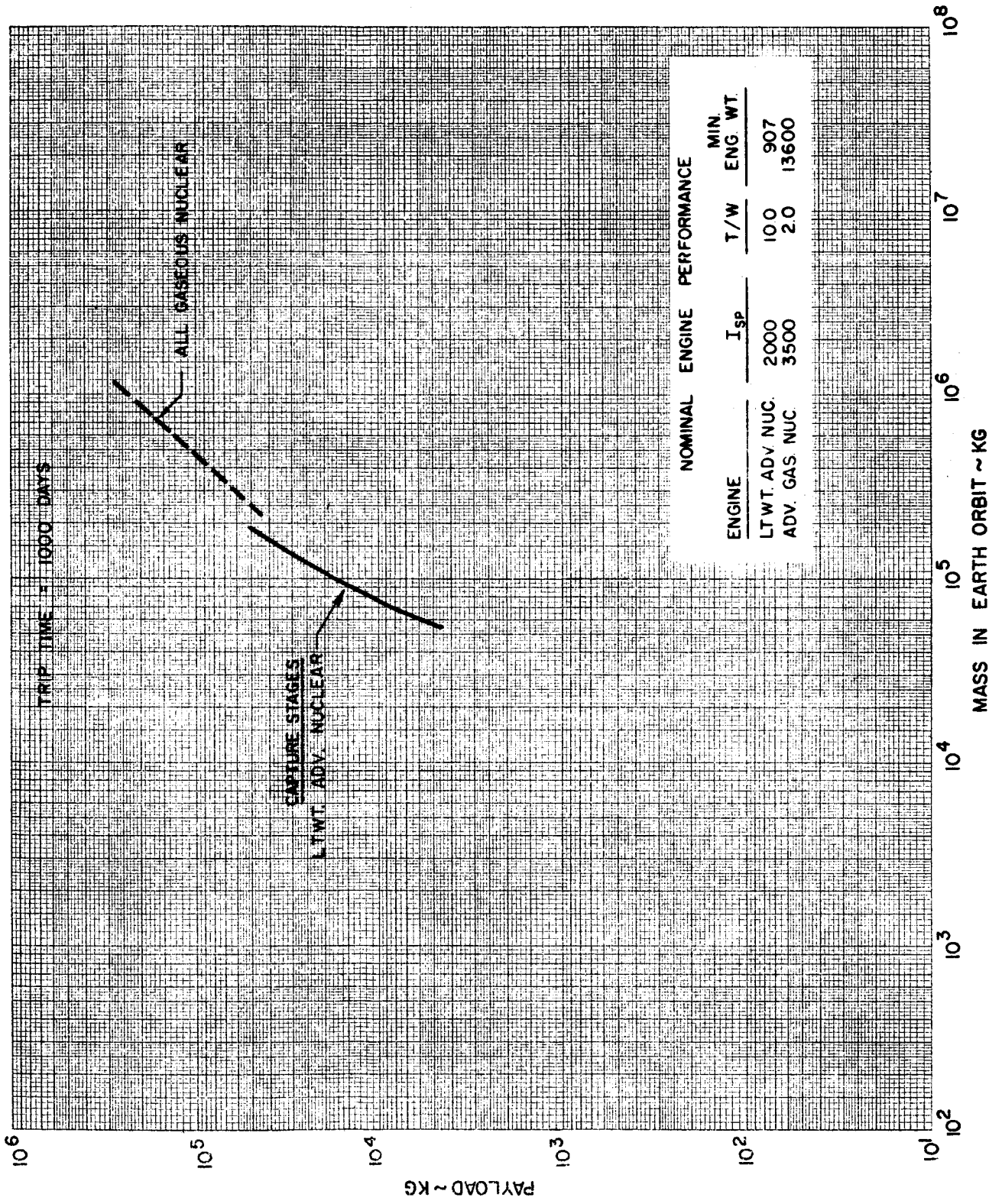


JUPITER ORBITAL STOPOVER

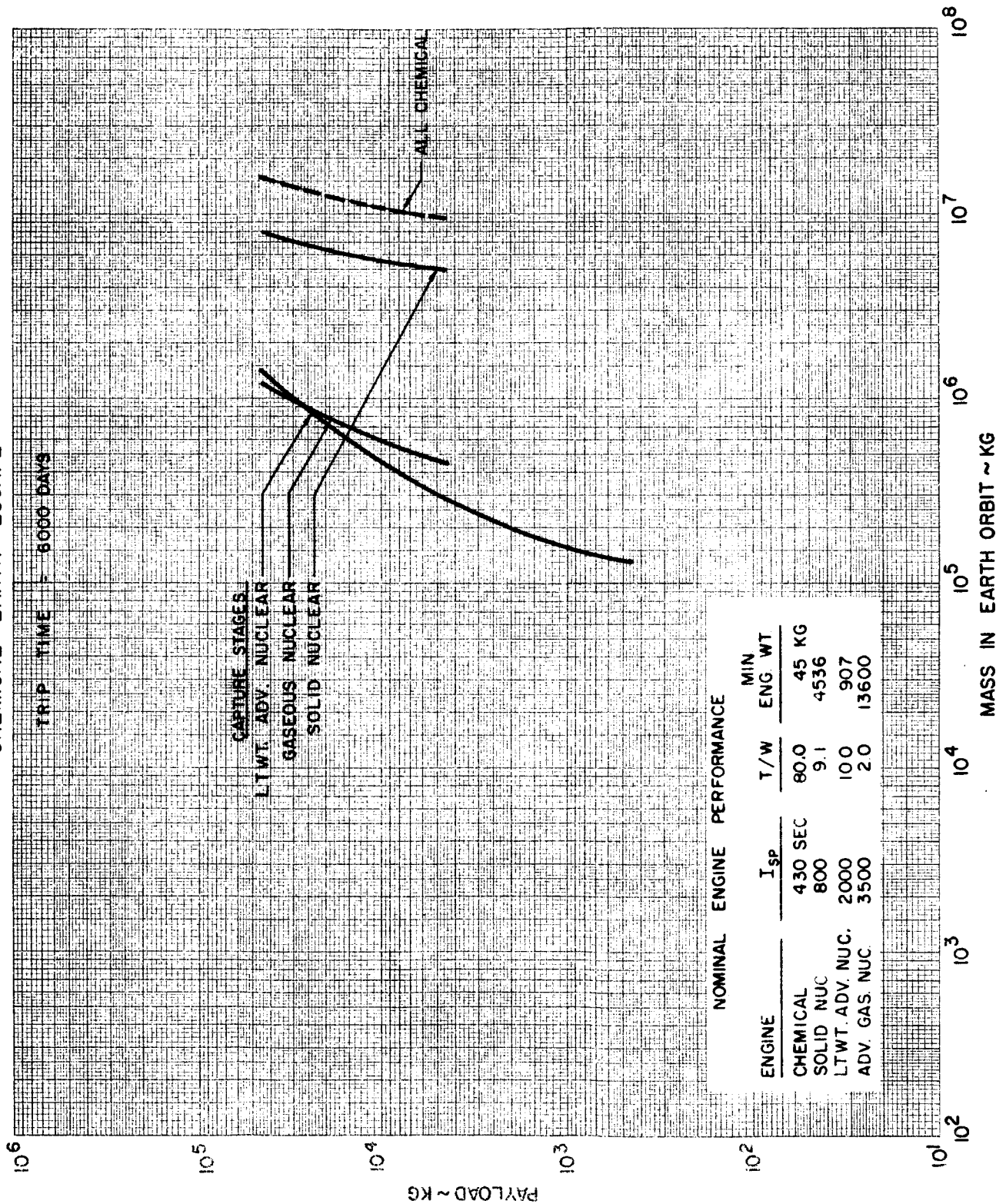
LIGHTWEIGHT ADVANCED NUCLEAR EARTH ESCAPE



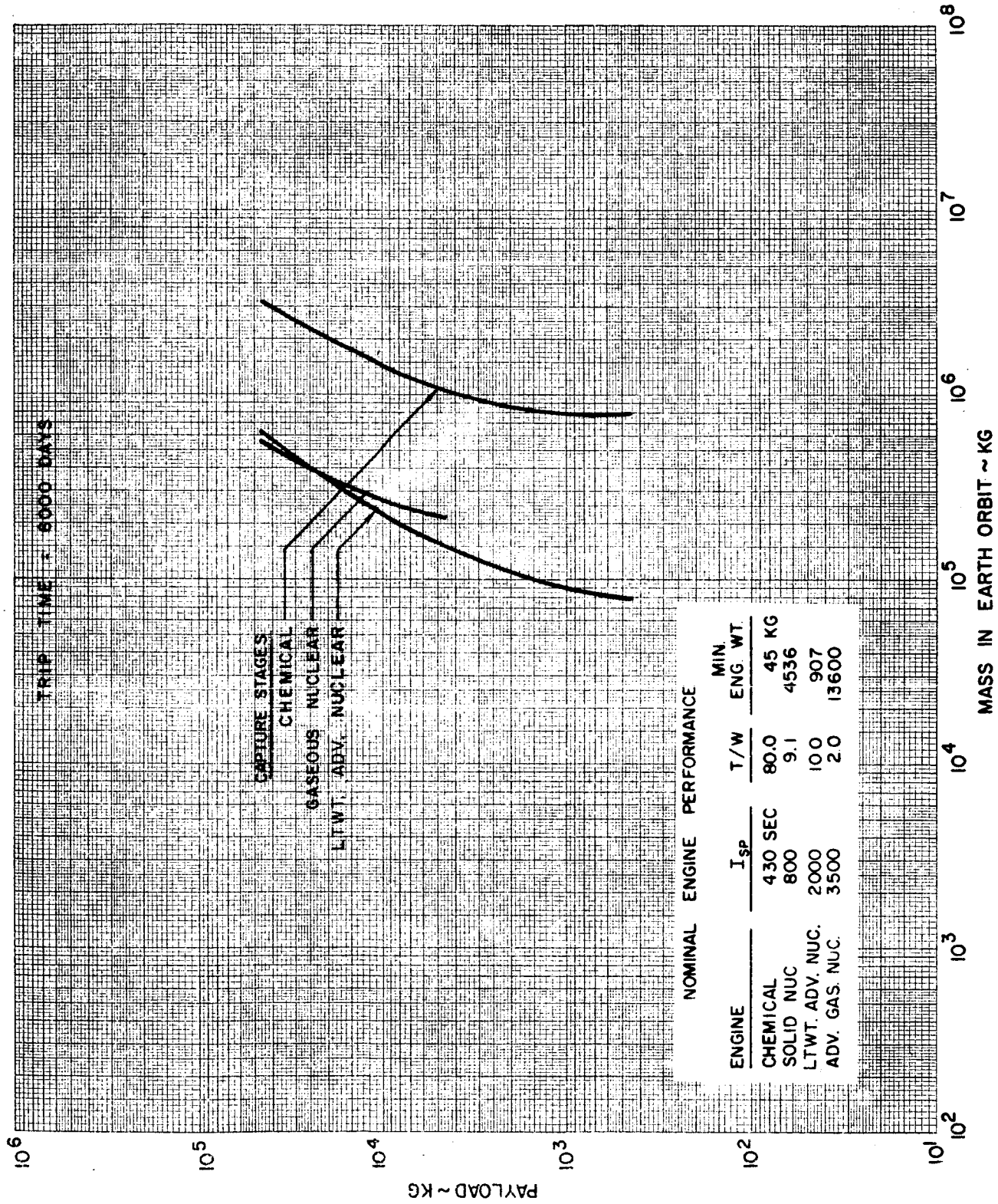
JUPITER ORBITAL STOPOVER
GASEOUS-CORE NUCLEAR EARTH ESCAPE



NEPTUNE ORBITAL STOPOVER
CHEMICAL EARTH ESCAPE

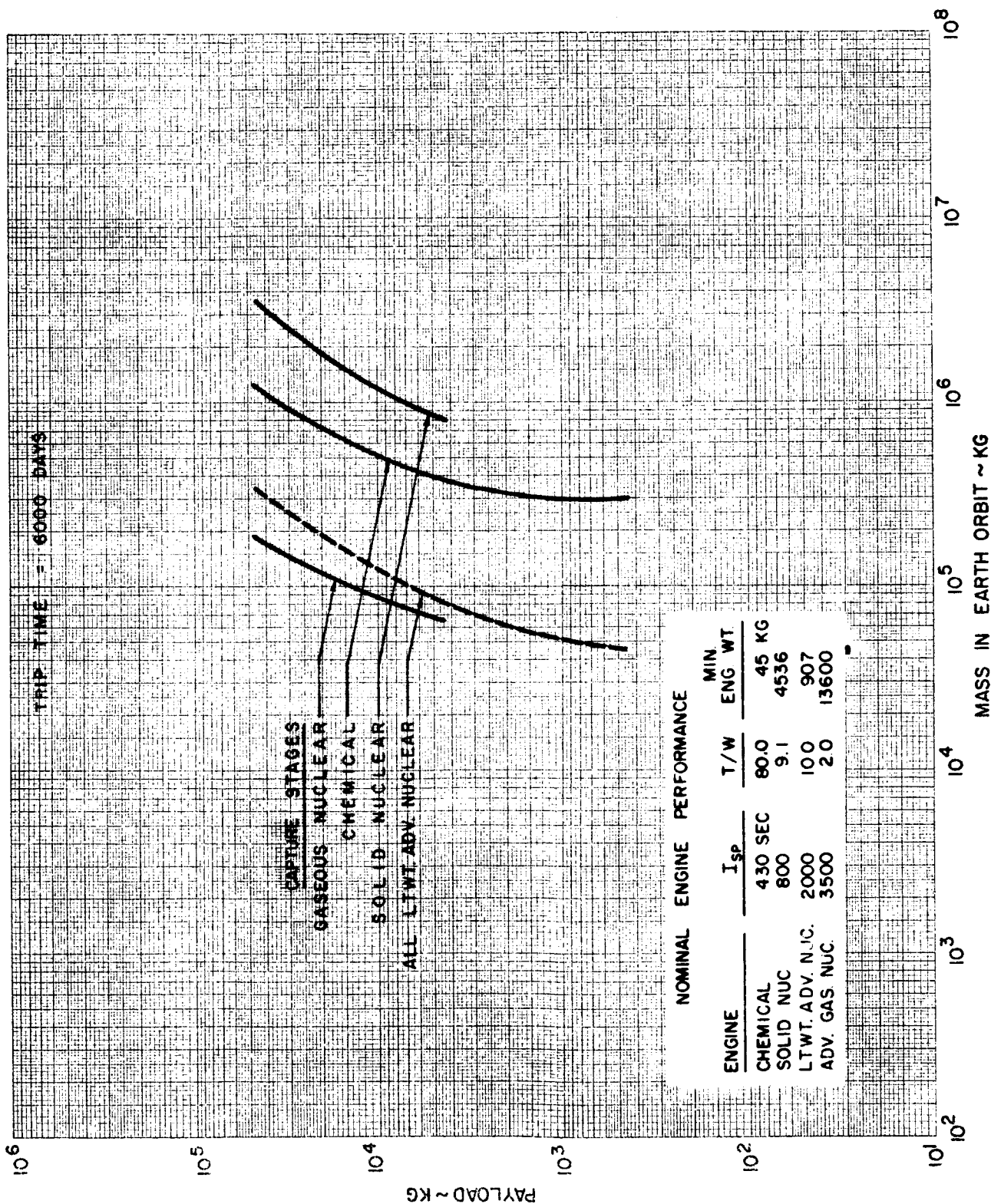


NEPTUNE ORBITAL STOPOVER
SOLID-CORE NUCLEAR EARTH ESCAPE



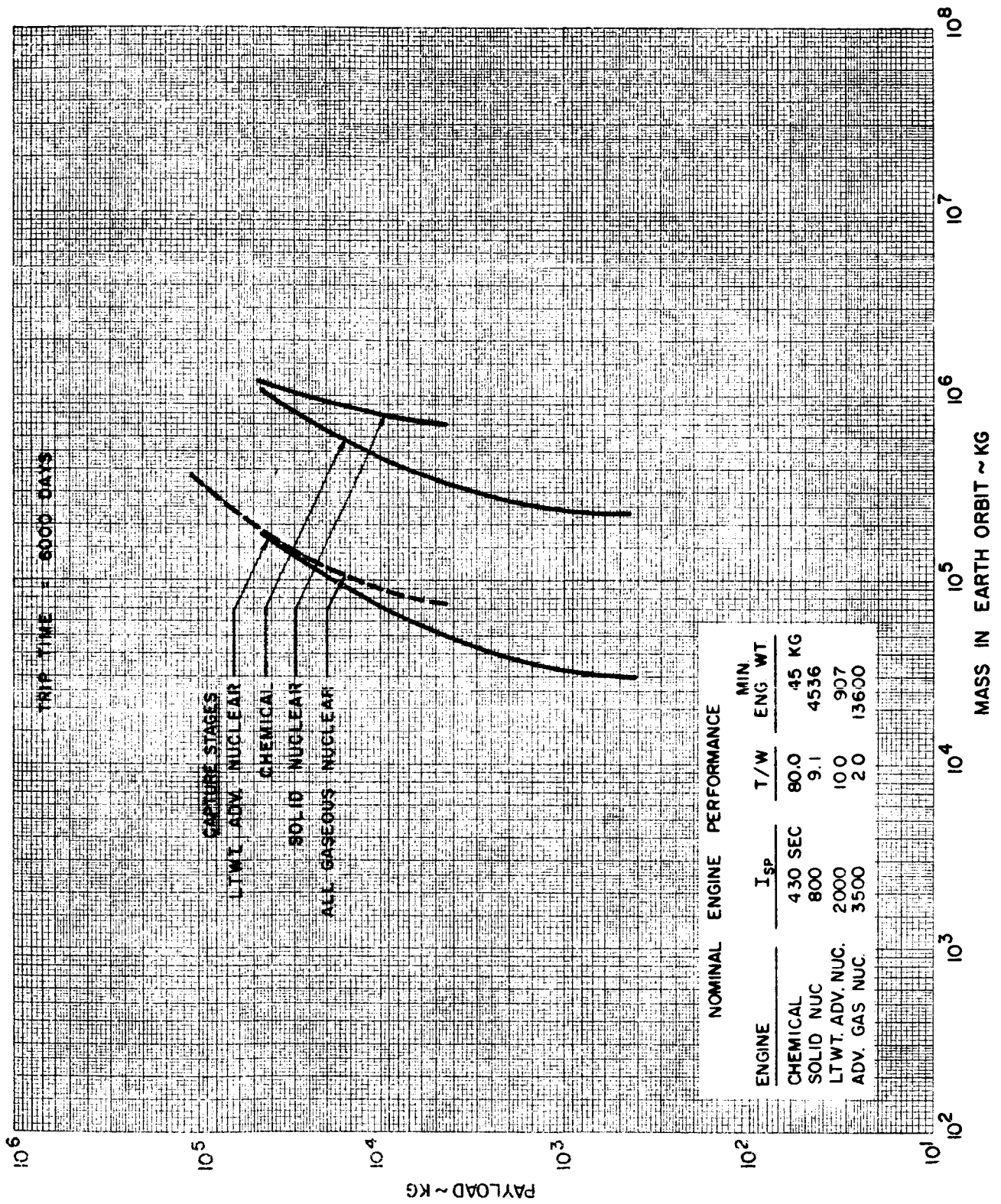
NEPTUNE ORBITAL STOPOVER

LIGHTWEIGHT ADVANCED NUCLEAR EARTH ESCAPE



NEPTUNE ORBITAL STOPOVER

GASEOUS-CORE NUCLEAR EARTH ESCAPE

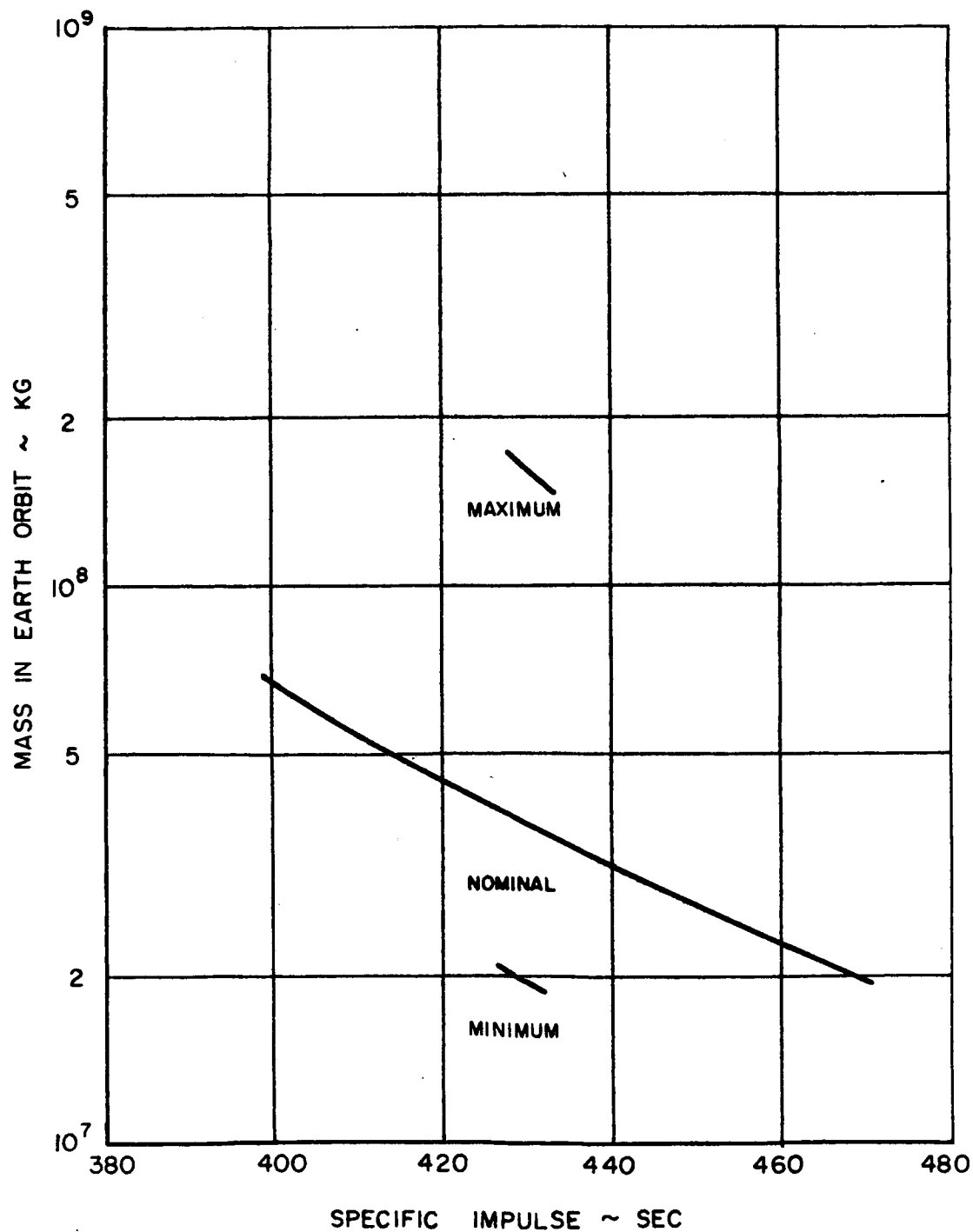


MERCURY MANNED ORBITAL STOPOVERS

CHEMICAL

TRIP TIME = 370 DAYS

STAY TIME = 80 DAYS

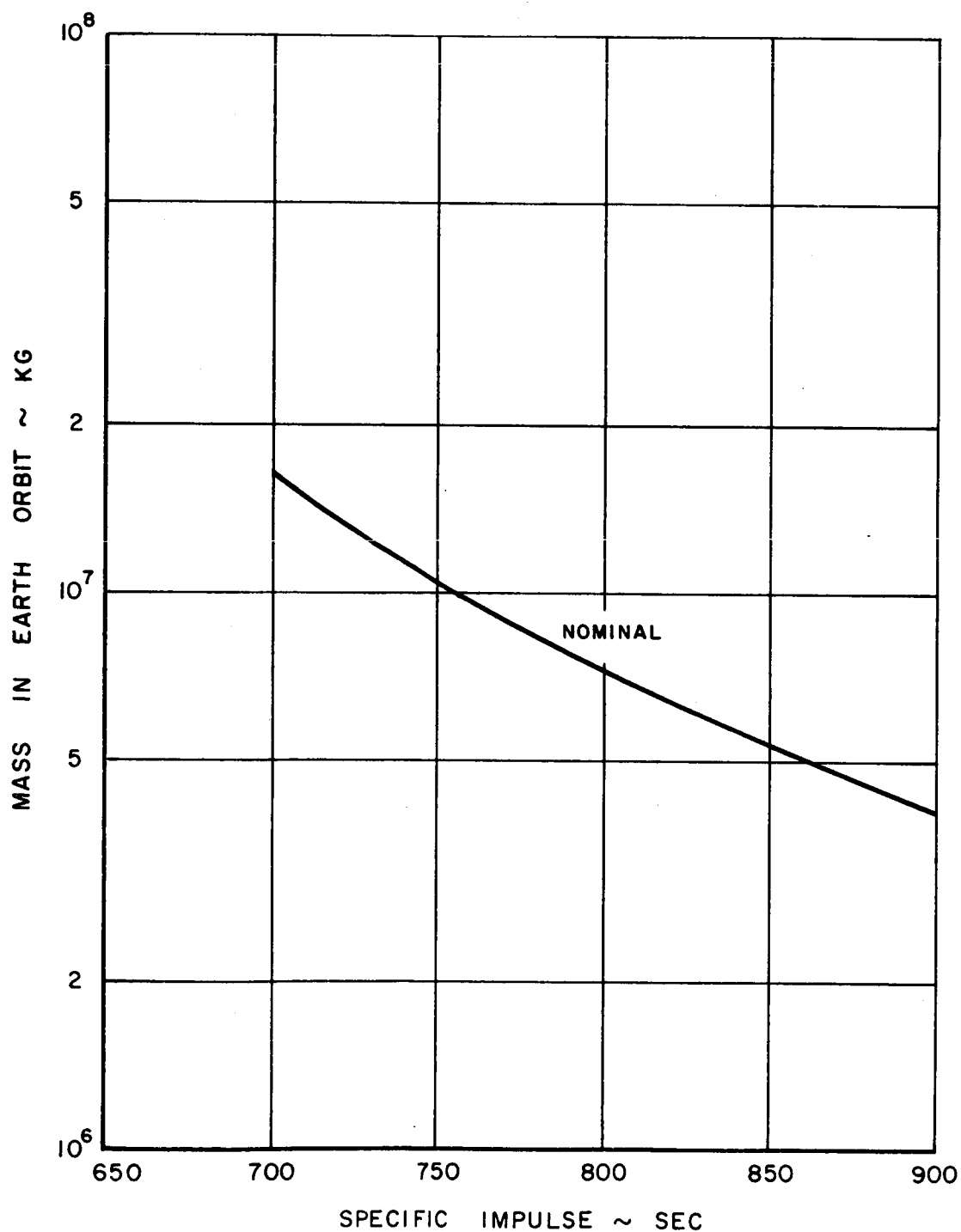


MERCURY MANNED ORBITAL STOPOVERS

SOLID - CORE NUCLEAR

TRIP TIME = 370 DAYS

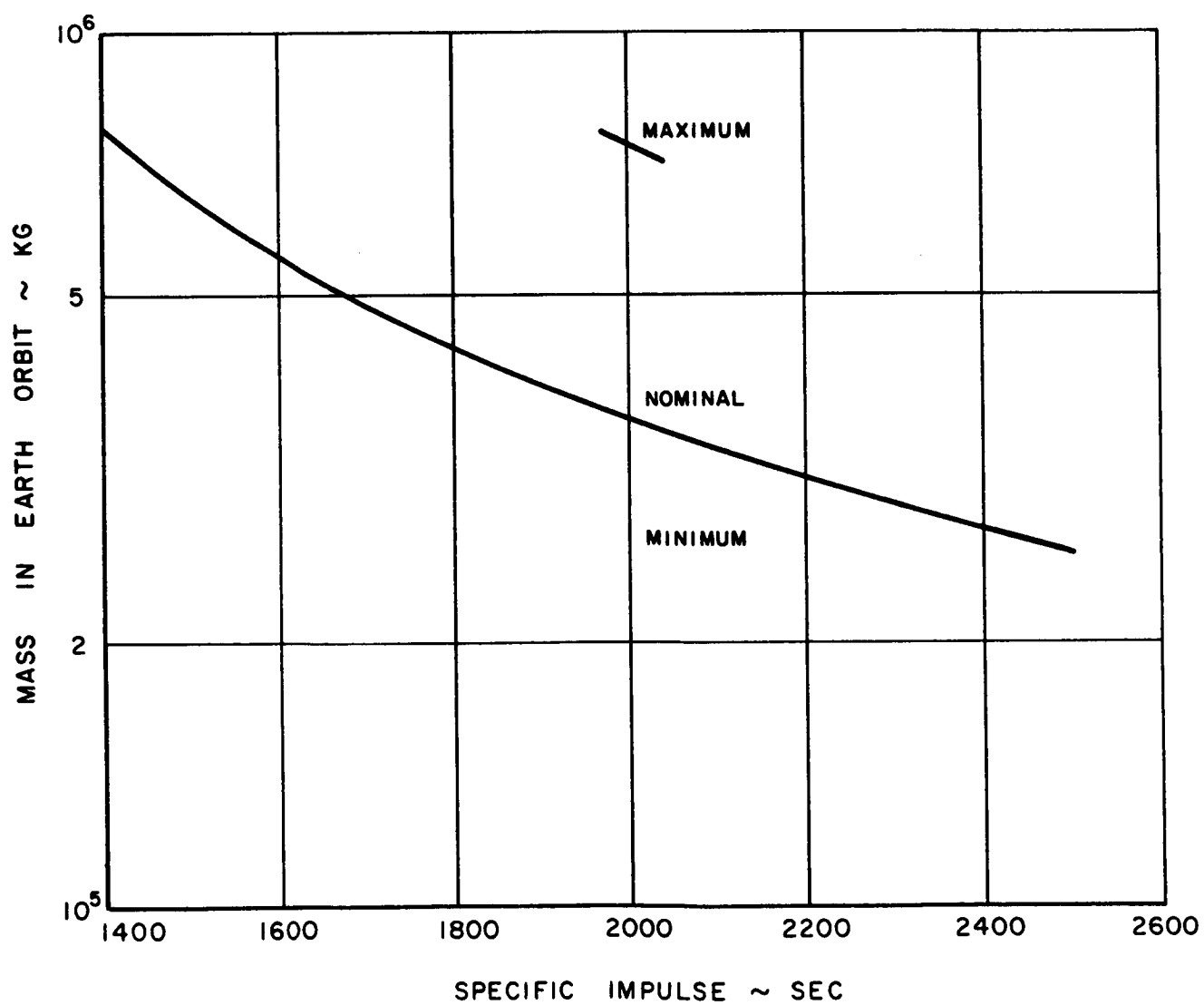
STAY TIME = 80



MERCURY MANNED ORBITAL STOPOVERS**LIGHTWEIGHT ADVANCED NUCLEAR**

TRIP TIME = 370 DAYS

STAY TIME = 80

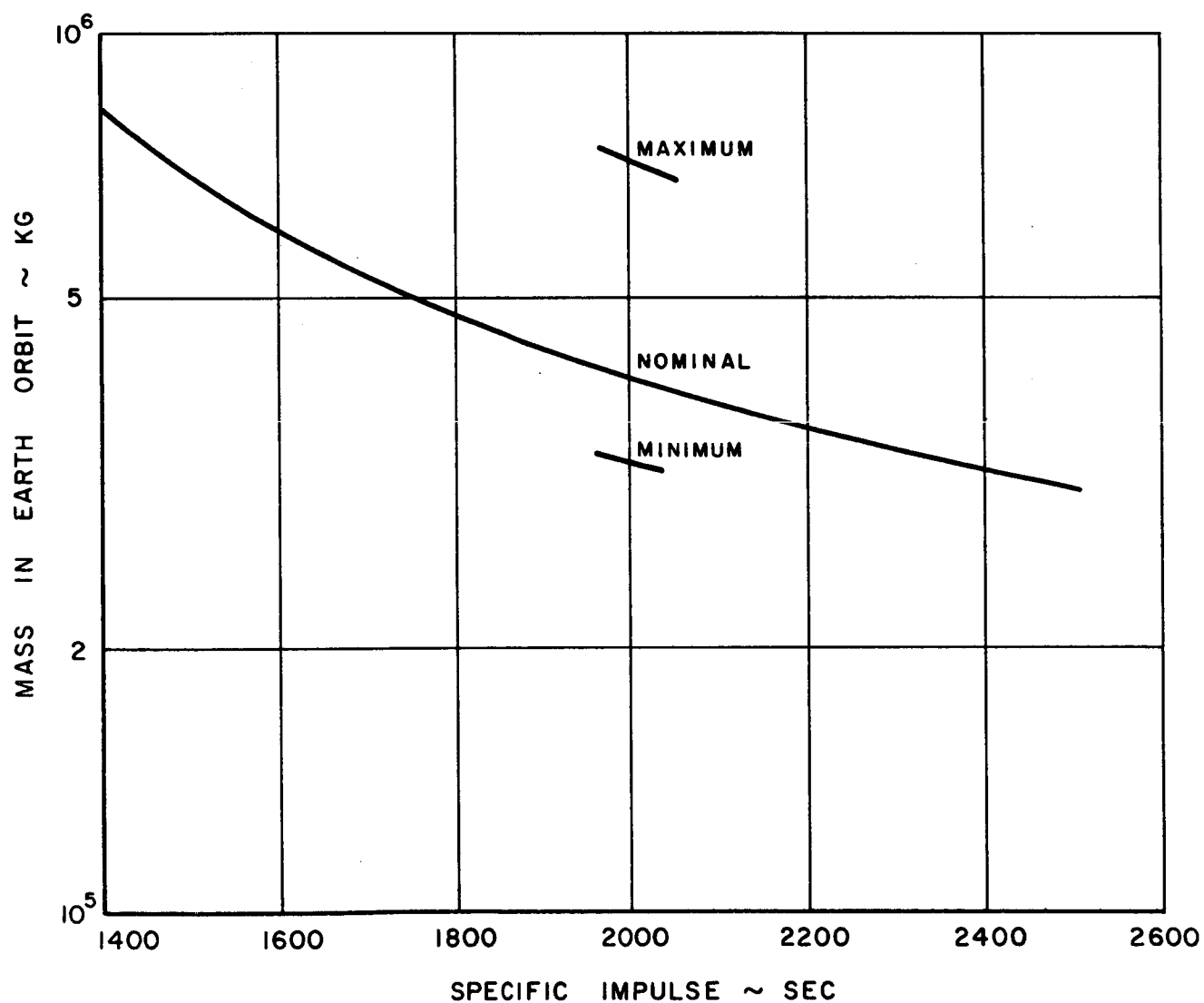


MERCURY MANNED ORBITAL STOPOVERS

GASEOUS - CORE NUCLEAR

TRIP TIME = 370 DAYS

STAY TIME = 80

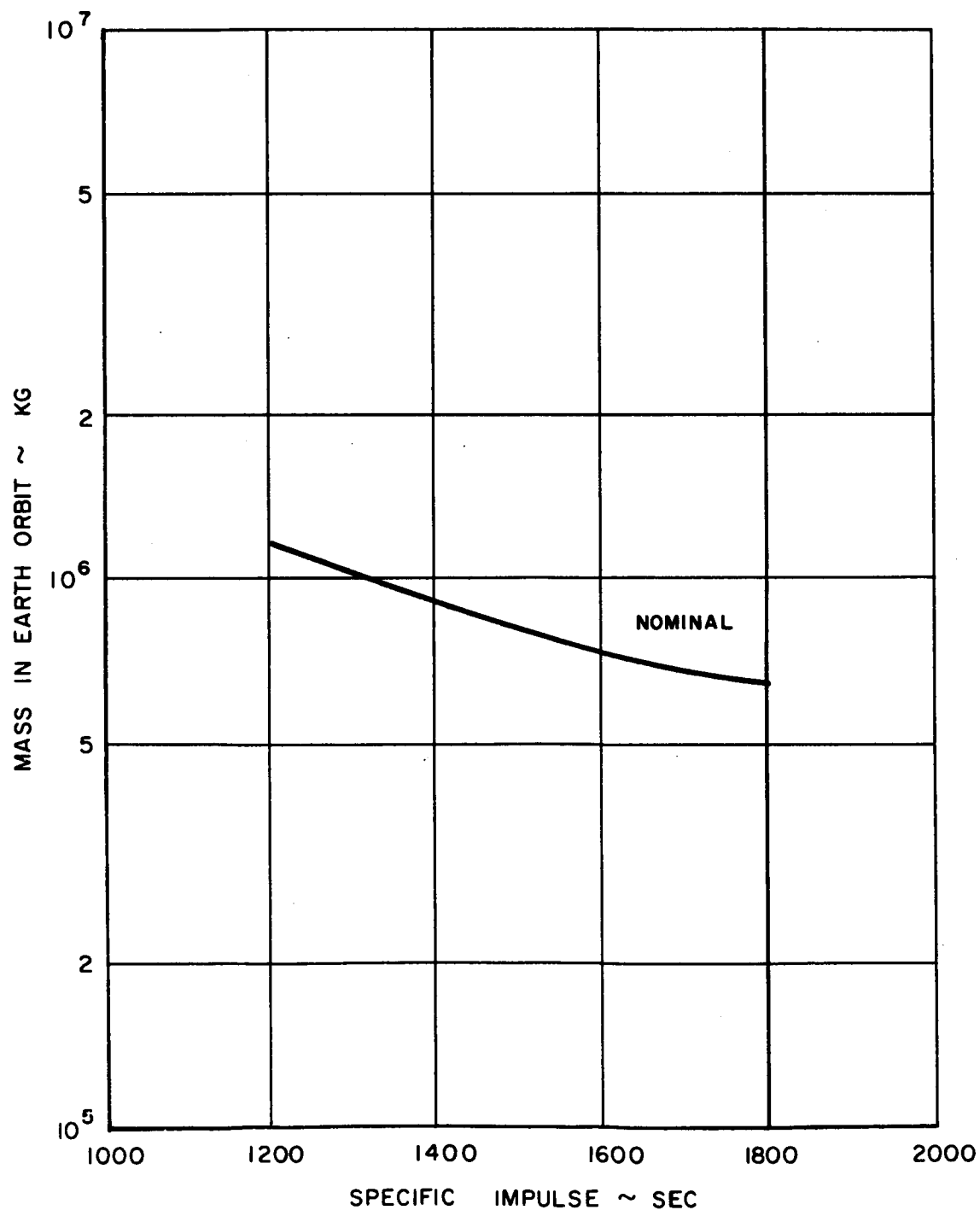


MERCURY MANNED ORBITAL STOPOVERS

PULSED NUCLEAR

TRIP TIME = 375 DAYS

STAY TIME = 80

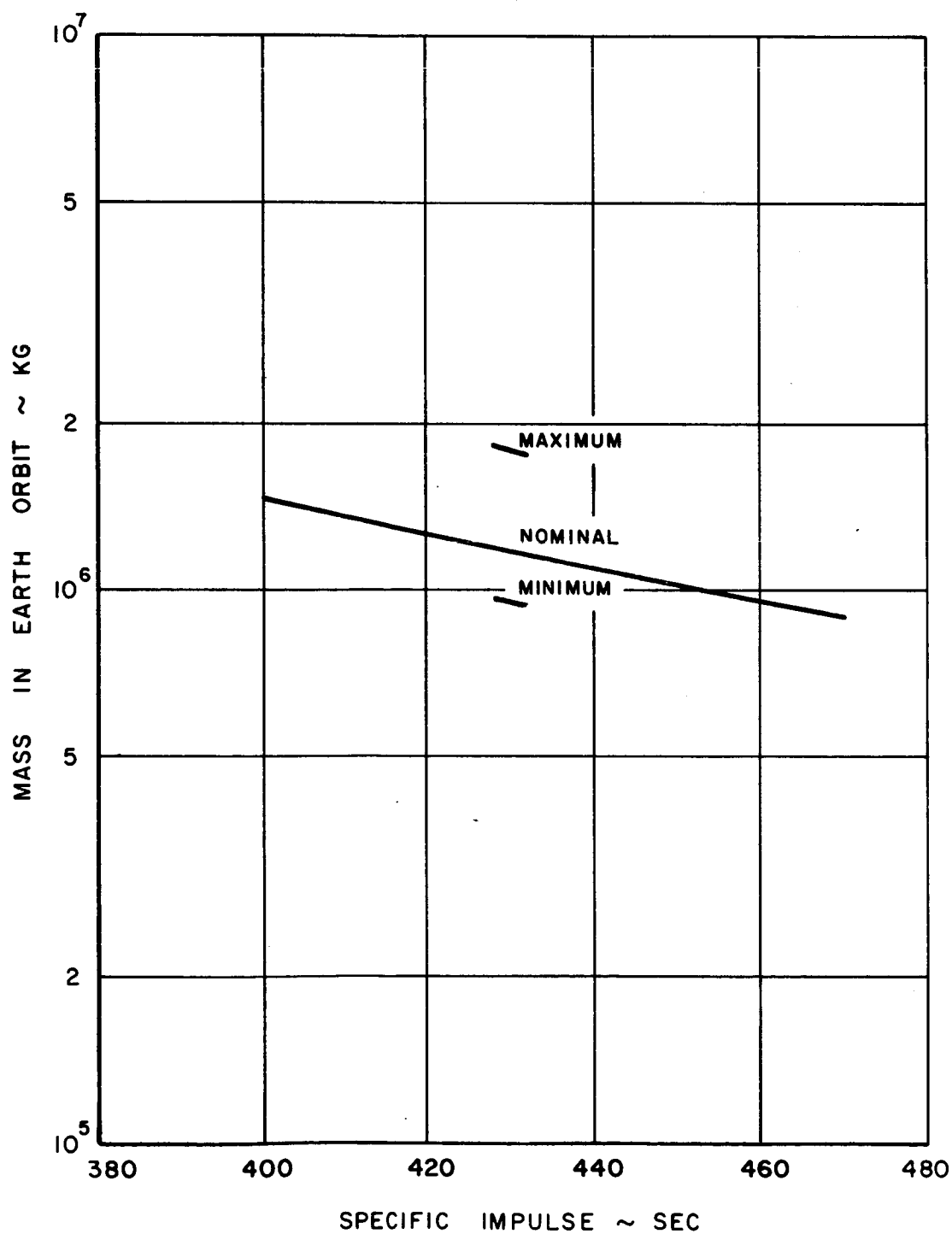


VENUS MANNED ORBITAL STOPOVERS

CHEMICAL

TRIP TIME = 450 DAYS

STAY TIME = 0

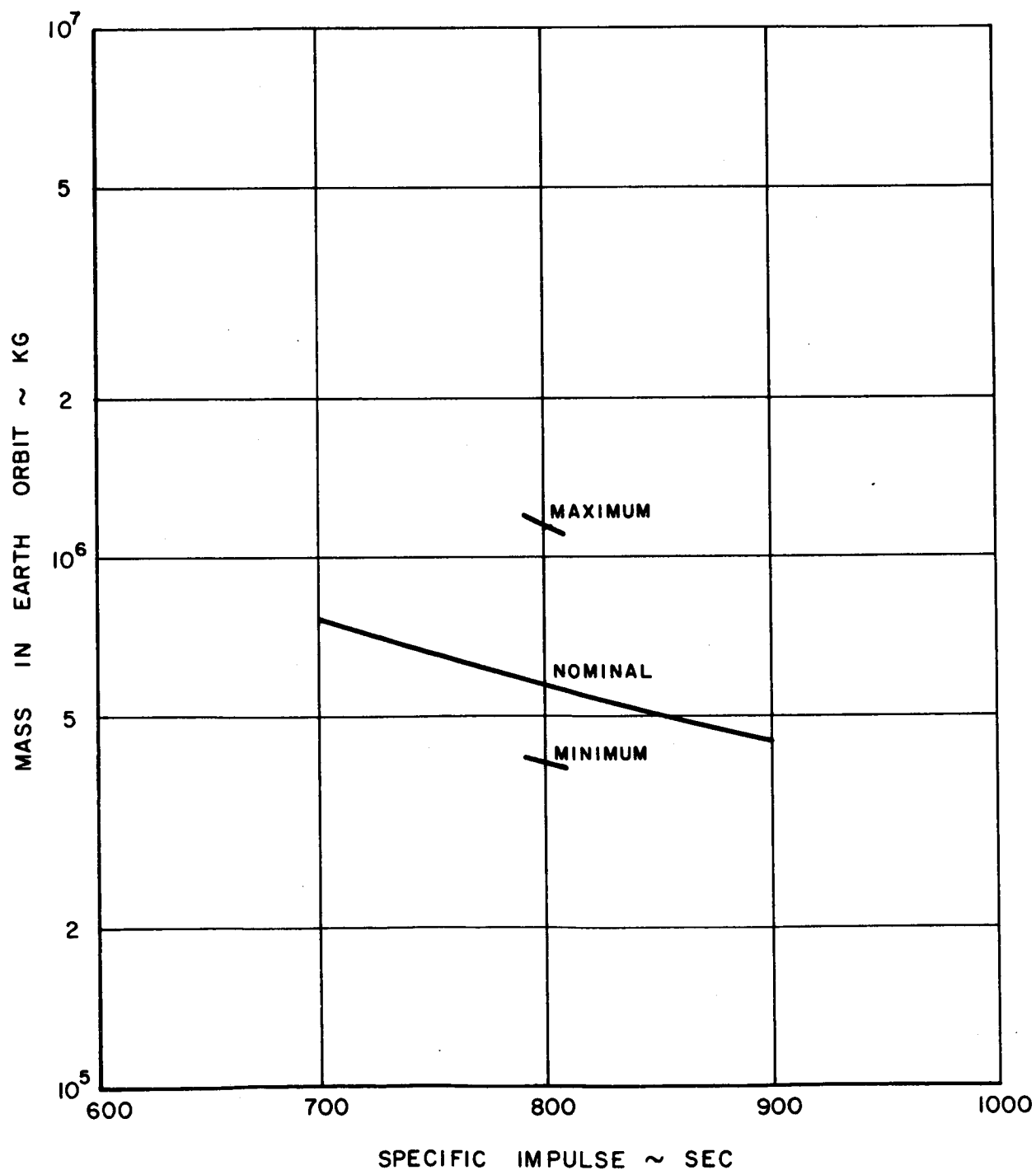


VENUS MANNED ORBITAL STOPOVERS

SOLID - CORE NUCLEAR

TRIP TIME = 450 DAYS

STAY TIME = 0

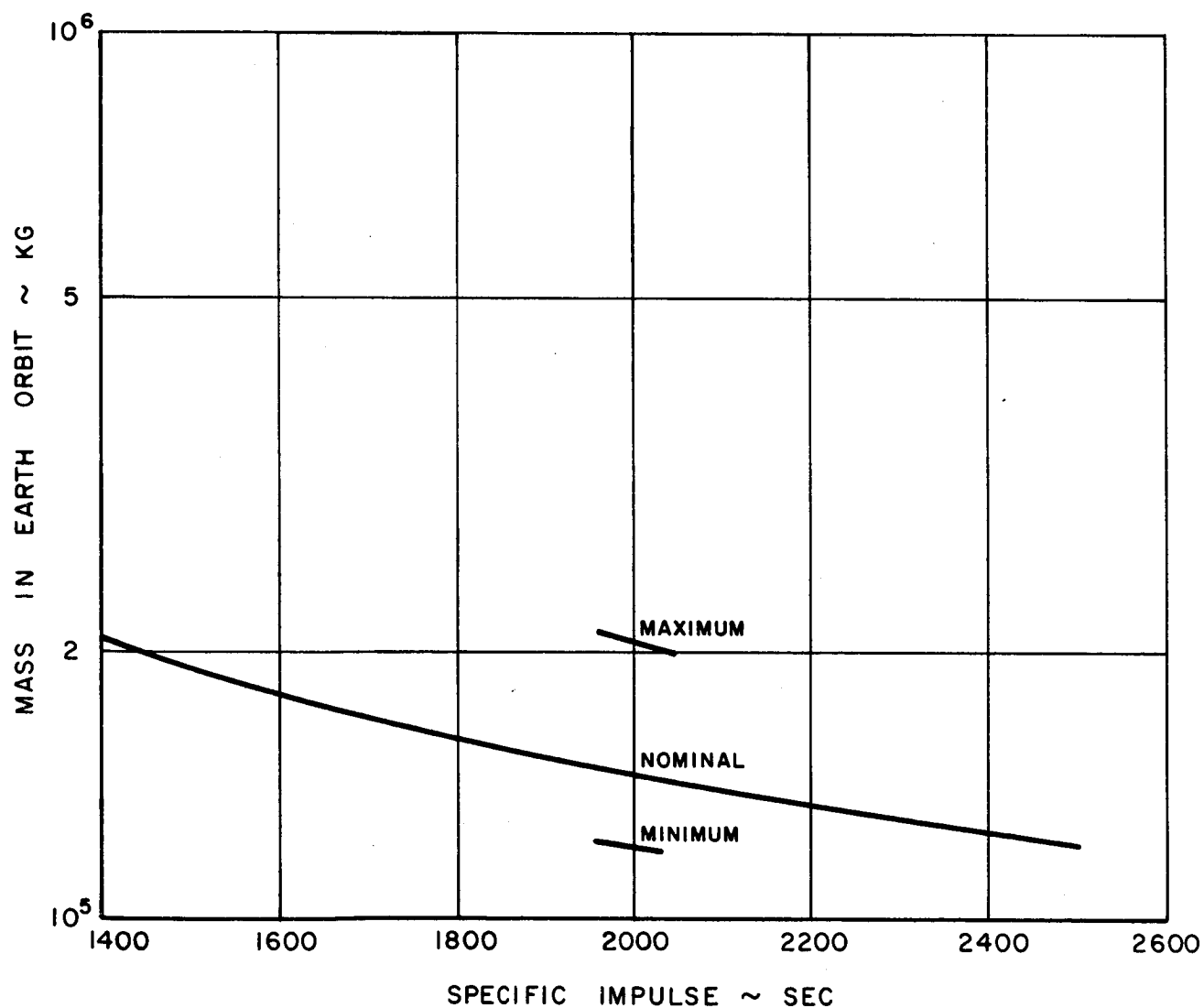


VENUS MANNED ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 470 DAYS

STAY TIME = 0

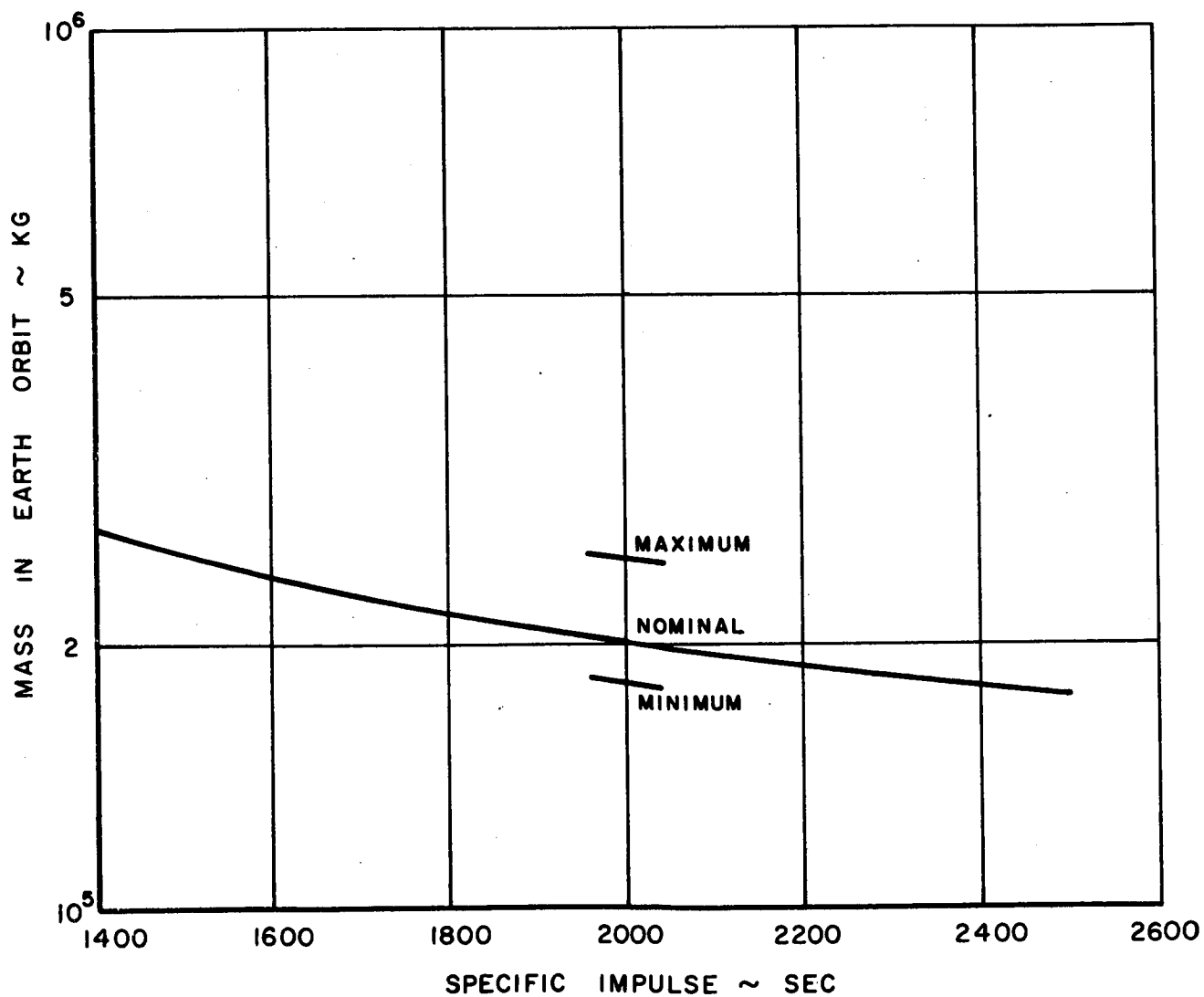


VENUS MANNED ORBITAL STOPOVERS

GASEOUS - CORE NUCLEAR

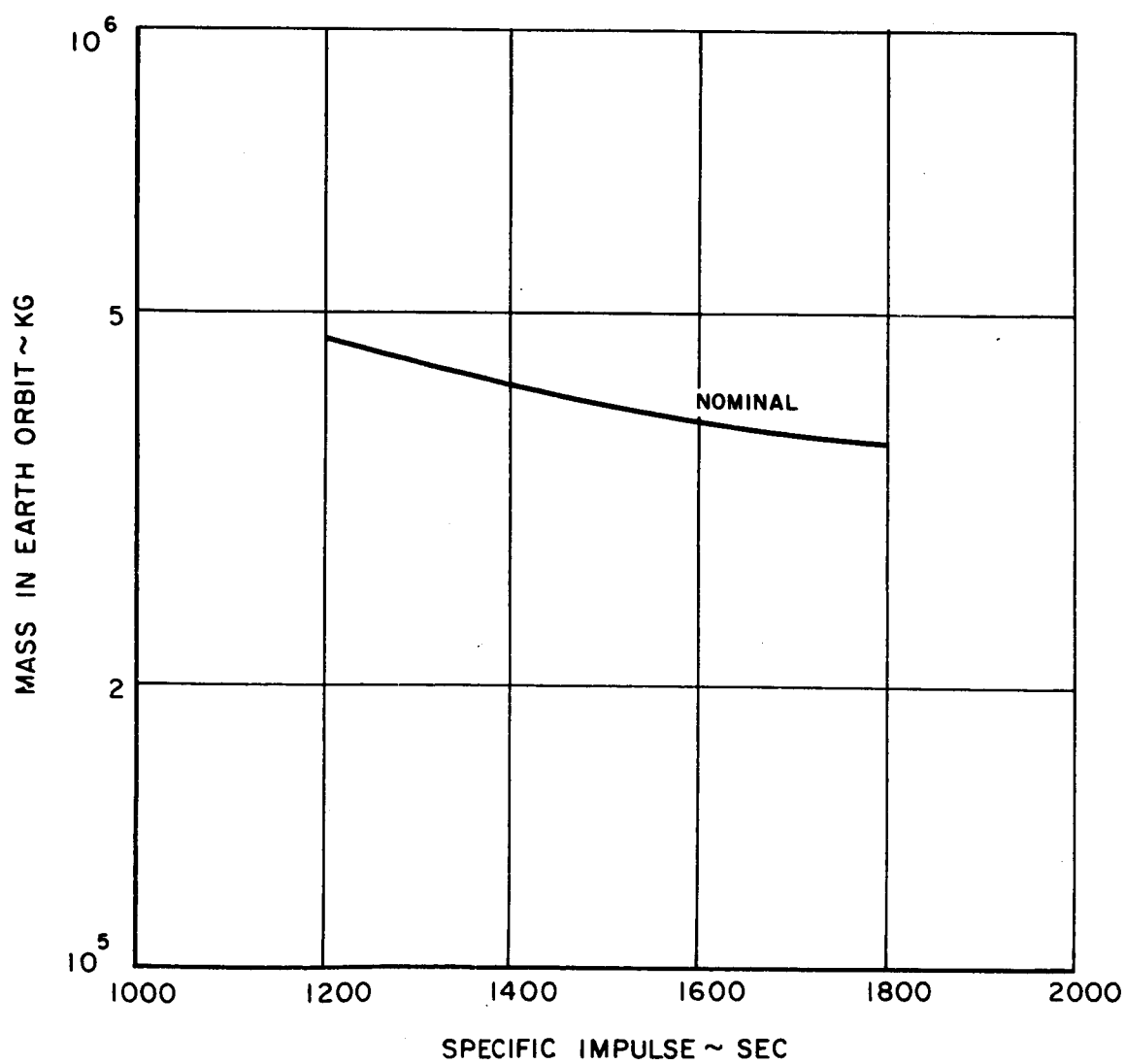
TRIP TIME = 450 DAYS

STAY TIME = 0



VENUS MANNED ORBITAL STOPOVERS

PULSED NUCLEAR

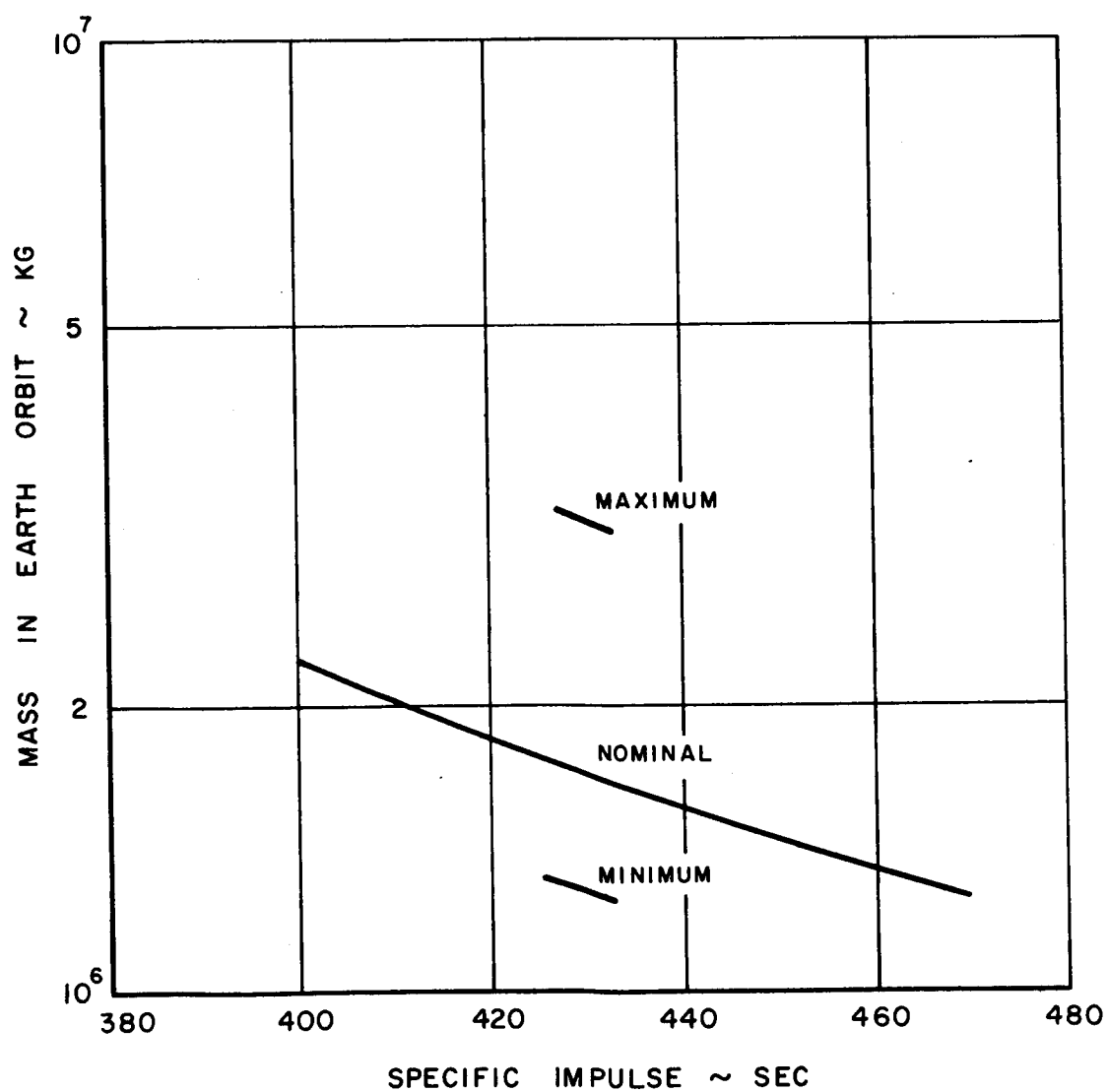
TRIP TIME = 450 DAYS
STAY TIME = 0

MARS MANNED ORBITAL STOPOVERS

CHEMICAL

TRIP TIME = 470 DAYS

STAY TIME = 0

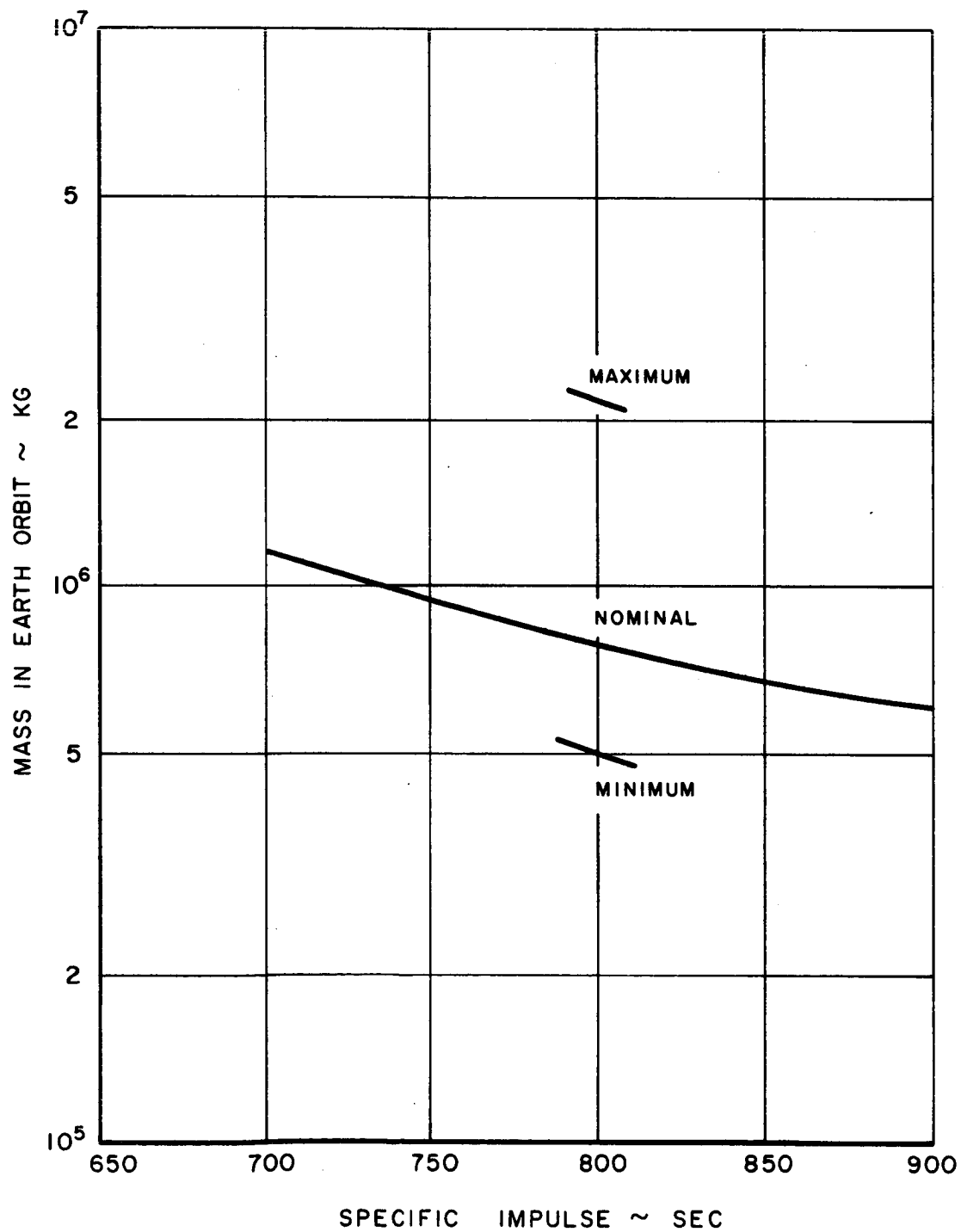


MARS MANNED ORBITAL STOPOVERS

SOLID - CORE NUCLEAR

TRIP TIME = 470 DAYS

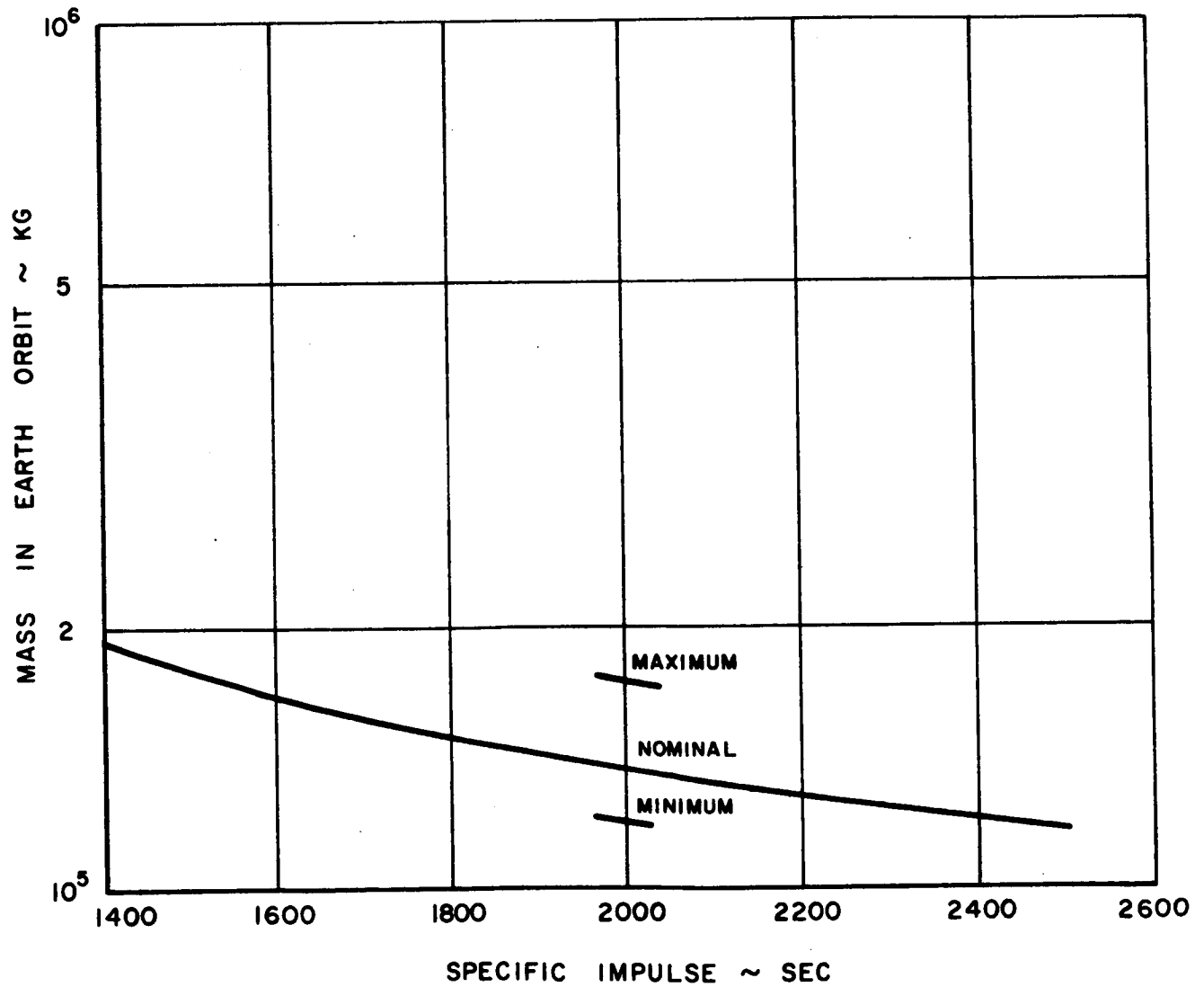
STAY TIME = 0



MARS MANNED ORBITAL STOPOVERS**LIGHTWEIGHT ADVANCED NUCLEAR**

TRIP TIME = 470 DAYS

STAY TIME = 0

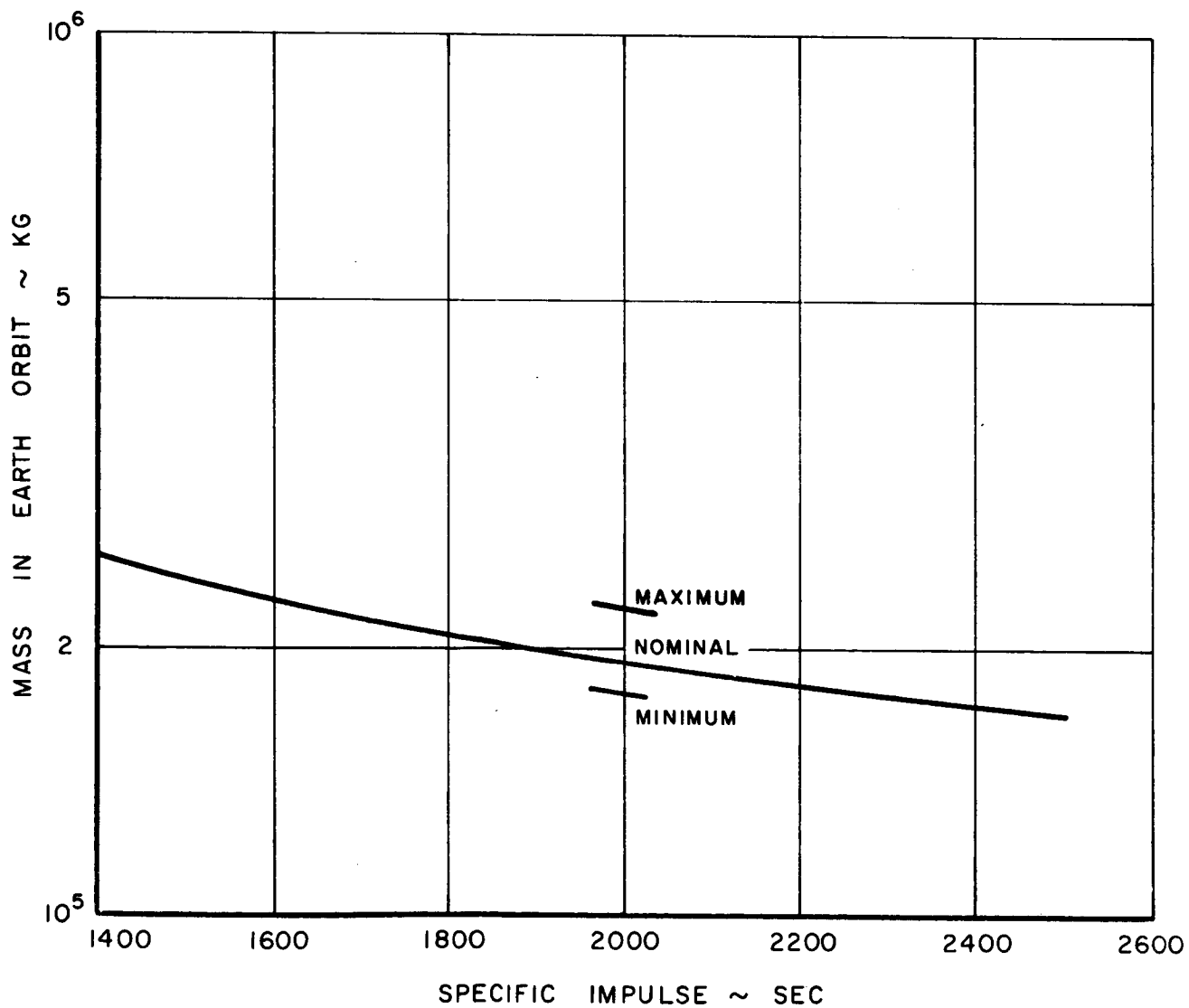


MARS MANNED ORBITAL STOPOVERS

GASEOUS - CORE NUCLEAR

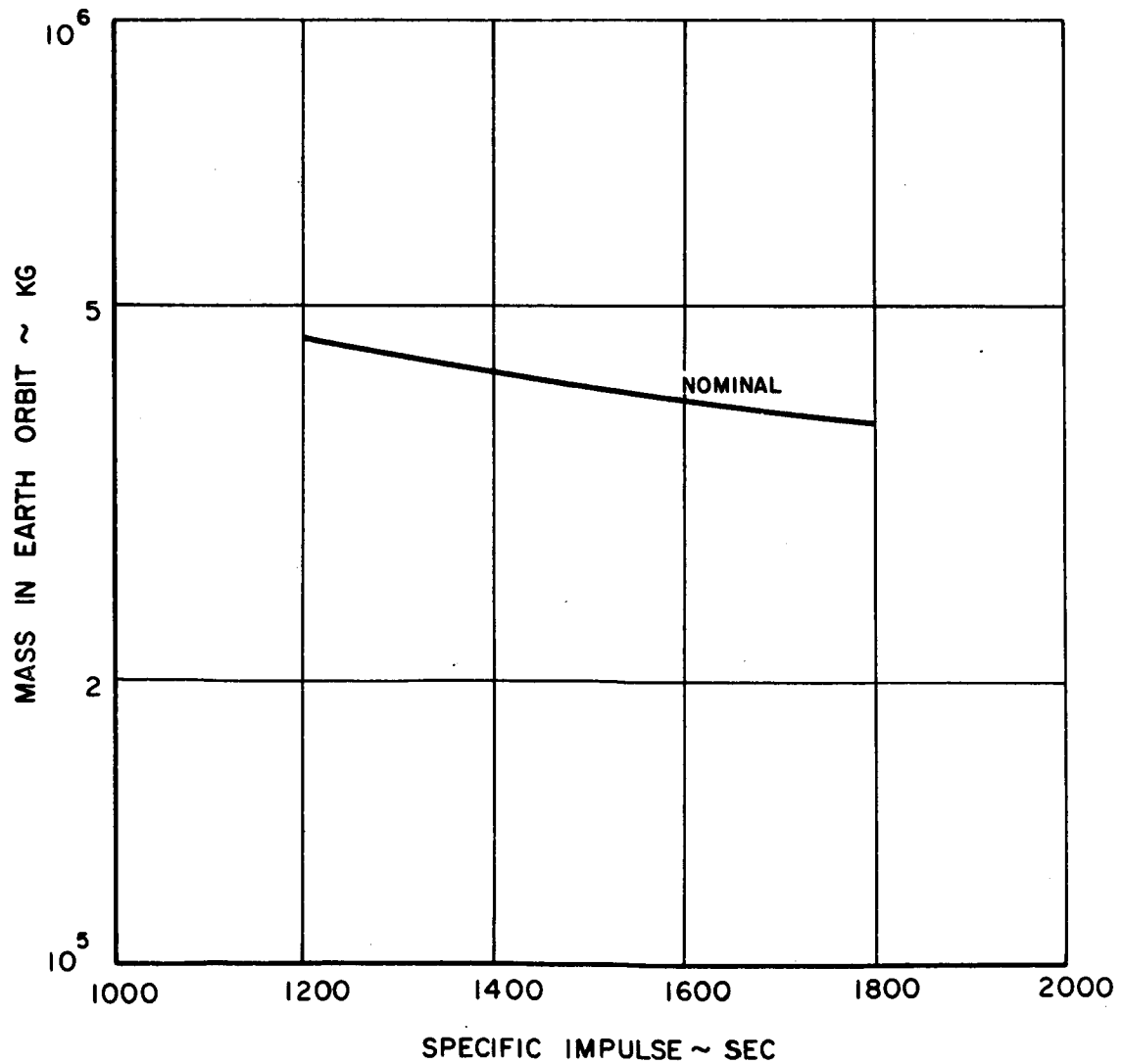
TRIP TIME = 470 DAYS

STAY TIME = 0



MARS MANNED ORBITAL STOPOVERS

PULSED NUCLEAR

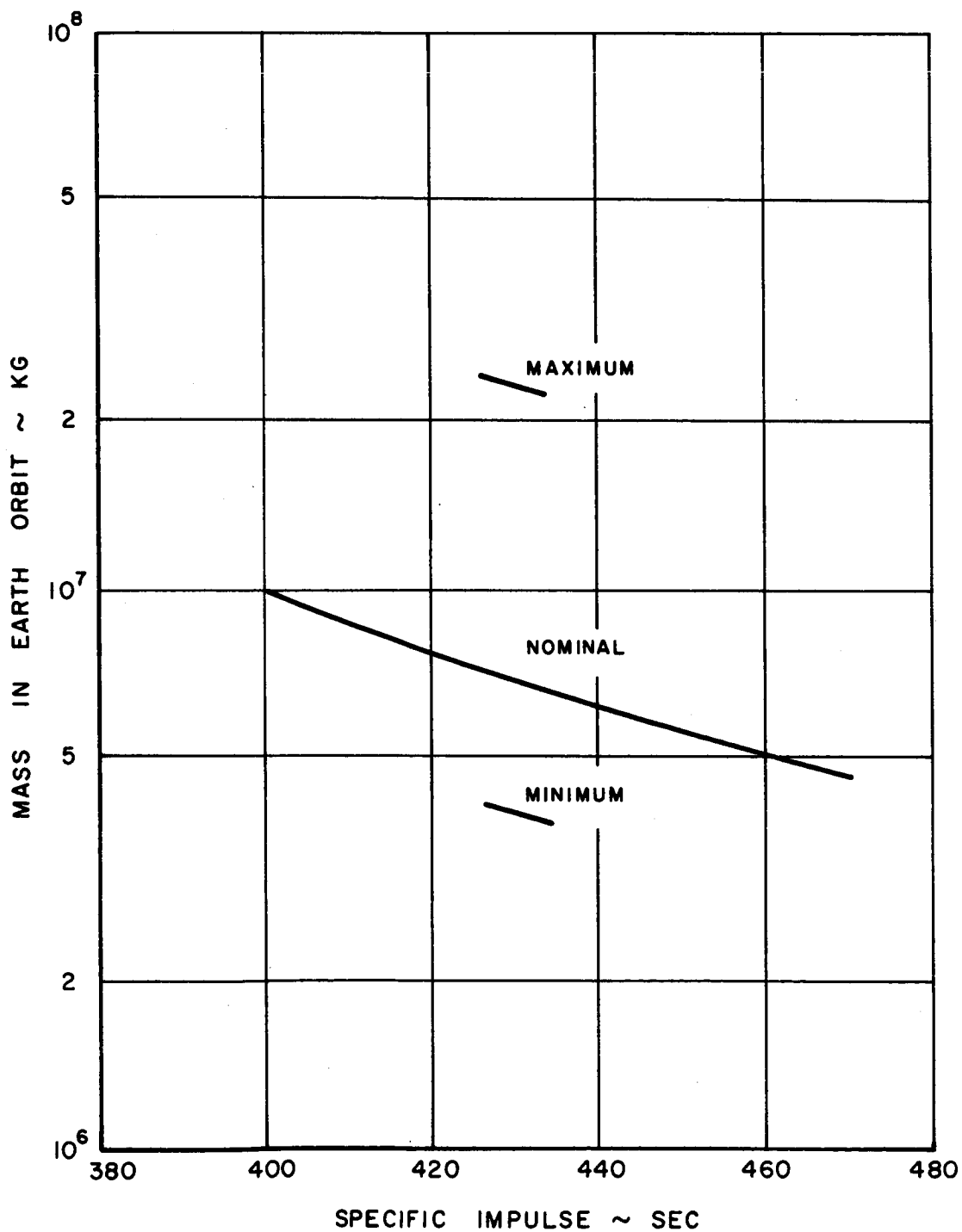
TRIP TIME = 470 DAYS
STAY TIME=0

GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

CHEMICAL

TRIP TIME = 1800 DAYS

STAY TIME = 0

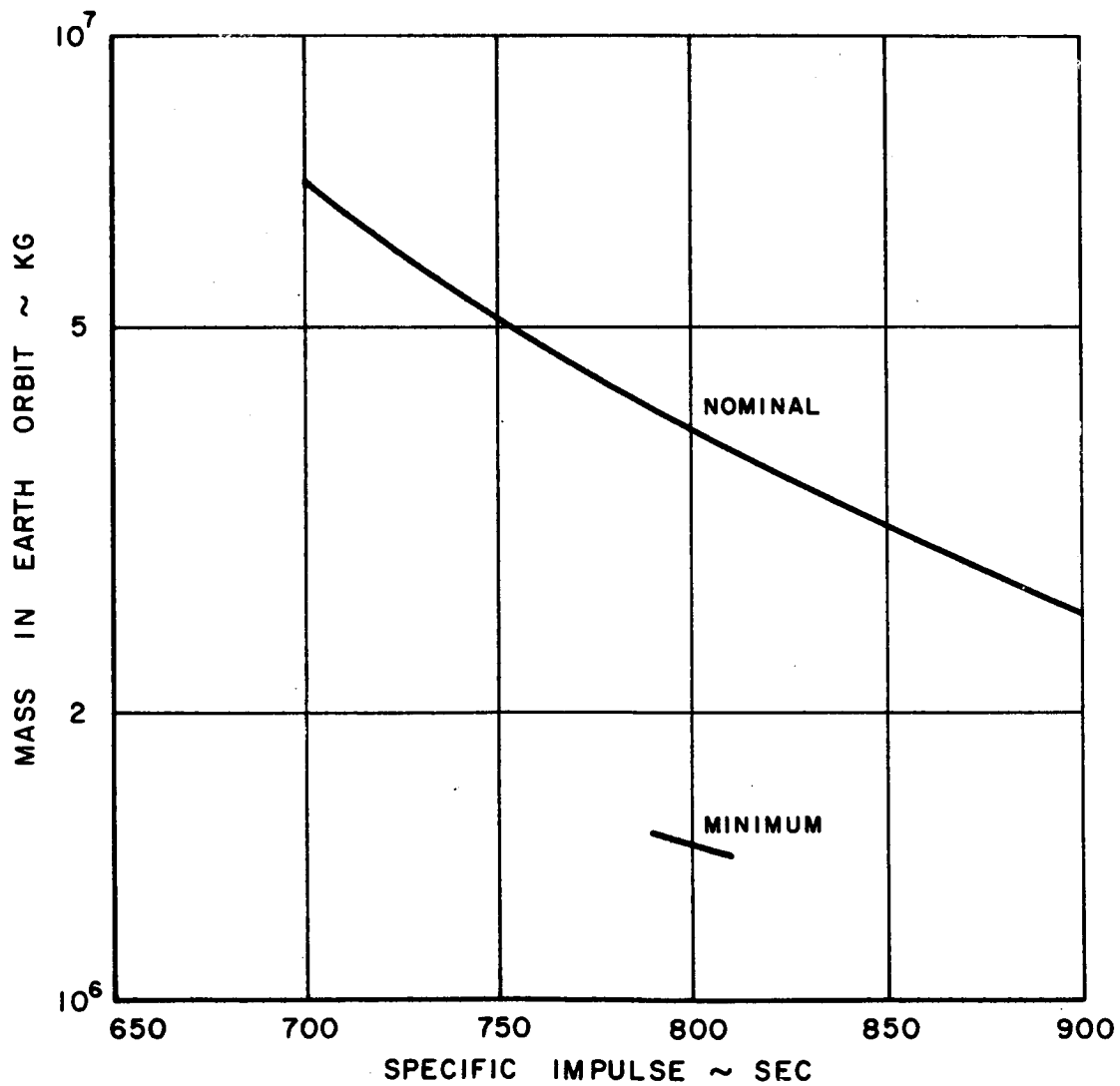


GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

SOLID - CORE NUCLEAR

TRIP TIME = 1800 DAYS

STAY TIME = 0

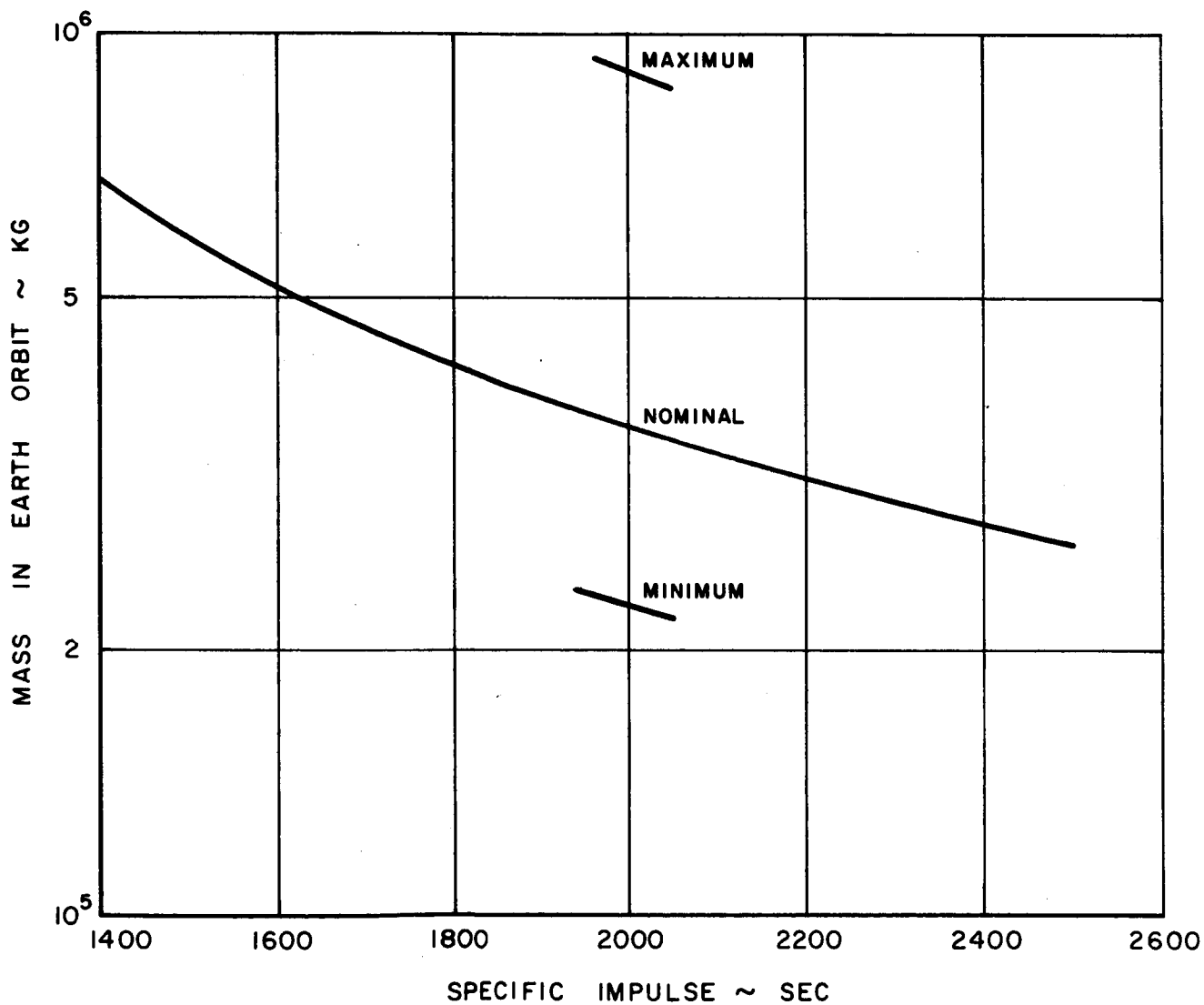


GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 1800 DAYS

STAY TIME = 0

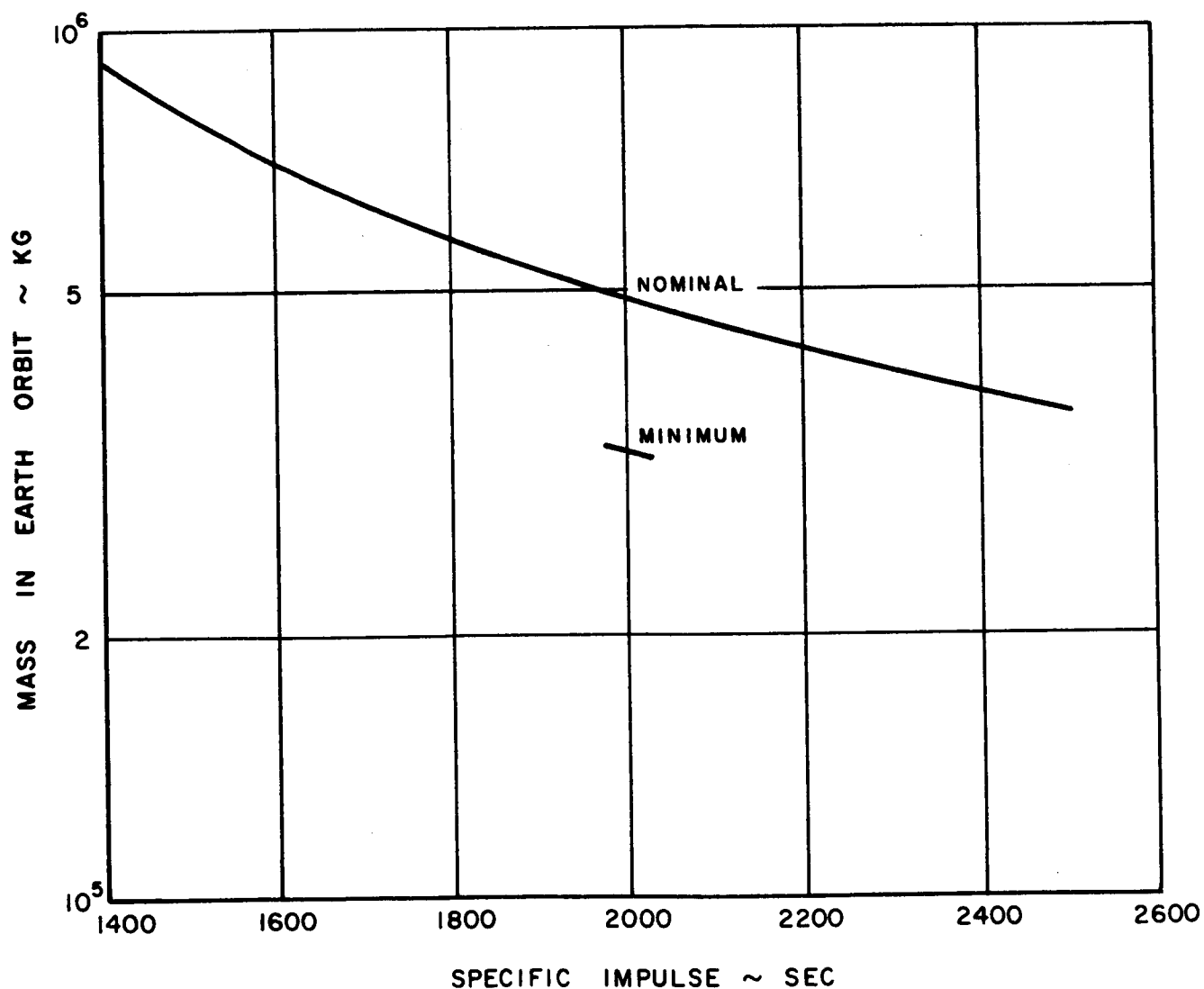


GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

GASEOUS - CORE NUCLEAR

TRIP TIME = 1800 DAYS

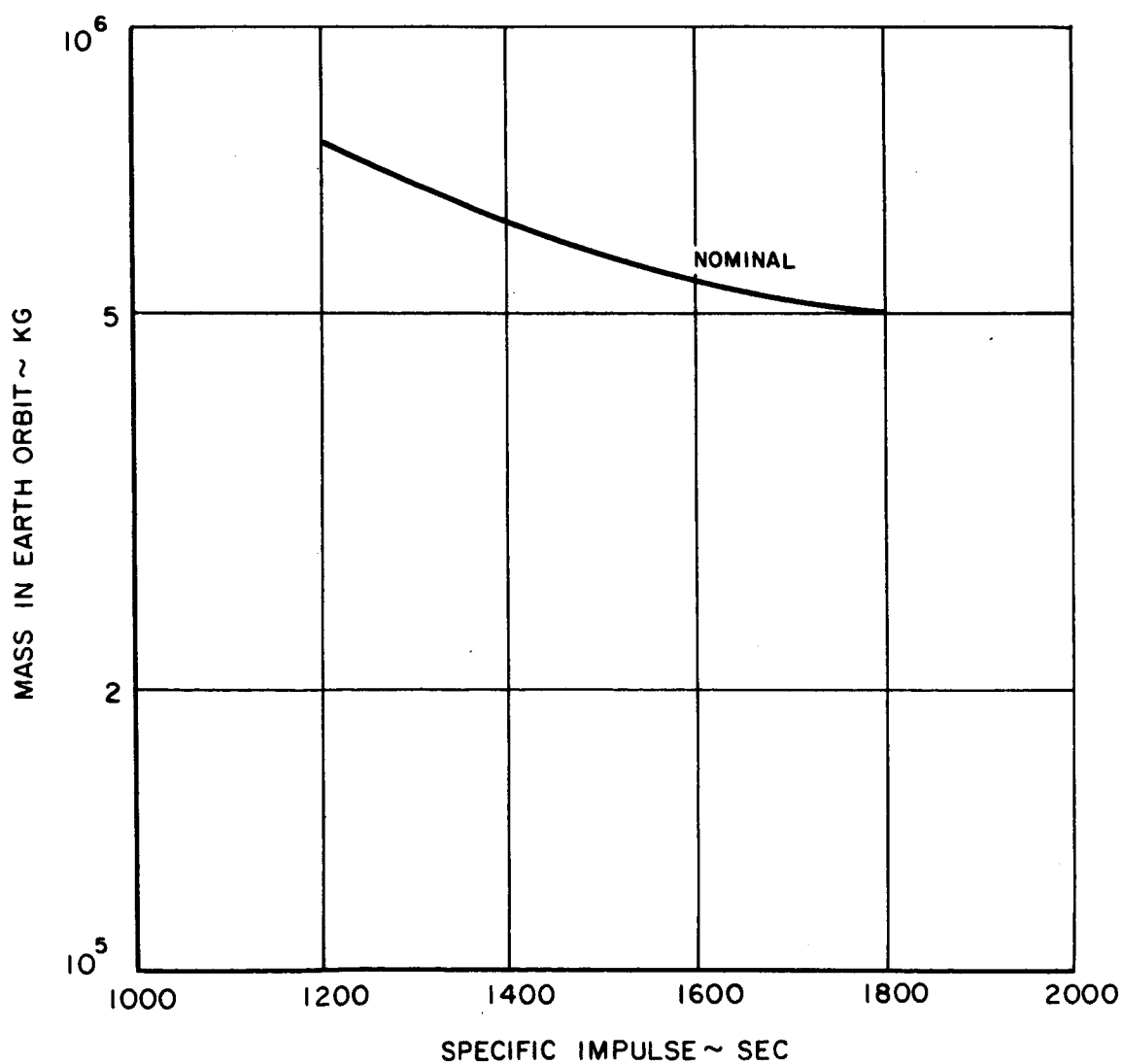
STAY TIME = 0



GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

PULSED NUCLEAR

TRIP TIME = 1800 DAYS
STAY TIME = 0

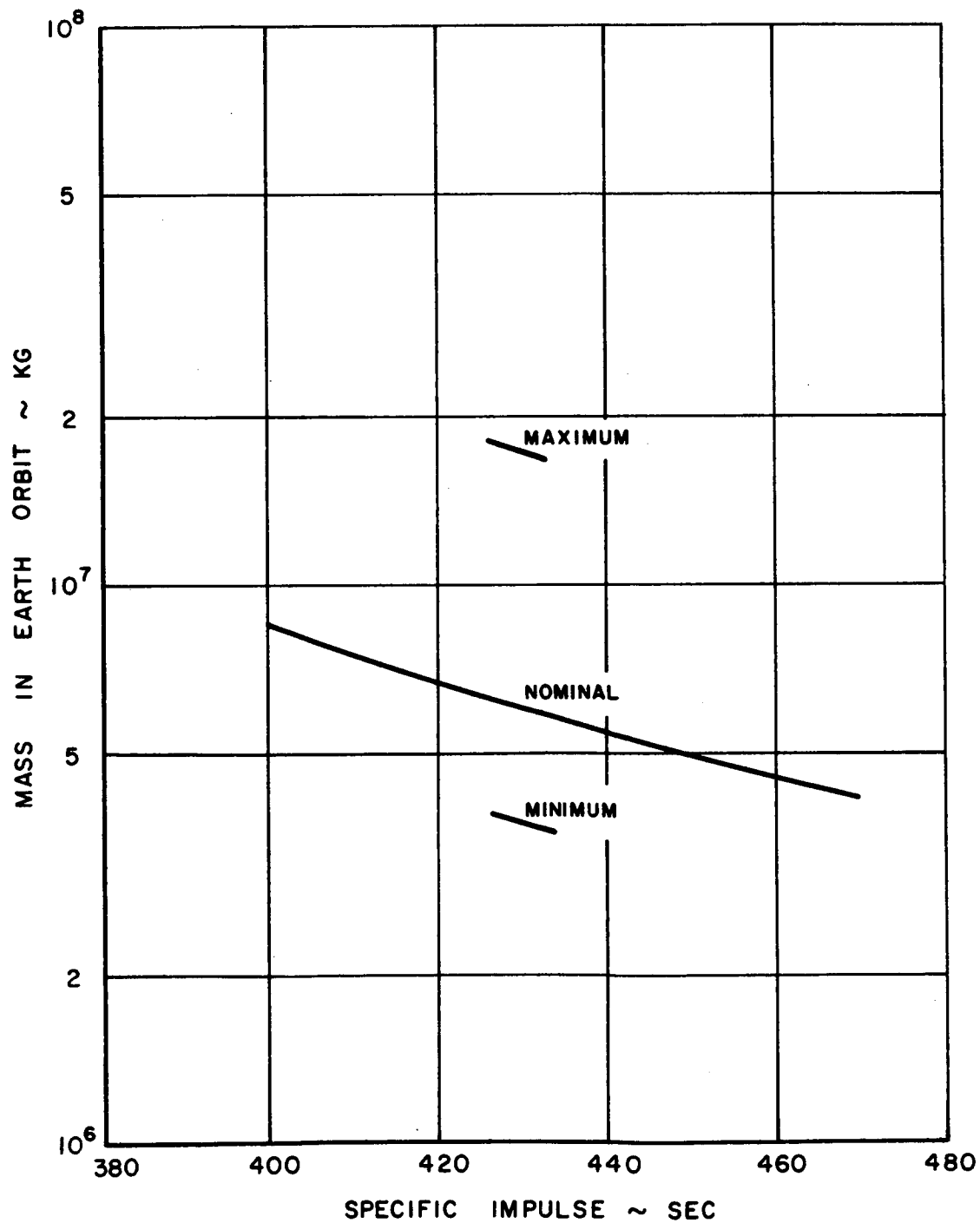


TITAN MANNED ORBITAL STOPOVERS (SATURN VI)

CHEMICAL

TRIP TIME = 4000 DAYS

STAY TIME = 0

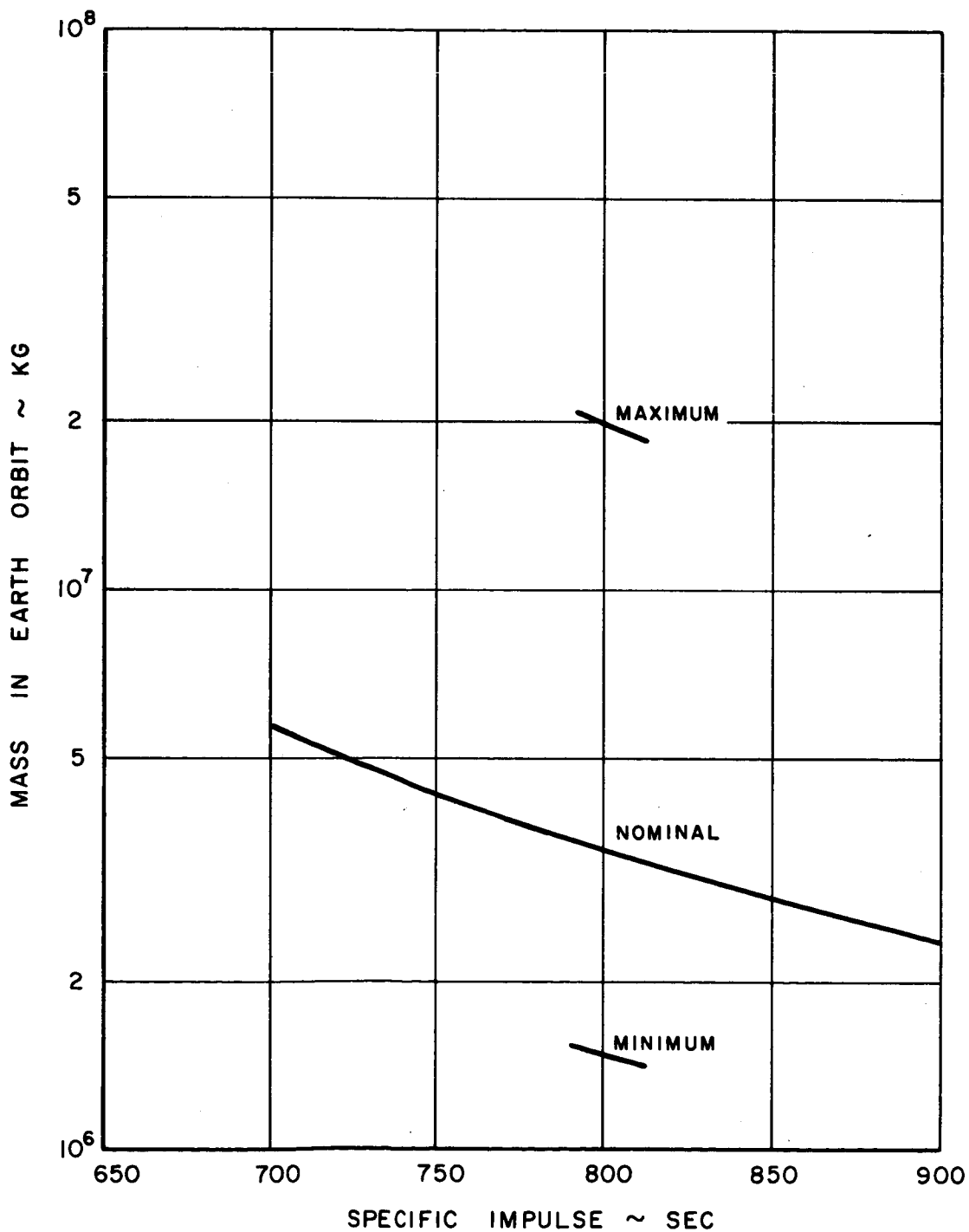


TITAN MANNED ORBITAL STOPOVERS (SATURN VI)

SOLID - CORE NUCLEAR

TRIP TIME = 4000 DAYS

STAY TIME = 0

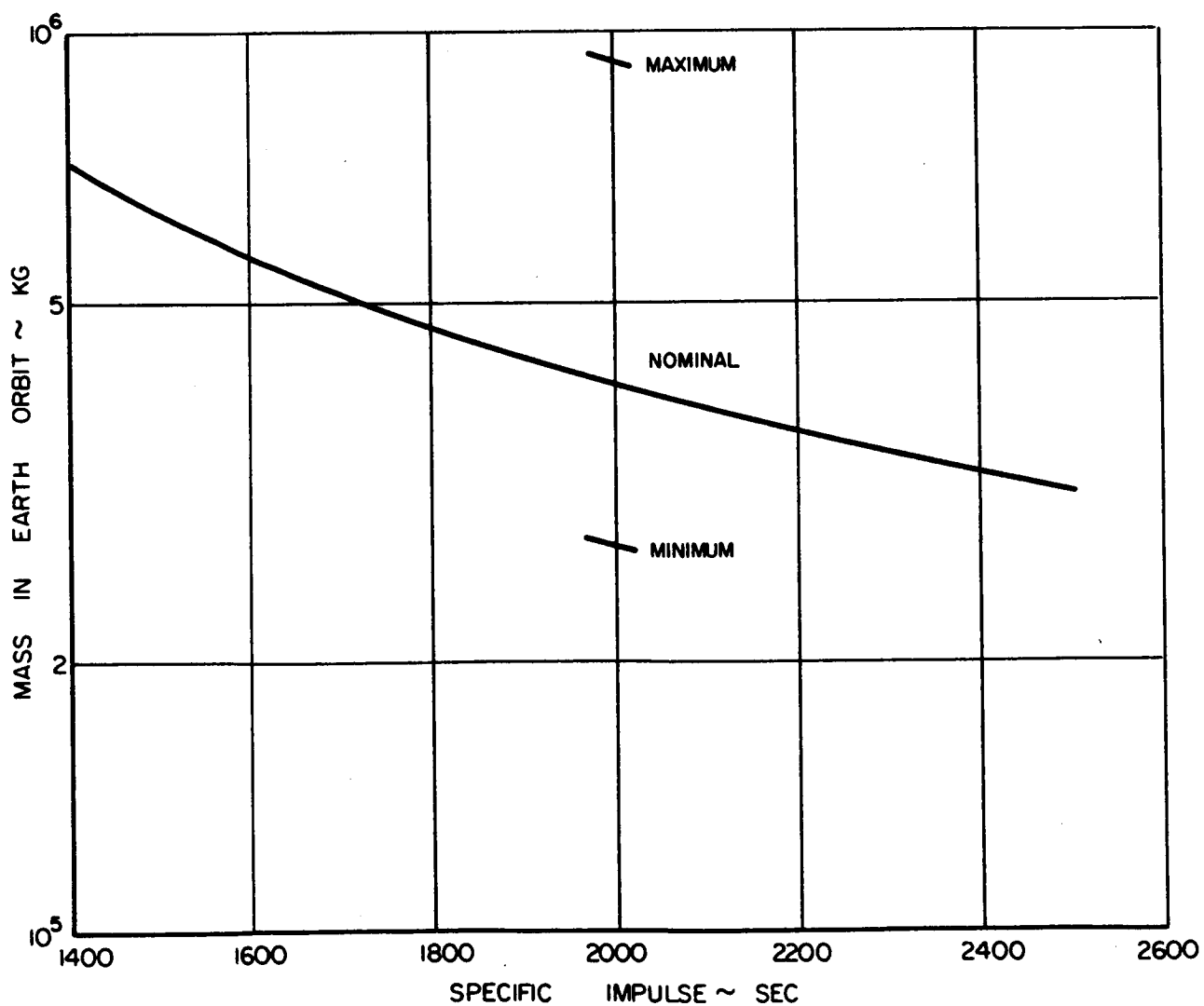


TITAN MANNED ORBITAL STOPOVERS (SATURN VI)

LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 4000 DAYS

STAY TIME = 0

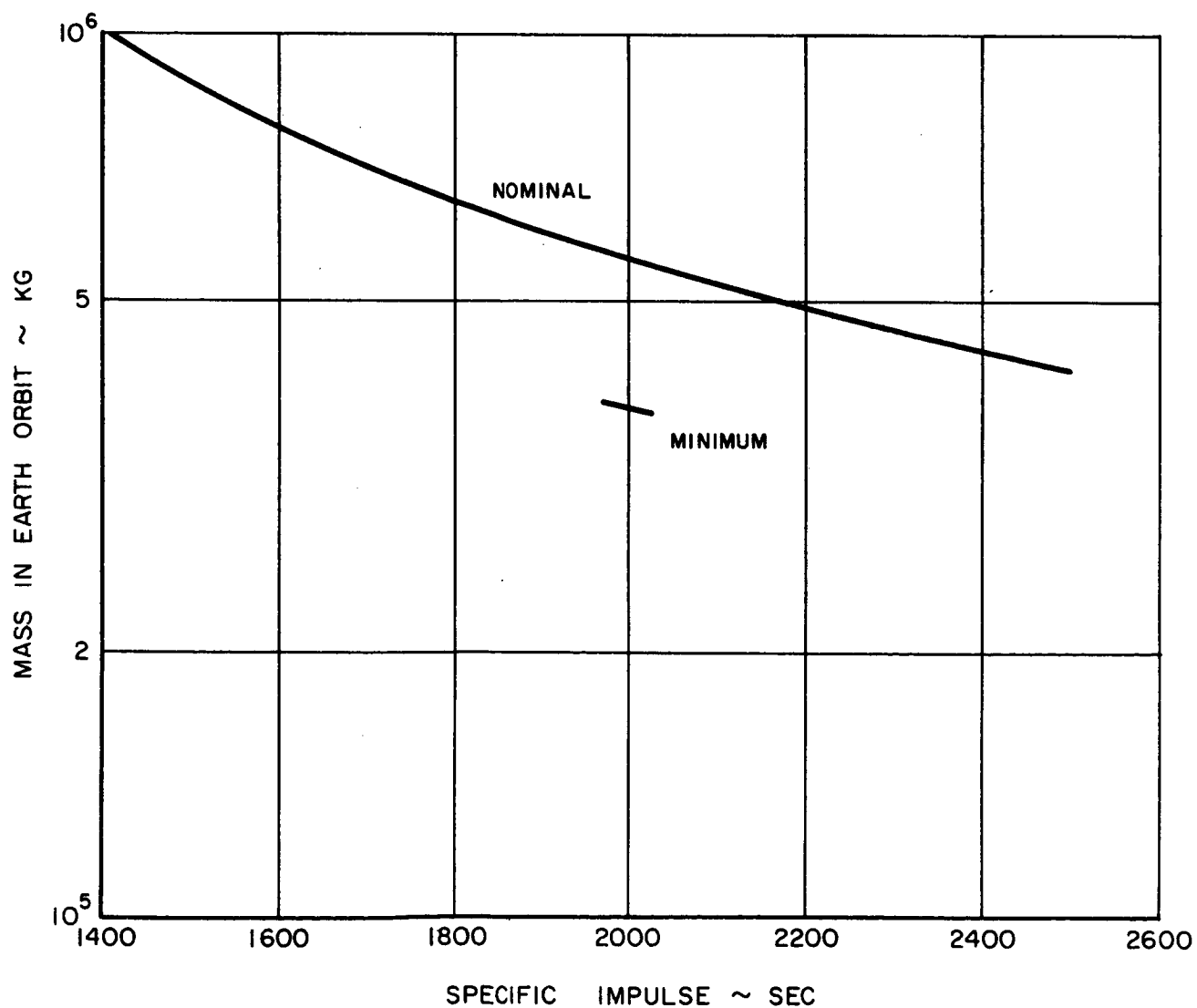


TITAN MANNED ORBITAL STOPOVERS (SATURN VI)

GASEOUS-CORE NUCLEAR

TRIP TIME = 4000 DAYS

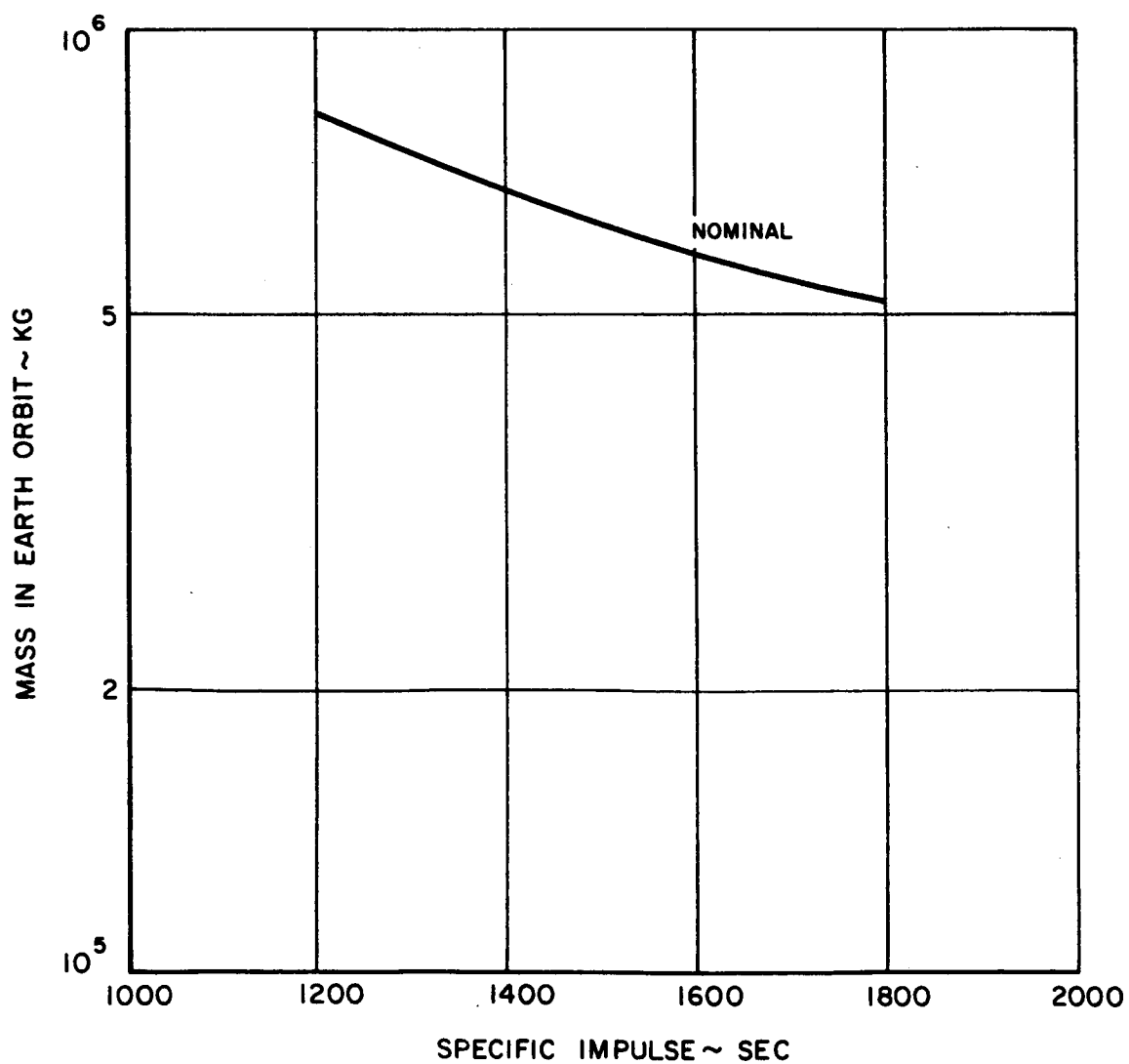
STAY TIME = 0



TITAN MANNED ORBITAL STOPOVERS (SATURN VI)

PULSED NUCLEAR

TRIP TIME = 4000 DAYS
STAY TIME = 0

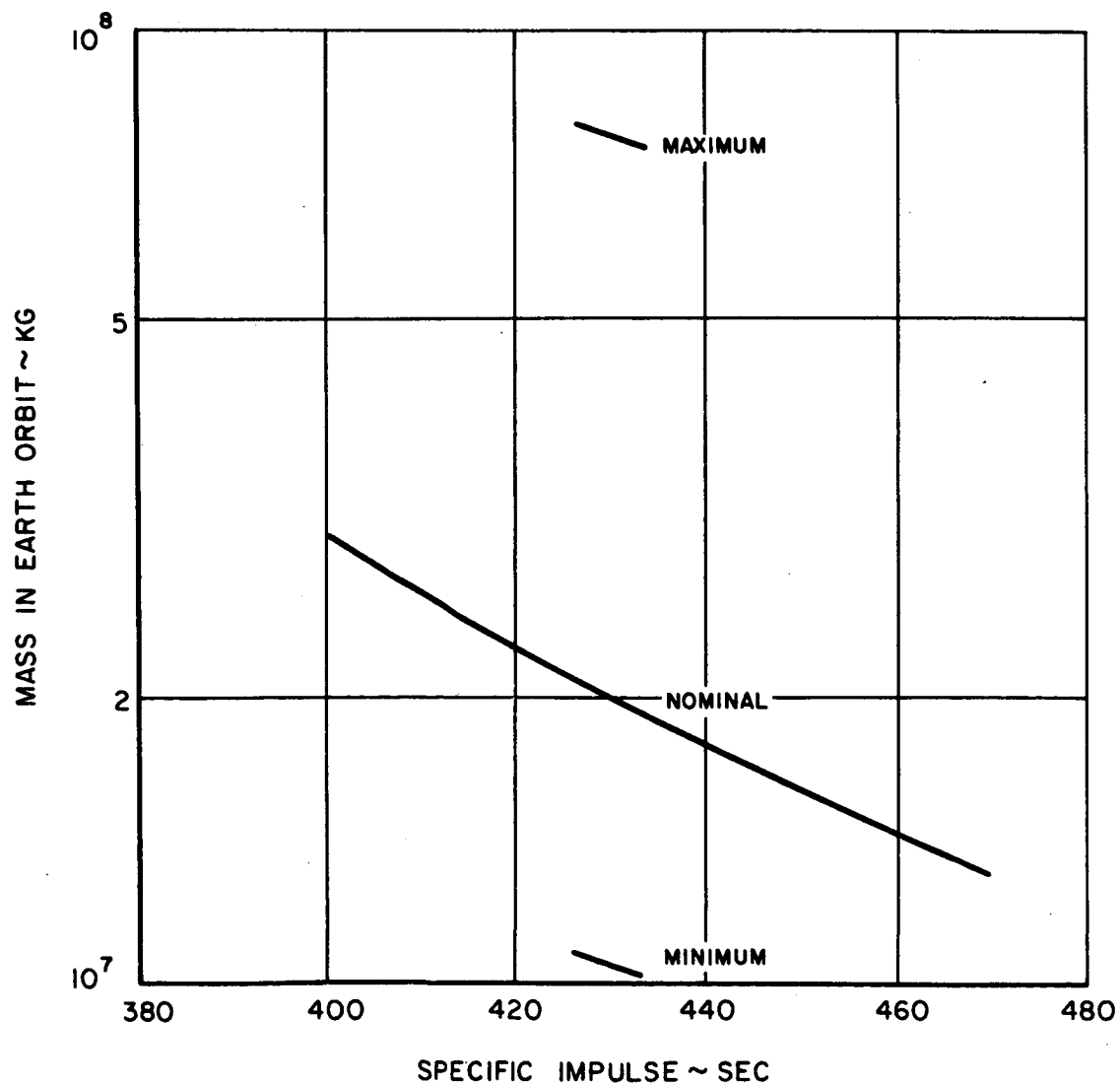


URANUS MANNED ORBITAL STOPOVERS

CHEMICAL

TRIP TIME = 12030 DAYS

STAY TIME = 0

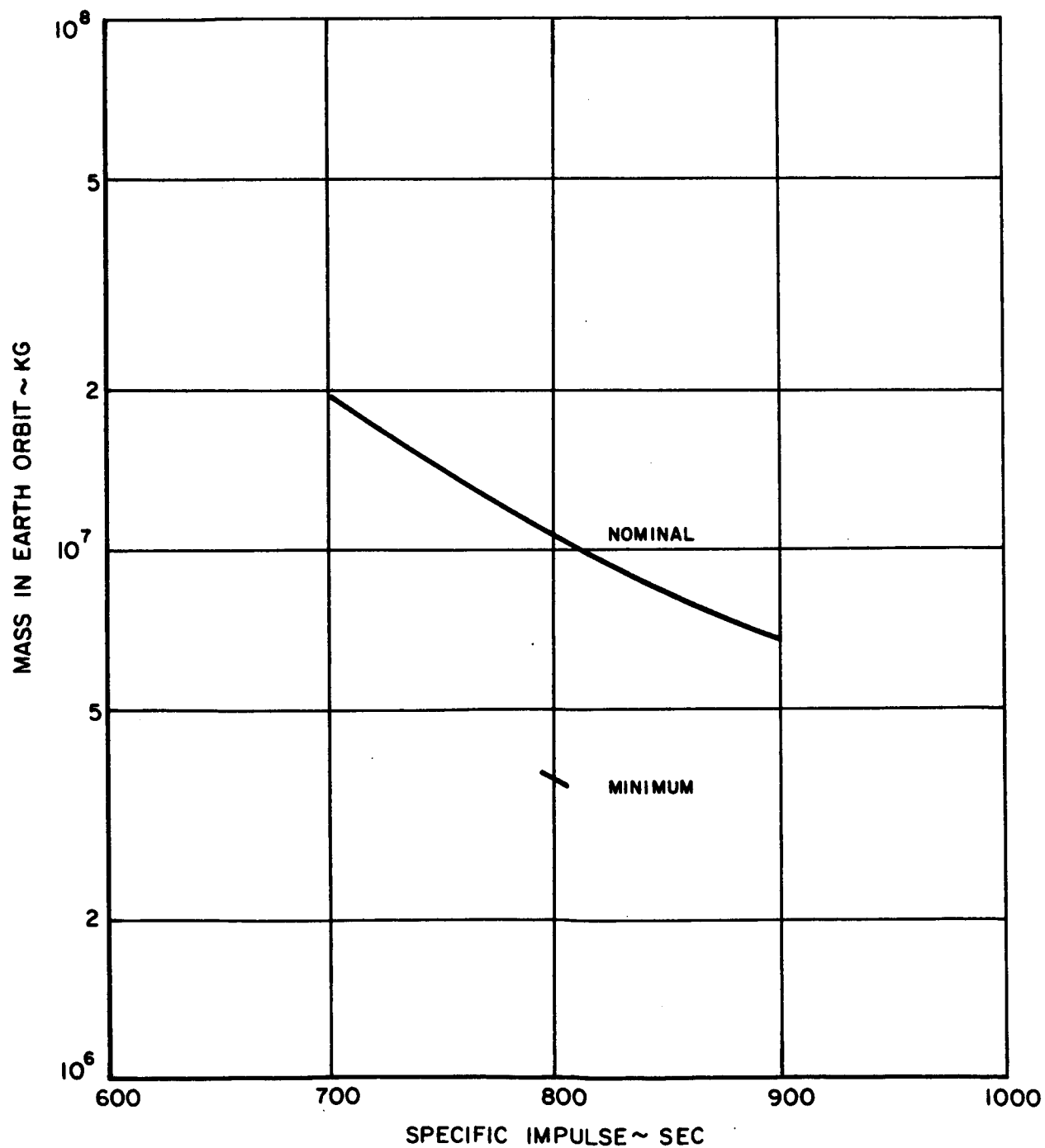


URANUS MANNED ORBITAL STOPOVERS

SOLID-CORE NUCLEAR

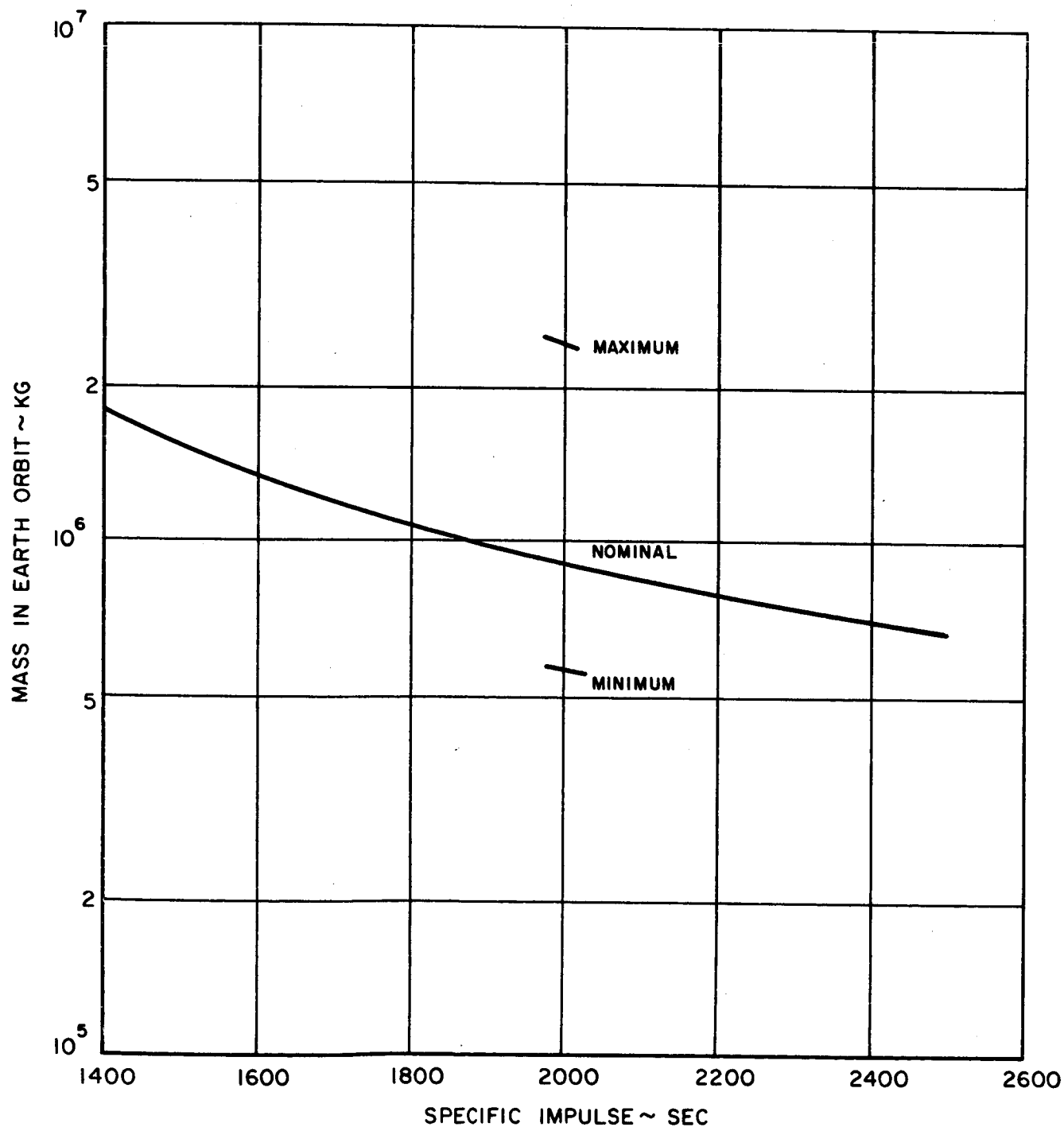
TRIP TIME = 12030 DAYS

STAY TIME = 0



URANUS MANNED ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR

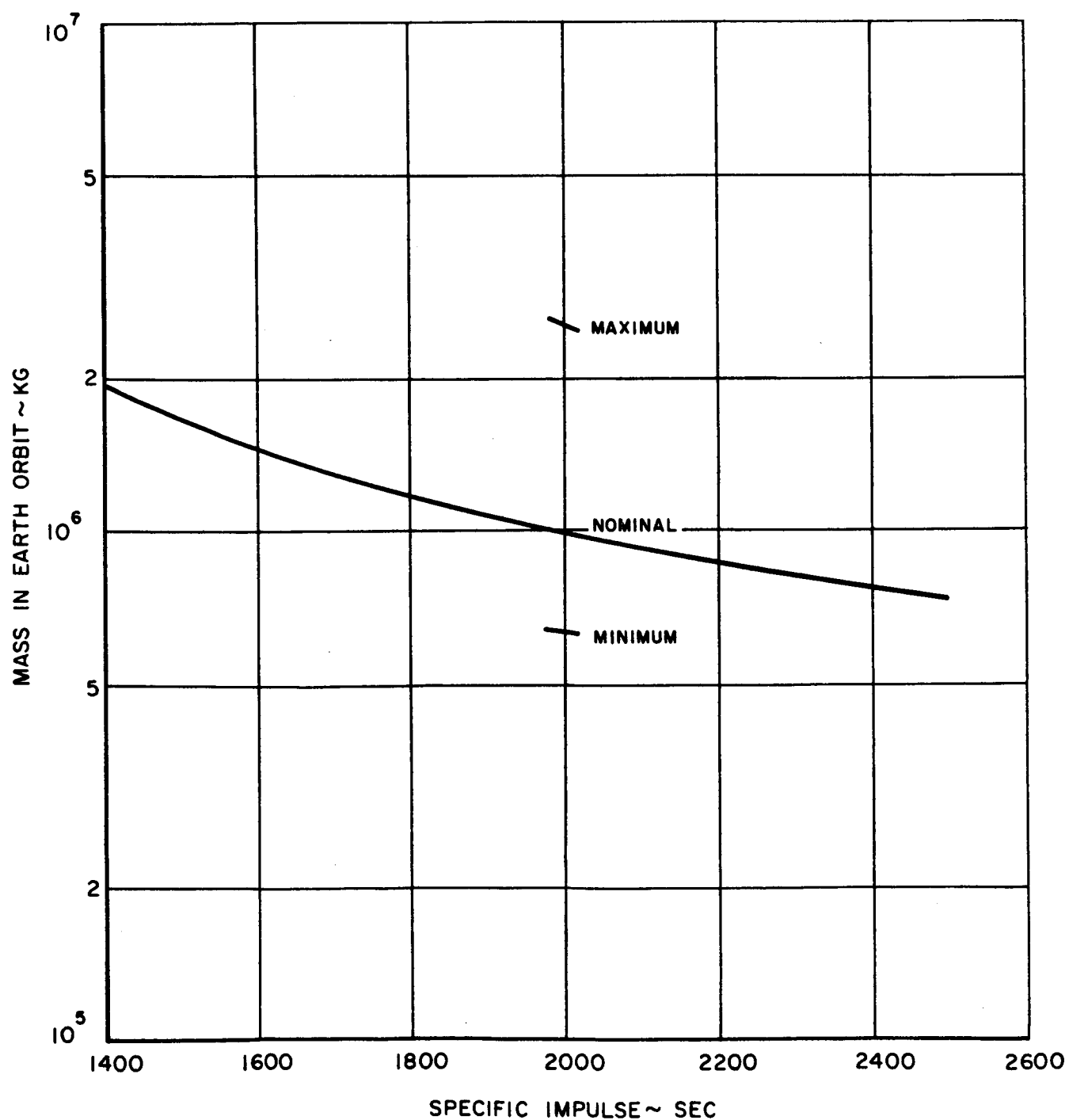
TRIP TIME = 12030 DAYS
STAY TIME = 0

URANUS MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR

TRIP TIME = 12030 DAYS

STAY TIME = 0

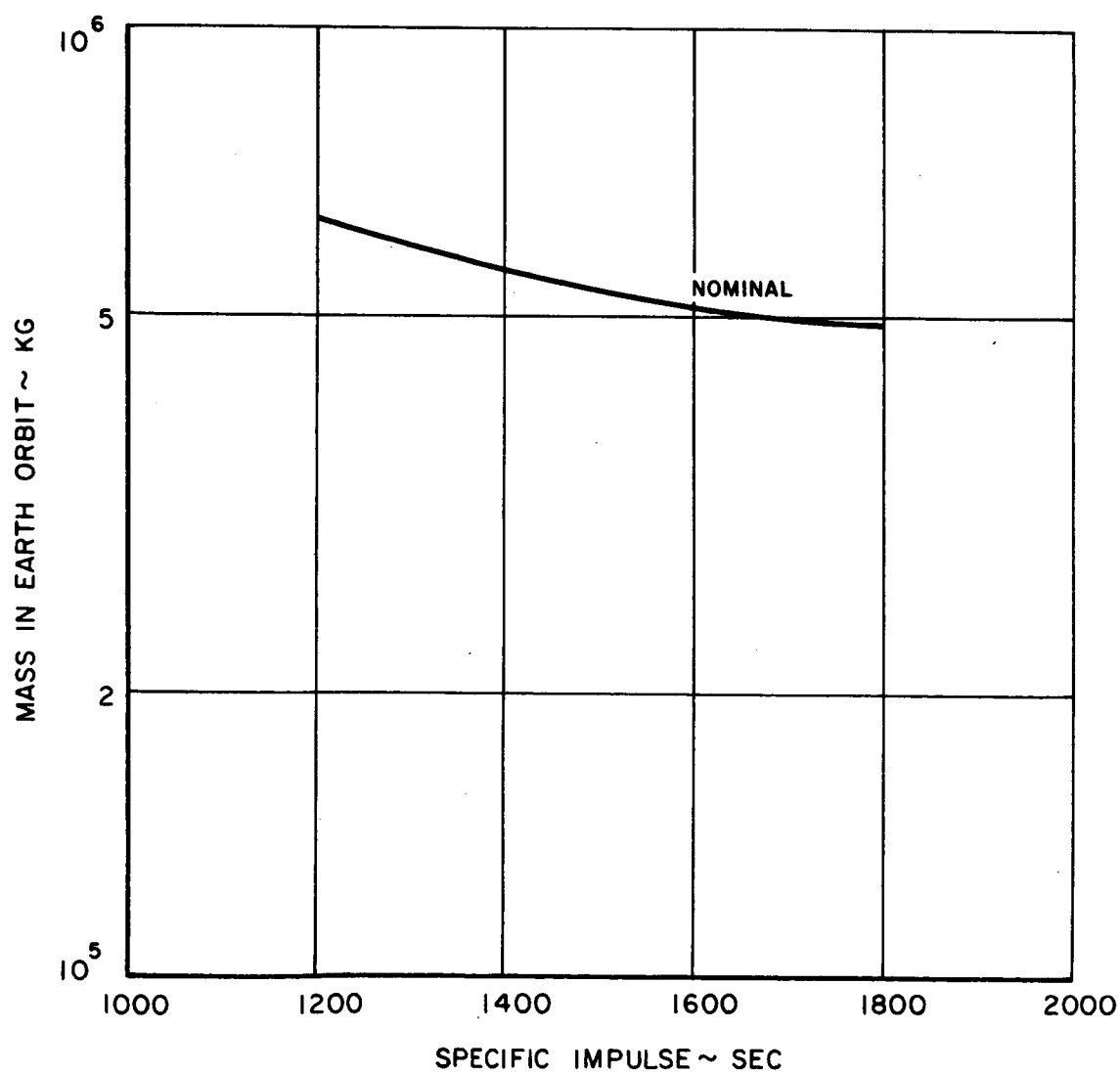


URANUS MANNED ORBITAL STOPOVERS

PULSED NUCLEAR

TRIP TIME = 12030 DAYS

STAY TIME = 0

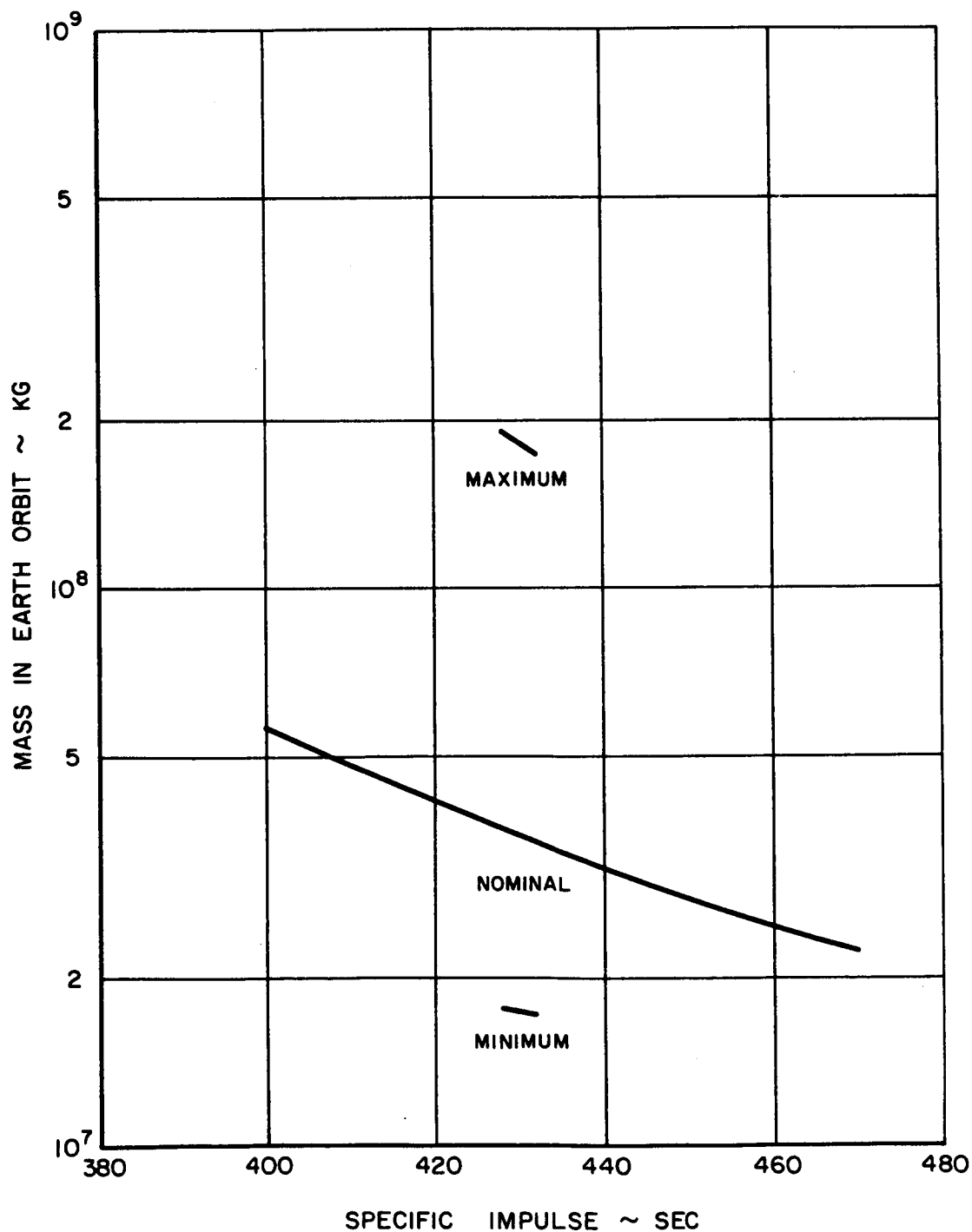


NEPTUNE MANNED ORBITAL STOPOVERS

CHEMICAL

TRIP TIME = 13,840 DAYS

STAY TIME = 0

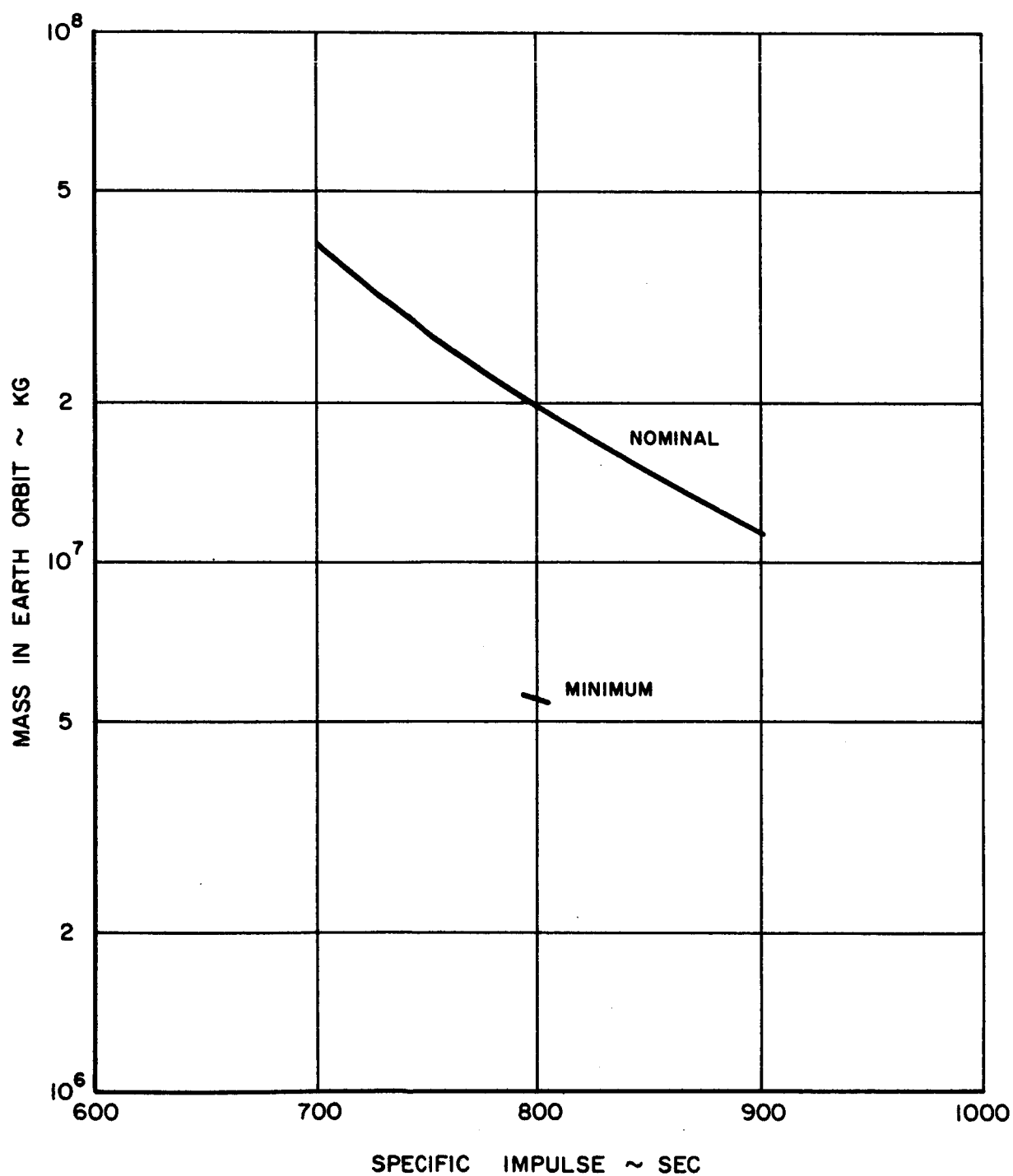


NEPTUNE MANNED ORBITAL STOPOVERS

SOLID-CORE NUCLEAR

TRIP TIME = 13,840 DAYS

STAY TIME = 0

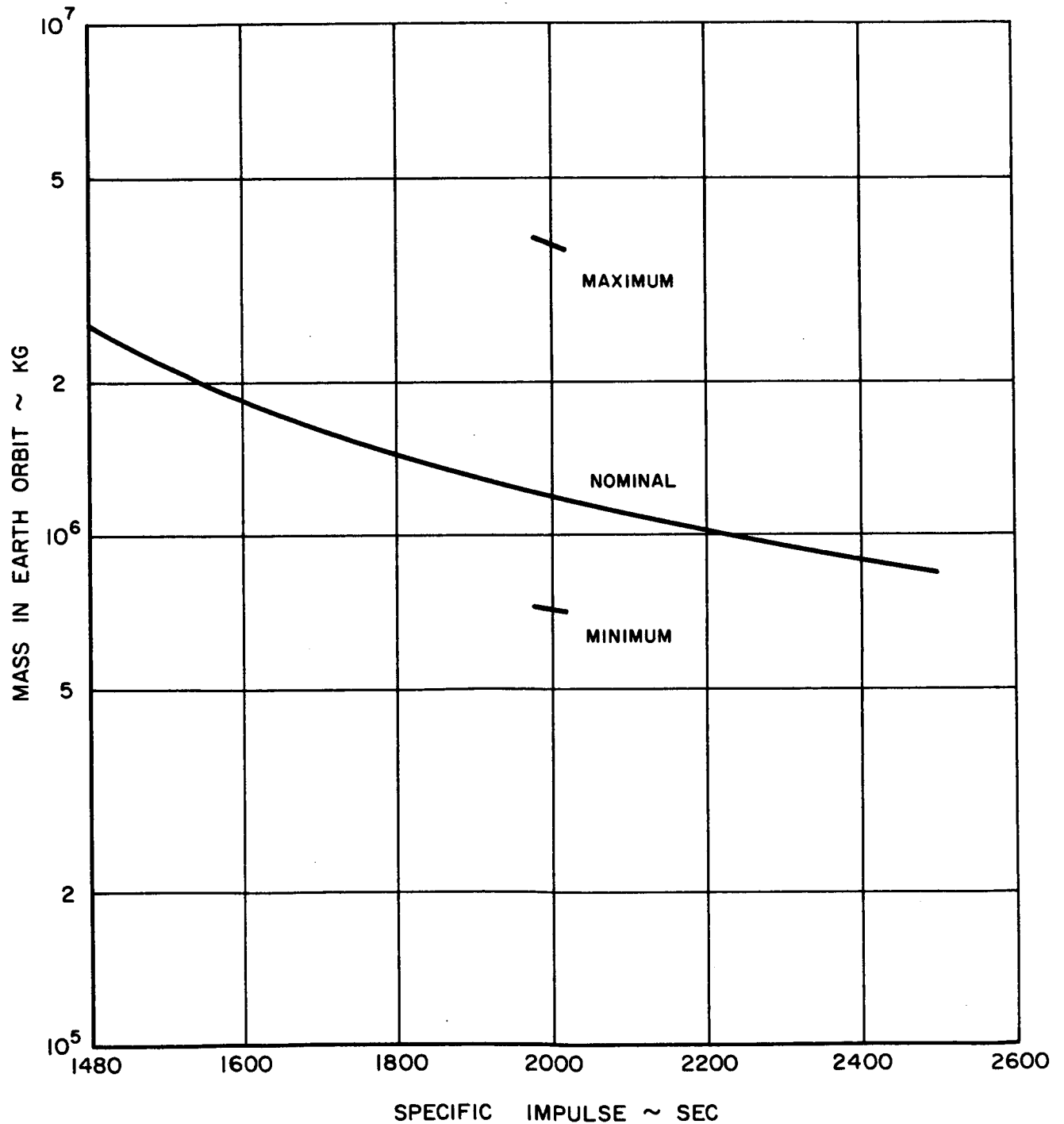


NEPTUNE MANNED ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 13,840 DAYS

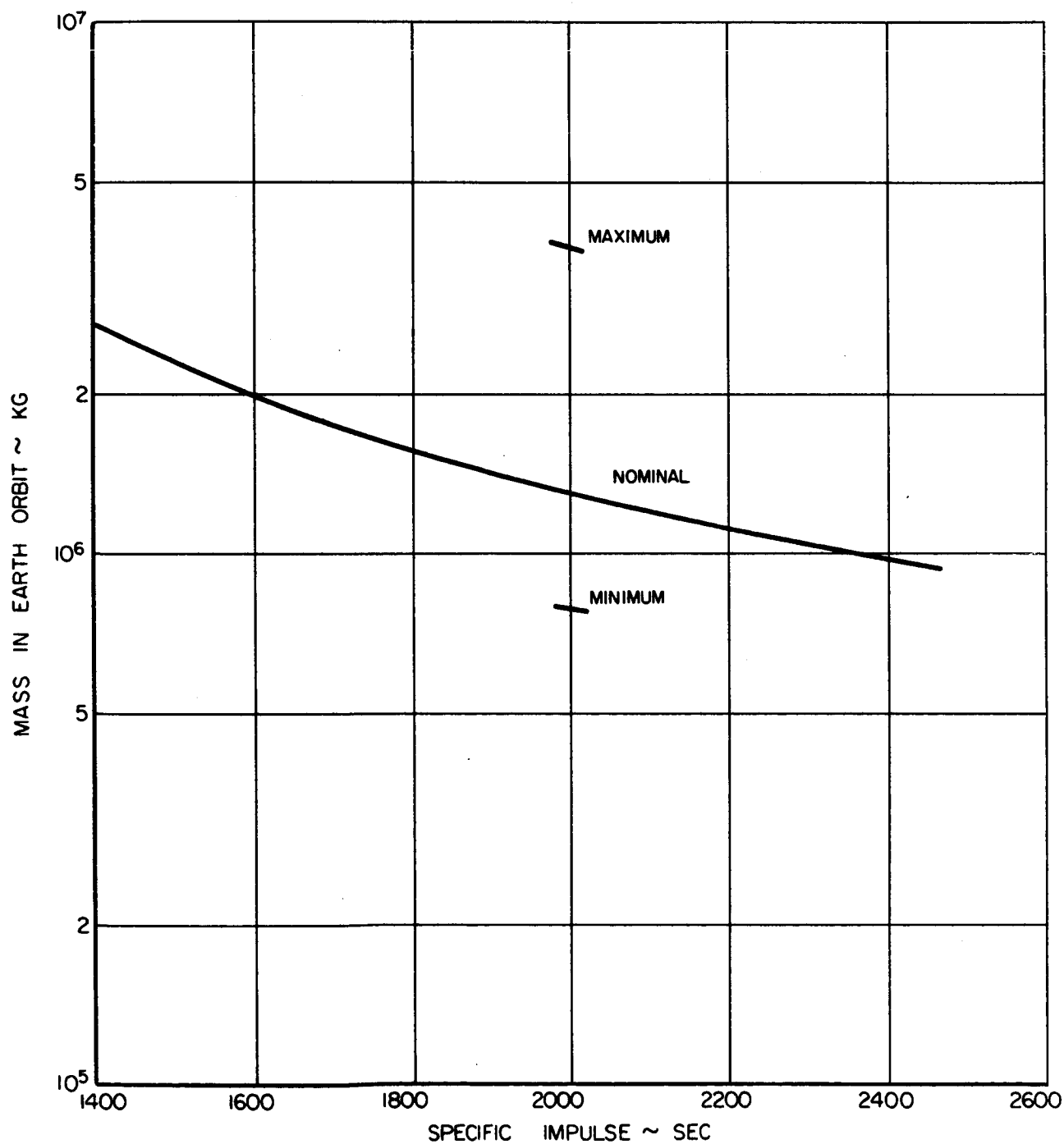
STAY TIME = 0



NEPTUNE MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR

TRIP TIME = 13840 DAYS
STAY TIME = 0

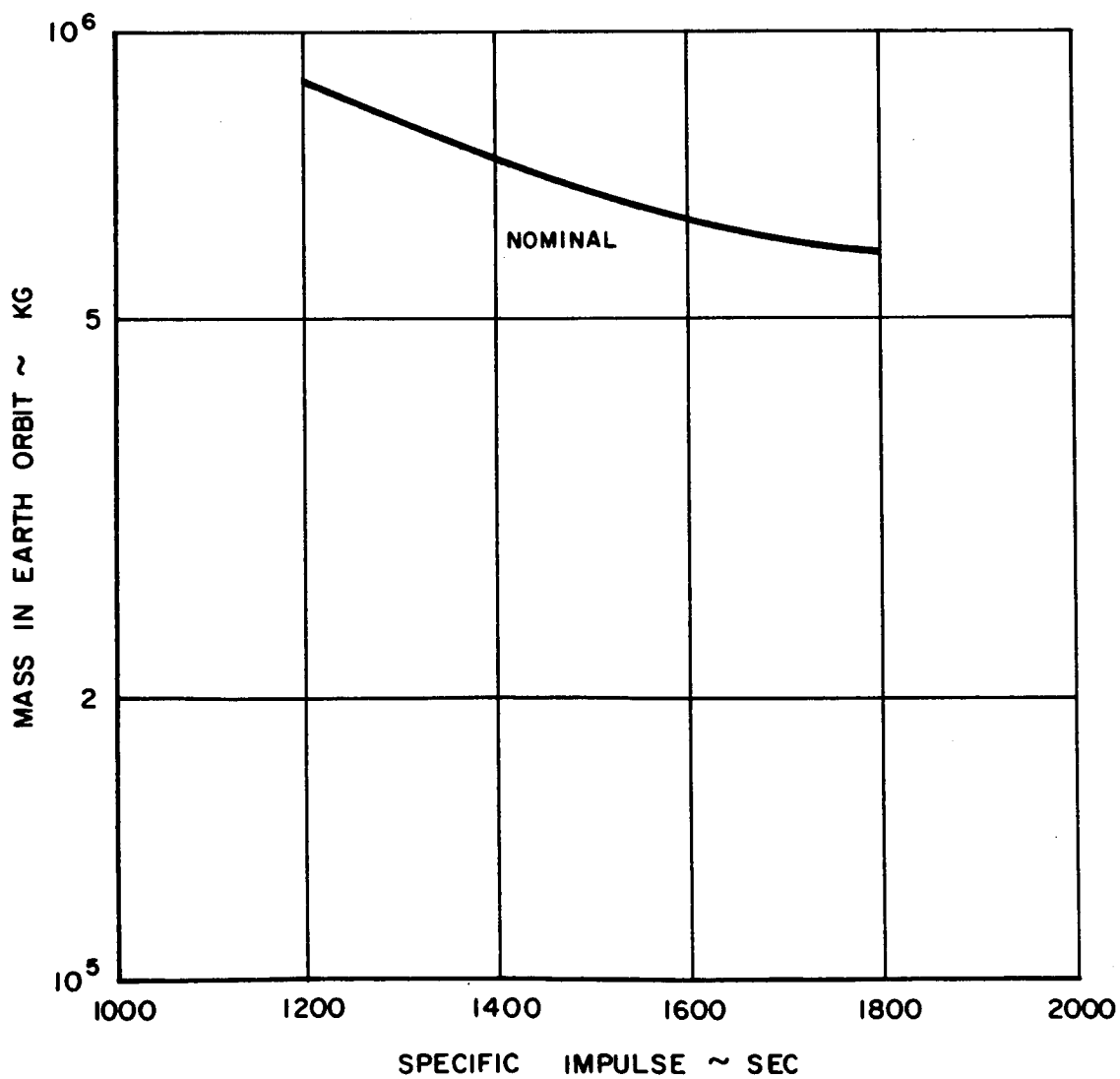


NEPTUNE MANNED ORBITAL STOPOVERS

PULSED NUCLEAR

TRIP TIME = 13840 DAYS

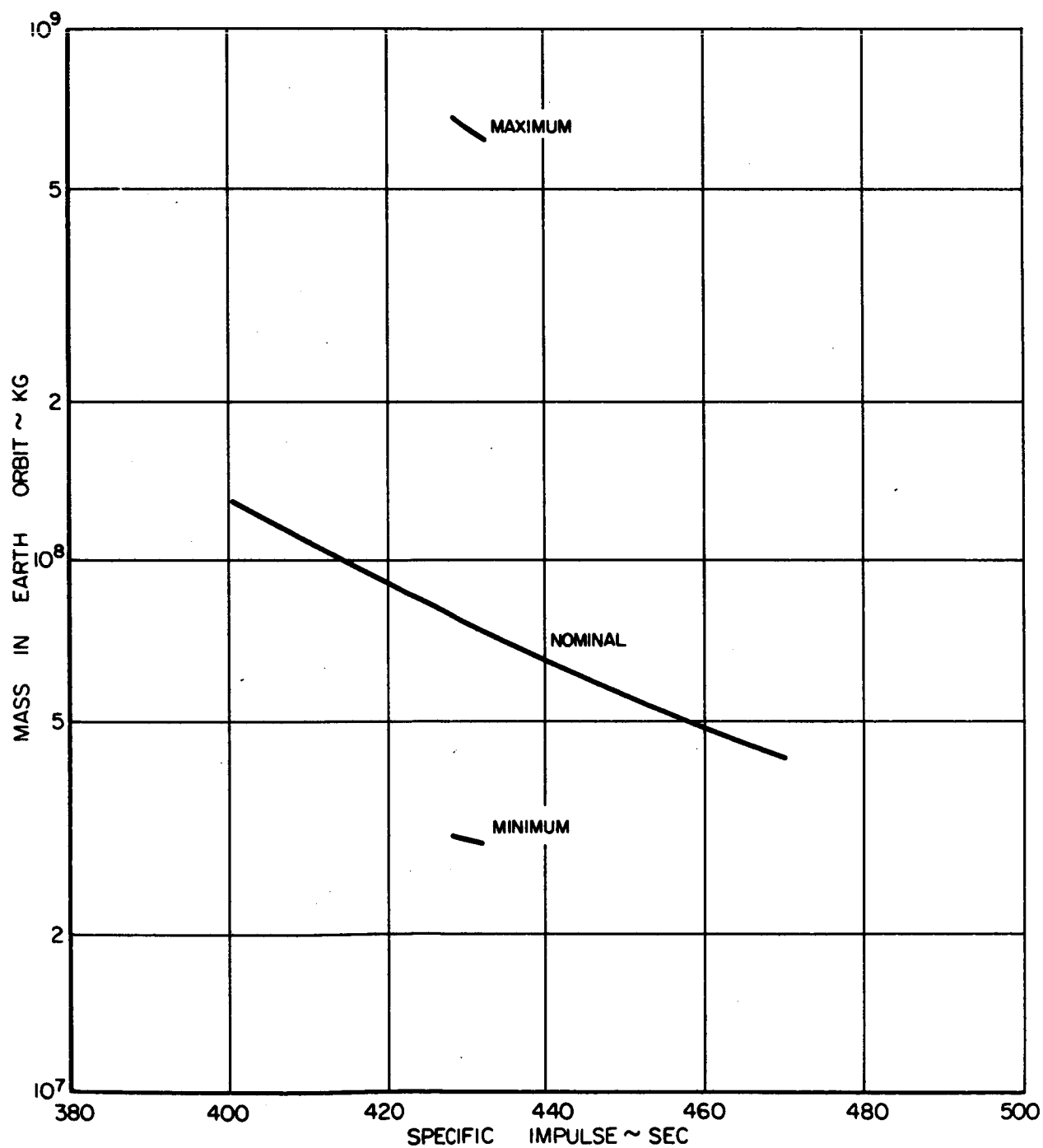
STAY TIME = 0



PLUTO MANNED ORBITAL STOPOVERS

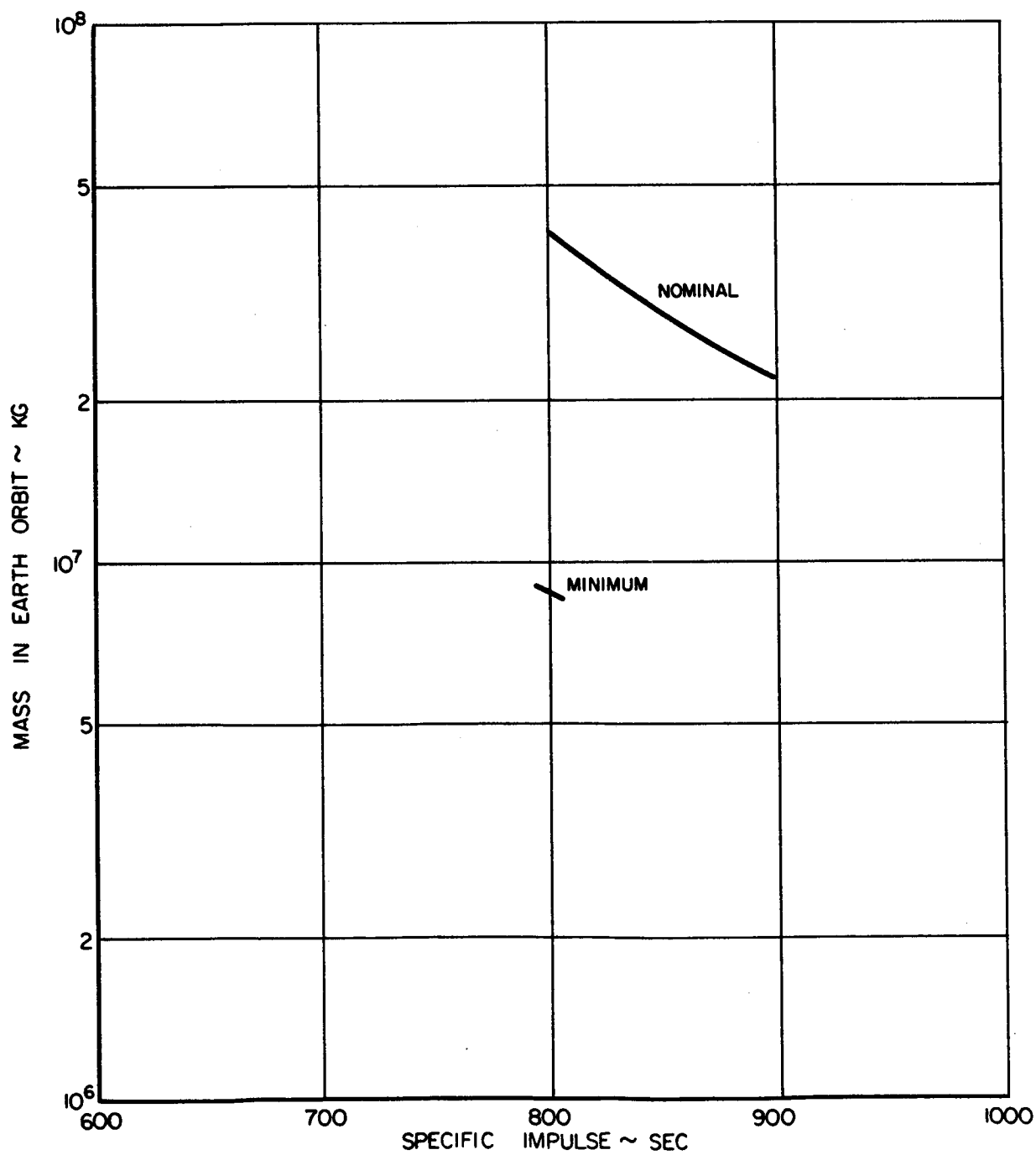
CHEMICAL

TRIP TIME = 13860 DAYS
STAY TIME = 0



PLUTO MANNED ORBITAL STOPOVERS

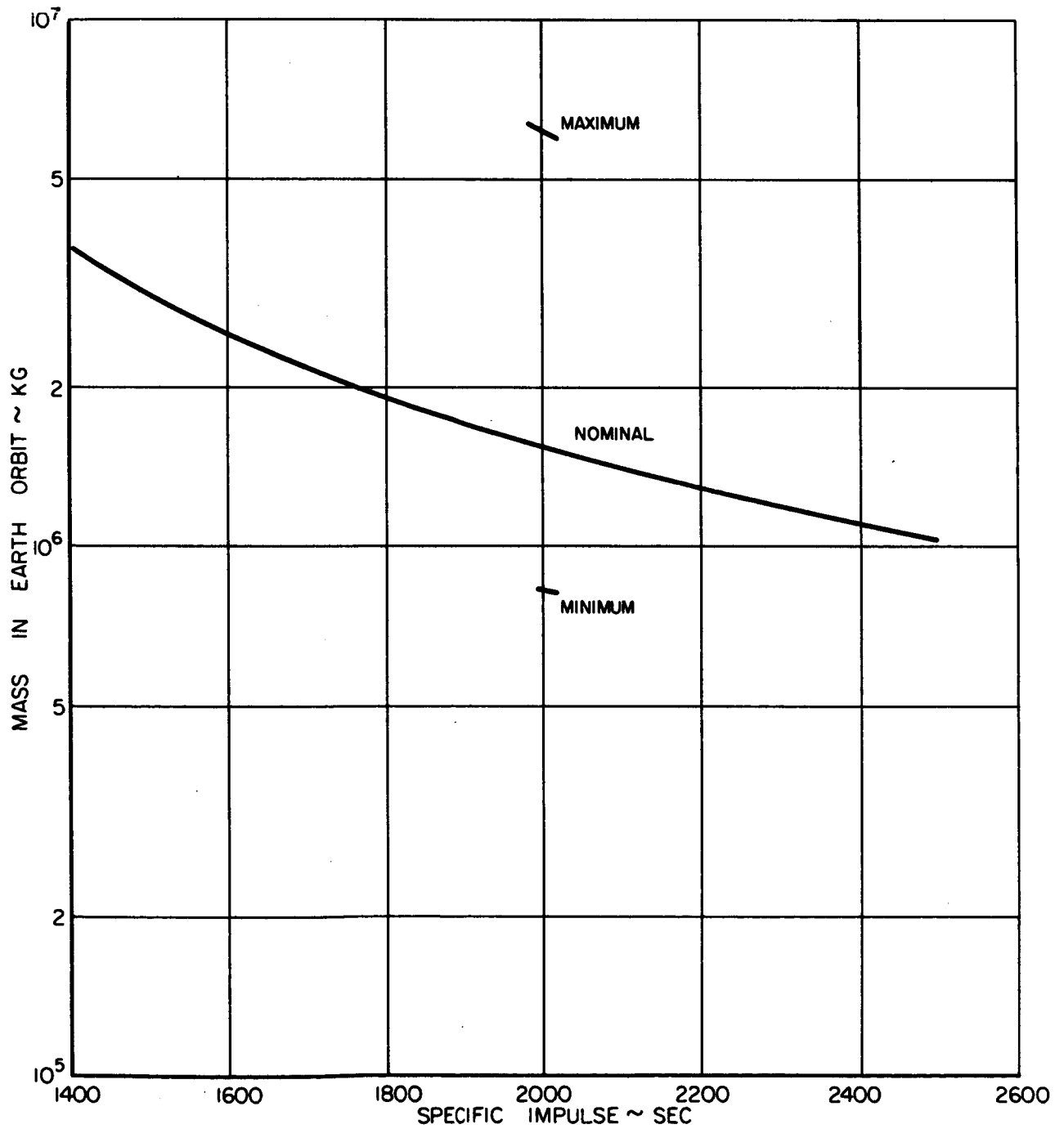
SOLID - CORE NUCLEAR

TRIP TIME = 13860 DAYS
STAY TIME = 0

PLUTO MANNED STOPOVER MISSIONS

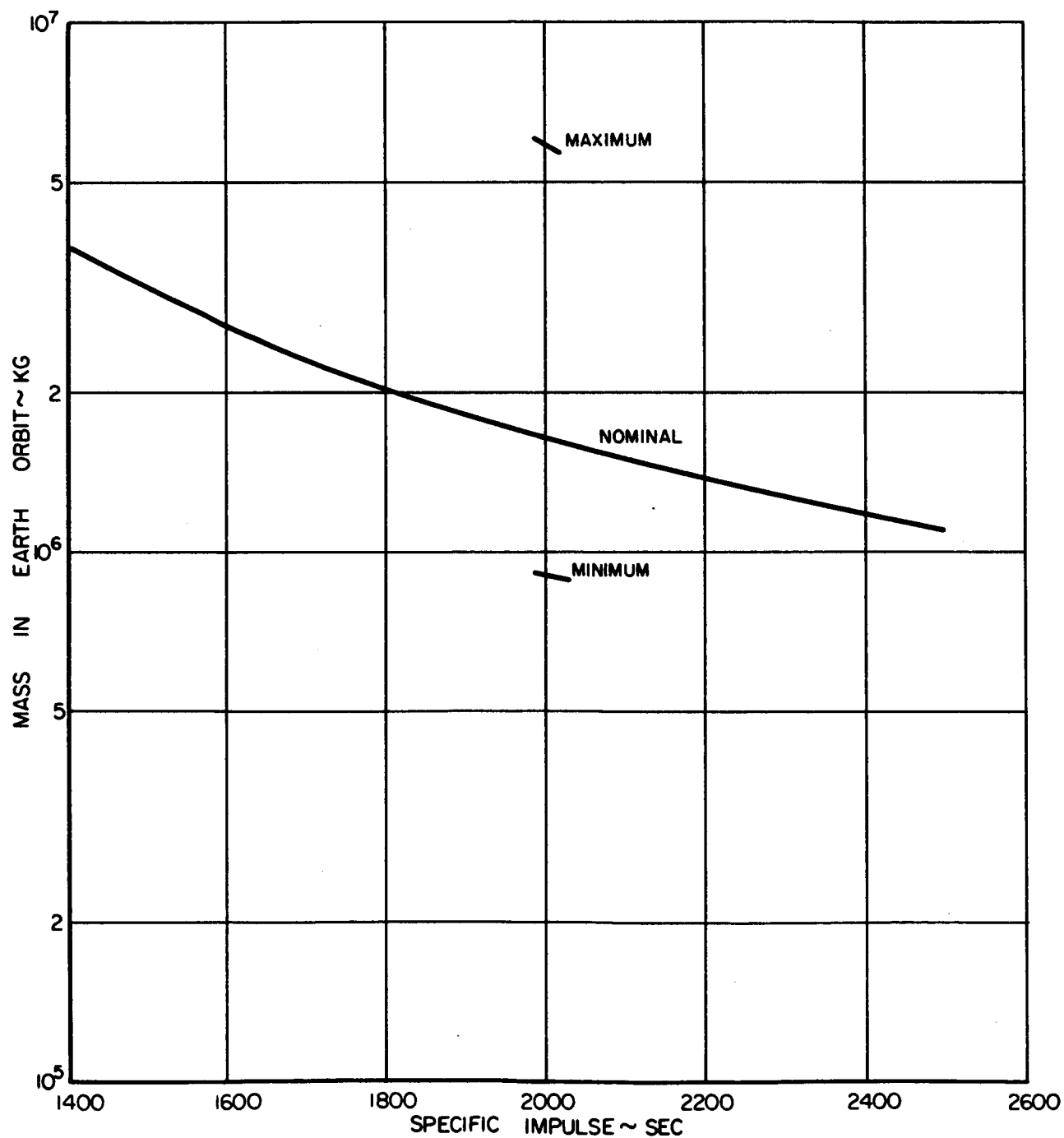
LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 13860 DAYS
STAY TIME = 0



PLUTO MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR

TRIP TIME = 13860 DAYS
STAY TIME = 0

PLUTO MANNED ORBITAL STOPOVERS

PULSED NUCLEAR

TRIP TIME = 13860 DAYS

STAY TIME = 0

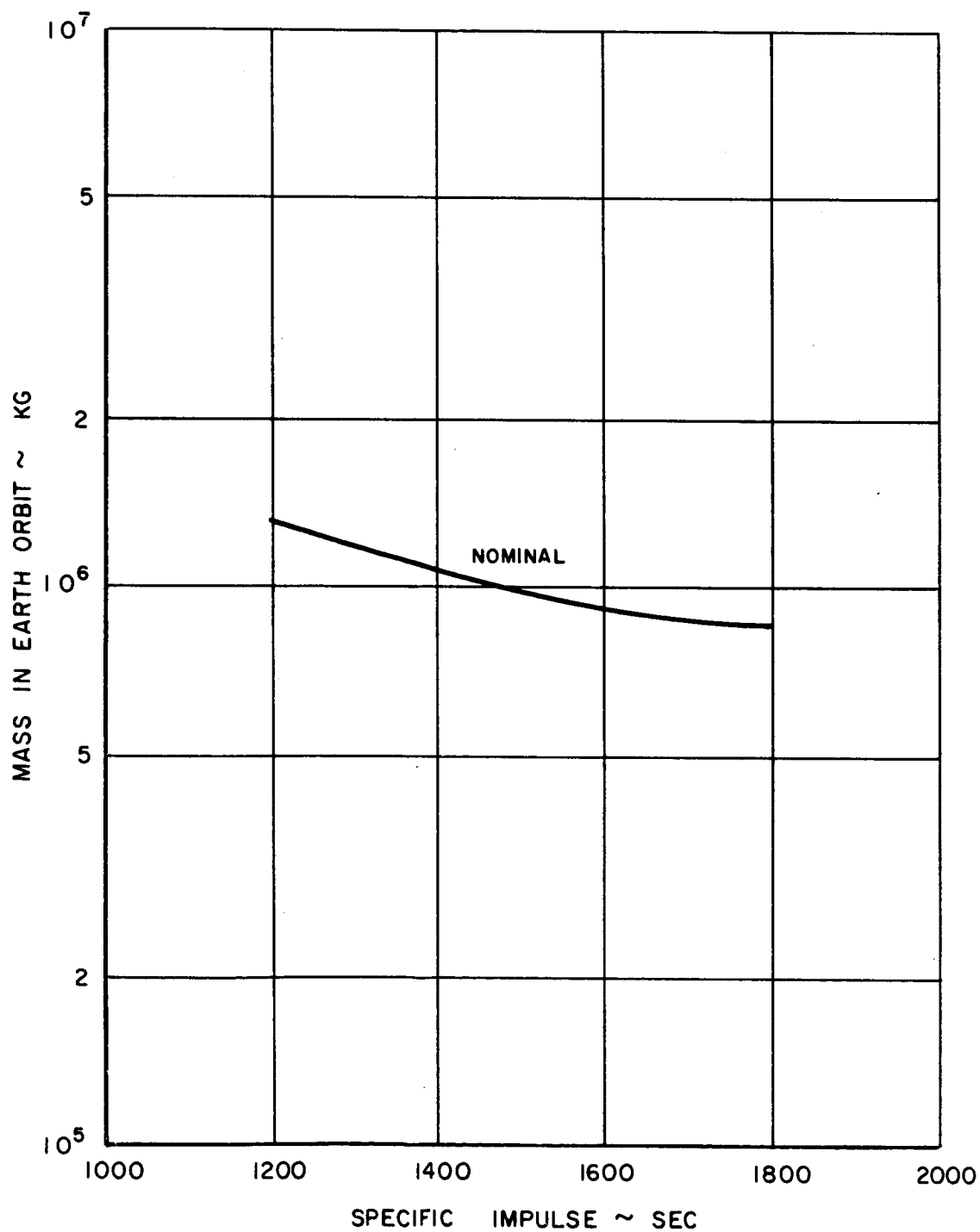


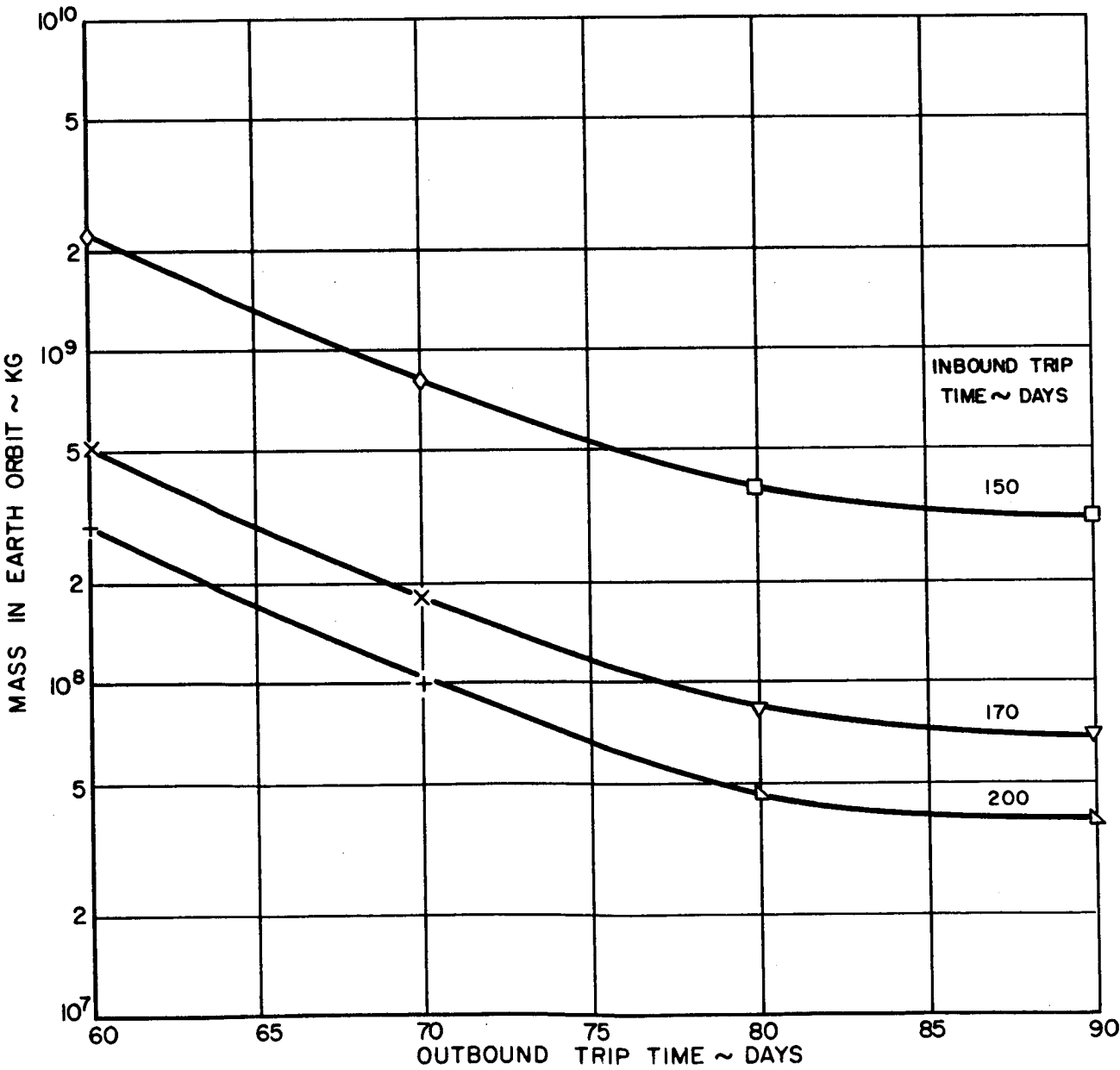
FIG. 300

E-910352-9

MERCURY MANNED ORBITAL STOPOVERS
CHEMICAL

$I_{SP} = 430 \text{ SEC}$

SYMBOL	□	◇	▽	×	△	+
STOPOVER TIME	60	65	70	75	80	85

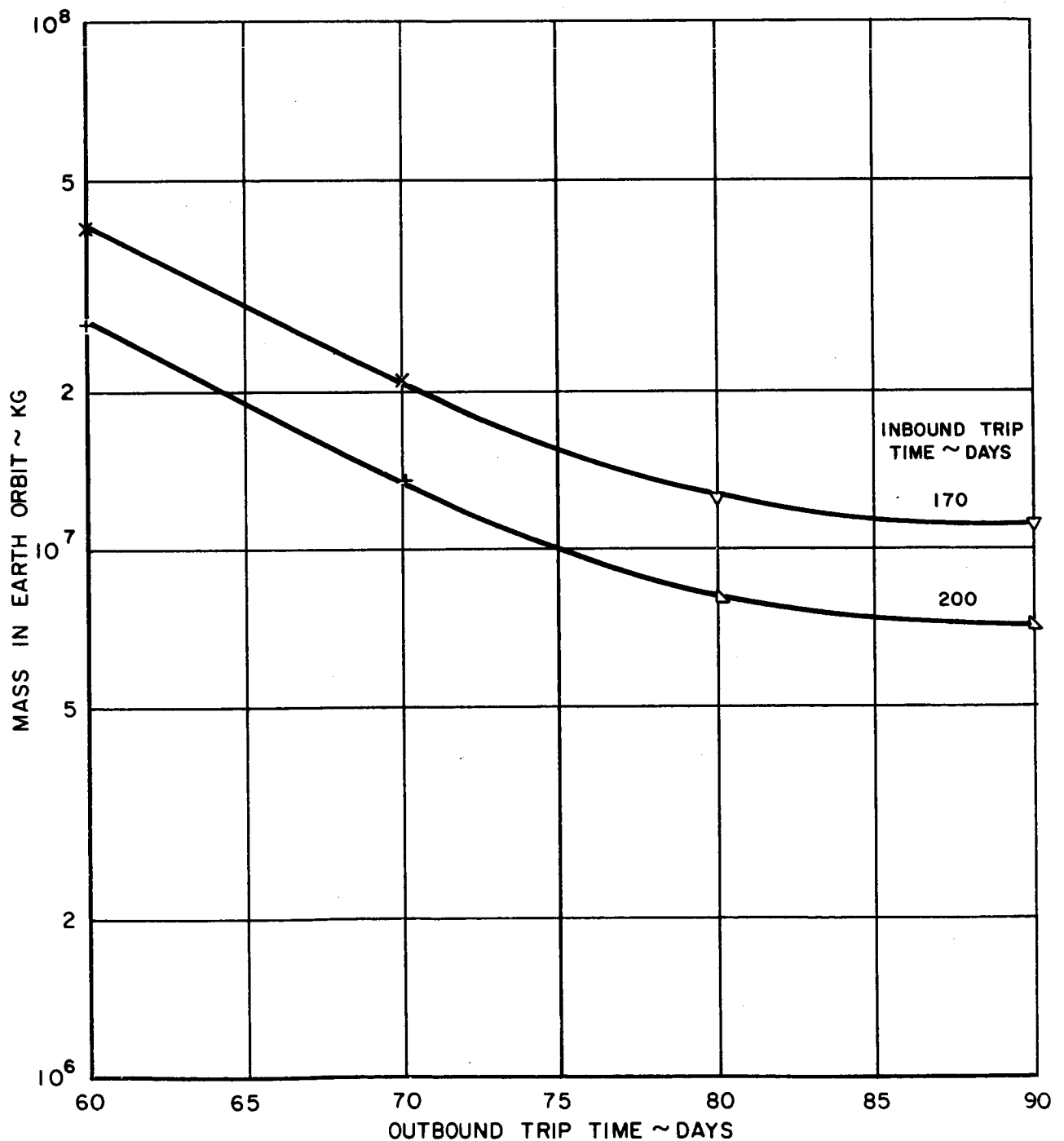


MERCURY MANNED ORBITAL STOPOVERS

SOLID-CORE NUCLEAR

 $I_{sp} = 800 \text{ SEC}$

SYMBOL	▽	×	△	+
STOPOVER TIME	70	75	80	85

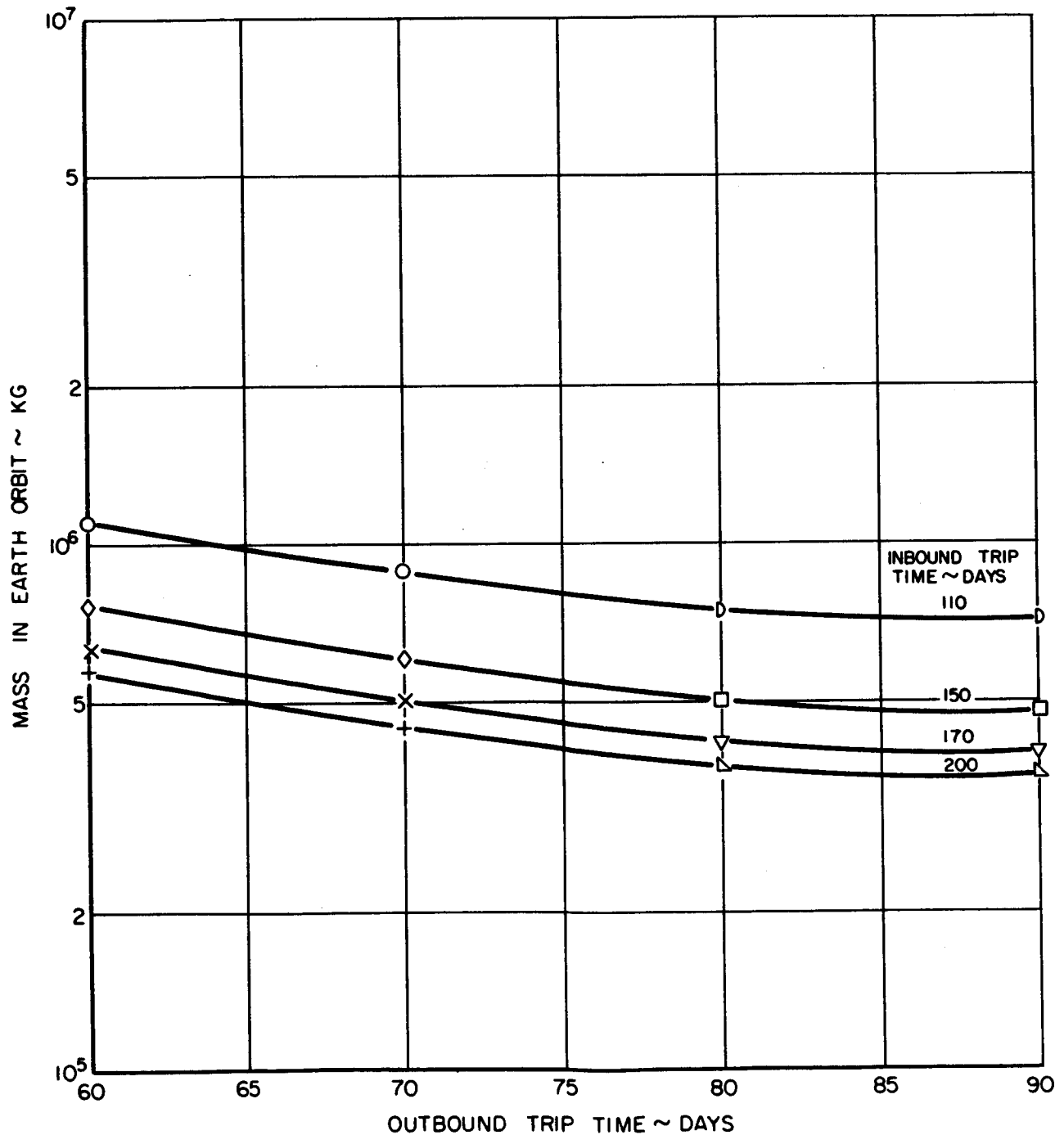


MERCURY MANNED ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{sp} = 2000 \text{ SEC}$

SYMBOL	D	O	□	◇	▽	X	△	+
STOPOVER TIME	50	55	60	65	70	75	80	85

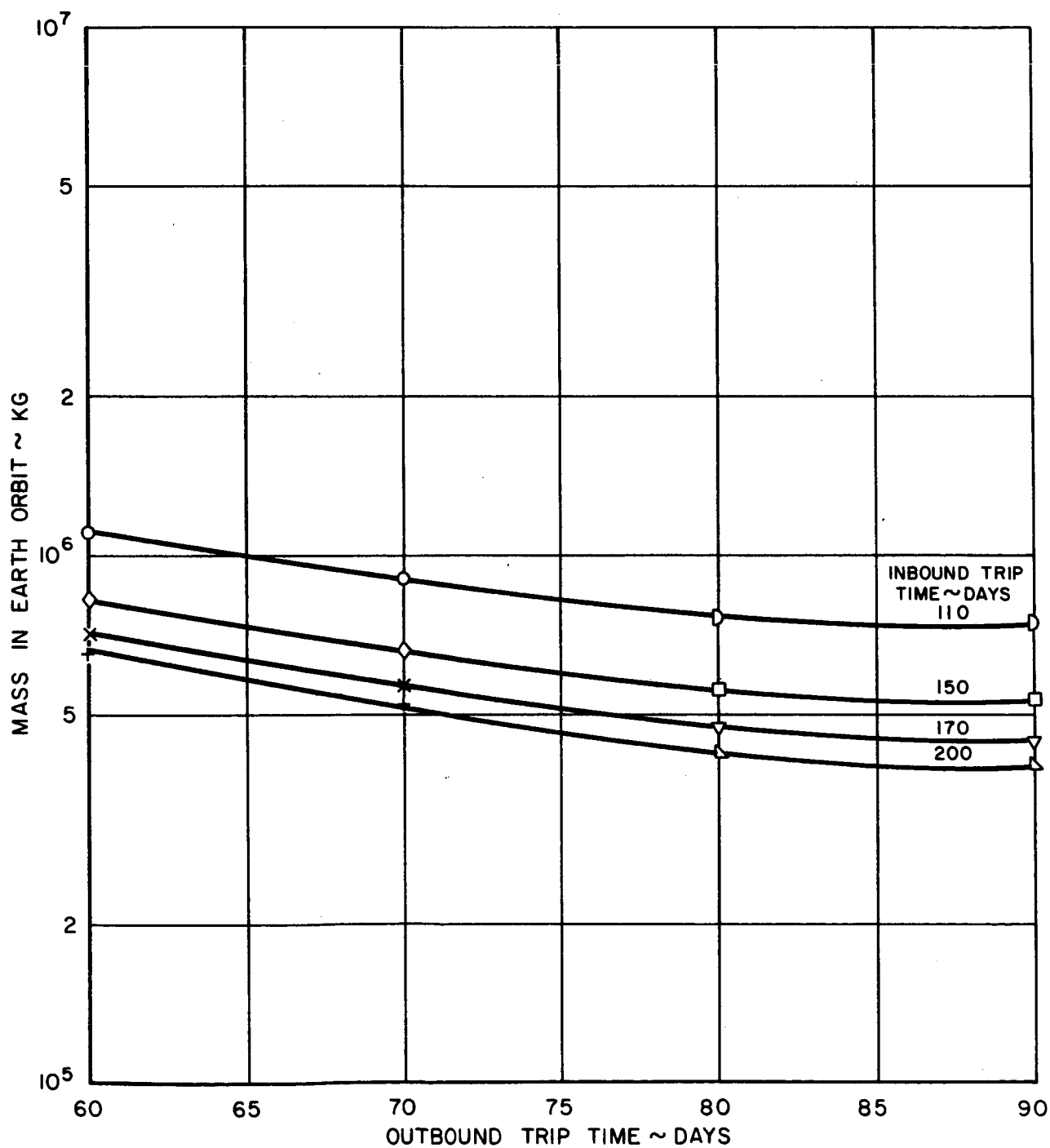


MERCURY MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR

 $I_{sp} = 2000 \text{ SEC}$

SYMBOL	D	O	□	◇	▽	×	△	+
STOPOVER TIME	50	55	60	65	70	75	80	85

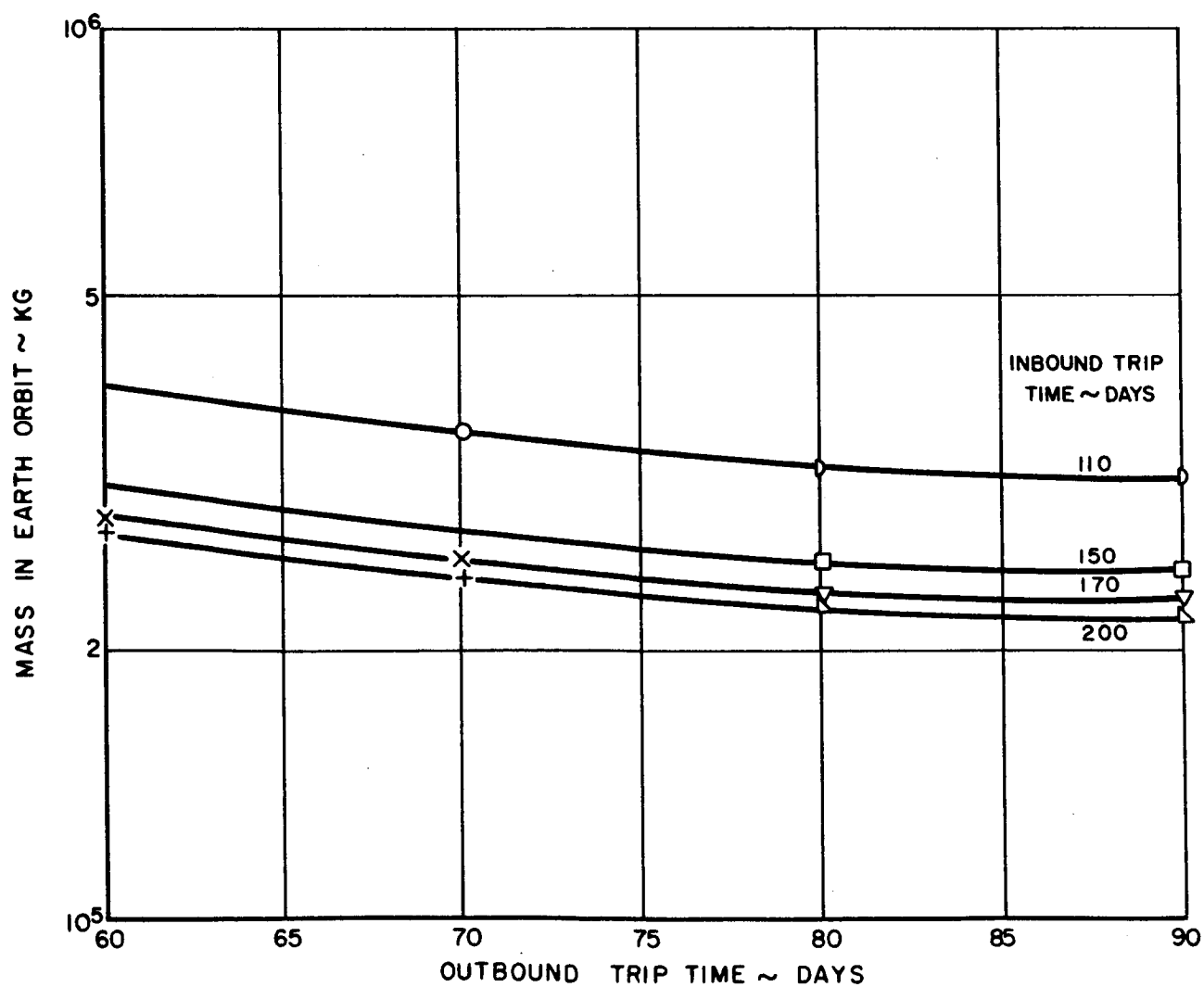


MERCURY MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR

 $I_{sp} = 3500 \text{ SEC}$

SYMBOL	D	O	□	▽	X	△	+
STOPOVER TIME	50	55	60	70	75	80	85

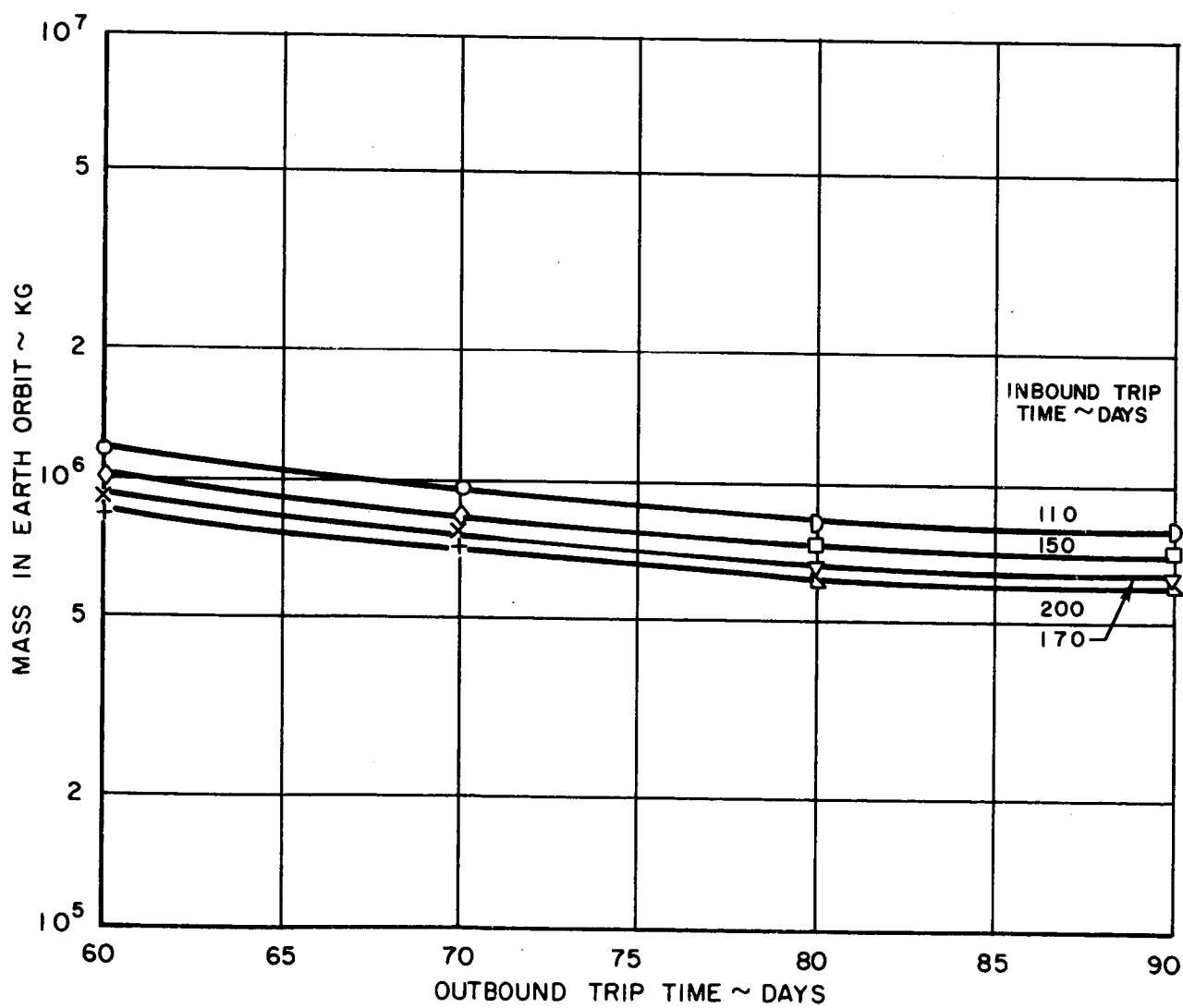


MERCURY MANNED ORBITAL STOPOVERS

PULSED NUCLEAR

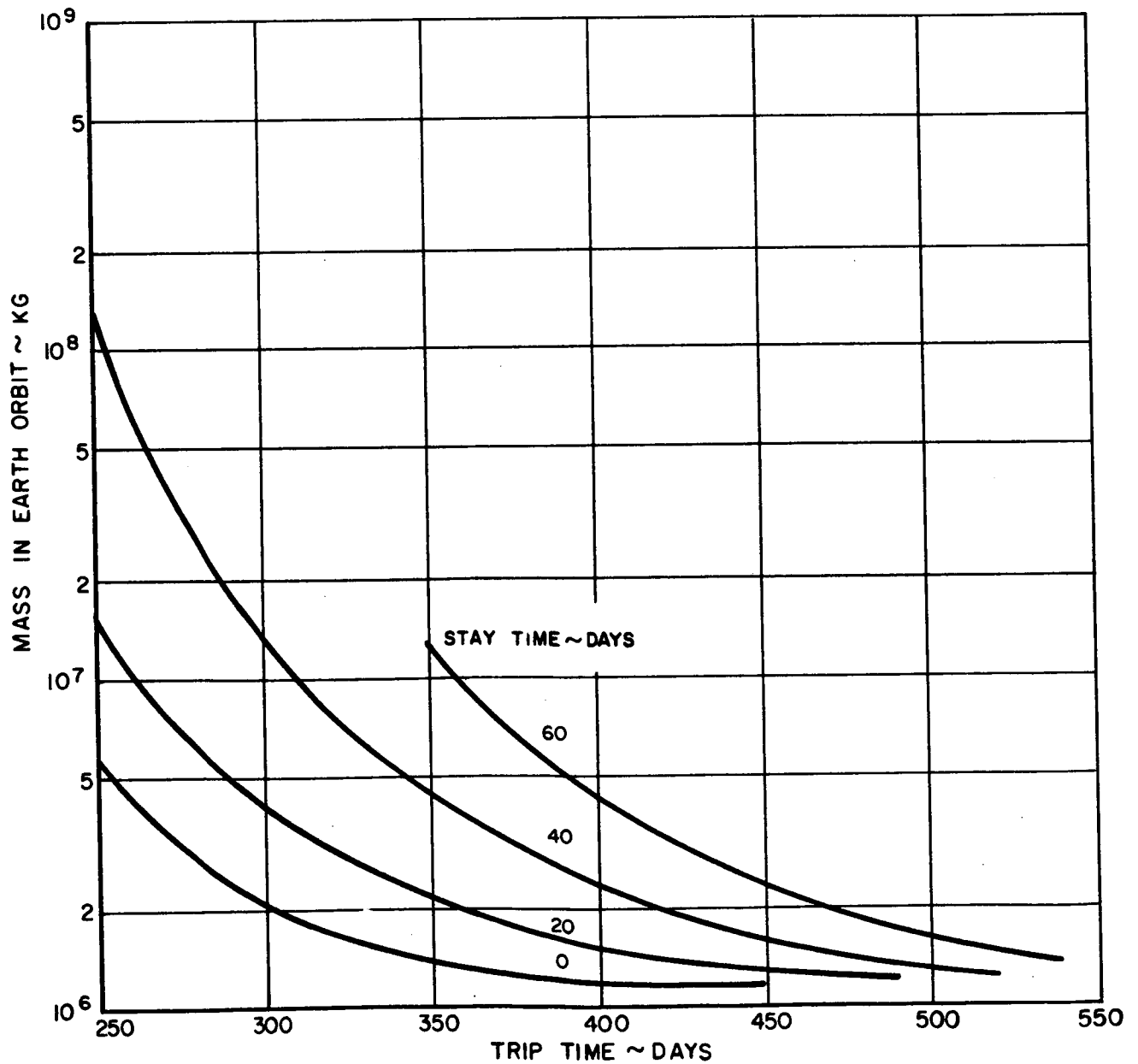
 $I_{sp} = 1800 \text{ SEC}$

SYMBOL	D	○	□	◇	▽	×	△	+
STOPOVER TIME	50	55	60	65	70	75	80	85



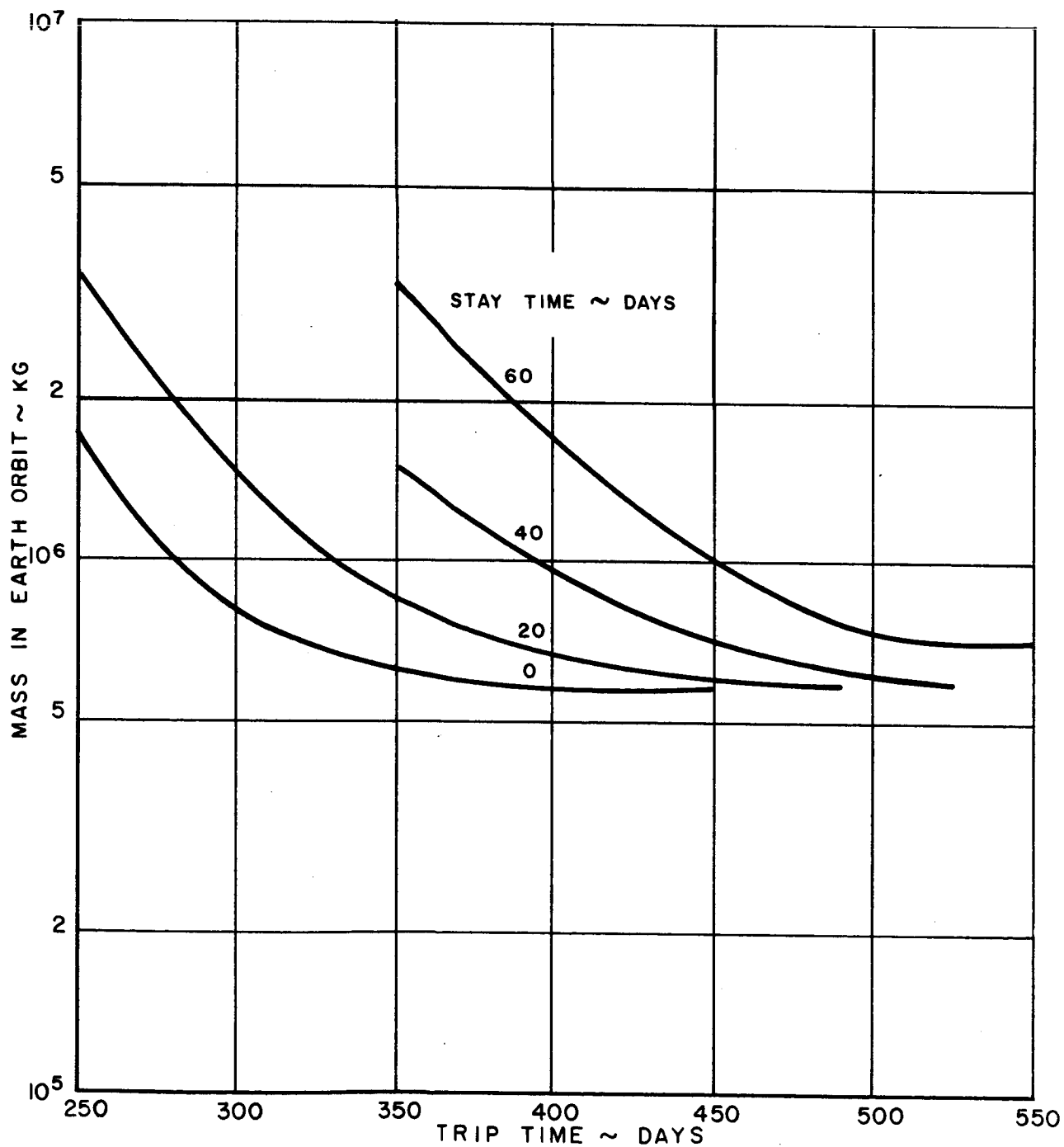
VENUS MANNED ORBITAL STOPOVERS

CHEMICAL

 $I_{SP} = 430 \text{ SEC}$ 

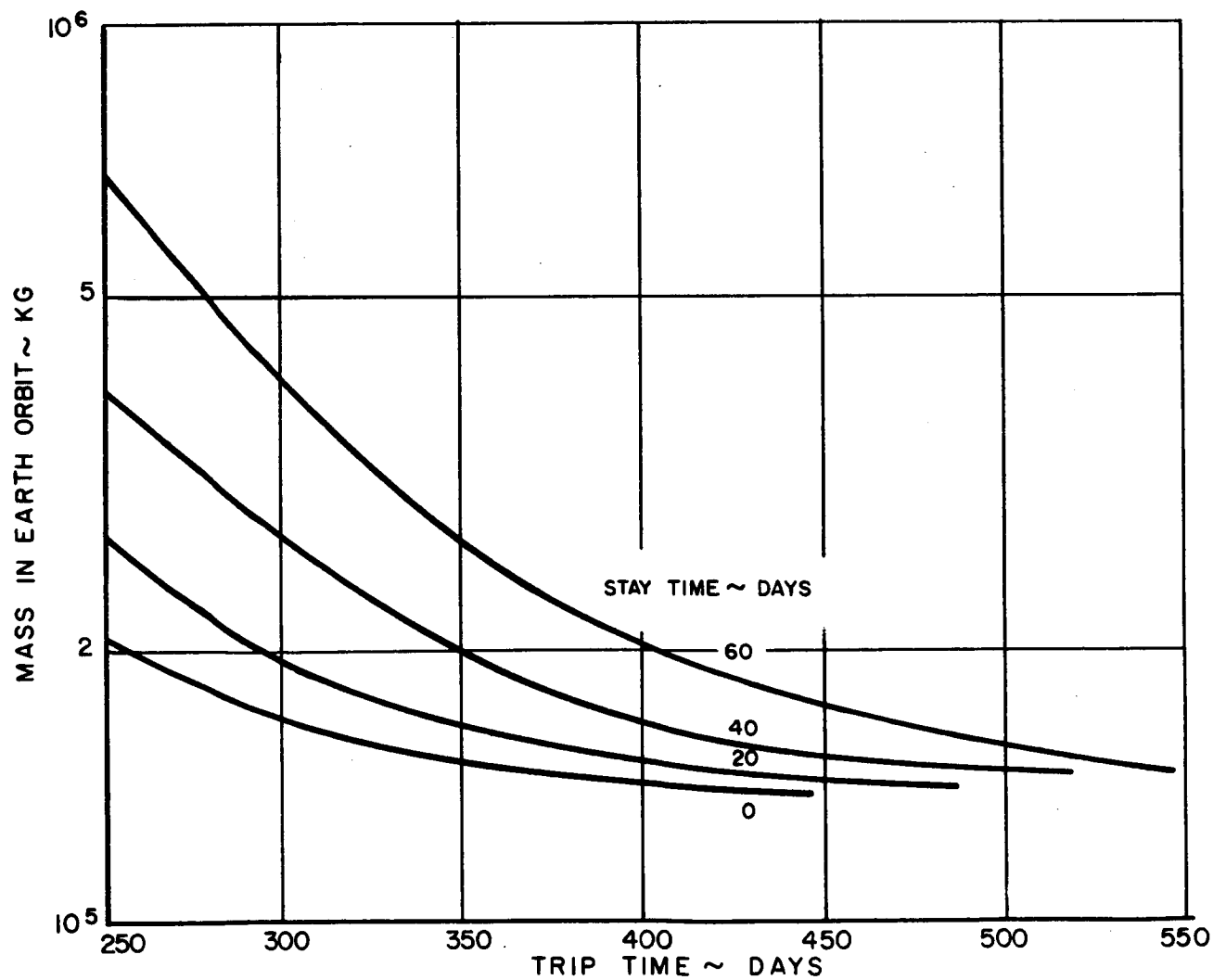
VENUS MANNED ORBITAL STOPOVERS

SOLID-CORE NUCLEAR

 $I_{SP} = 800 \text{ SEC}$ 

VENUS MANNED ORBITAL STOPOVERS

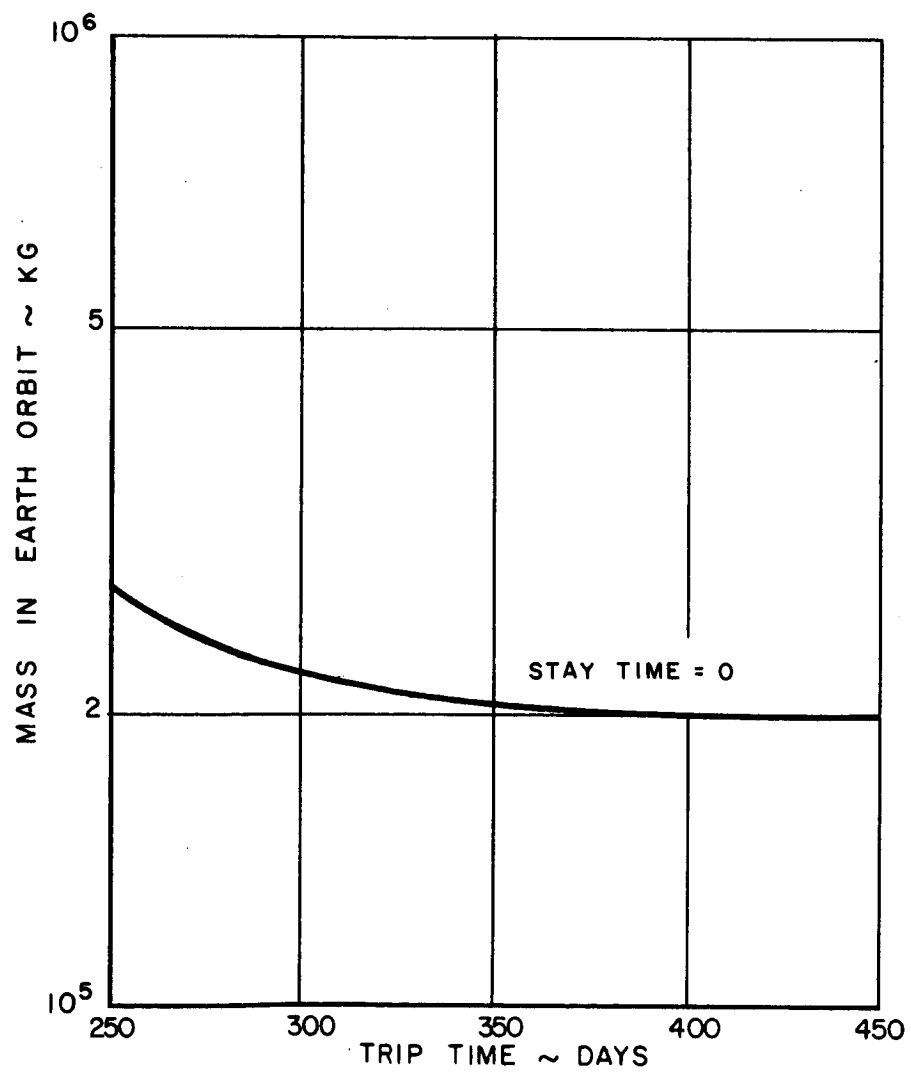
LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ 

VENUS MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR

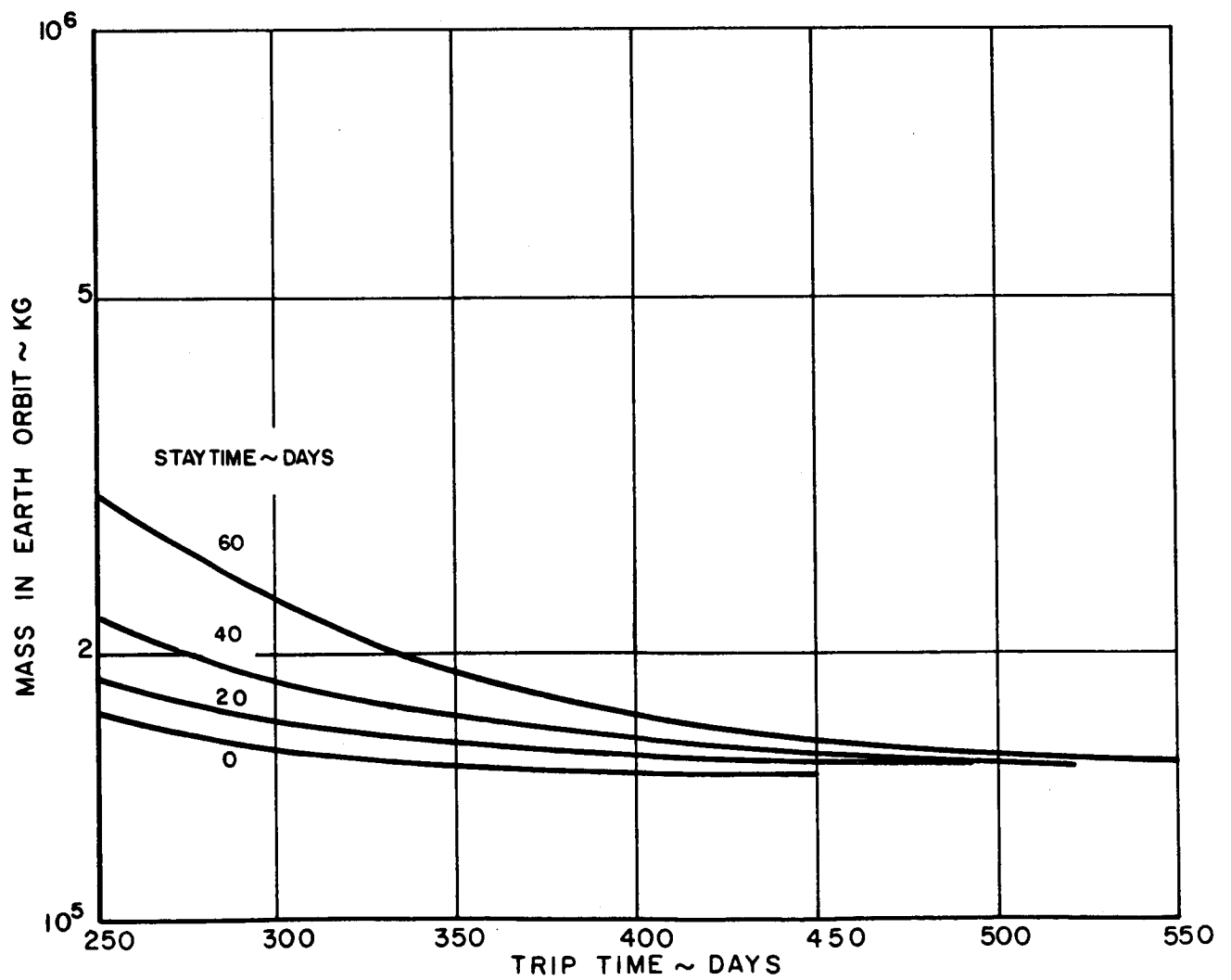
$$I_{SP} = 2000 \text{ SEC}$$



VENUS MANNED ORBITAL STOPOVERS

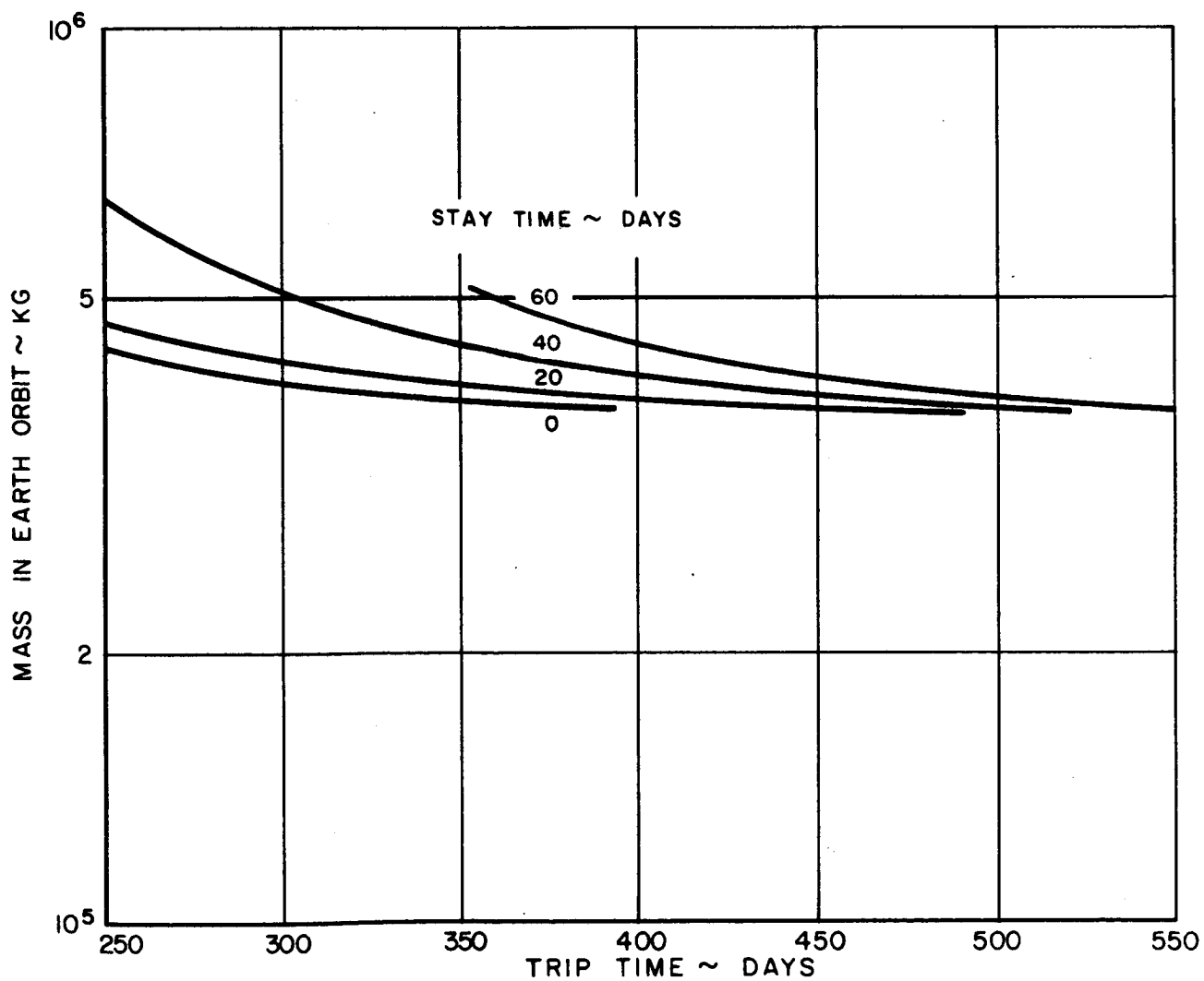
GASEOUS-CORE NUCLEAR

$$I_{SP} = 3500 \text{ SEC}$$



VENUS MANNED ORBITAL STOPOVERS

PULSED NUCLEAR

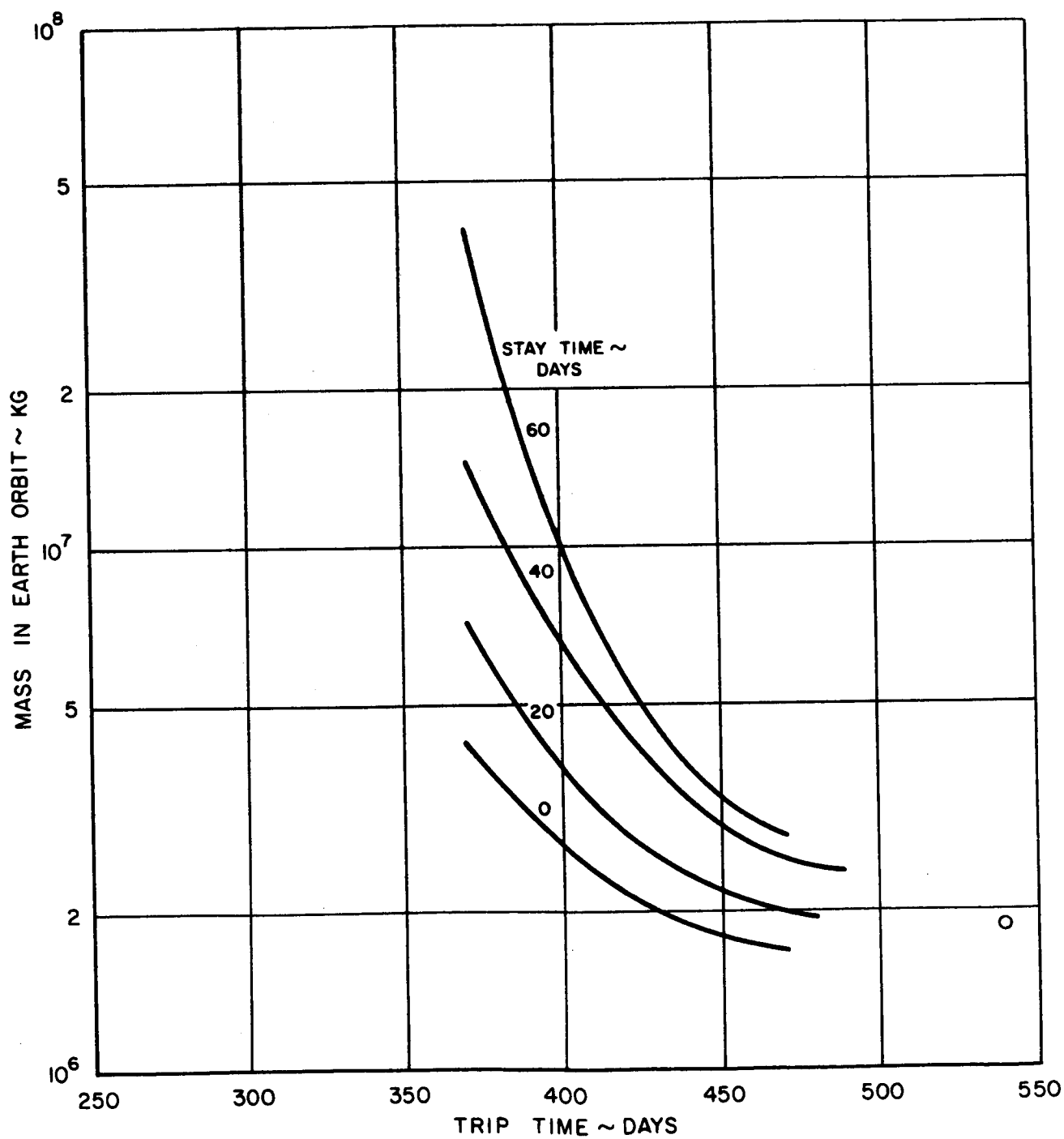
 $I_{sp} = 1800 \text{ SEC}$ 

MARS MANNED ORBITAL STOPOVERS

CHEMICAL

 $I_{sp} = 430 \text{ SEC}$

O VENUS SWINGBY, 10-DAY STOPOVER

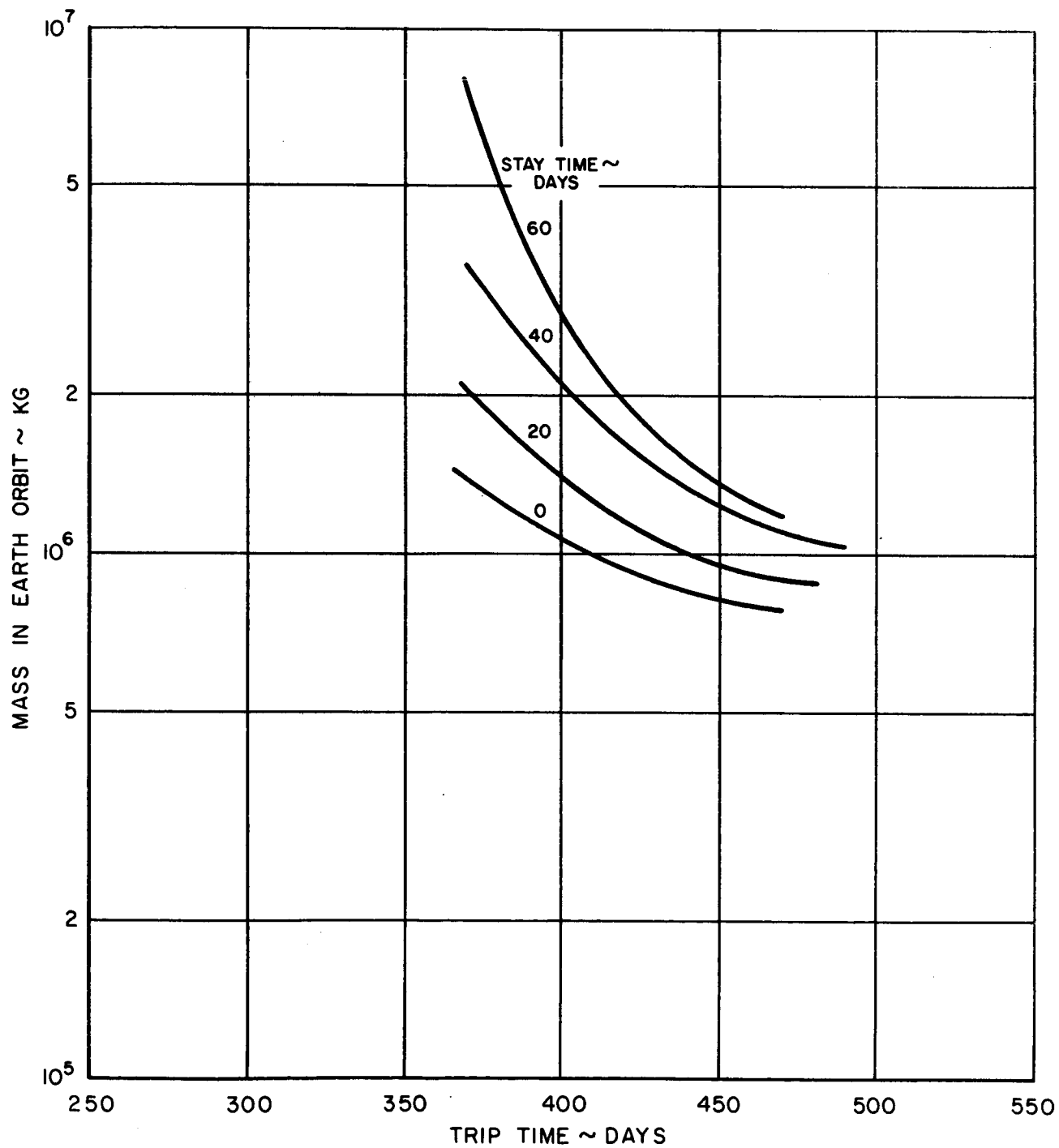


MARS MANNED ORBITAL STOPOVERS

SOLID-CORE NUCLEAR

 $I_{SP} = 800 \text{ SEC}$

O VENUS SWINGBY, 10-DAY STOPOVER

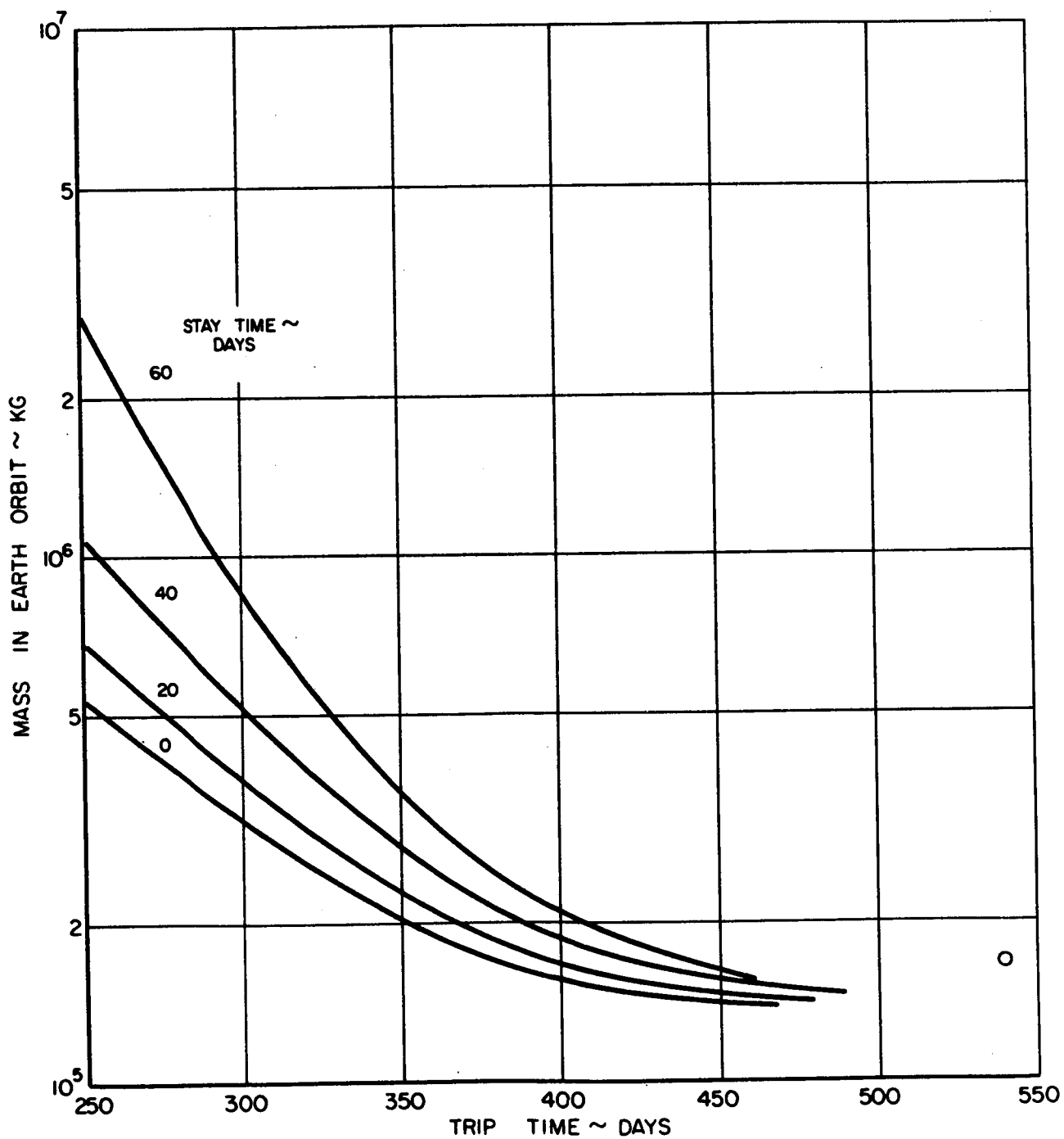


MARS MANNED ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{sp} = 2000 \text{ SEC}$

O VENUS SWINGBY, 10-DAY STOPOVER

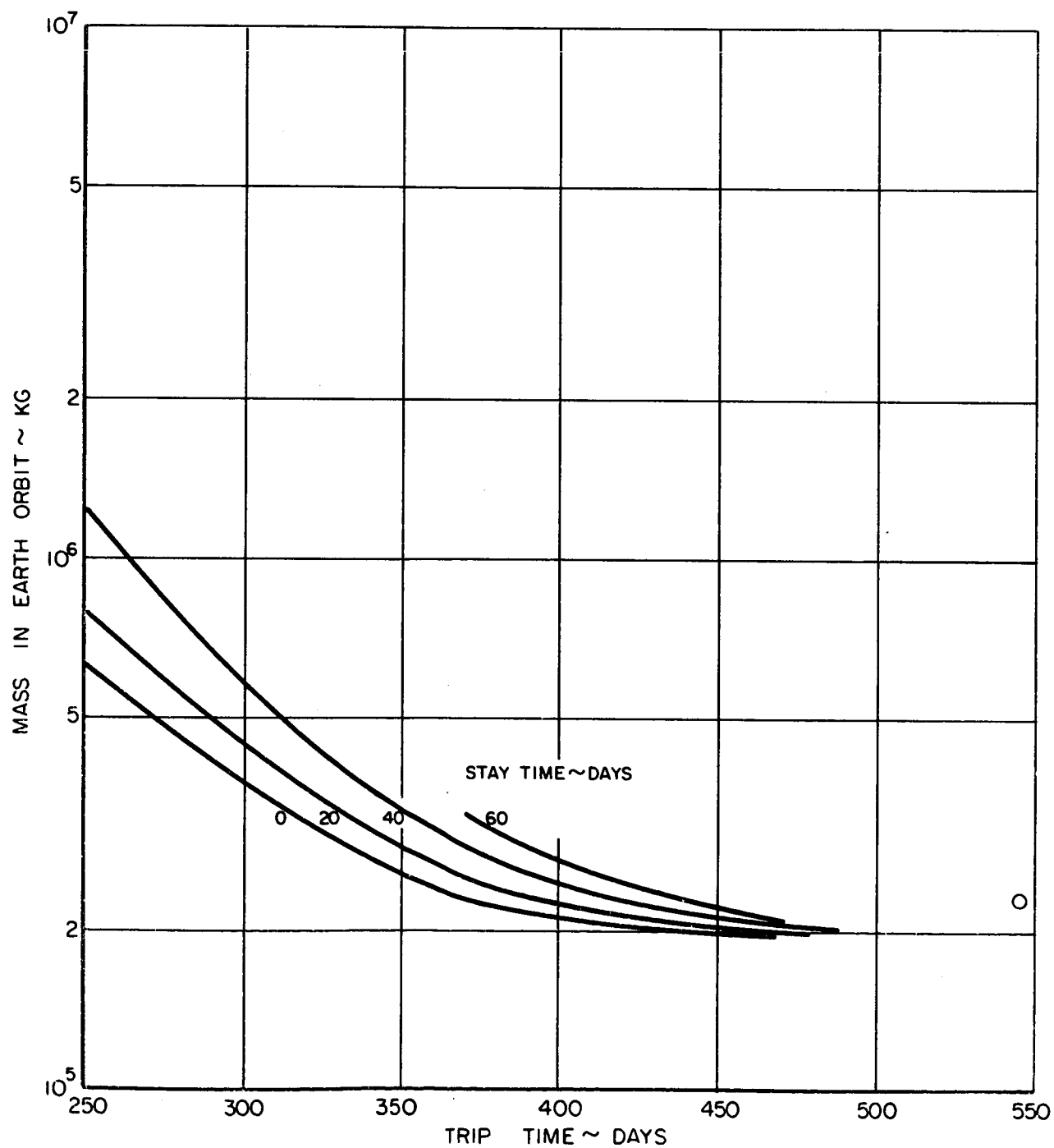


MARS MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR

 $I_{sp} = 2000 \text{ SEC}$

O VENUS SWINGBY, 10-DAY STOPOVER

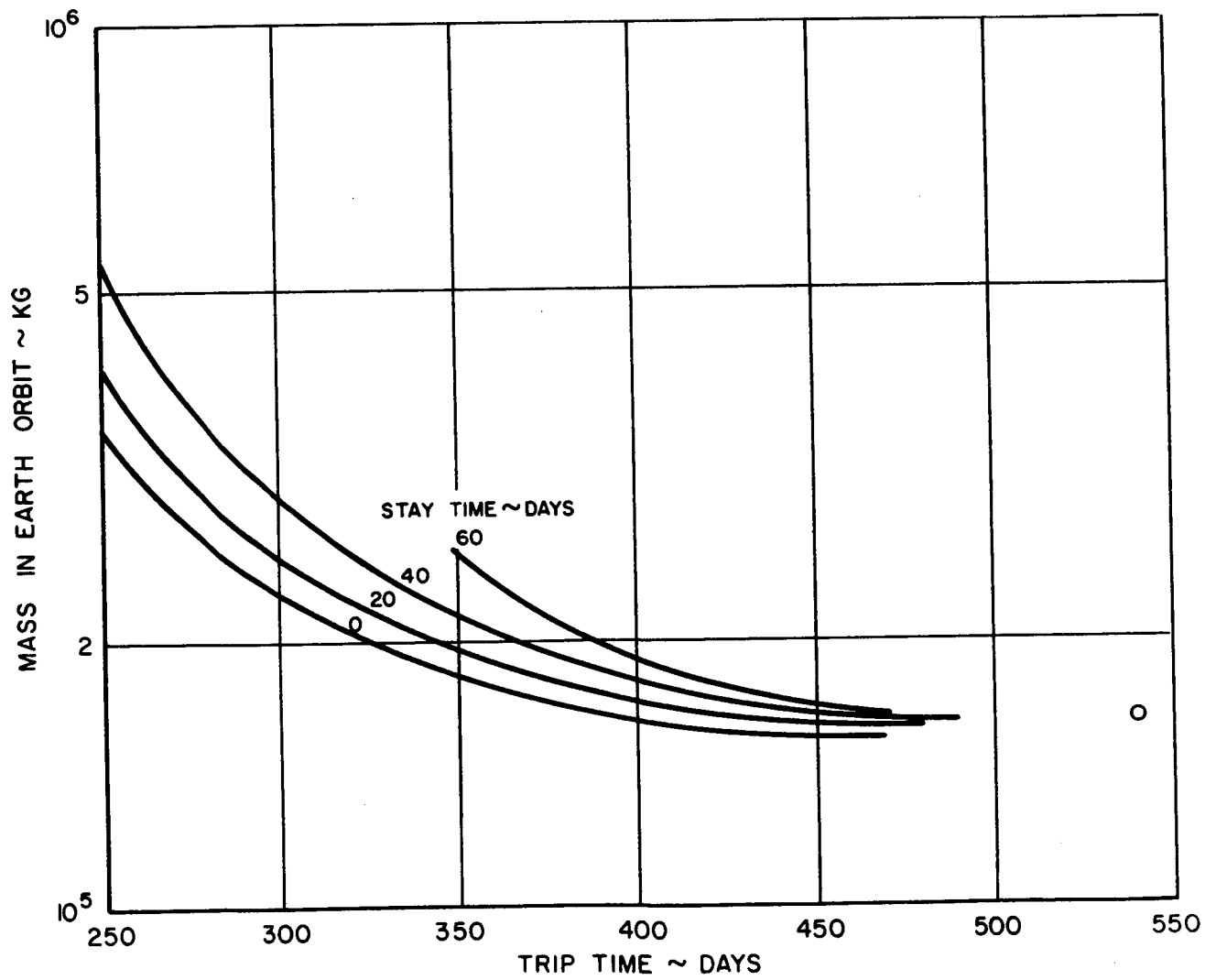


MARS MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR

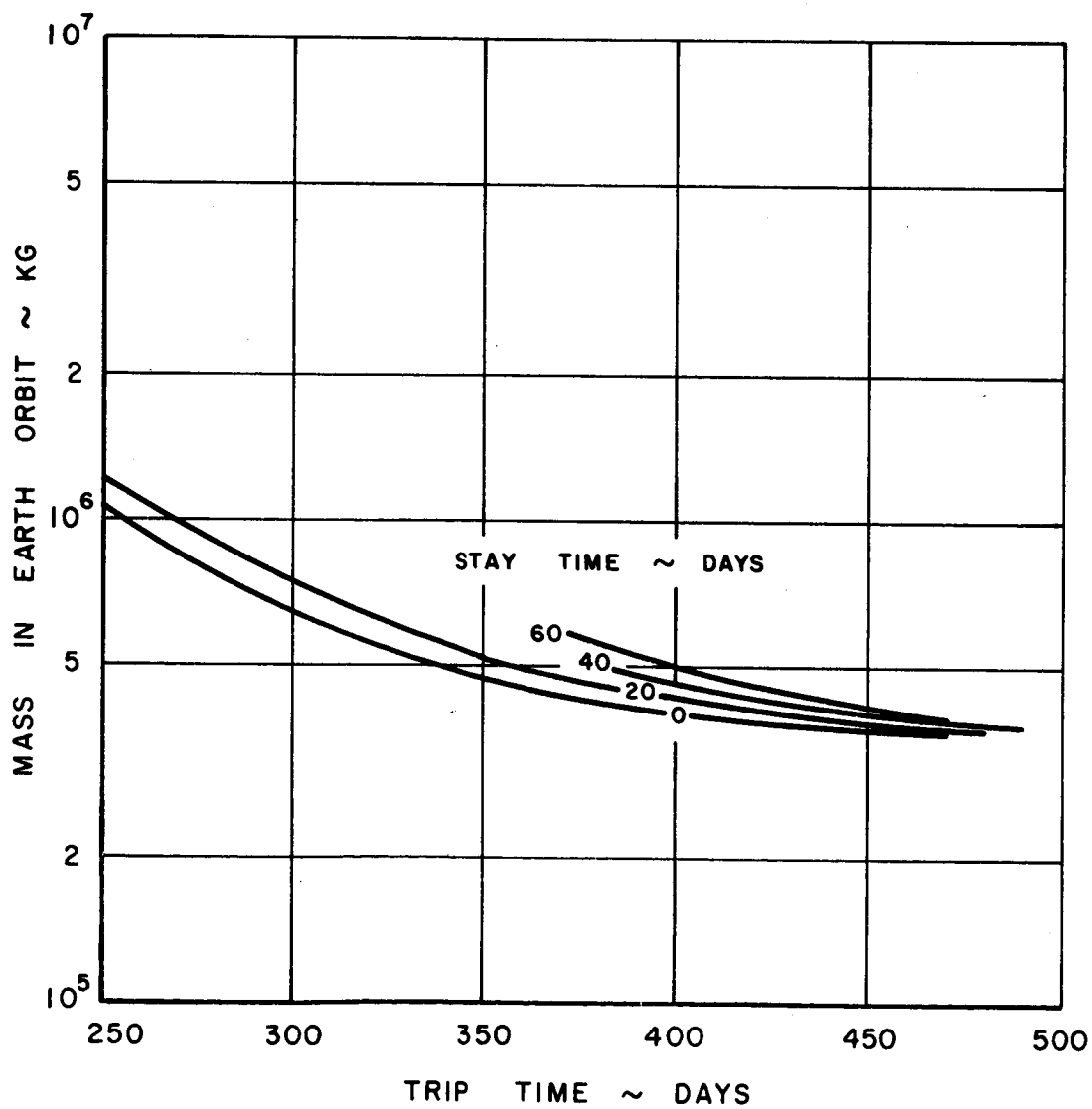
 $I_{SP} = 3500 \text{ SEC}$

O VENUS SWINGBY, 10-DAY STOPOVER



MARS MANNED ORBITAL STOPOVERS

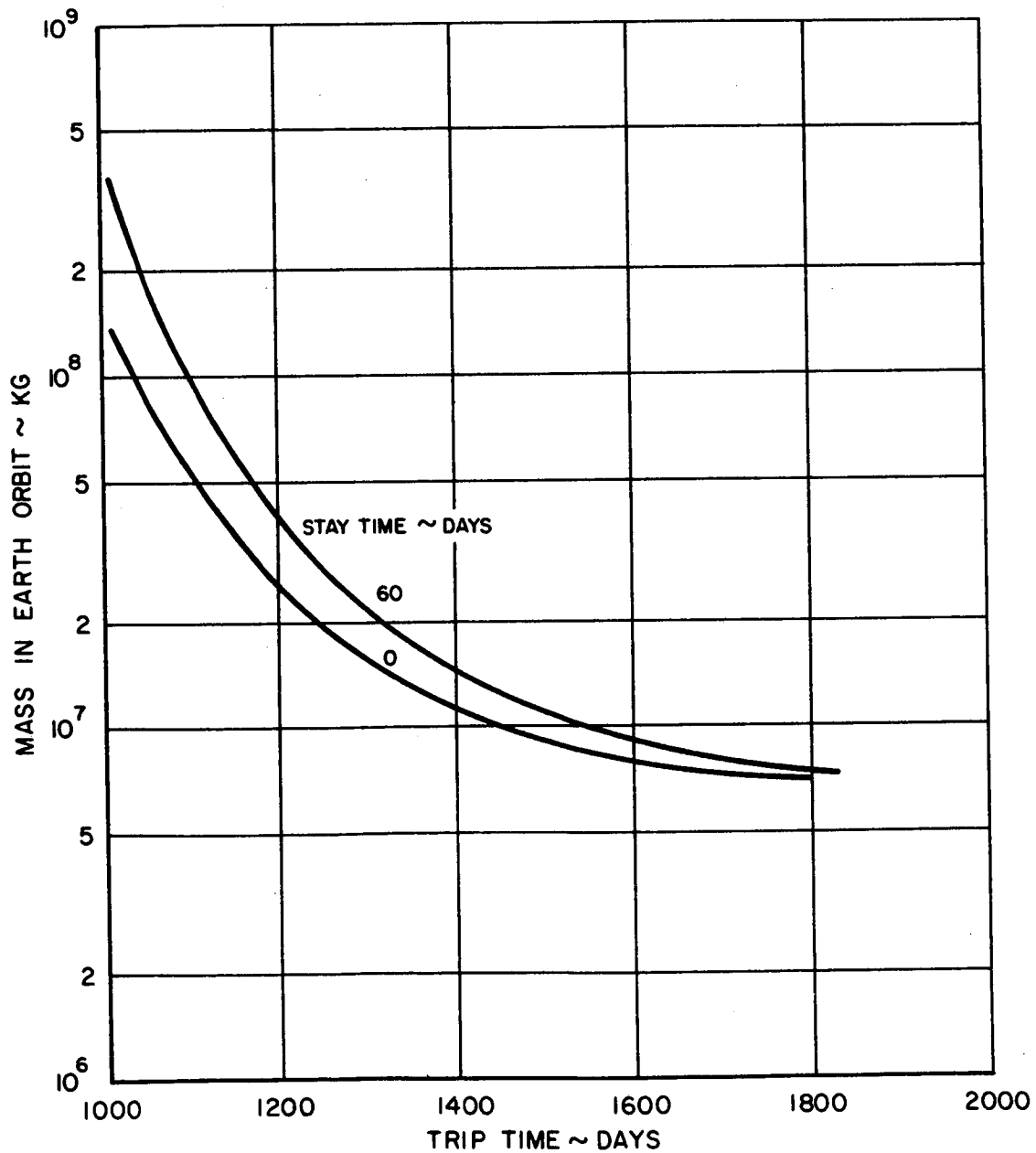
PULSED NUCLEAR

 $I_{sp} = 1800 \text{ SEC}$ 

GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

CHEMICAL

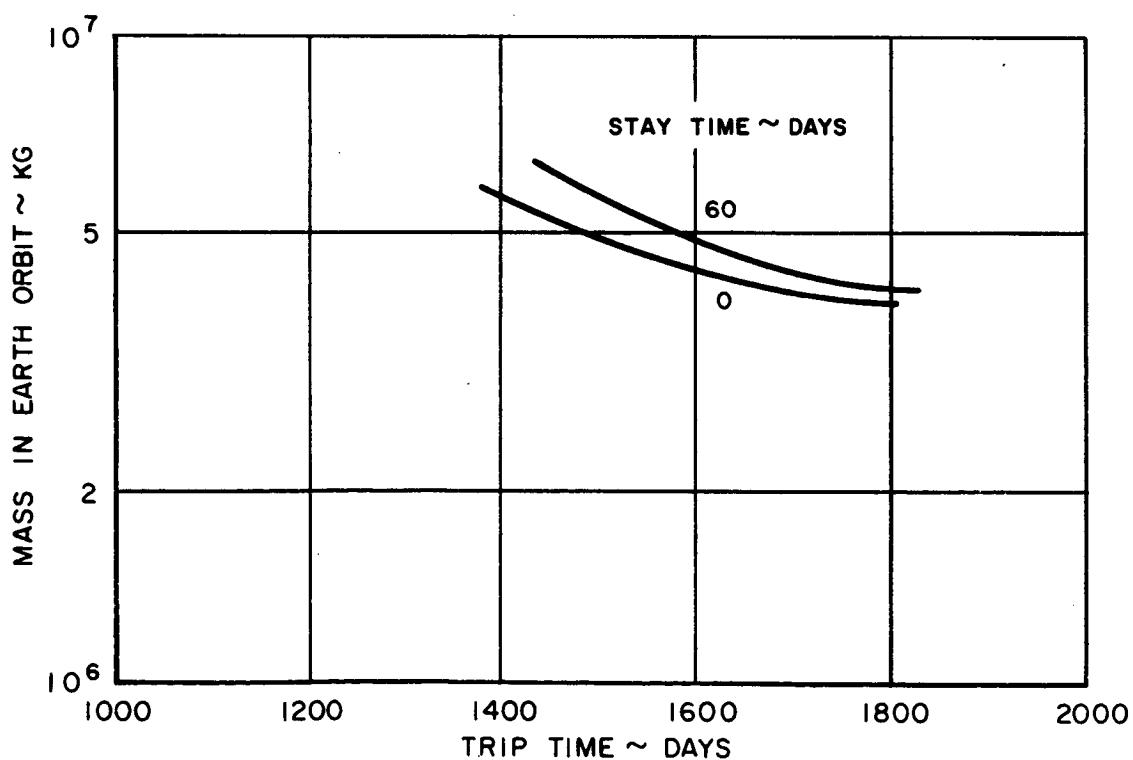
$I_{SP} = 430 \text{ SEC}$



GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

SOLID-CORE NUCLEAR

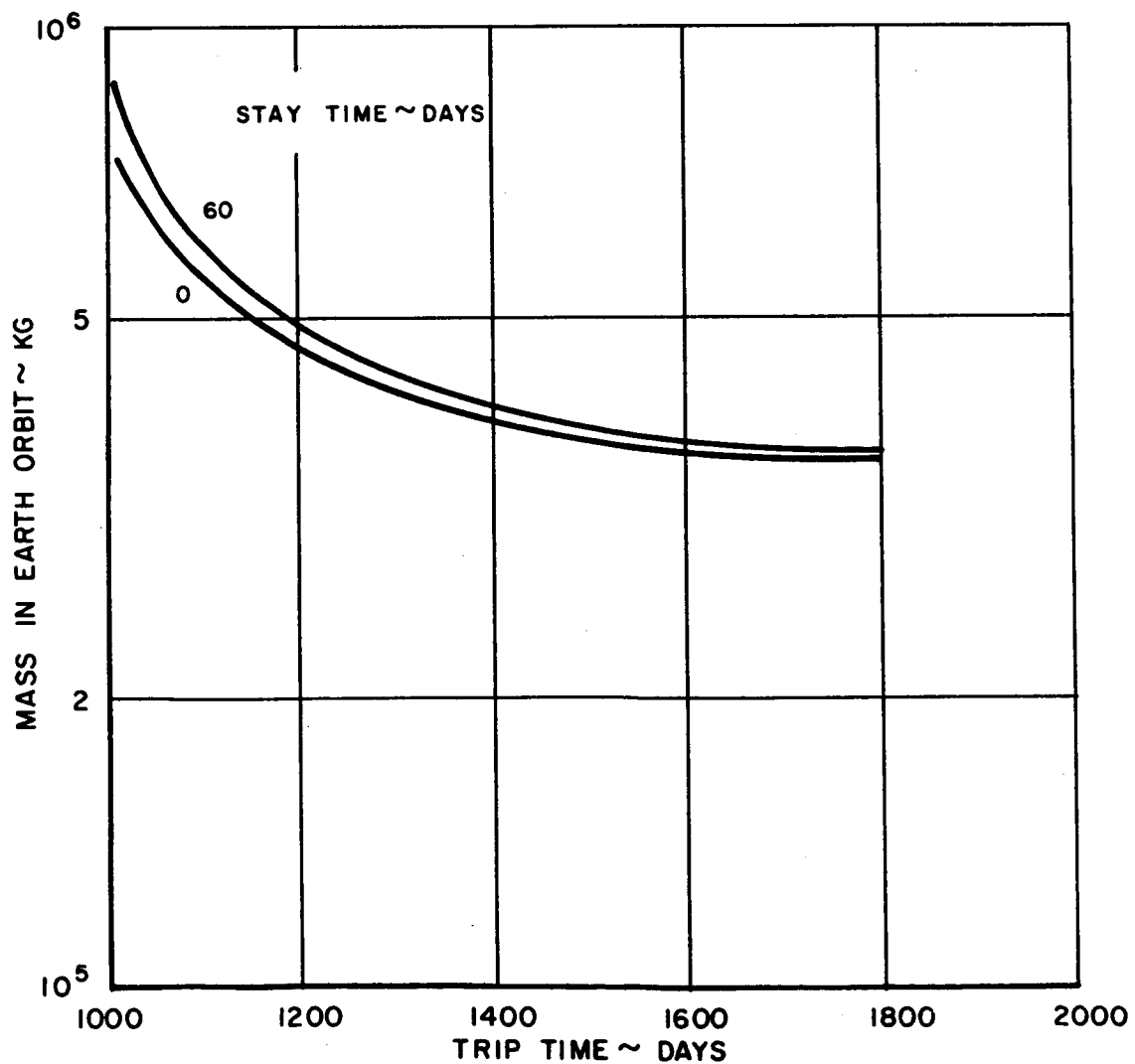
$I_{sp} = 800 \text{ SEC}$



GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

LIGHTWEIGHT ADVANCED NUCLEAR

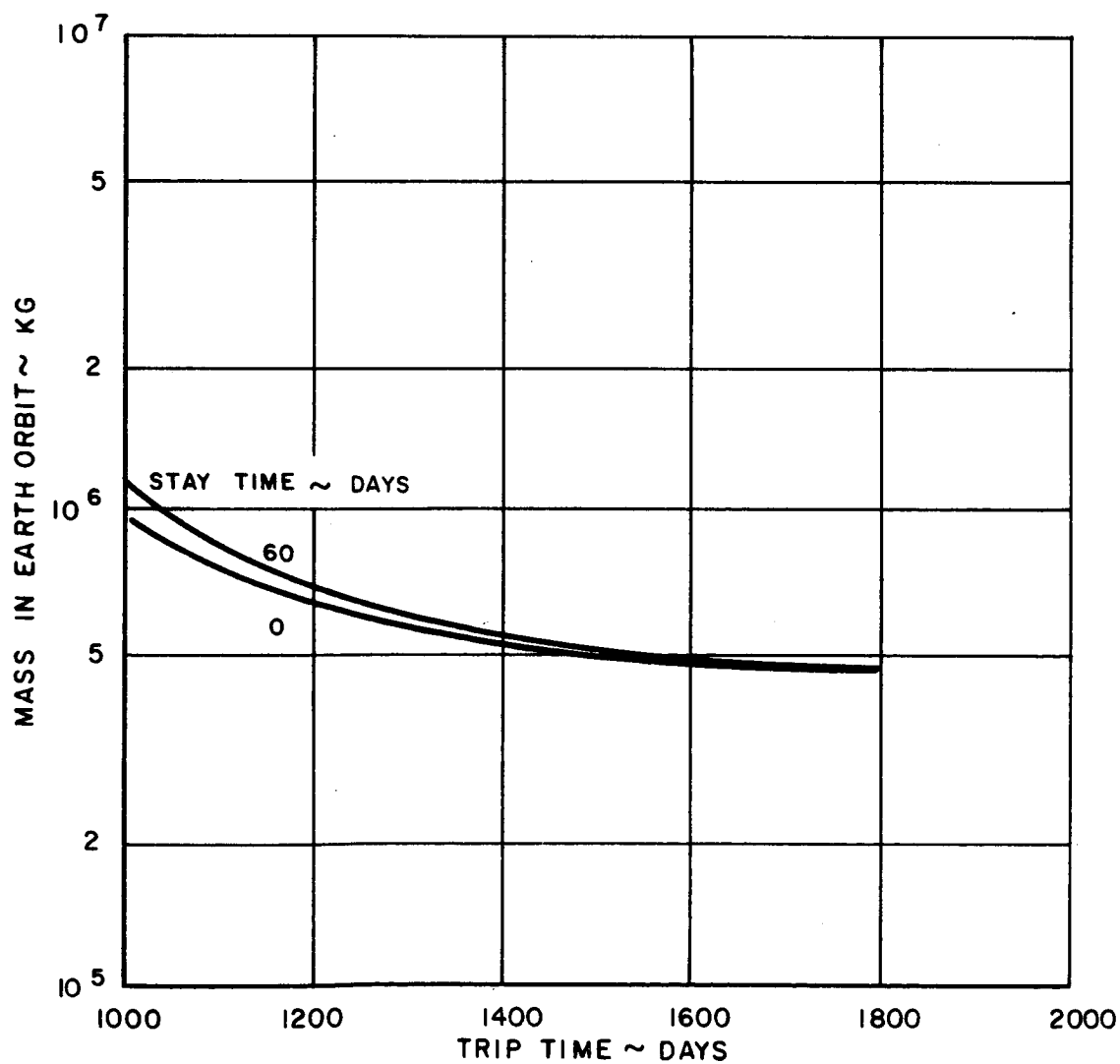
$I_{SP} = 2000 \text{ SEC}$



GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

GASEOUS-CORE NUCLEAR

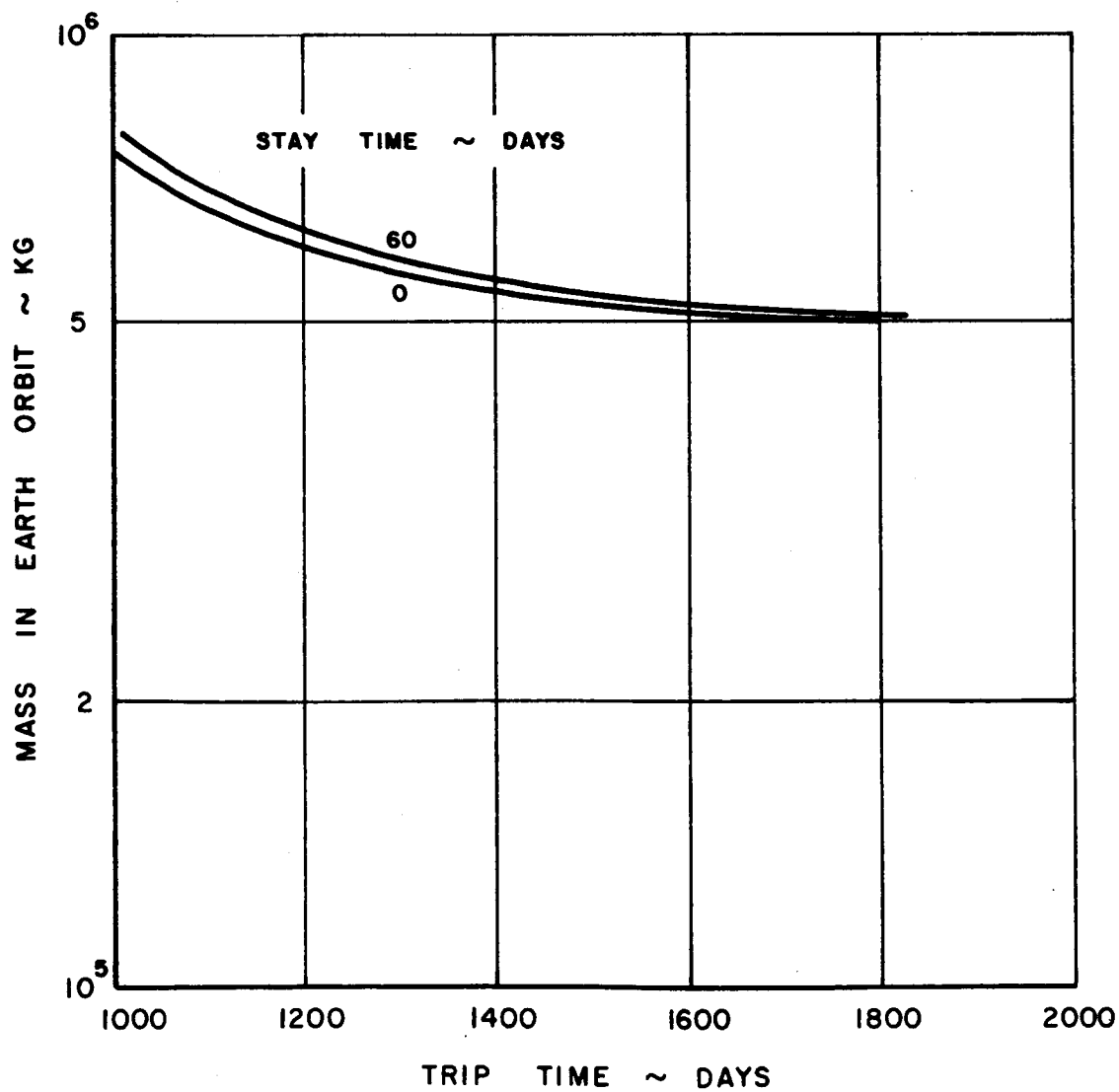
$$I_{SP} = 2000 \text{ SEC}$$



GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

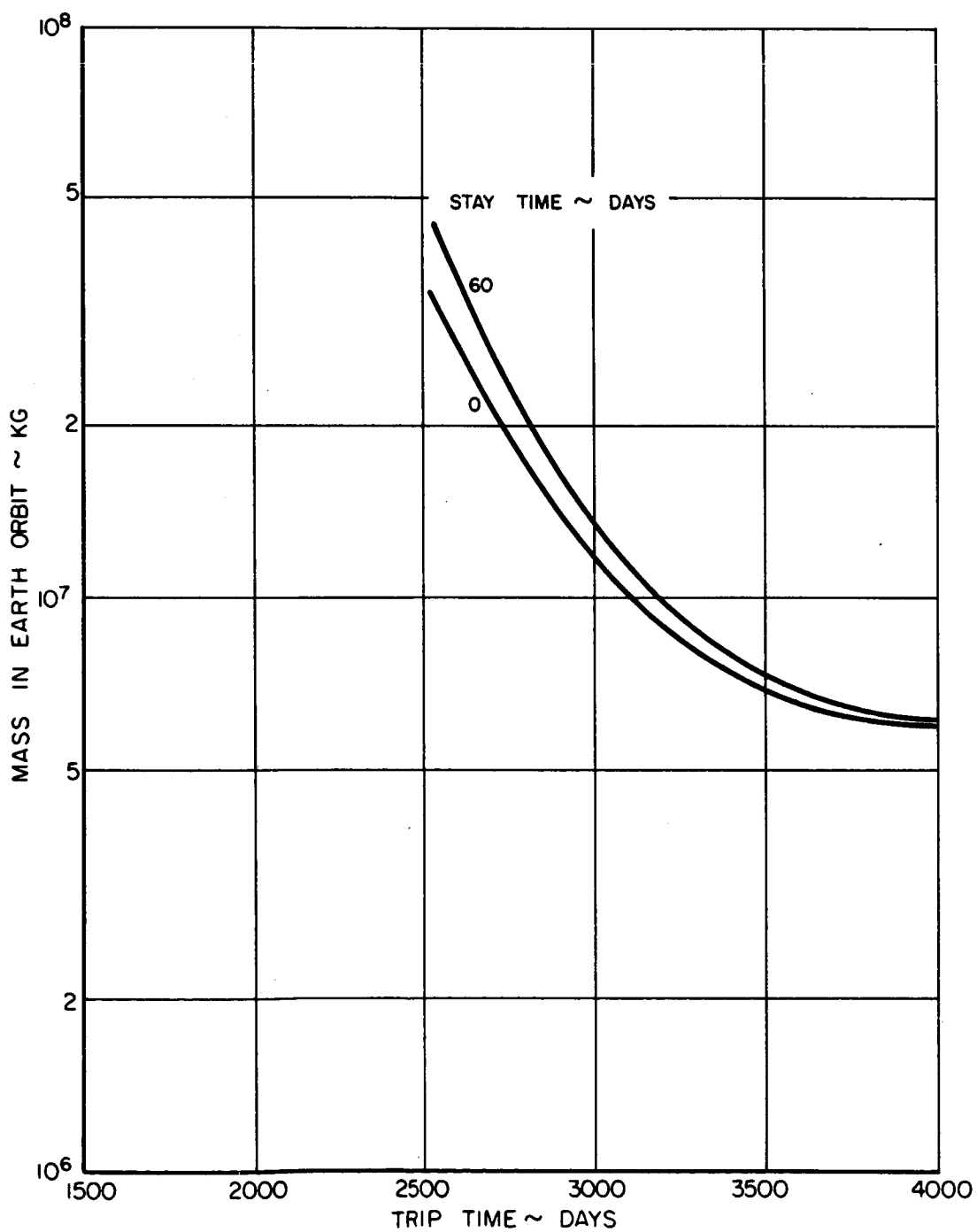
PULSED NUCLEAR

$I_{sp} = 1800 \text{ SEC}$



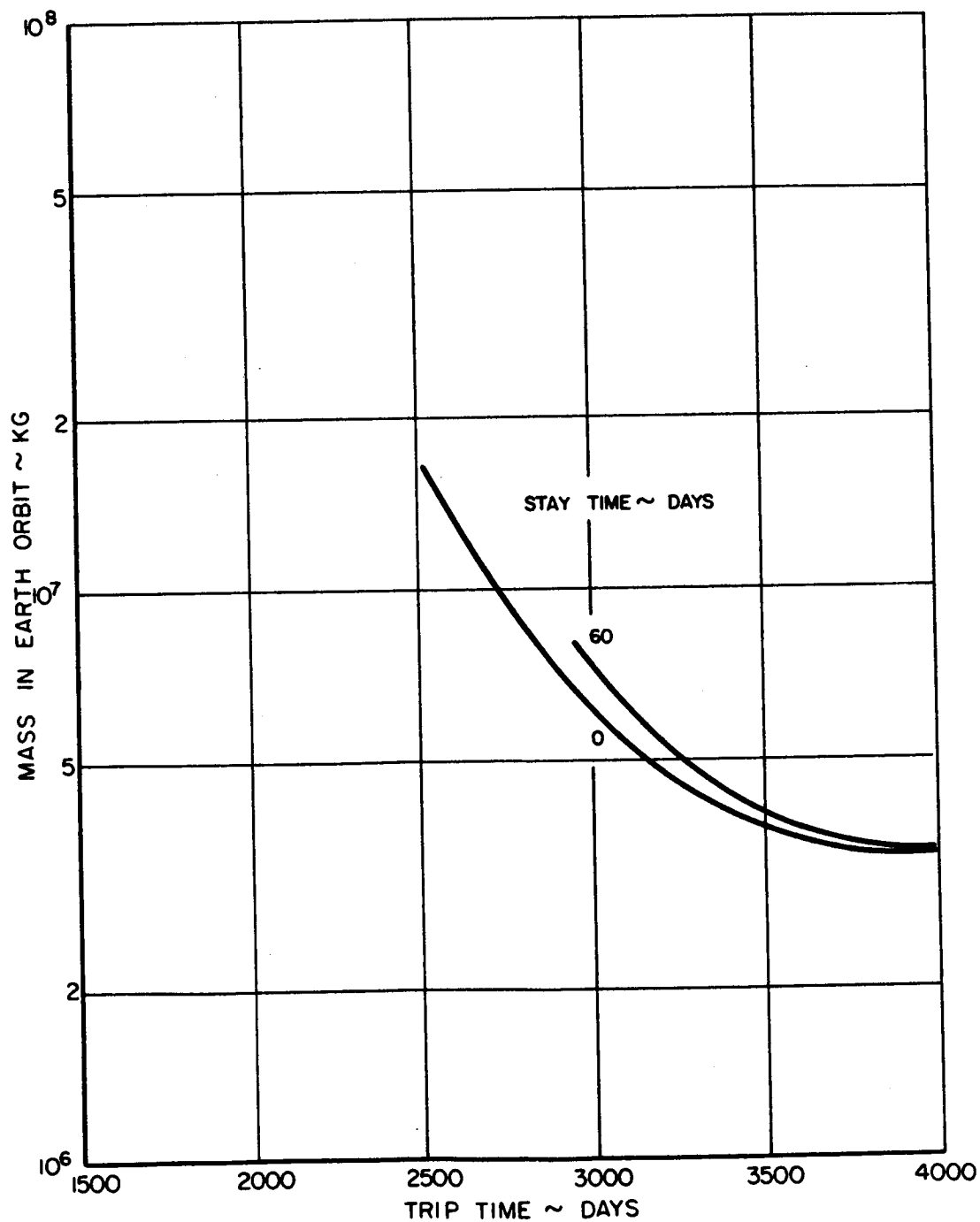
TITAN MANNED ORBITAL STOPOVERS
(SATURN VI)
CHEMICAL

$I_{sp} = 430 \text{ SEC}$



TITAN MANNED ORBITAL STOPOVERS
(SATURN VI)
SOLID-CORE NUCLEAR

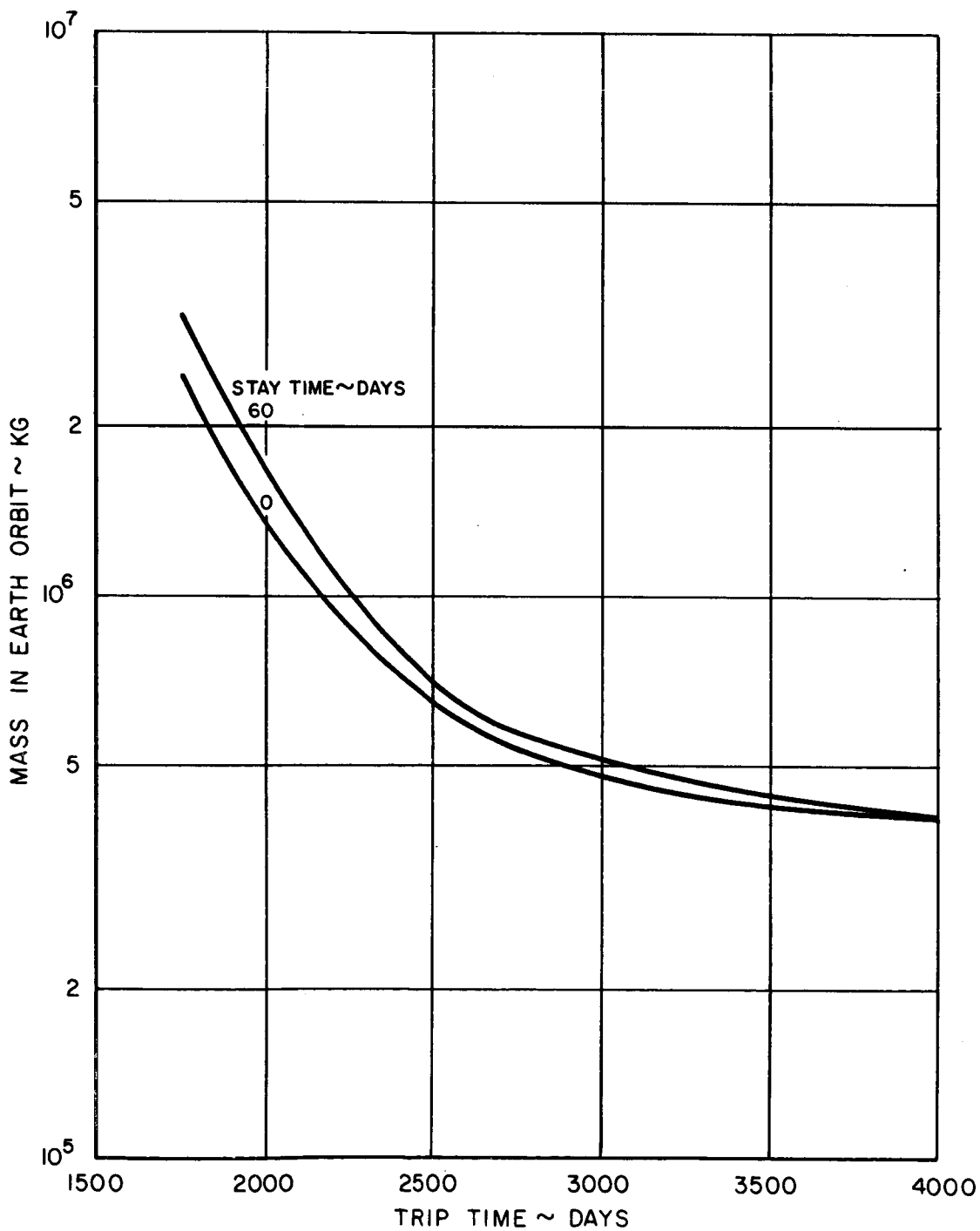
$I_{sp} = 800 \text{ SEC}$



TITAN MANNED ORBITAL STOPOVERS (SATURN VI)

LIGHTWEIGHT ADVANCED NUCLEAR

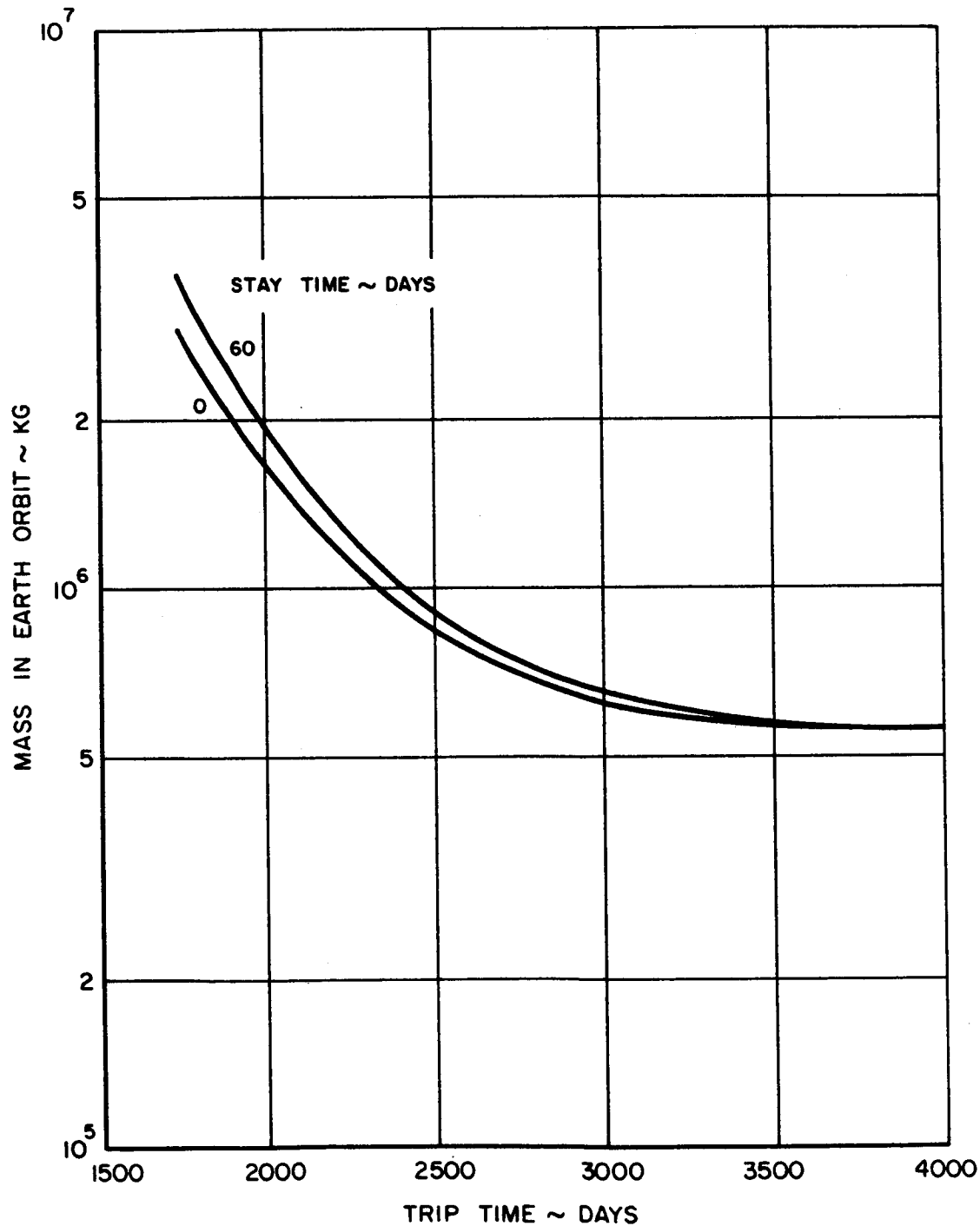
$I_{SP} = 2000 \text{ SEC}$



TITAN MANNED ORBITAL STOPOVERS (SATURN VI)

GASEOUS - CORE NUCLEAR

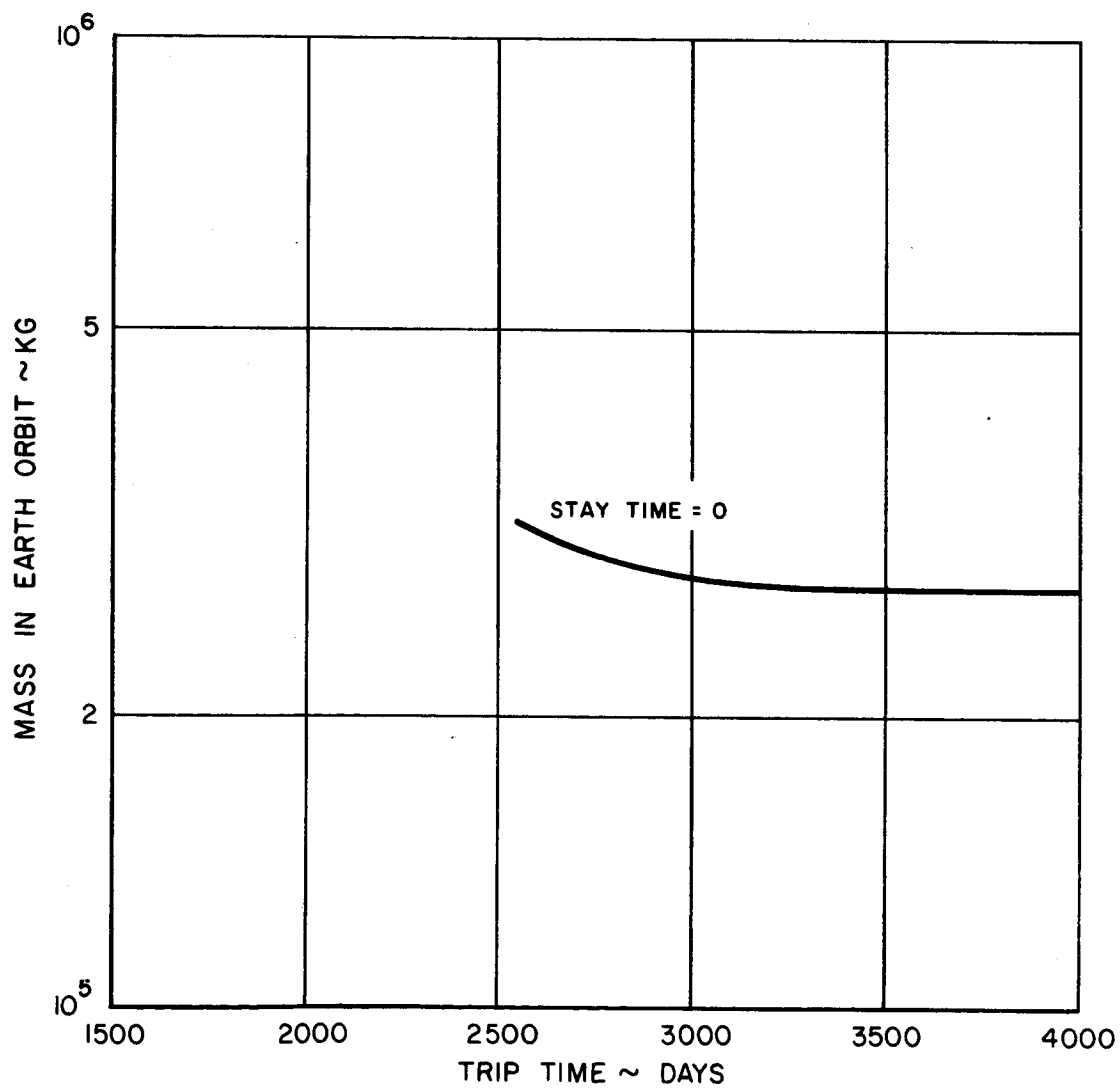
$I_{SP} = 2000 \text{ SEC}$



TITAN MANNED ORBITAL STOPOVERS (SATURN VI)

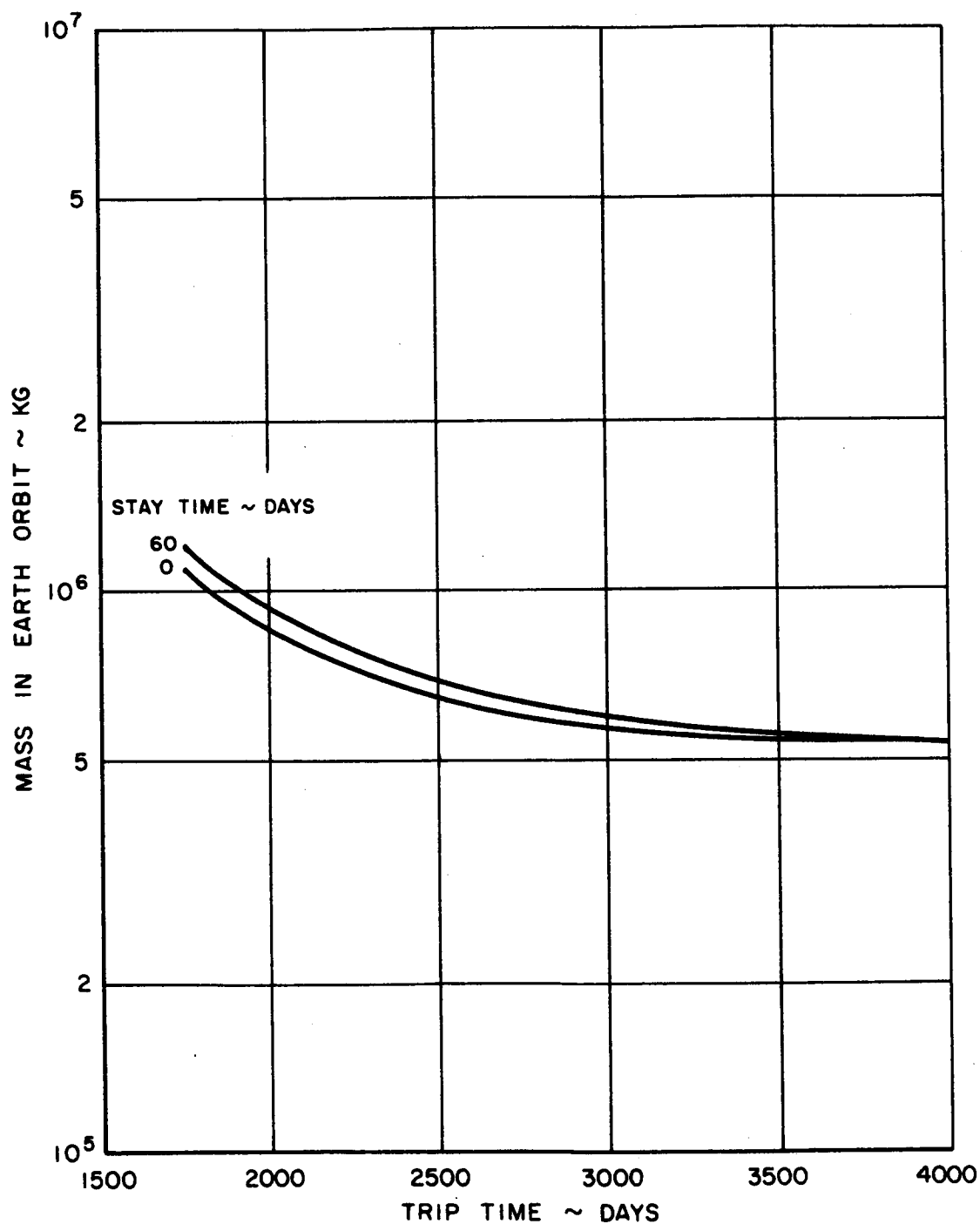
GASEOUS-CORE NUCLEAR

$I_{sp} = 3500 \text{ SEC}$



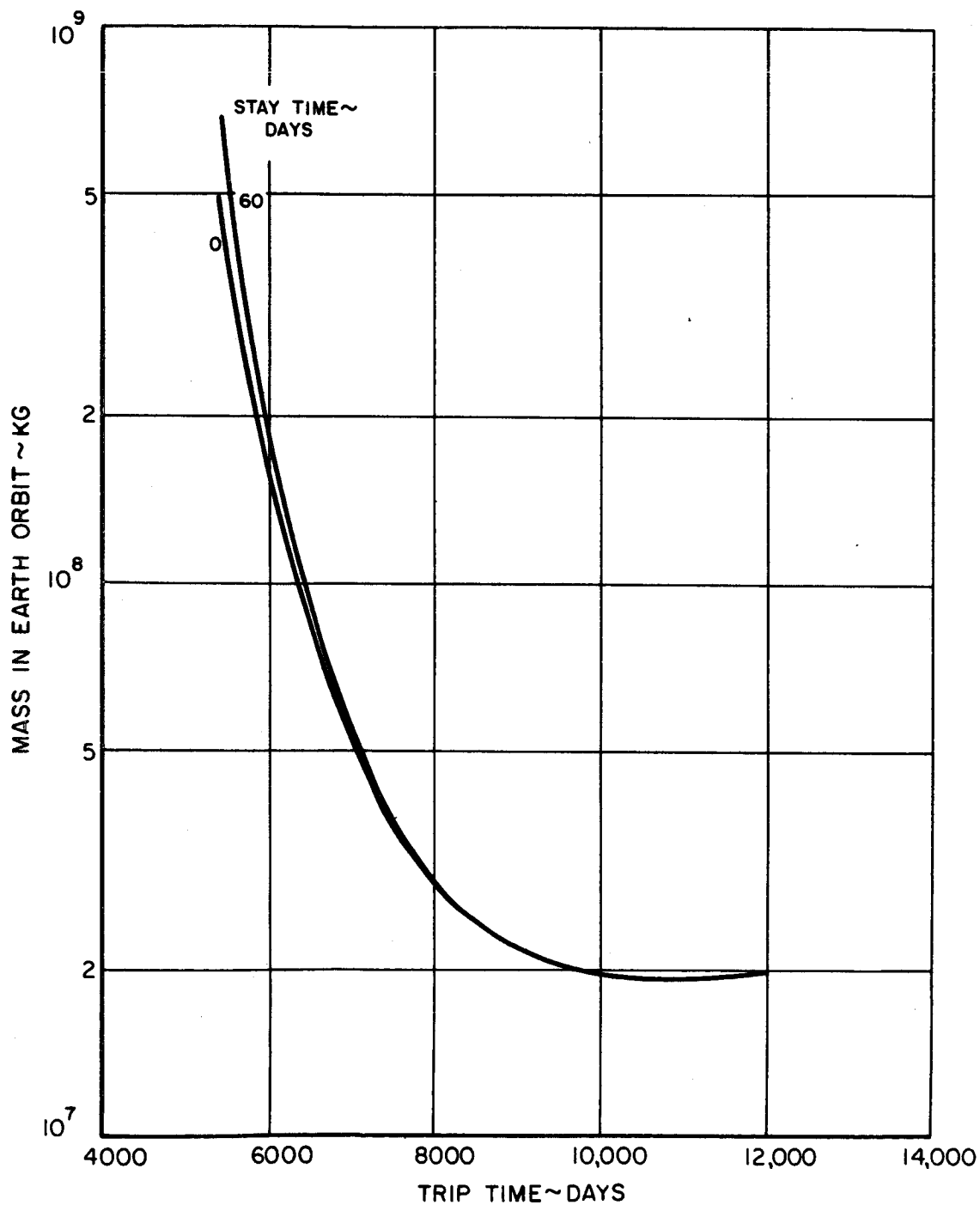
TITAN MANNED ORBITAL STOPOVERS
(SATURN VI)
PULSED NUCLEAR

$$I_{SP} = 1800 \text{ SEC}$$



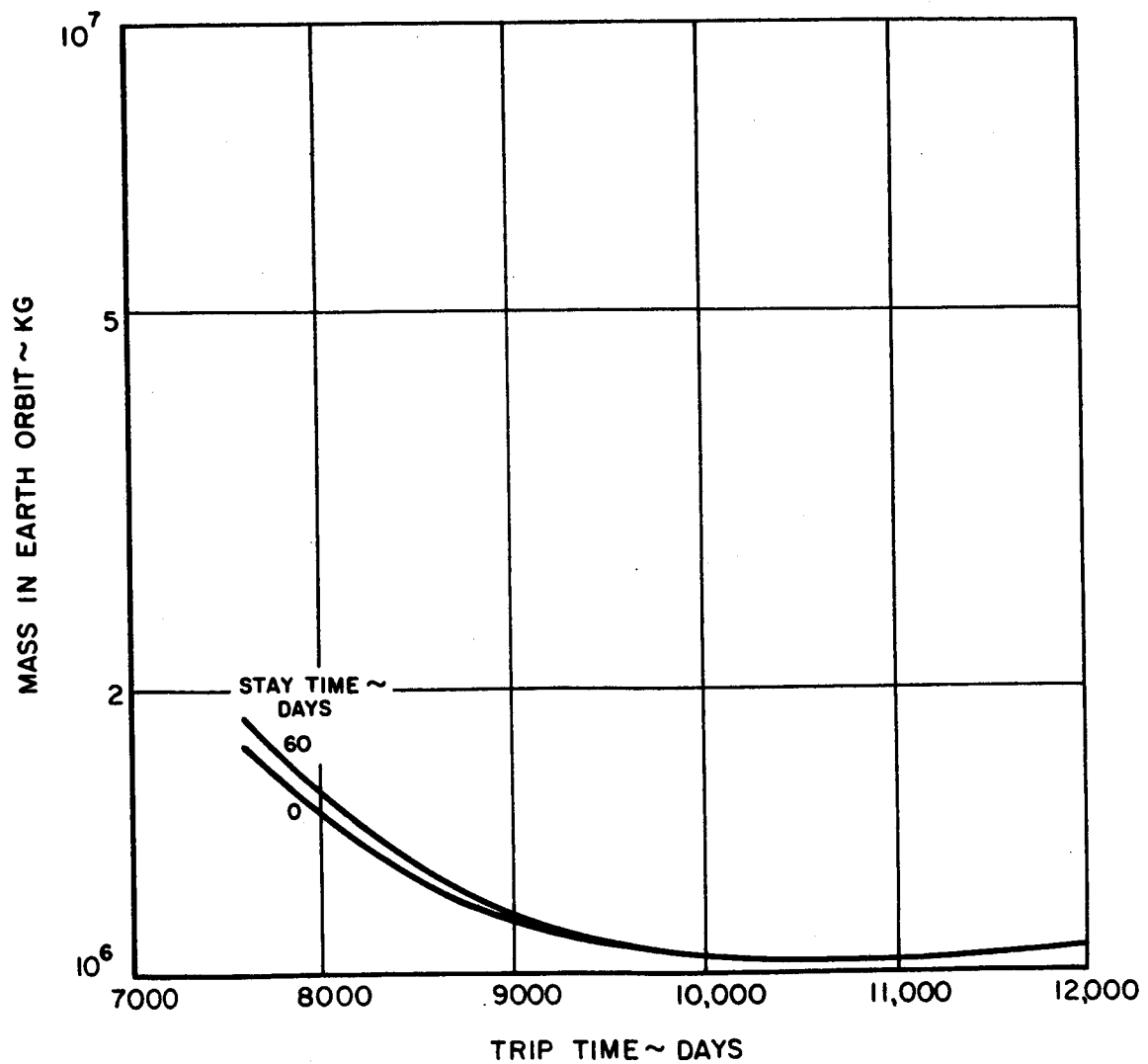
URANUS MANNED ORBITAL STOPOVERS

CHEMICAL

 $I_{sp} = 430 \text{ SEC}$ 

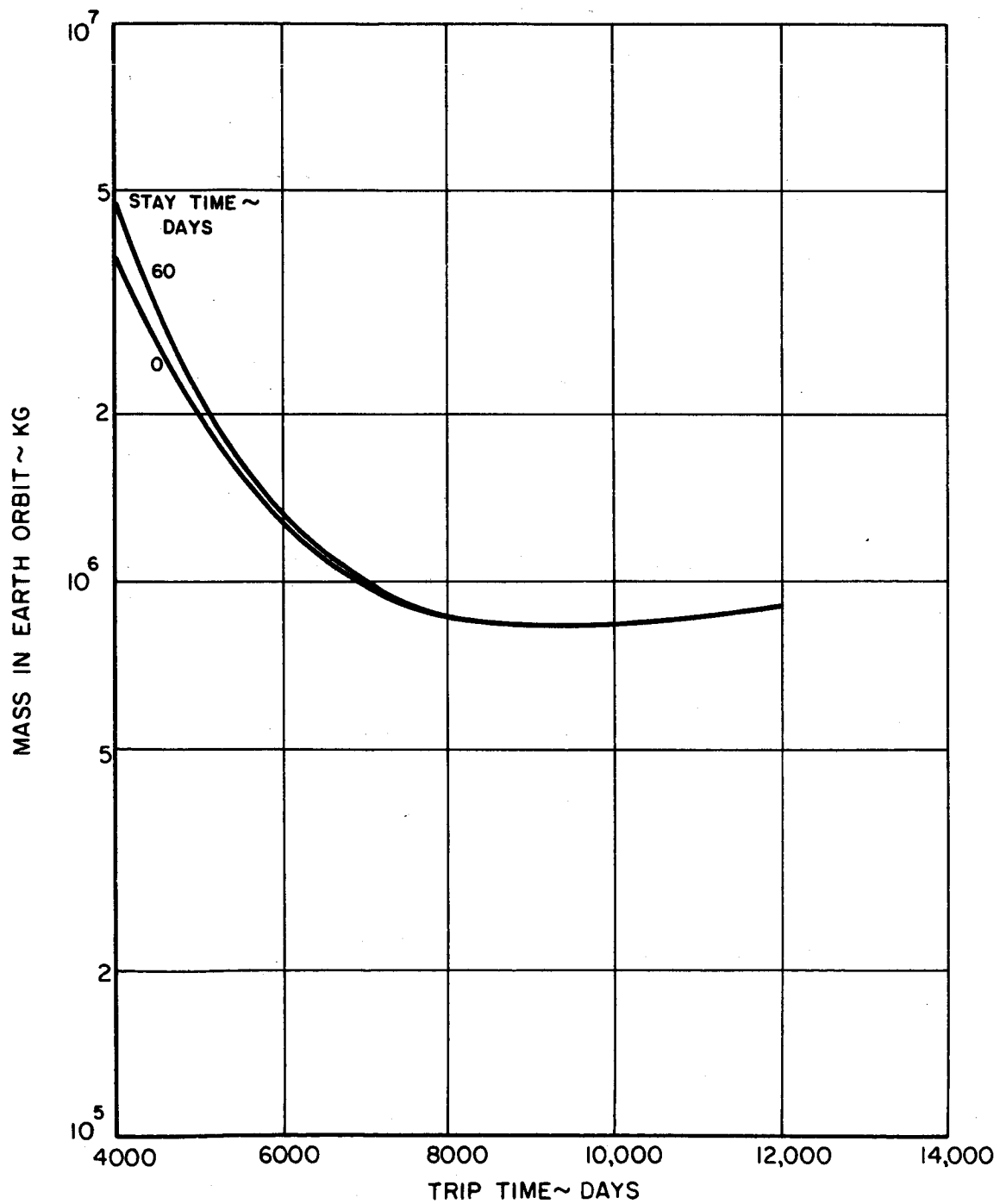
URANUS MANNED ORBITAL STOPOVERS

SOLID-CORE NUCLEAR

 $I_{SP} = 800 \text{ SEC}$ 

URANUS MANNED ORBITAL STOPOVERS

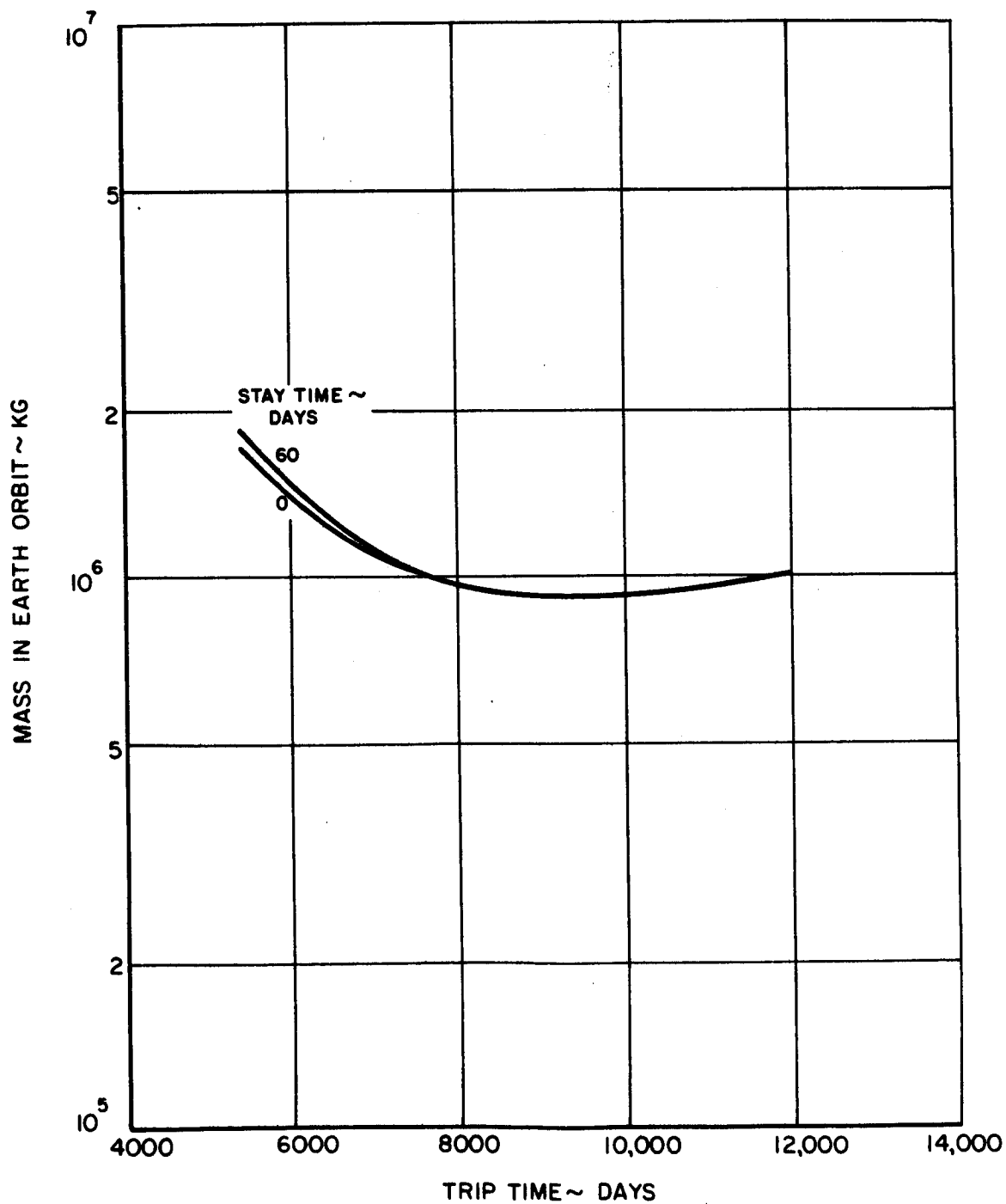
LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ 

URANUS MANNED ORBITAL STOPOVERS

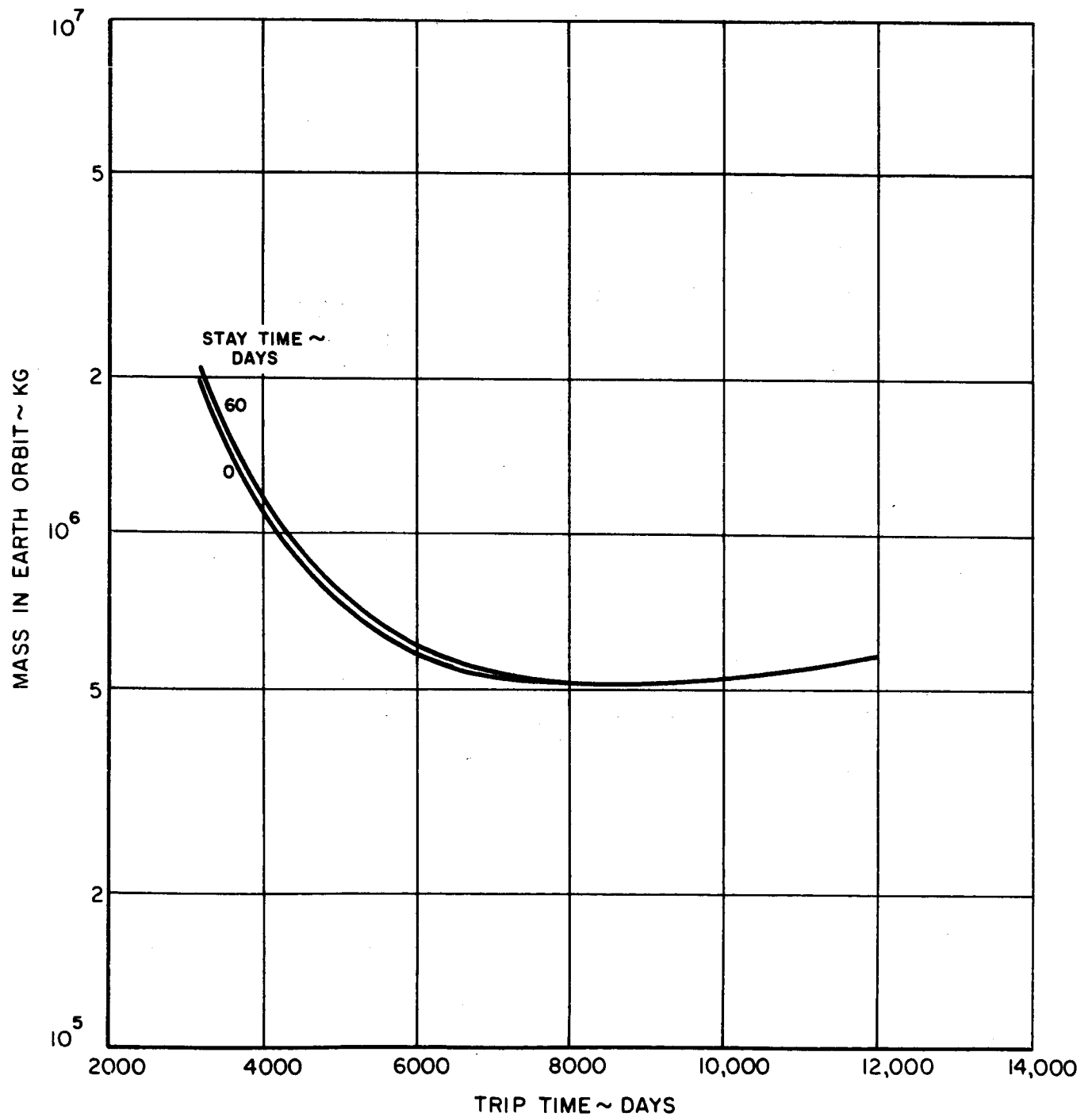
E-910352-9

GASEOUS-CORE NUCLEAR

 $I_{sp} = 2000 \text{ SEC}$ 

URANUS MANNED ORBITAL STOPOVERS

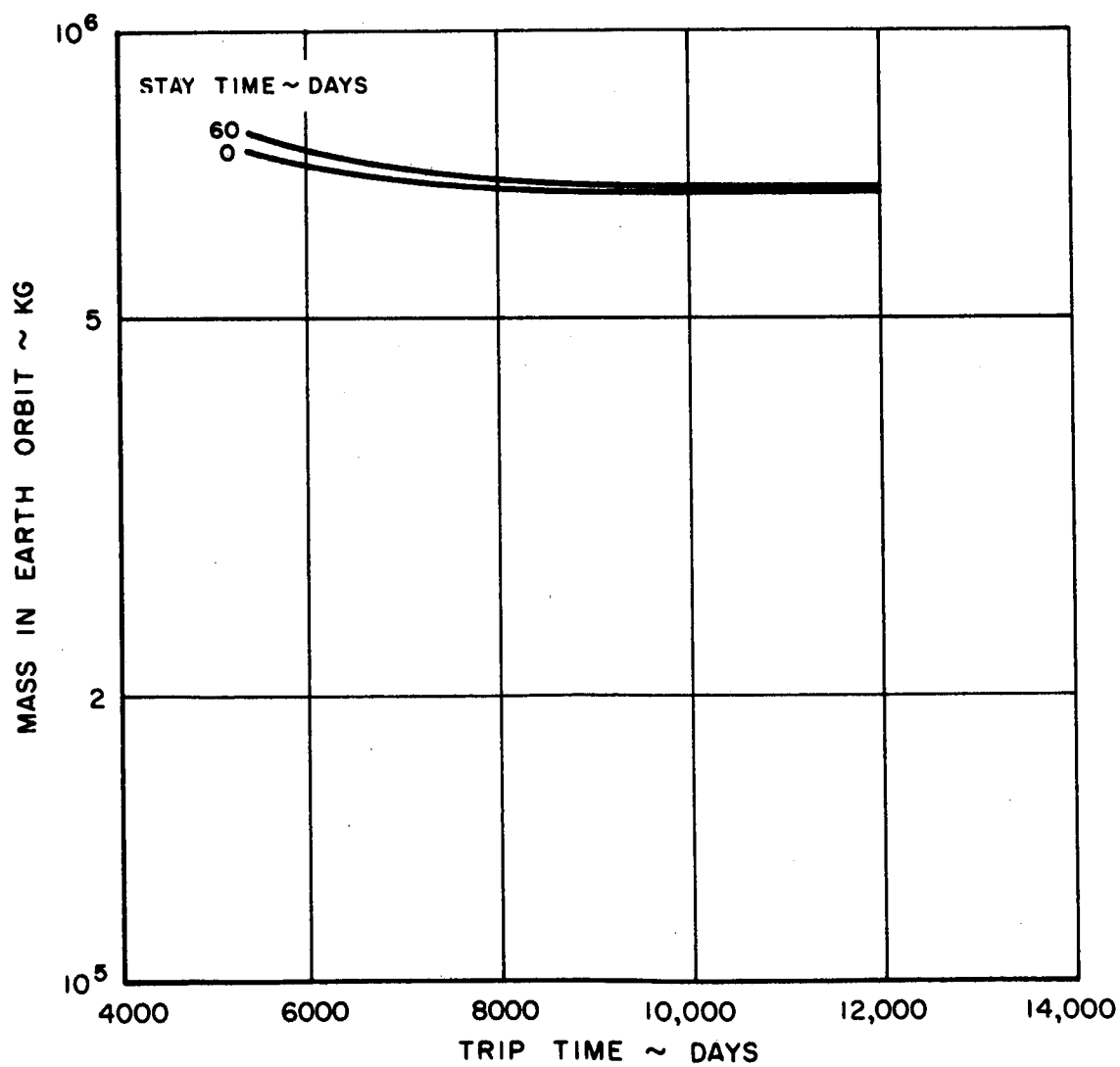
GASEOUS-CORE NUCLEAR

 $I_{sp} = 3500 \text{ SEC}$ 

URANUS MANNED ORBITAL STOPOVERS

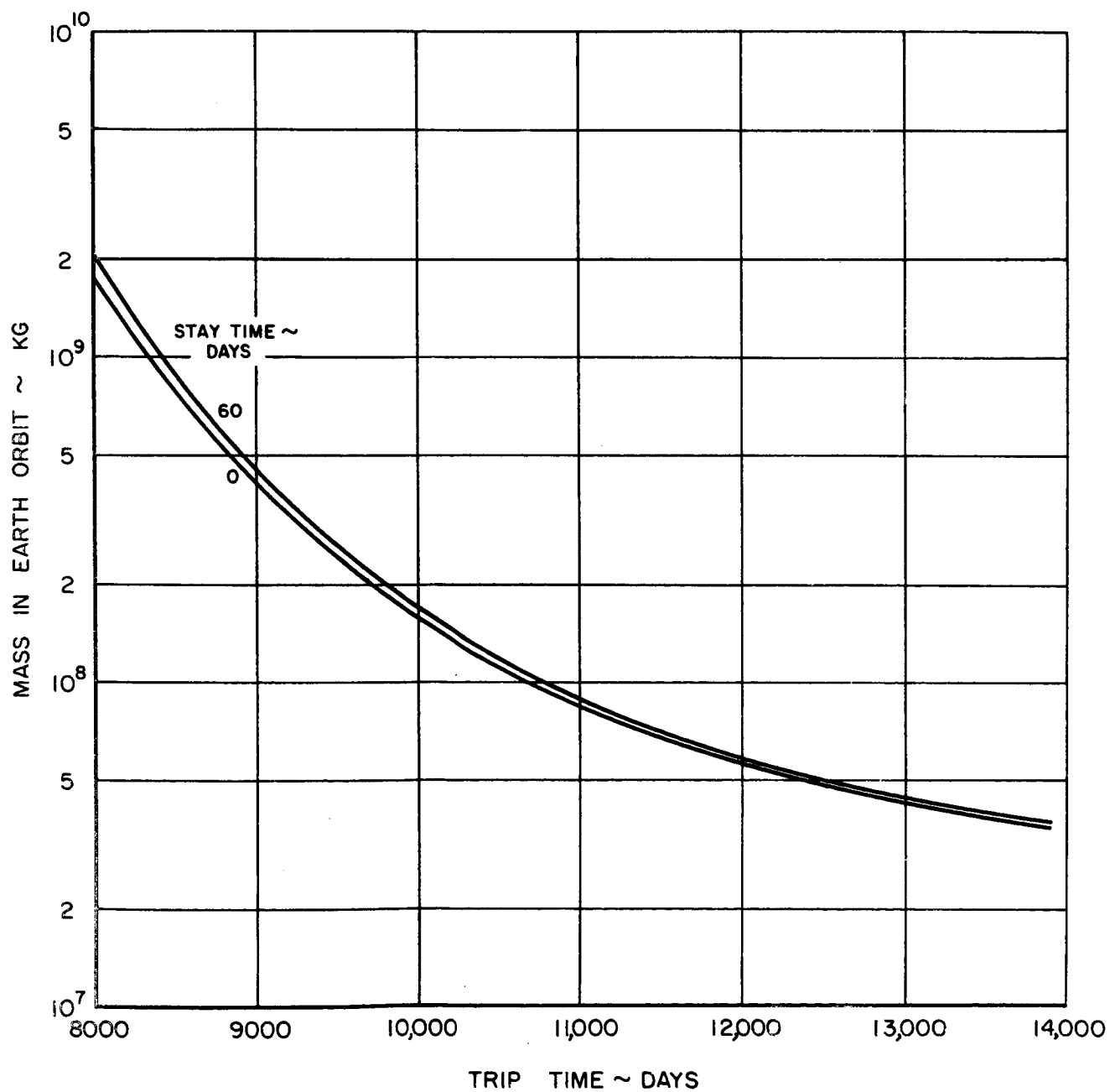
PULSED NUCLEAR

$$I_{sp} = 1800 \text{ SEC}$$



NEPTUNE MANNED ORBITAL STOPOVERS

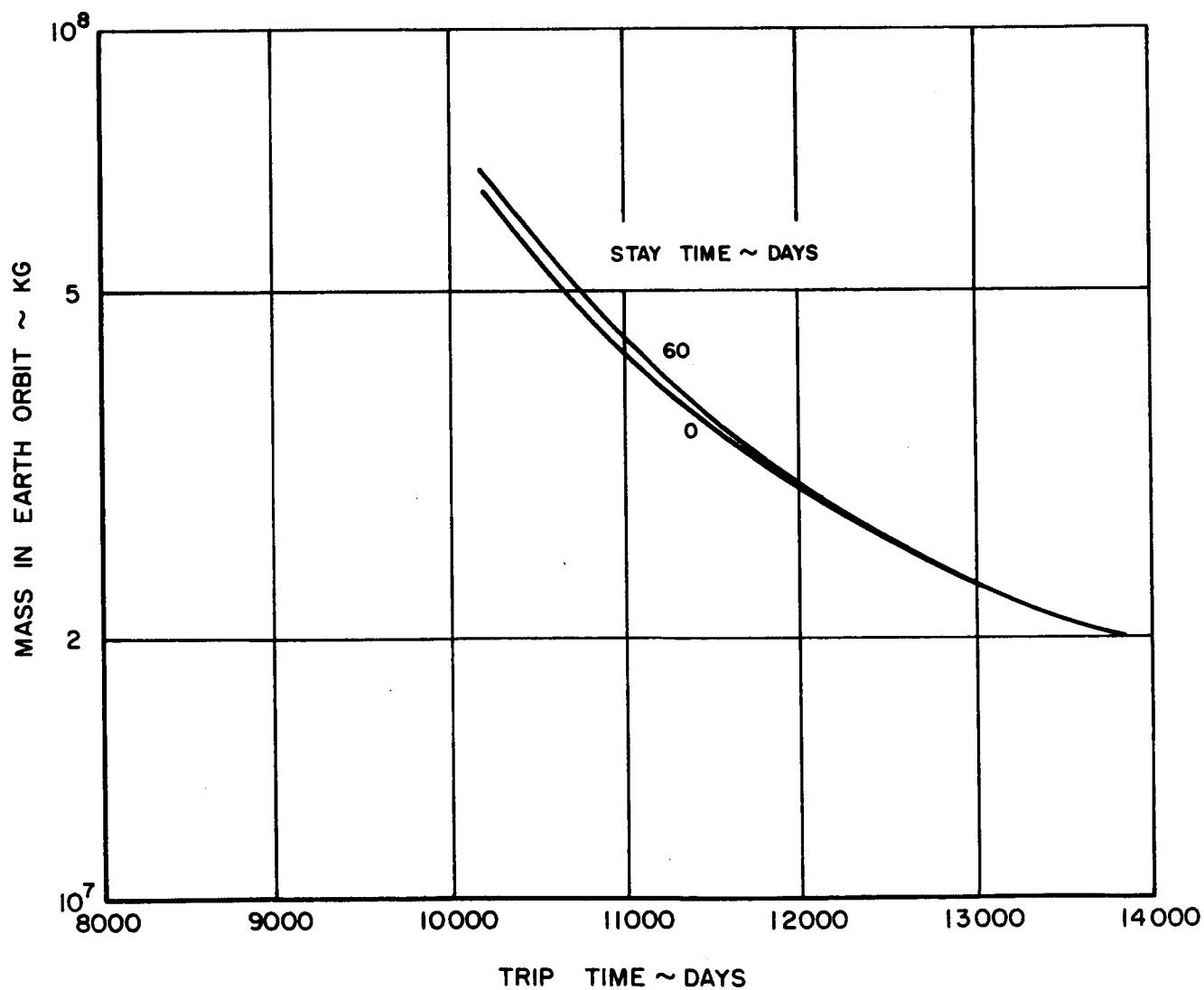
CHEMICAL

 $I_{SP} = 430 \text{ SEC}$ 

NEPTUNE MANNED ORBITAL STOPOVERS

SOLID-CORE NUCLEAR

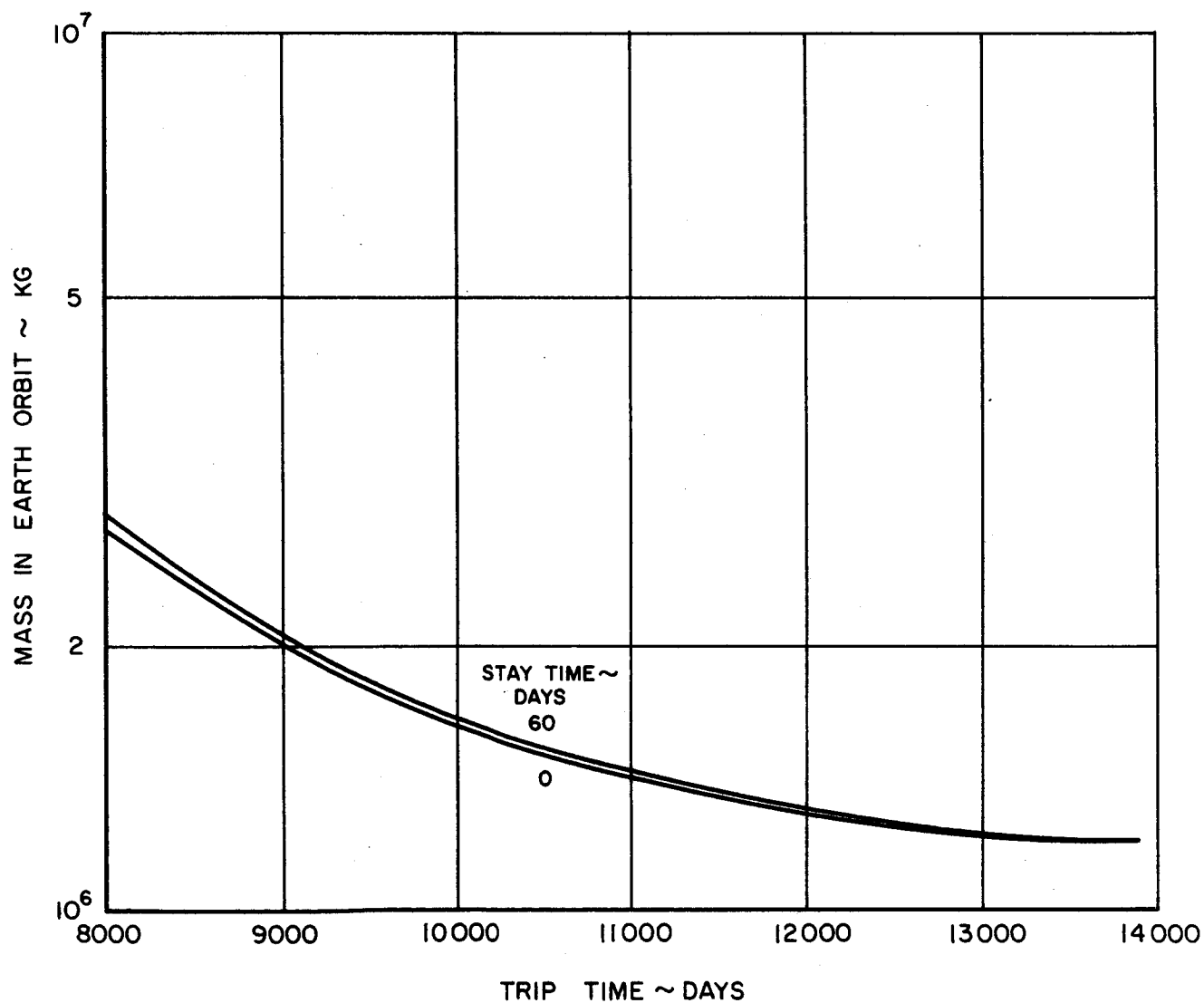
$$I_{SP} = 800 \text{ SEC}$$



NEPTUNE MANNED ORBITAL STOPOVERS

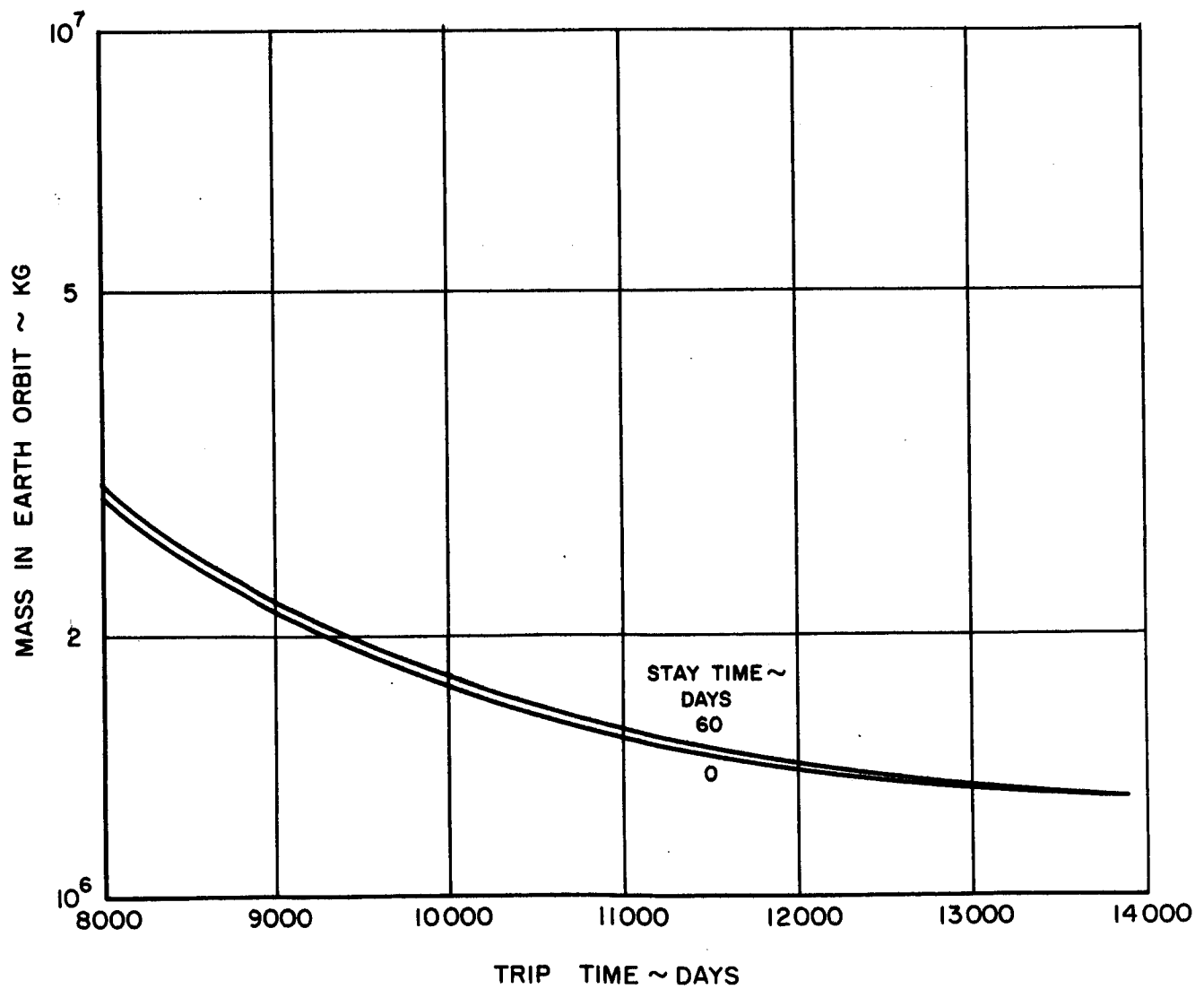
LIGHTWEIGHT ADVANCED NUCLEAR

$$I_{SP} = 2000 \text{ SEC}$$



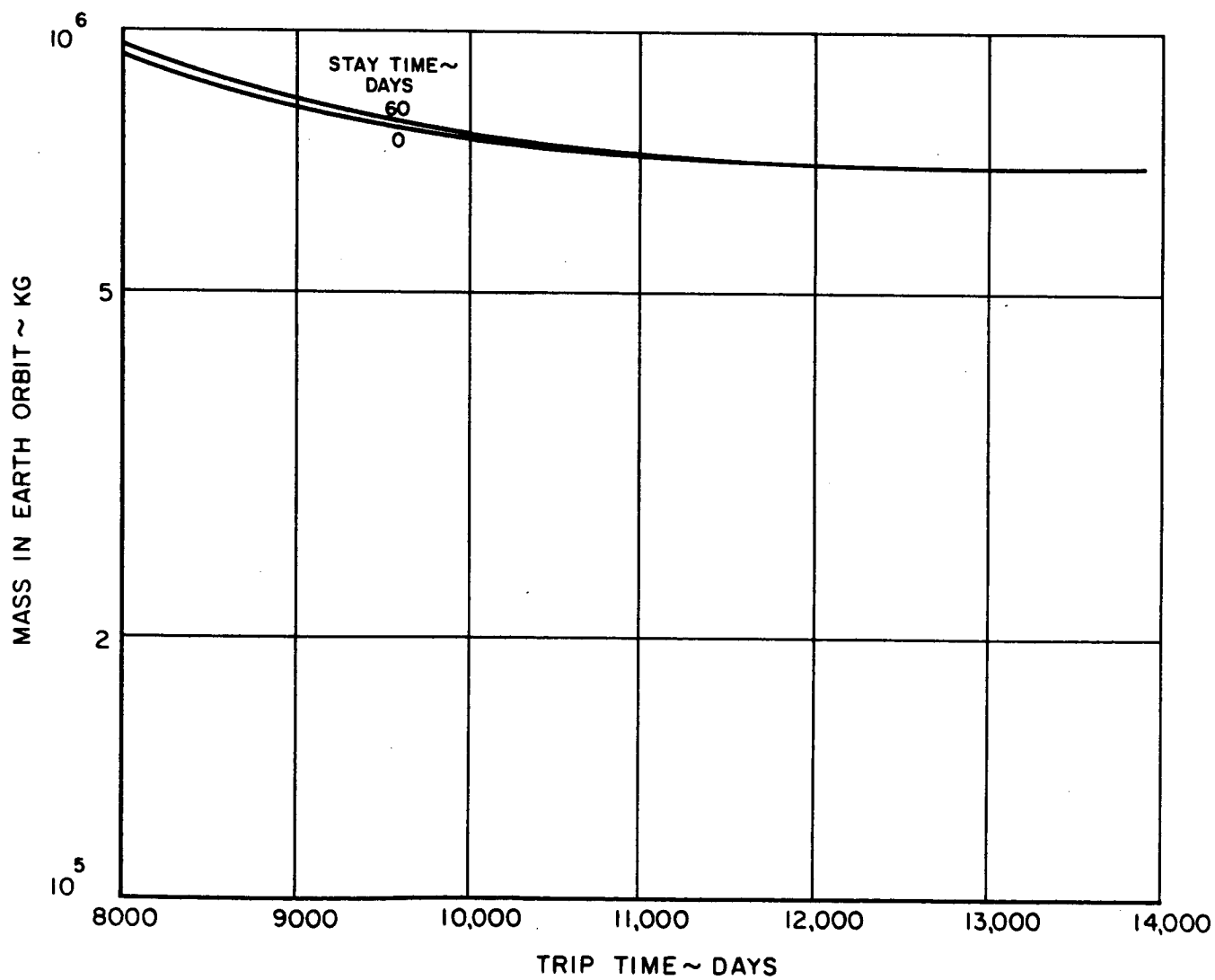
NEPTUNE MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ 

NEPTUNE MANNED ORBITAL STOPOVERS

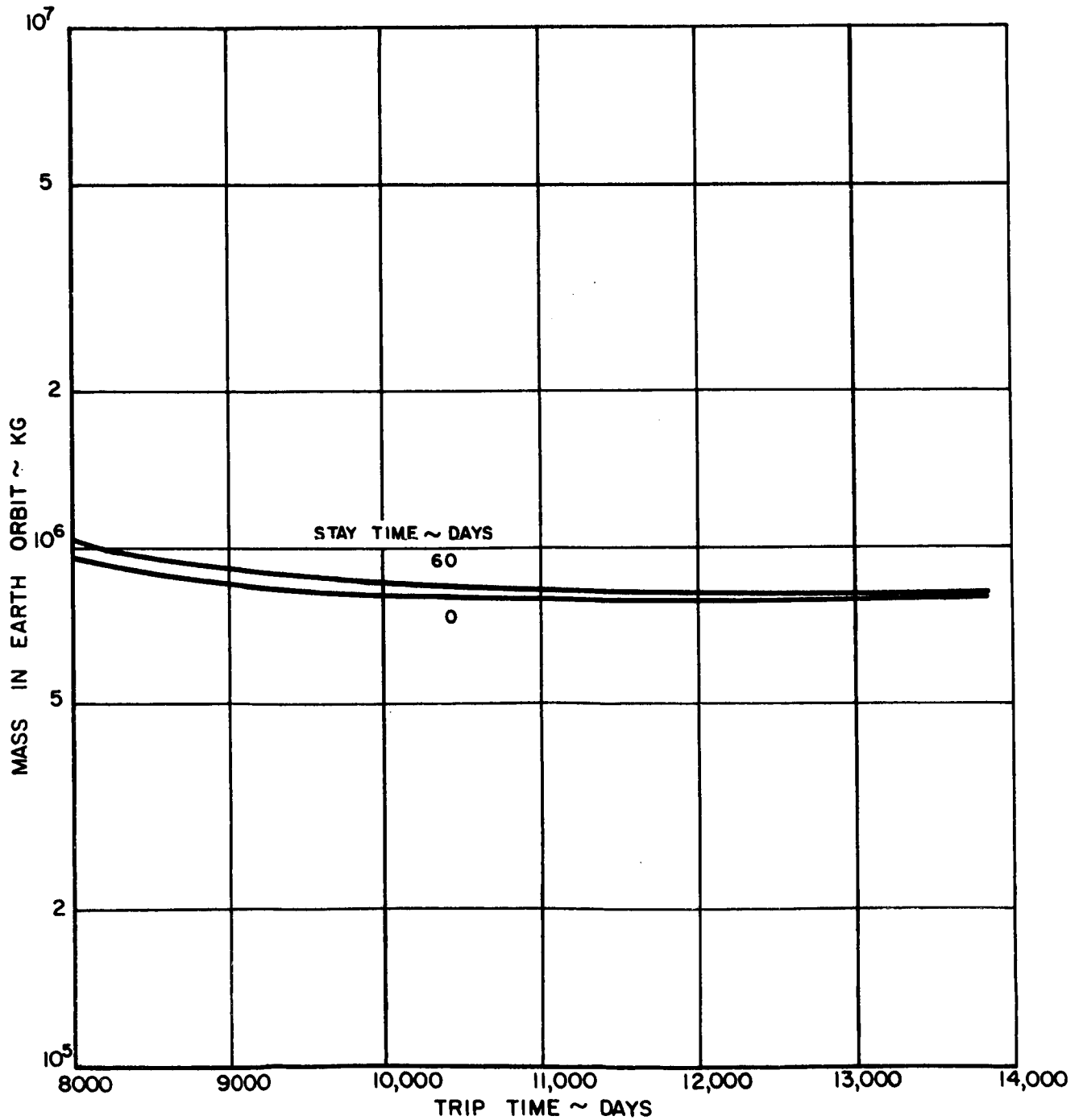
GASEOUS-CORE NUCLEAR

 $I_{sp} = 3500 \text{ SEC}$ 

NEPTUNE MANNED ORBITAL STOPOVERS

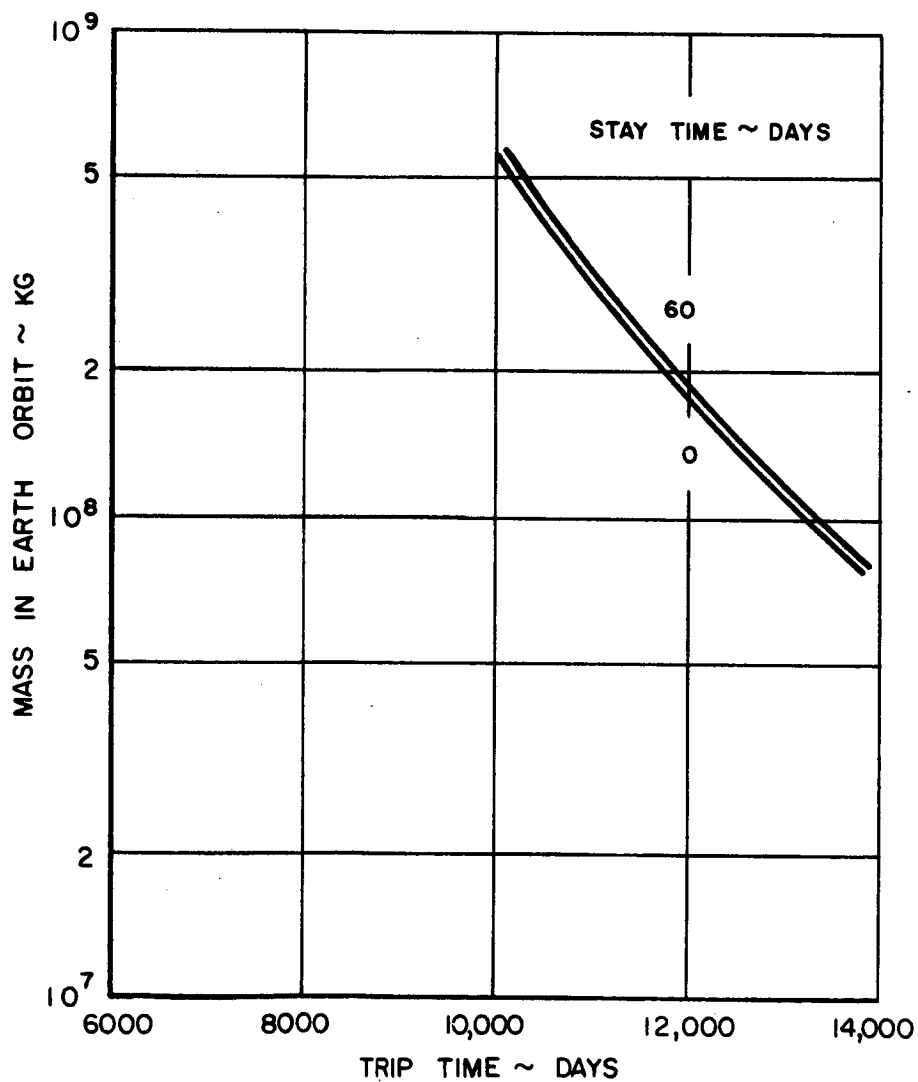
PULSED NUCLEAR

$$I_{sp} = 1800 \text{ SEC}$$



PLUTO MANNED ORBITAL STOPOVERS

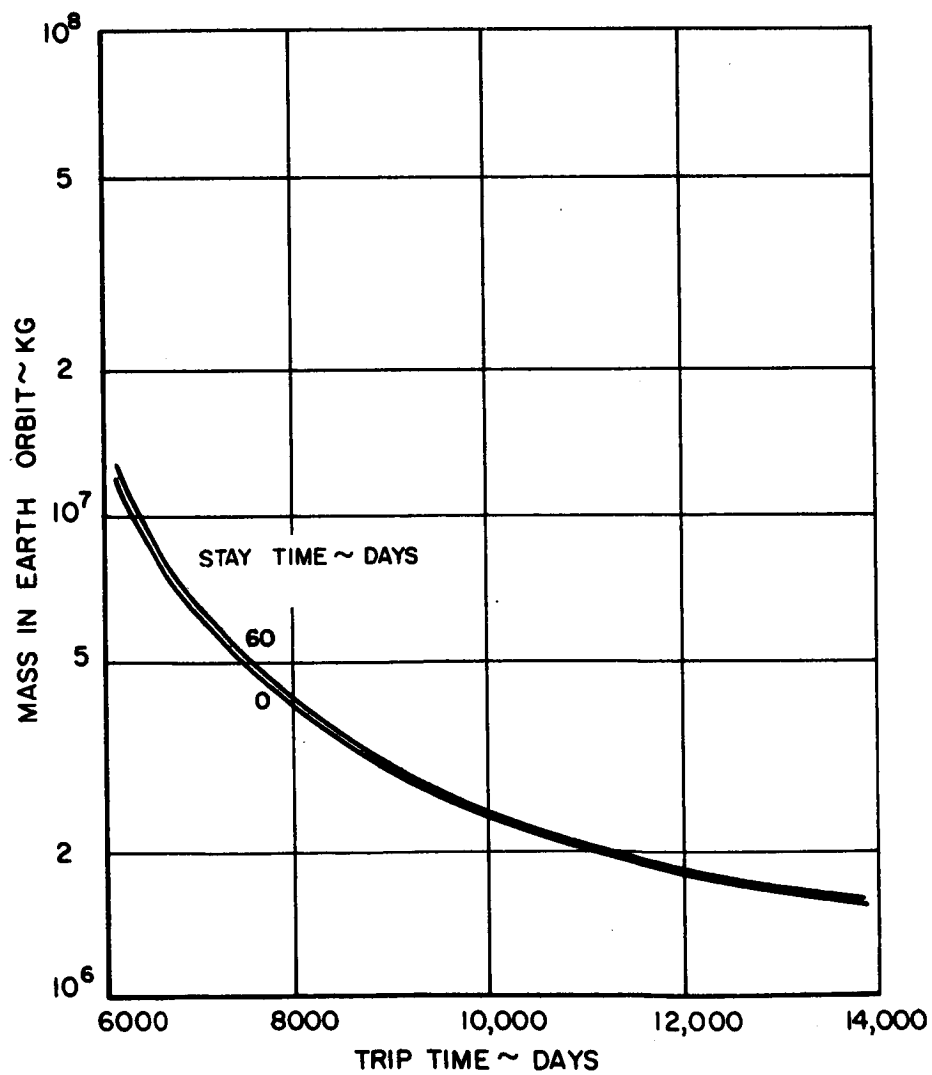
CHEMICAL

 $I_{SP} = 430 \text{ SEC}$ 

PLUTO MANNED ORBITAL STOPOVERS

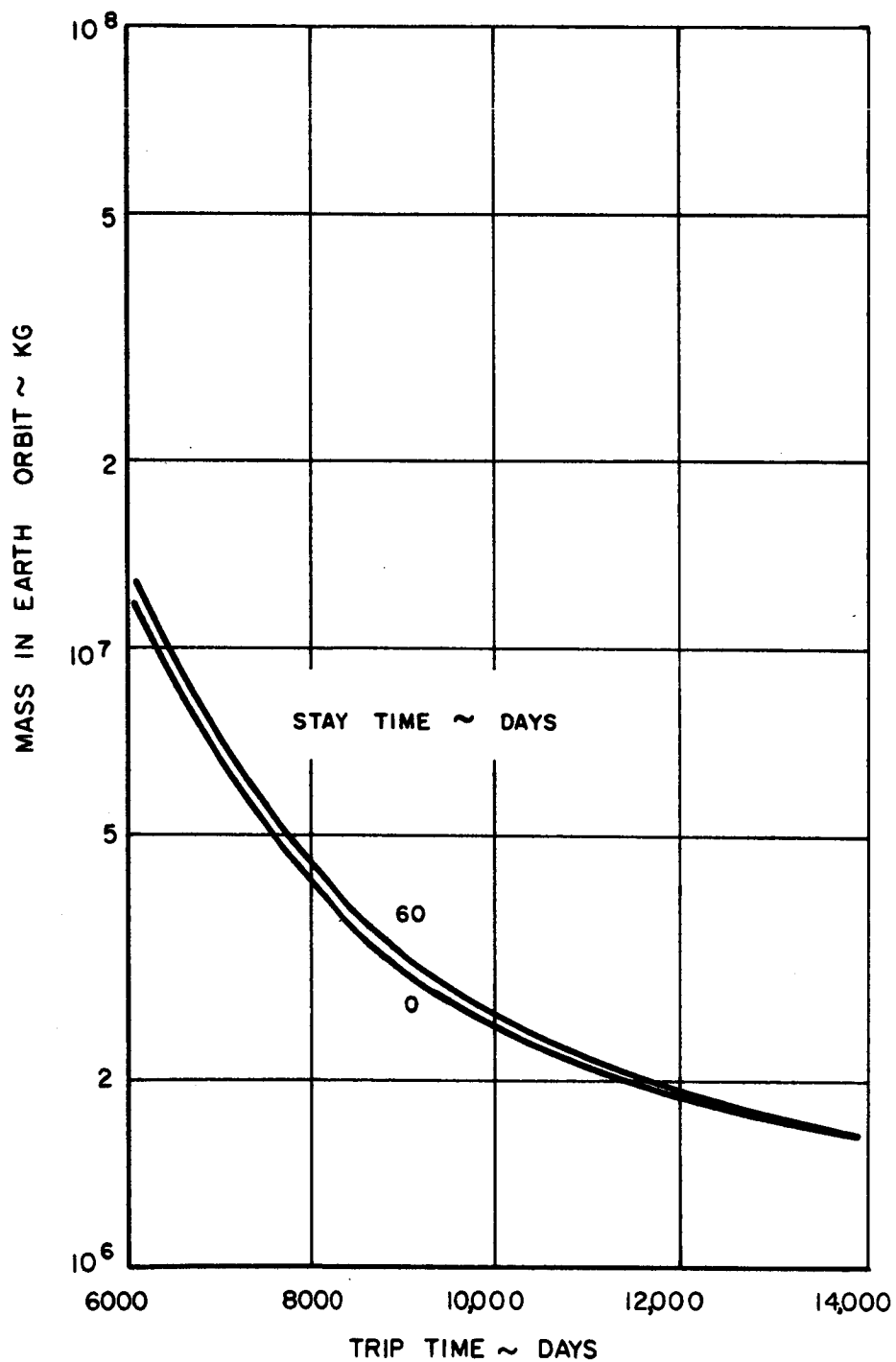
LIGHTWEIGHT ADVANCED NUCLEAR

$$I_{sp} = 2000 \text{ SEC}$$



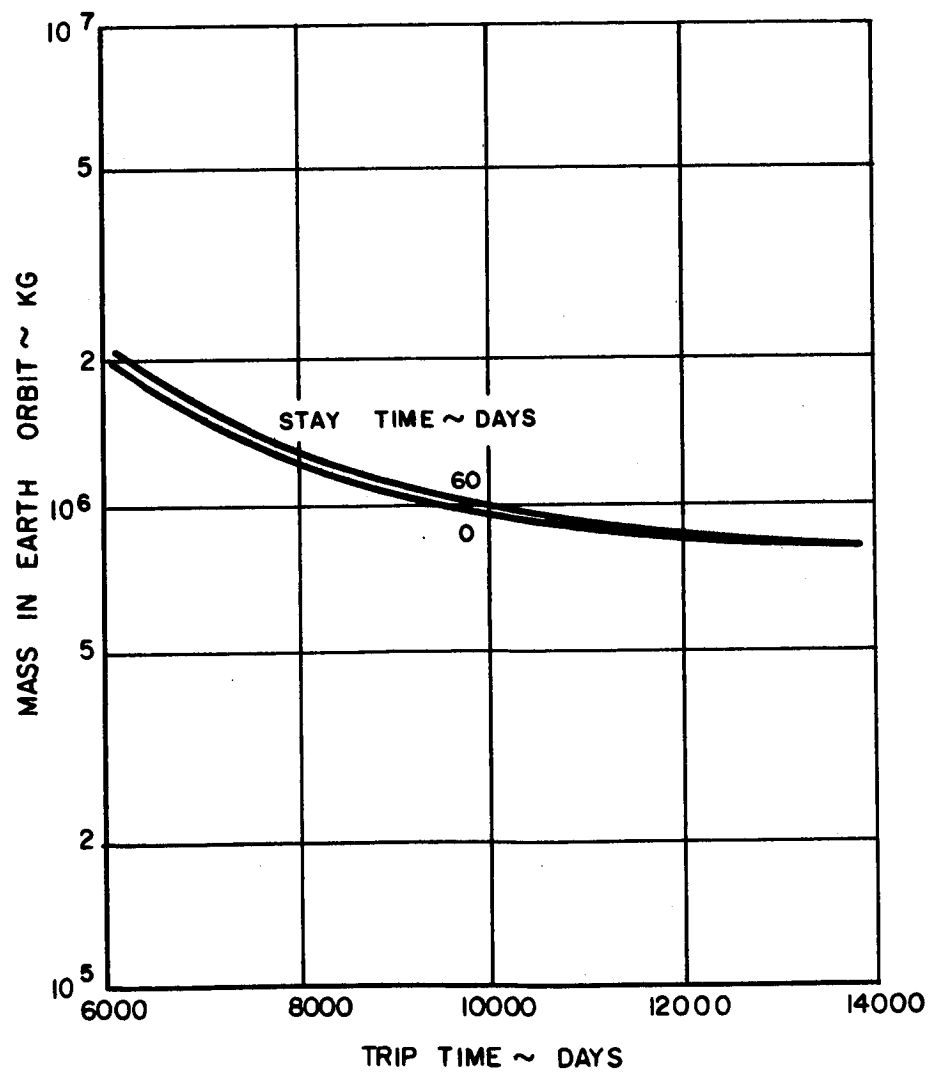
PLUTO MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR

 $I_{sp} = 2000 \text{ SEC}$ 

PLUTO MANNED ORBITAL STOPOVERS

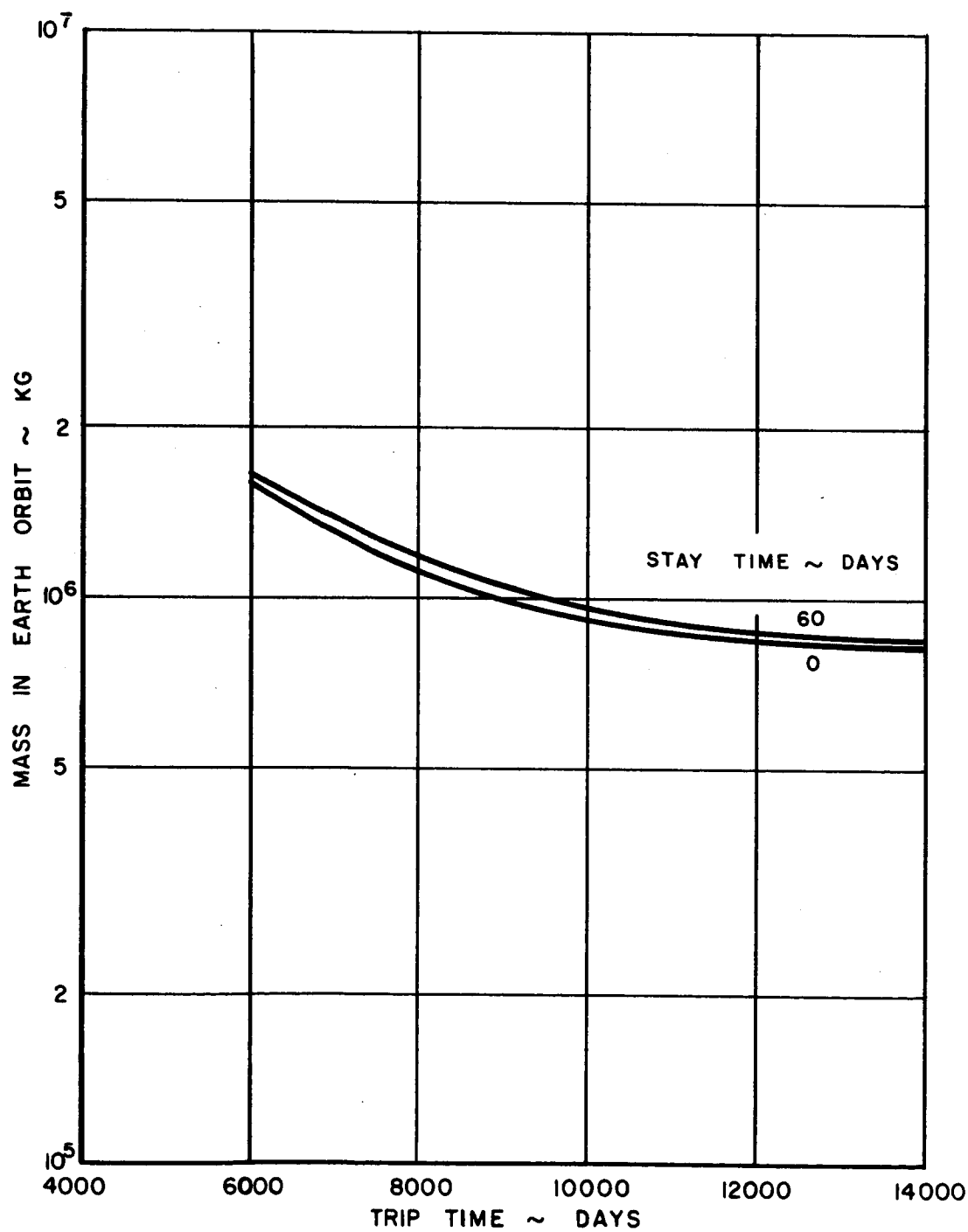
GASEOUS-CORE NUCLEAR

 $I_{SP} = 3500 \text{ SEC}$ 

PLUTO MANNED ORBITAL STOPOVERS

PULSED NUCLEAR

$$I_{SP} = 1800 \text{ SEC}$$



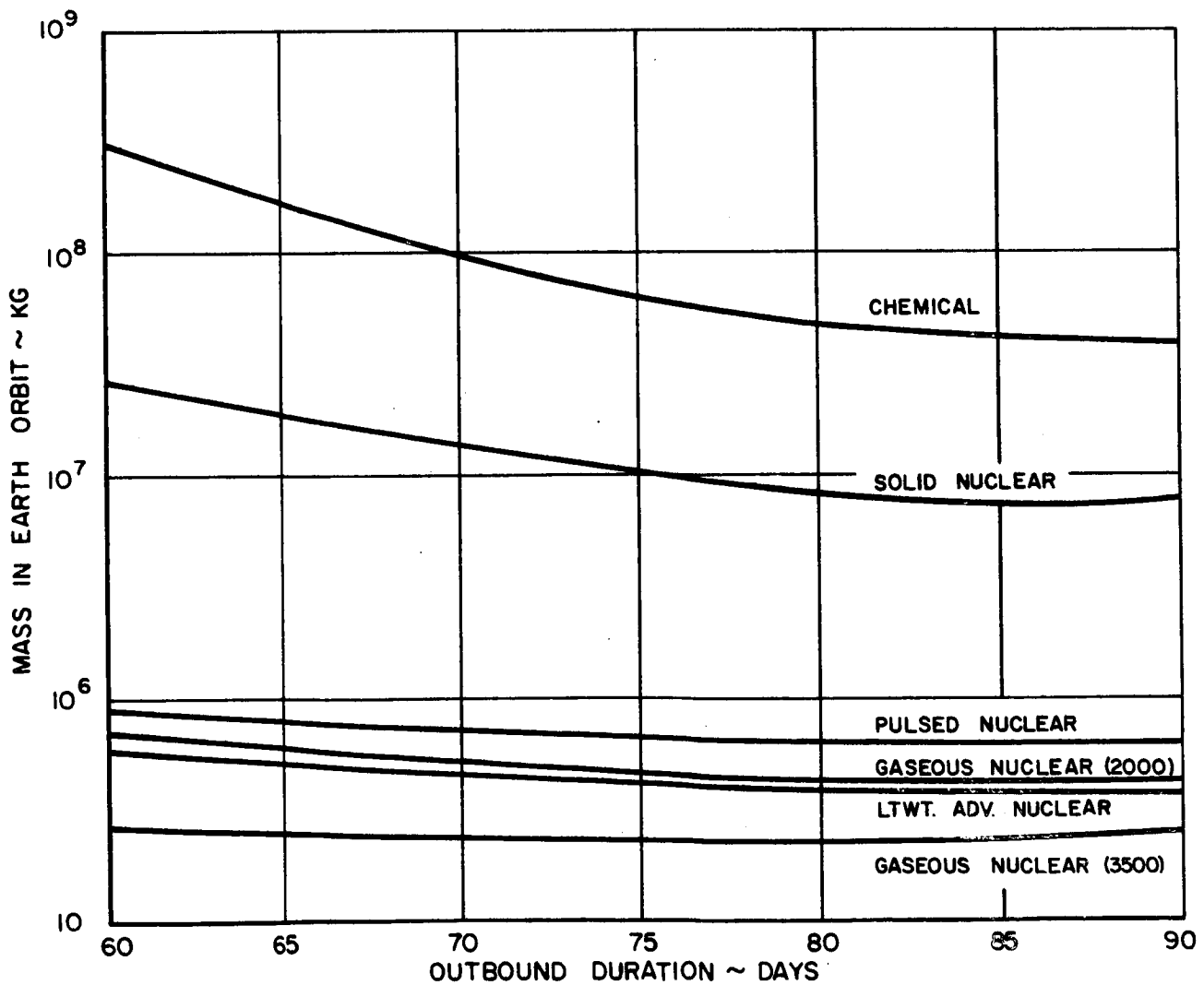
MERCURY MANNED ORBITAL STOPOVERS

STAY TIME = 80 + 85 DAYS

INBOUND DURATION = 200 DAYS

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

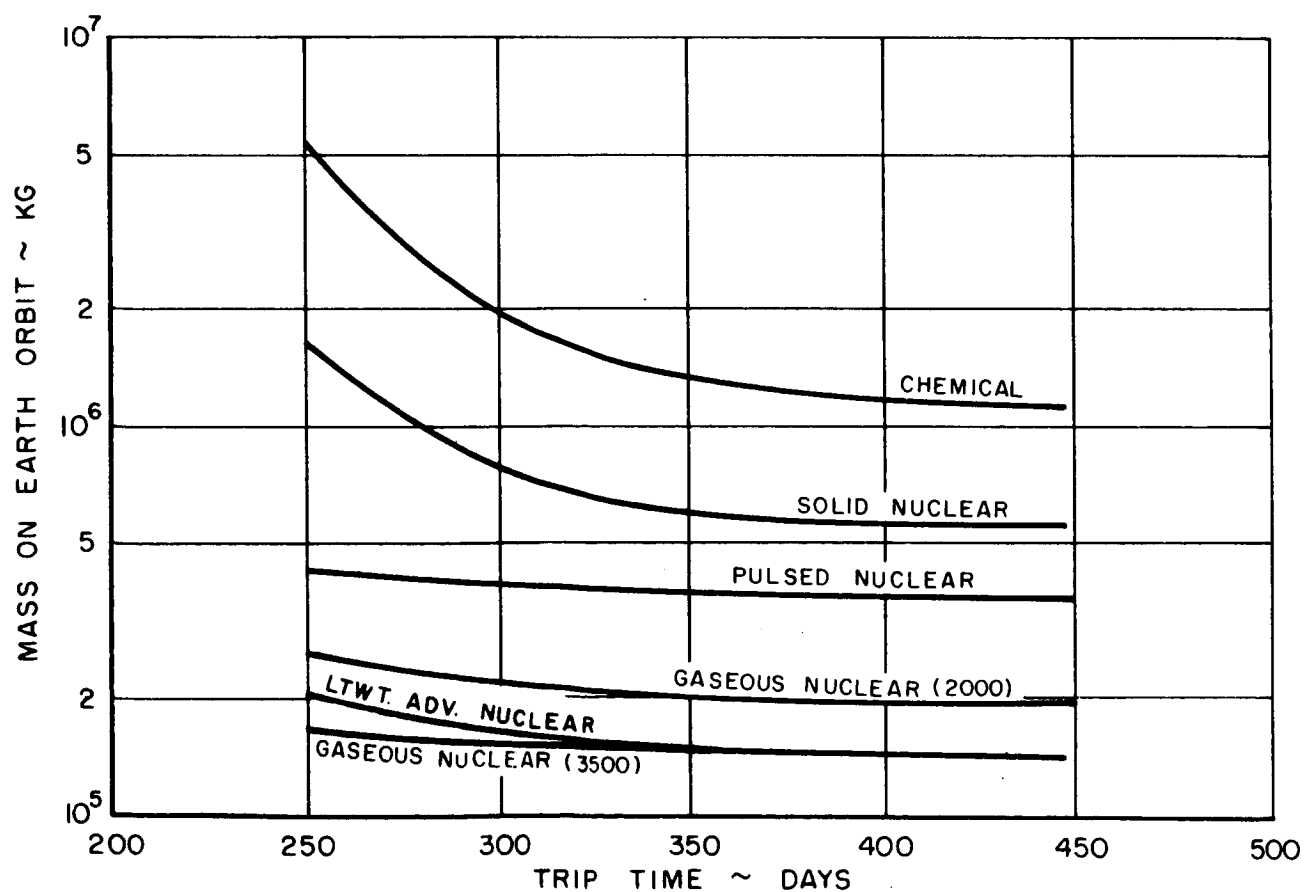


VENUS MANNED ORBITAL STOPOVERS

STAY TIME = 0

NOMINAL ENGINE PERFORMANCE

ENGINE	I _{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

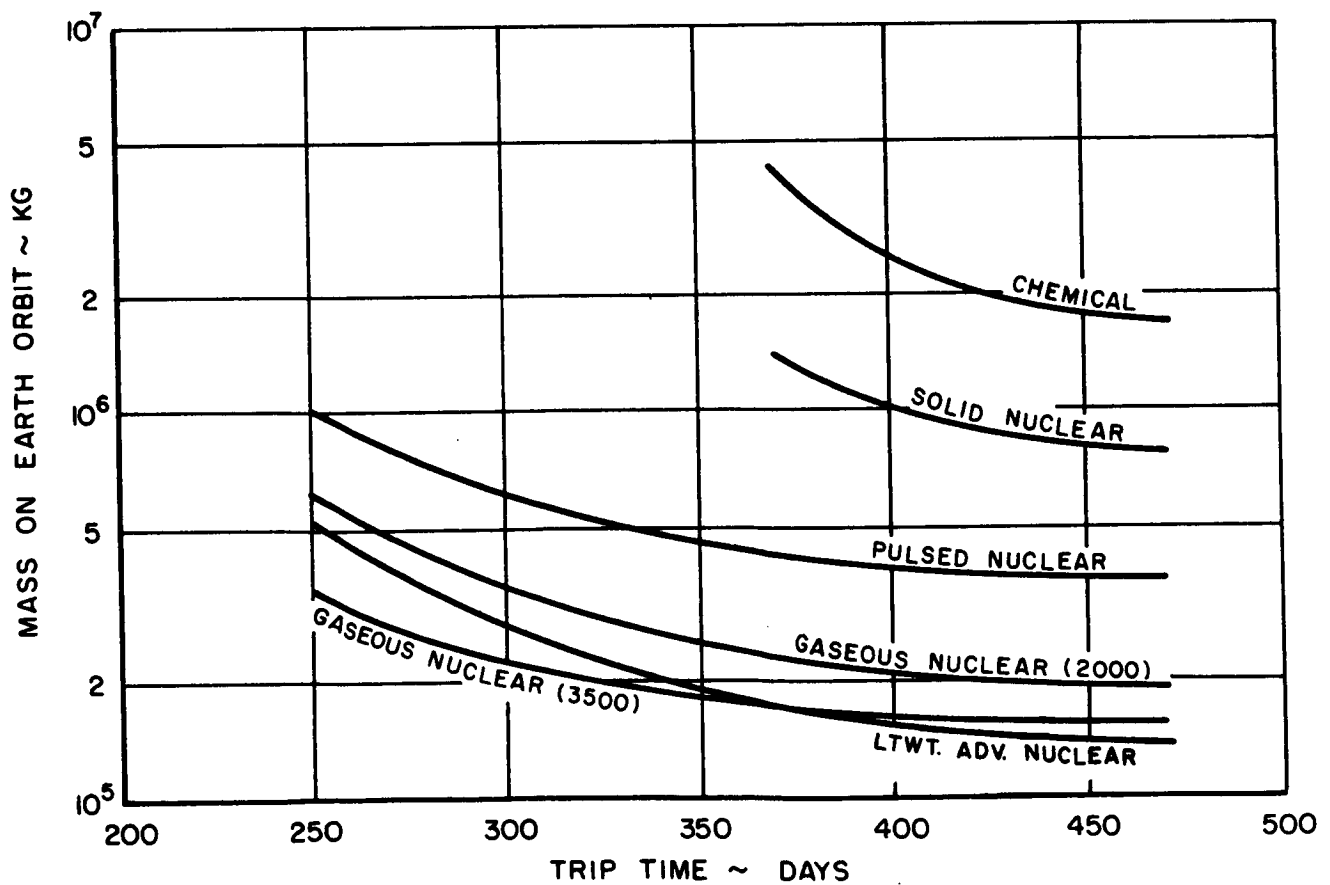


MARS MANNED ORBITAL STOPOVERS

STAY TIME = 0

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

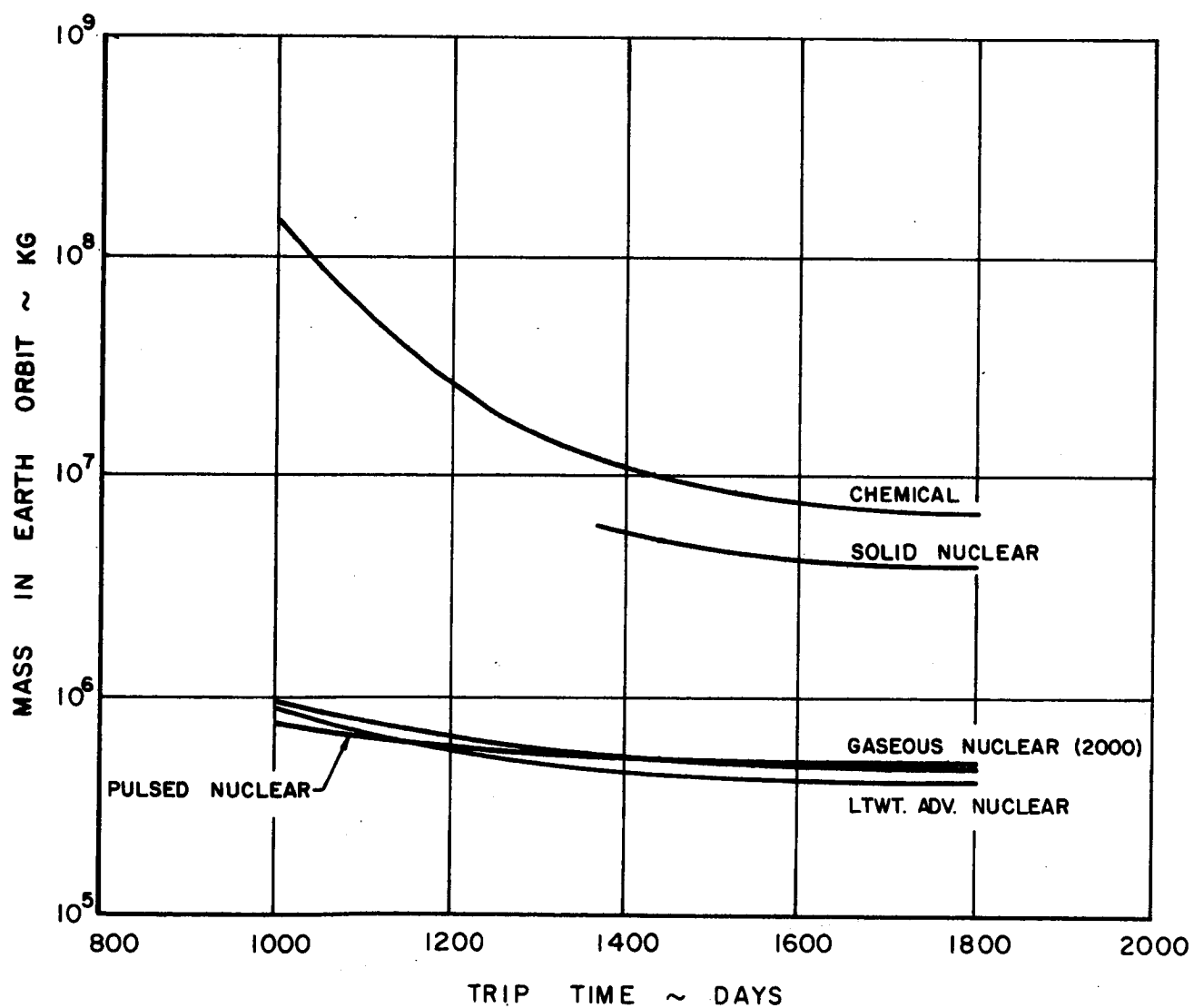


GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

STAY TIME = 0

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{SP}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1600	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600

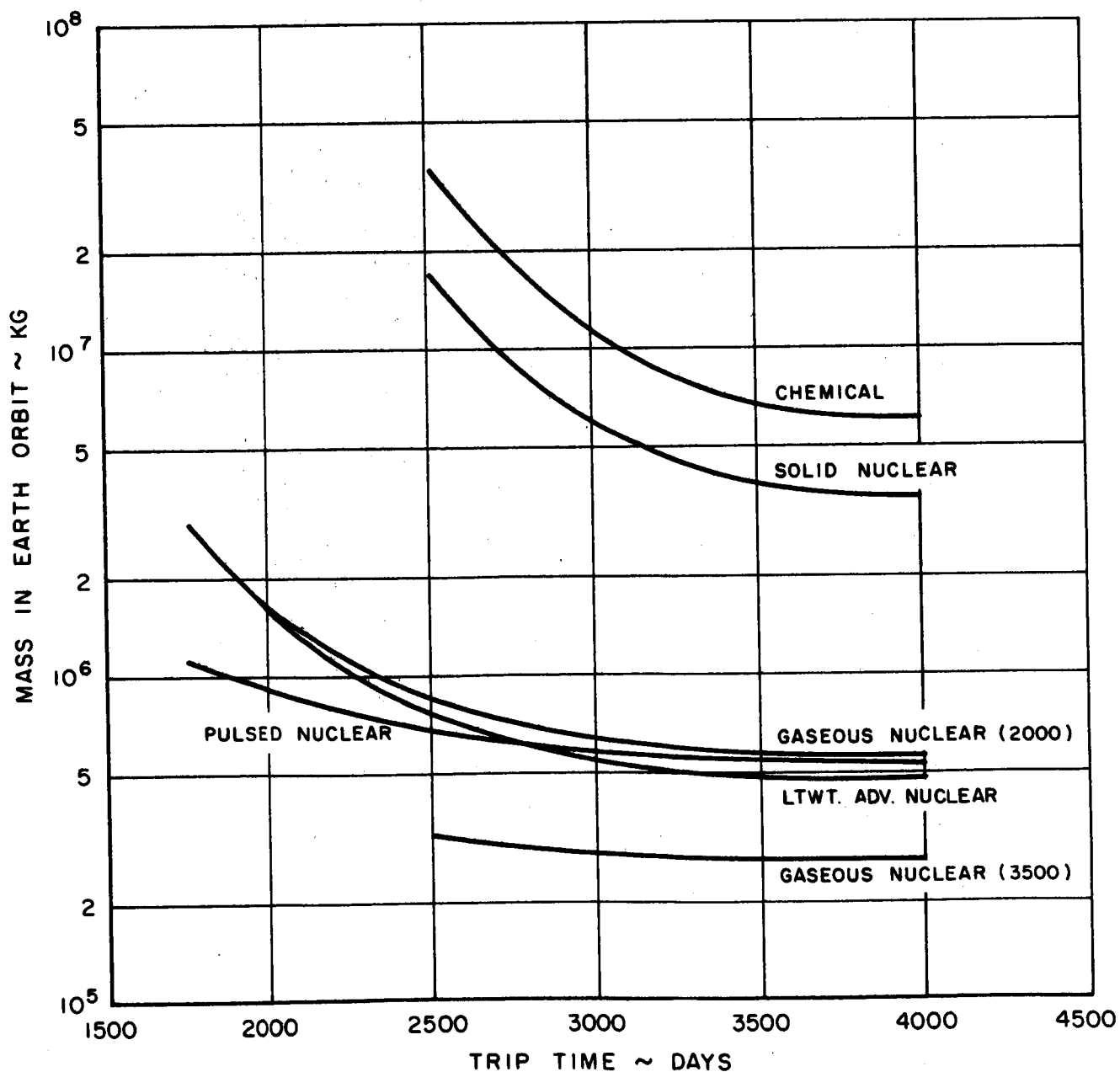


TITAN MANNED ORBITAL STOPOVERS (SATURN VI)

STAY TIME = 0

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

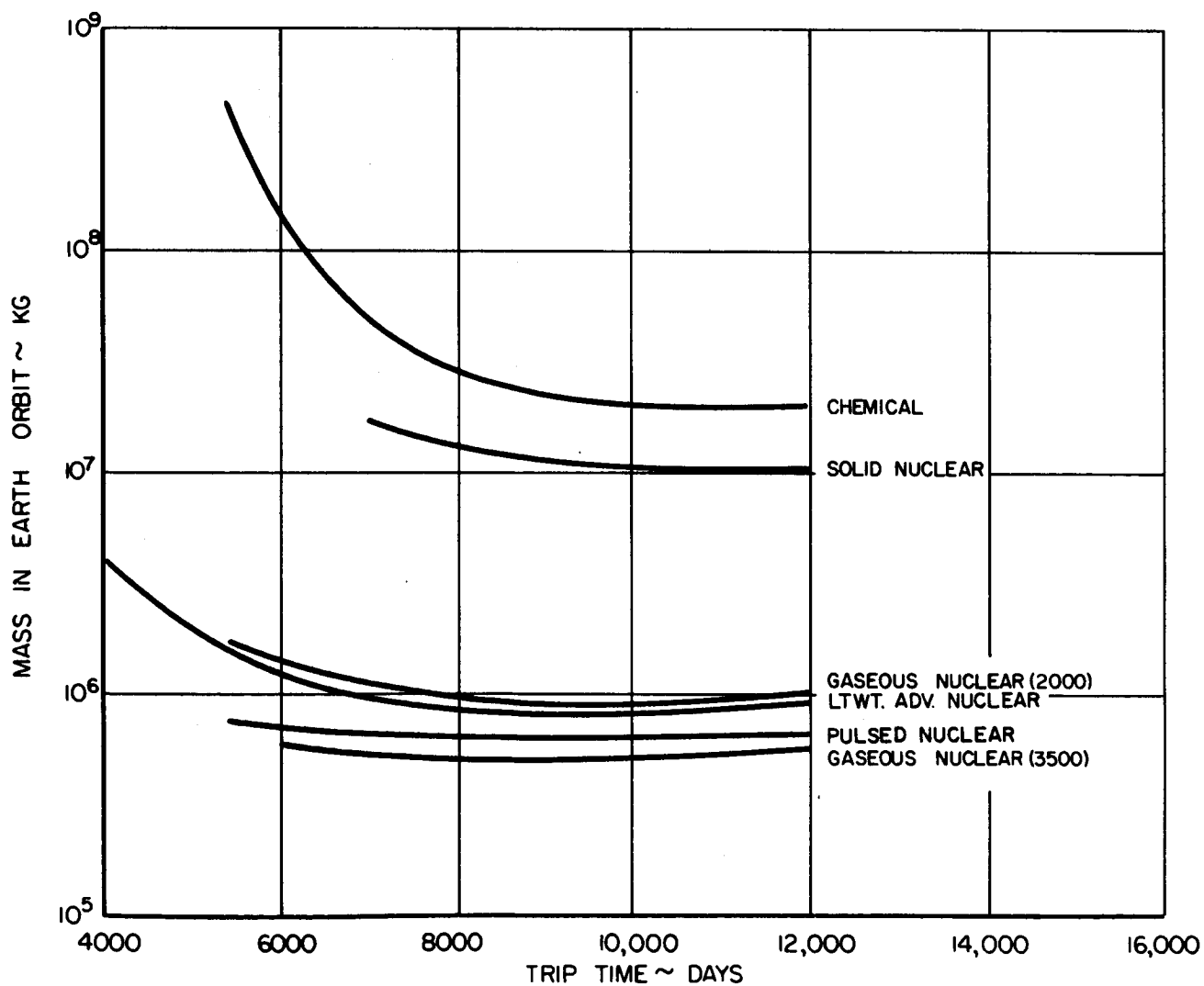


URANUS MANNED ORBITAL STOPOVERS

STAY TIME = 0

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600

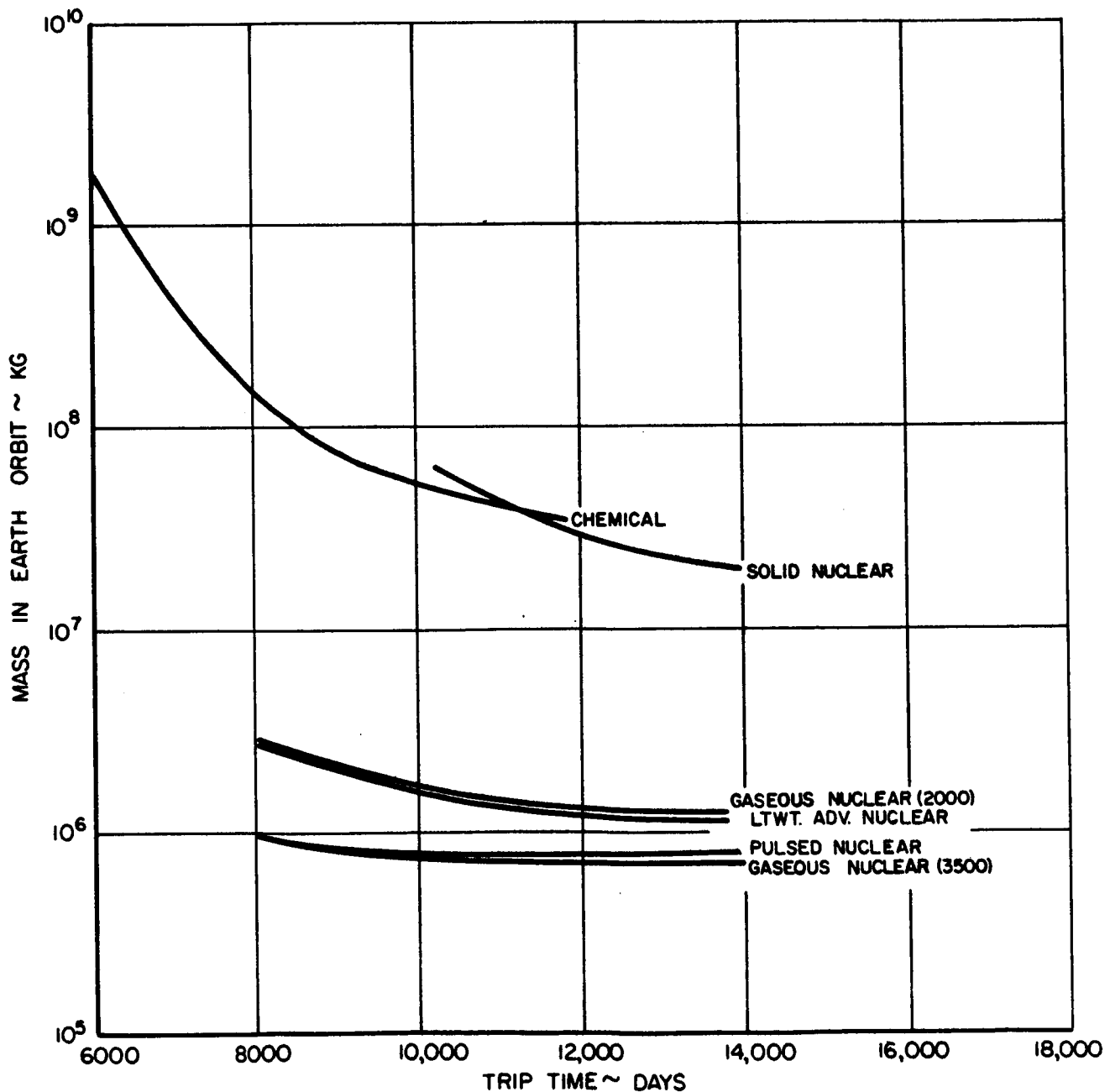


NEPTUNE MANNED ORBITAL STOPOVERS

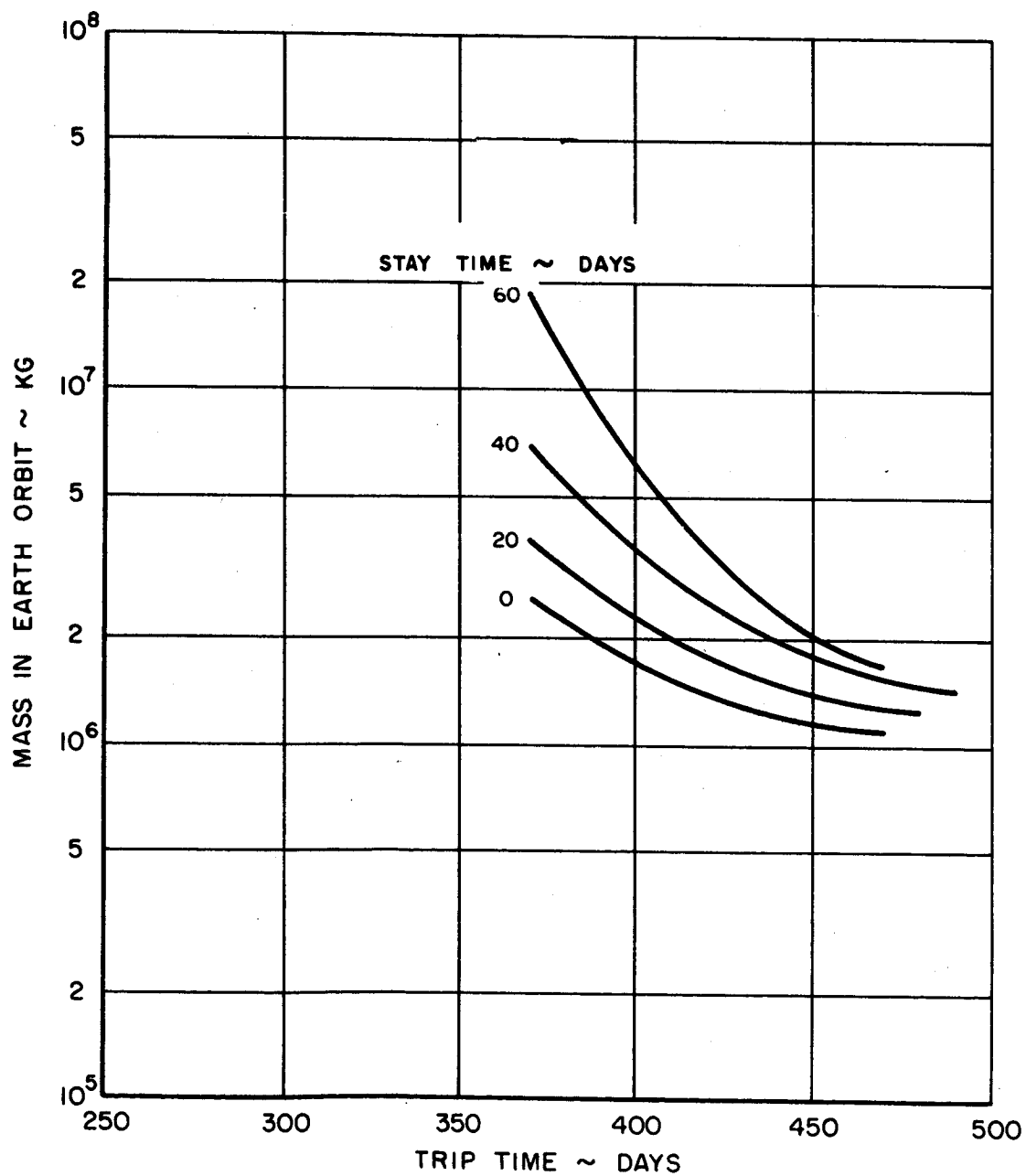
STAY TIME = 0

NOMINAL ENGINE PERFORMANCE

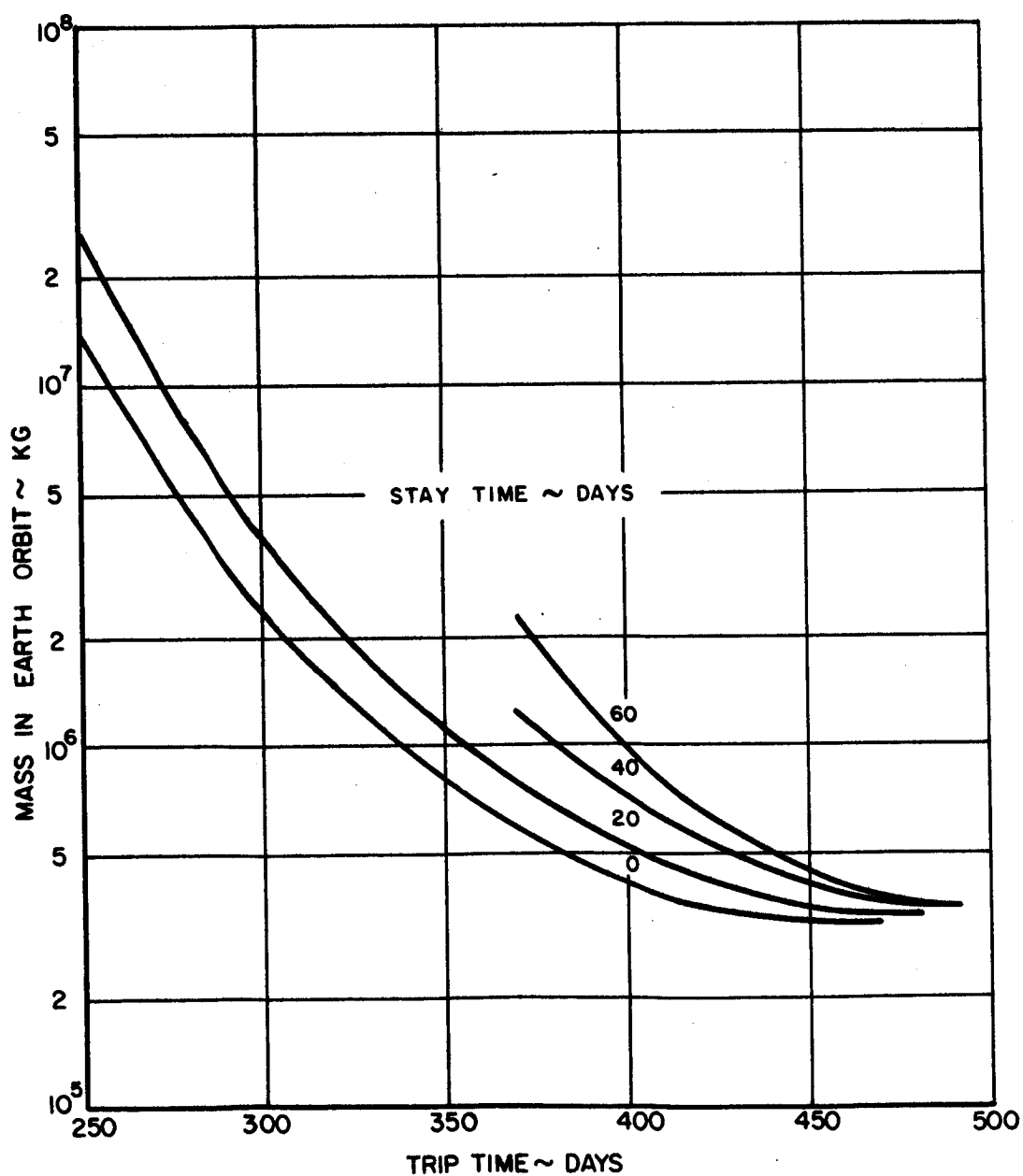
ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600



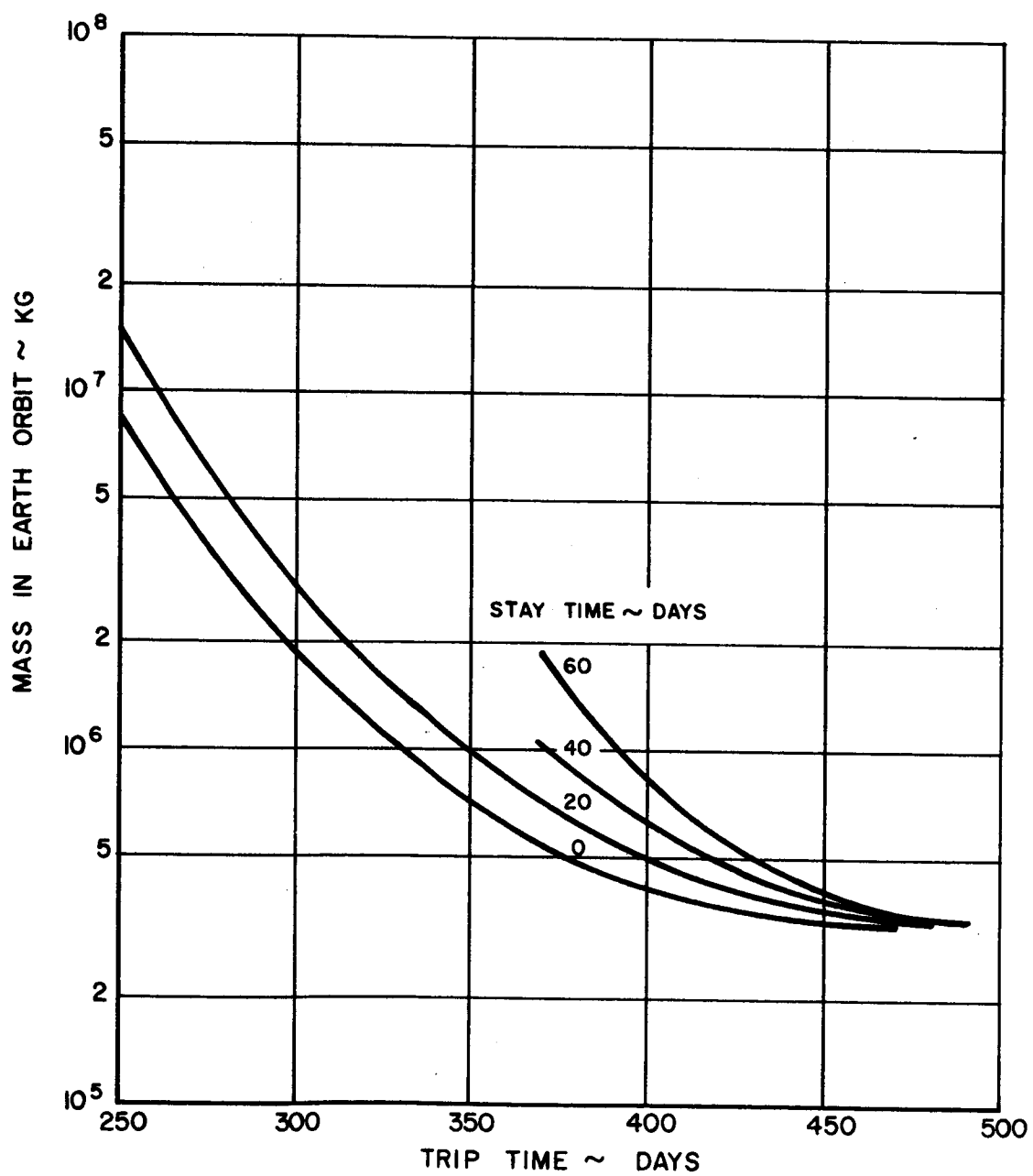
MARS MANNED ORBITAL STOPOVERS

CHEMICAL EARTH ESCAPE ($I_{sp} = 430$ SEC)SOLID NUCLEAR PLANETARY STAGES ($I_{sp} = 800$ SEC)

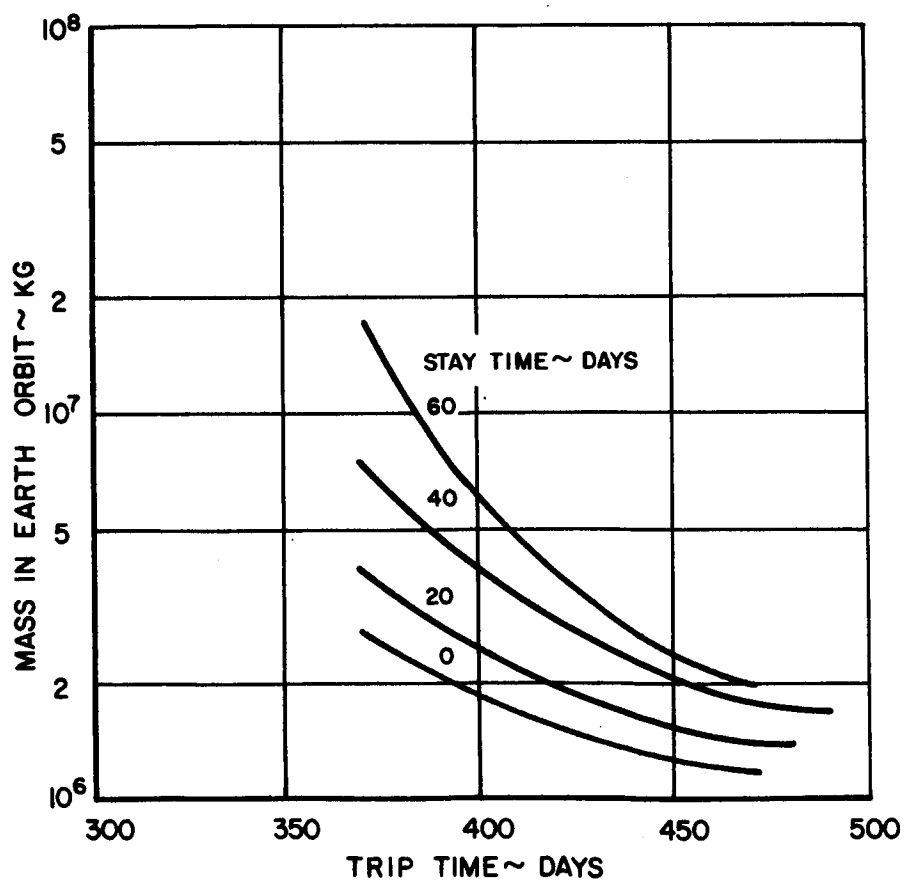
MARS MANNED ORBITAL STOPOVERS

CHEMICAL EARTH ESCAPE ($I_{sp}=430$ SEC)LIGHTWEIGHT ADVANCED NUCLEAR PLANETARY STAGES ($I_{sp} = 2000$ SEC)

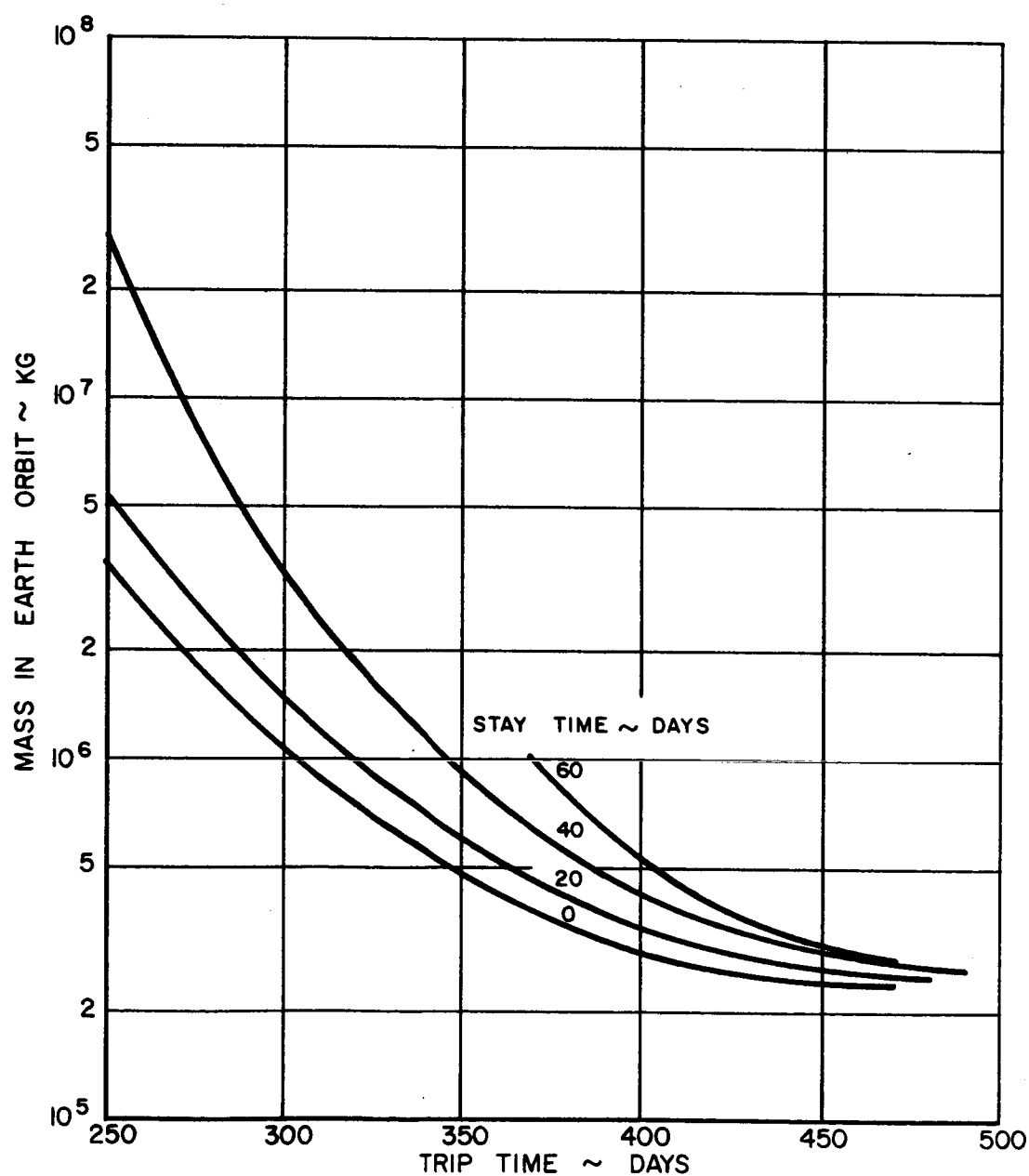
MARS MANNED ORBITAL STOPOVERS

CHEMICAL EARTH ESCAPE ($I_{sp}=430$ SEC)GASEOUS-CORE NUCLEAR UPPER STAGES ($I_{sp}=3500$ SEC)

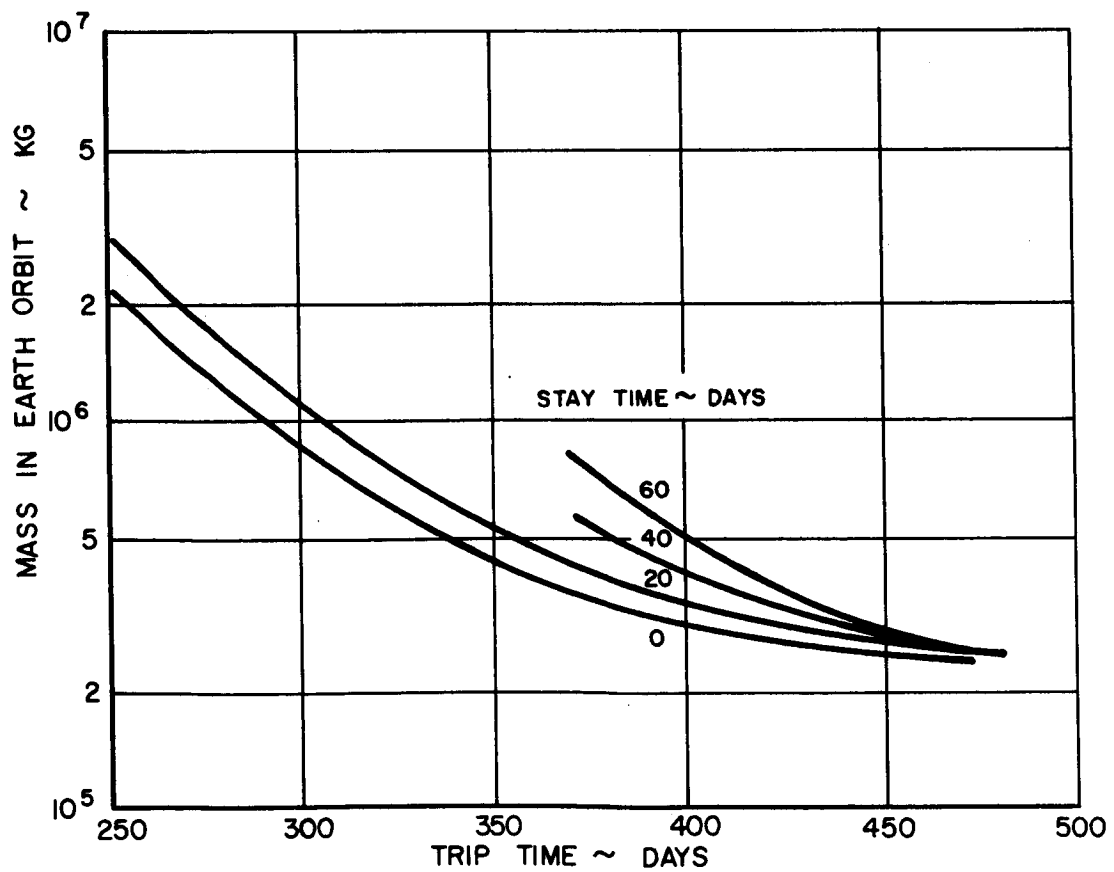
MARS MANNED ORBITAL STOPOVERS

SOLID-CORE NUCLEAR EARTH ESCAPE ($I_{sp} = 800$ SEC)CHEMICAL UPPER STAGES ($I_{sp} = 430$ SEC)

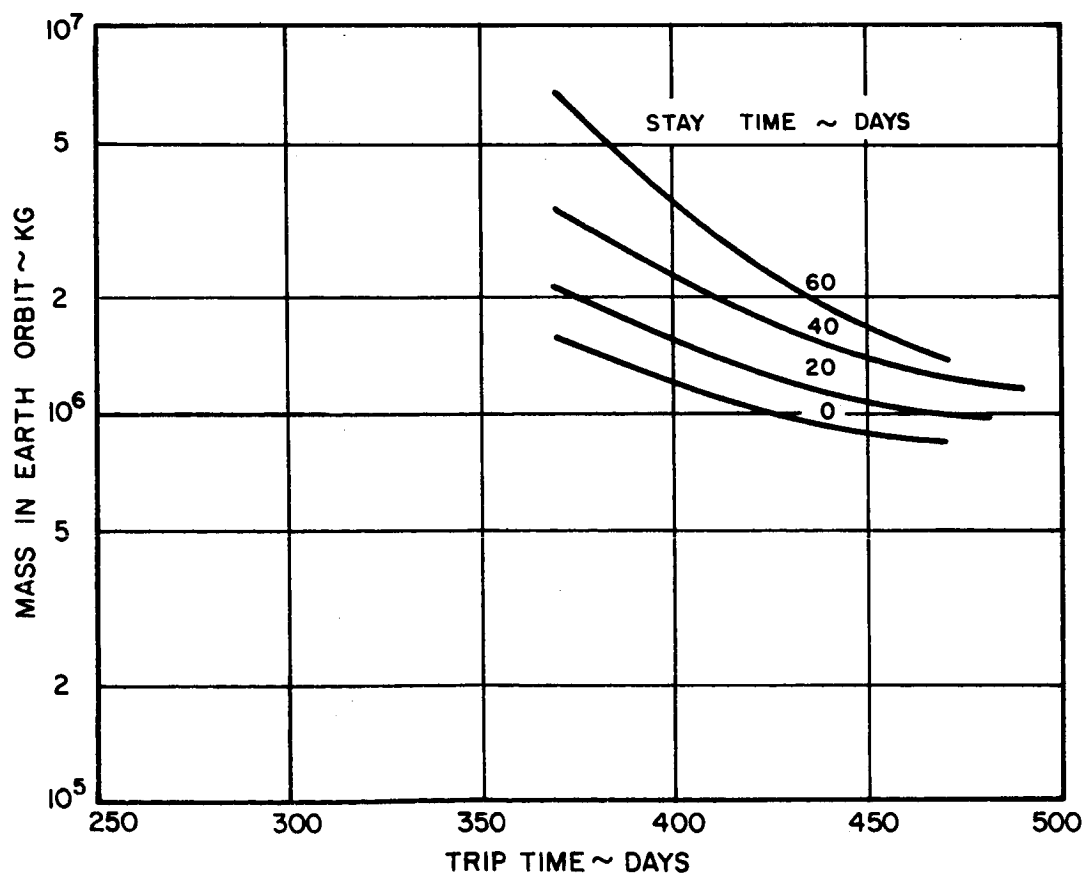
MARS MANNED ORBITAL STOPOVERS

SOLID-CORE NUCLEAR EARTH ESCAPE ($I_{sp}=800$ SEC)LIGHTWEIGHT ADVANCED NUCLEAR PLANETARY STAGES ($I_{sp} = 2000$ SEC)

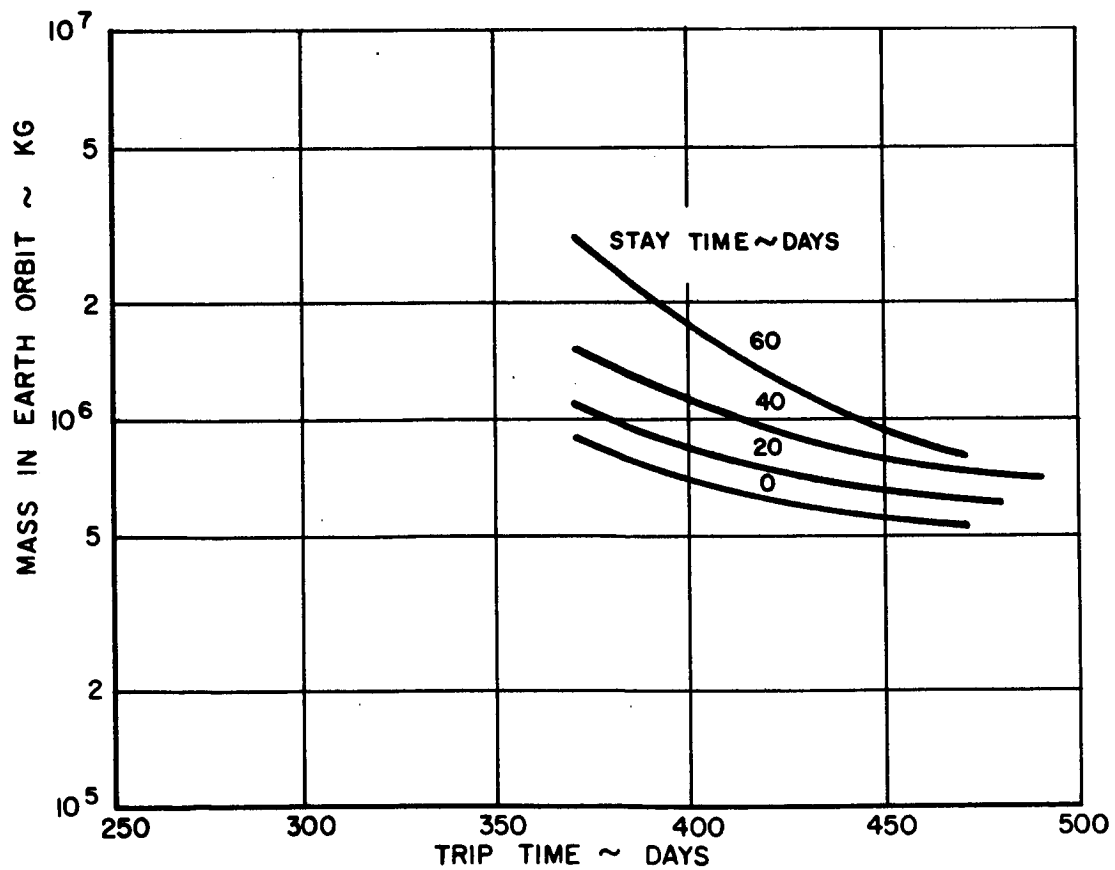
MARS MANNED ORBITAL STOPOVERS

SOLID-CORE NUCLEAR EARTH ESCAPE (I_{sp} 800 SEC)GASEOUS-CORE NUCLEAR UPPER STAGES (I_{sp} = 3500 SEC)

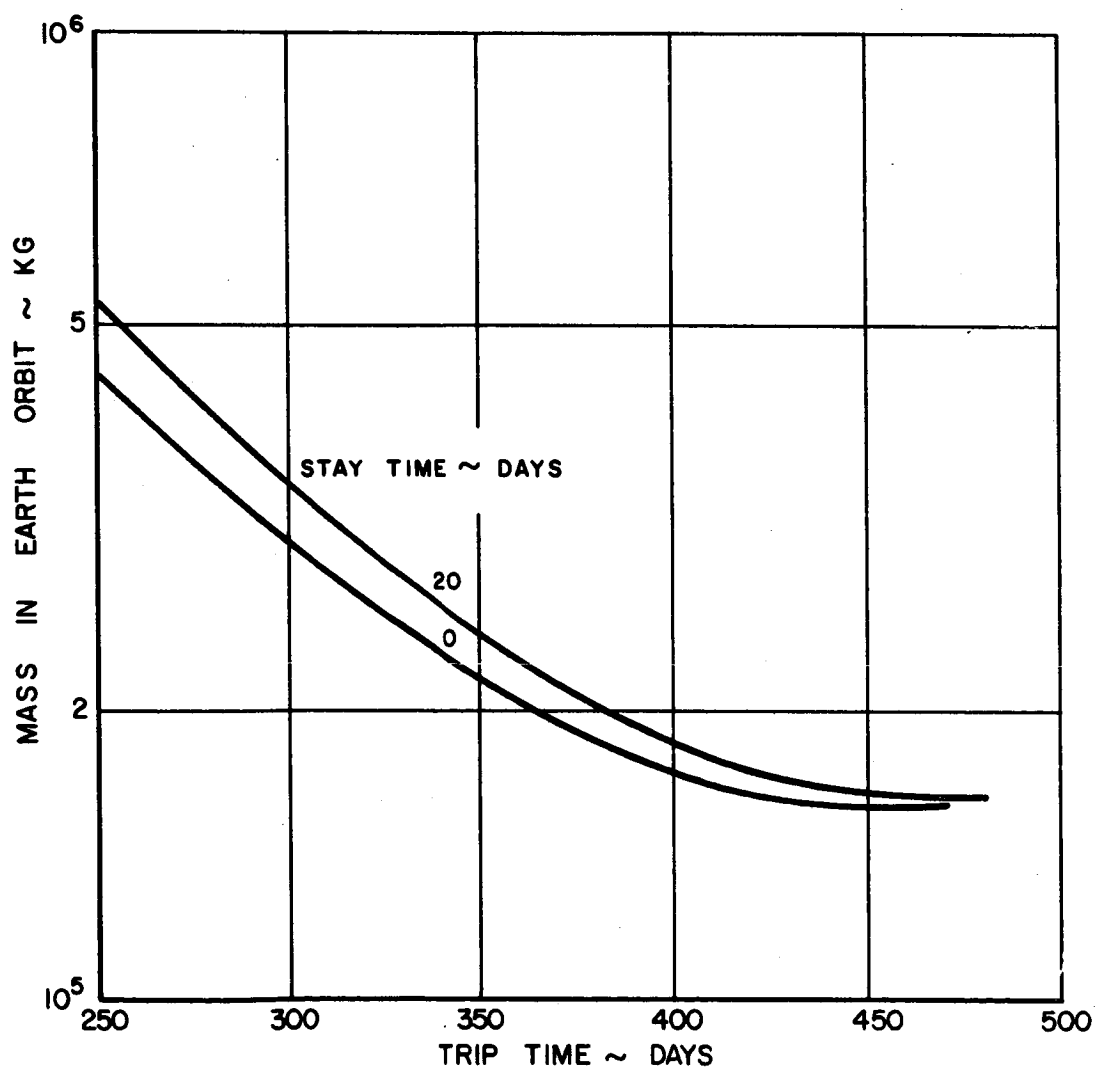
MARS MANNED ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR EARTH ESCAPE ($I_{sp} = 2000$ SEC)CHEMICAL PLANETARY STAGES ($I_{sp} = 430$ SEC)

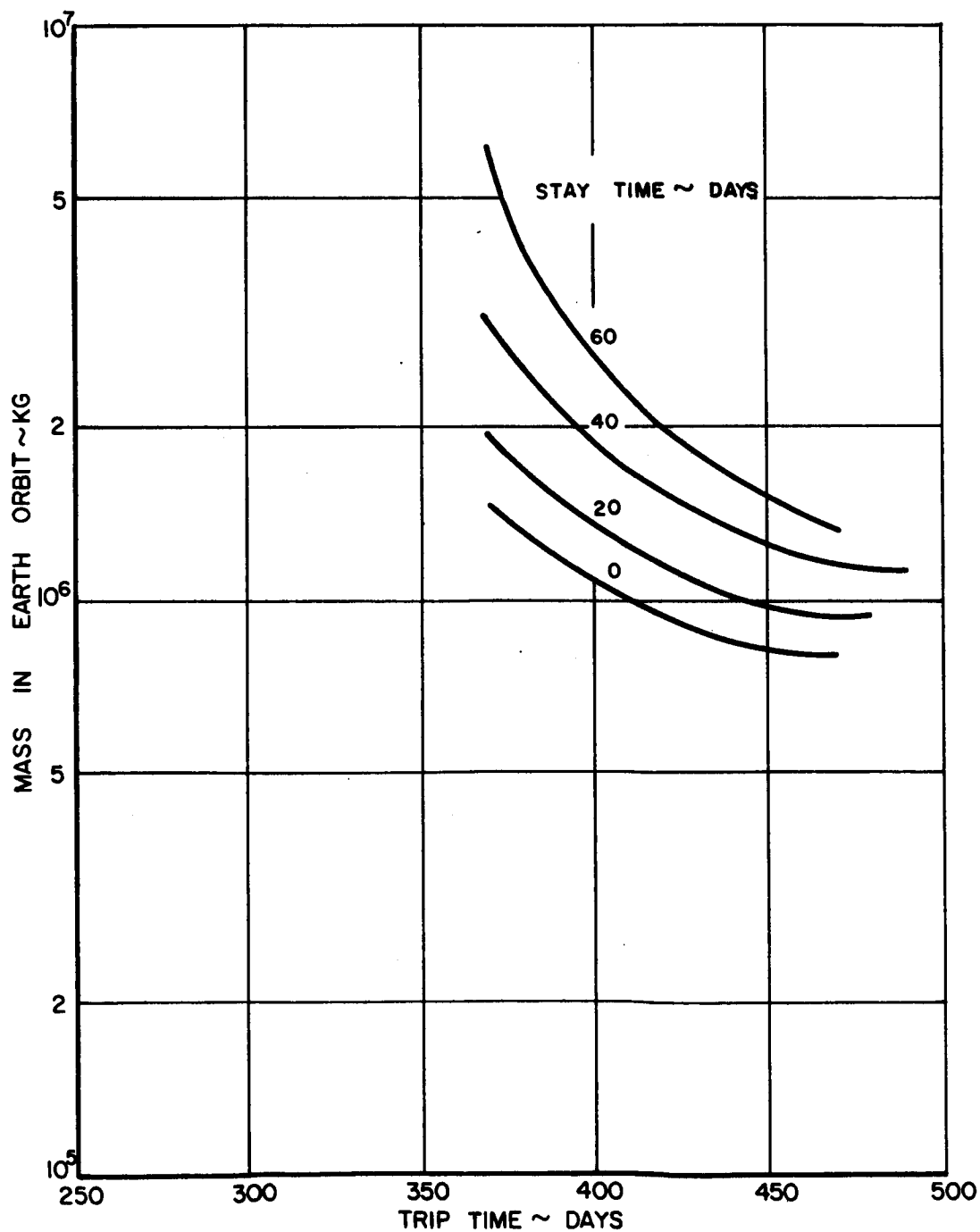
MARS MANNED ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR EARTH ESCAPE ($I_{sp} = 2000$ SEC)SOLID-CORE NUCLEAR PLANETARY STAGES ($I_{sp} = 800$ SEC)

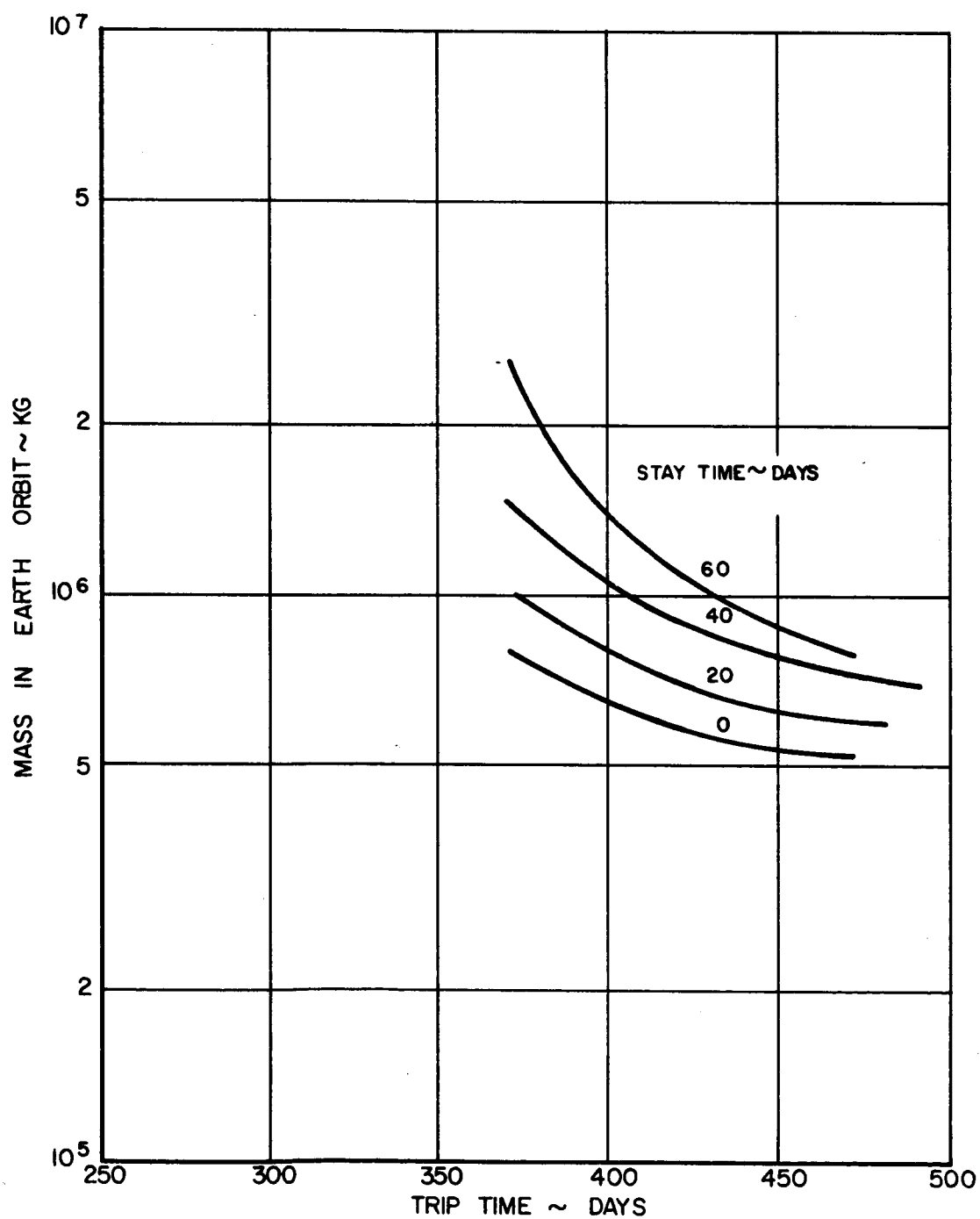
MARS MANNED ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR EARTH ESCAPE ($I_{sp} = 2000$ SEC)GASEOUS - CORE NUCLEAR PLANETARY STAGES ($I_{sp} = 3500$ SEC)

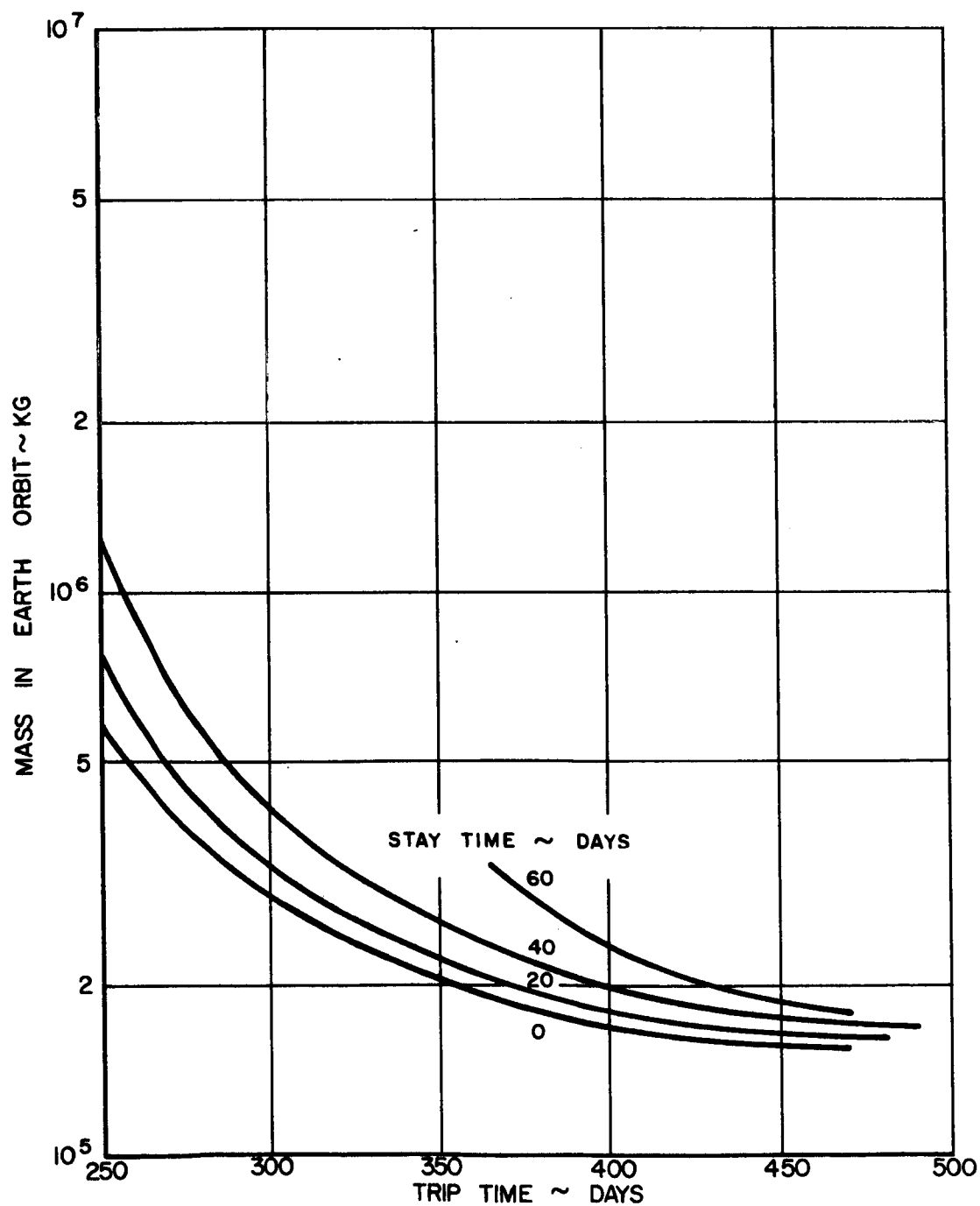
MARS MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR EARTH ESCAPE ($I_{sp}=3500$ SEC)CHEMICAL UPPER STAGES ($I_{sp}=430$ SEC)

MARS MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR EARTH ESCAPE ($I_{sp}=3500$ SEC)SOLID-CORE NUCLEAR UPPER STAGES ($I_{sp} = 800$ SEC)

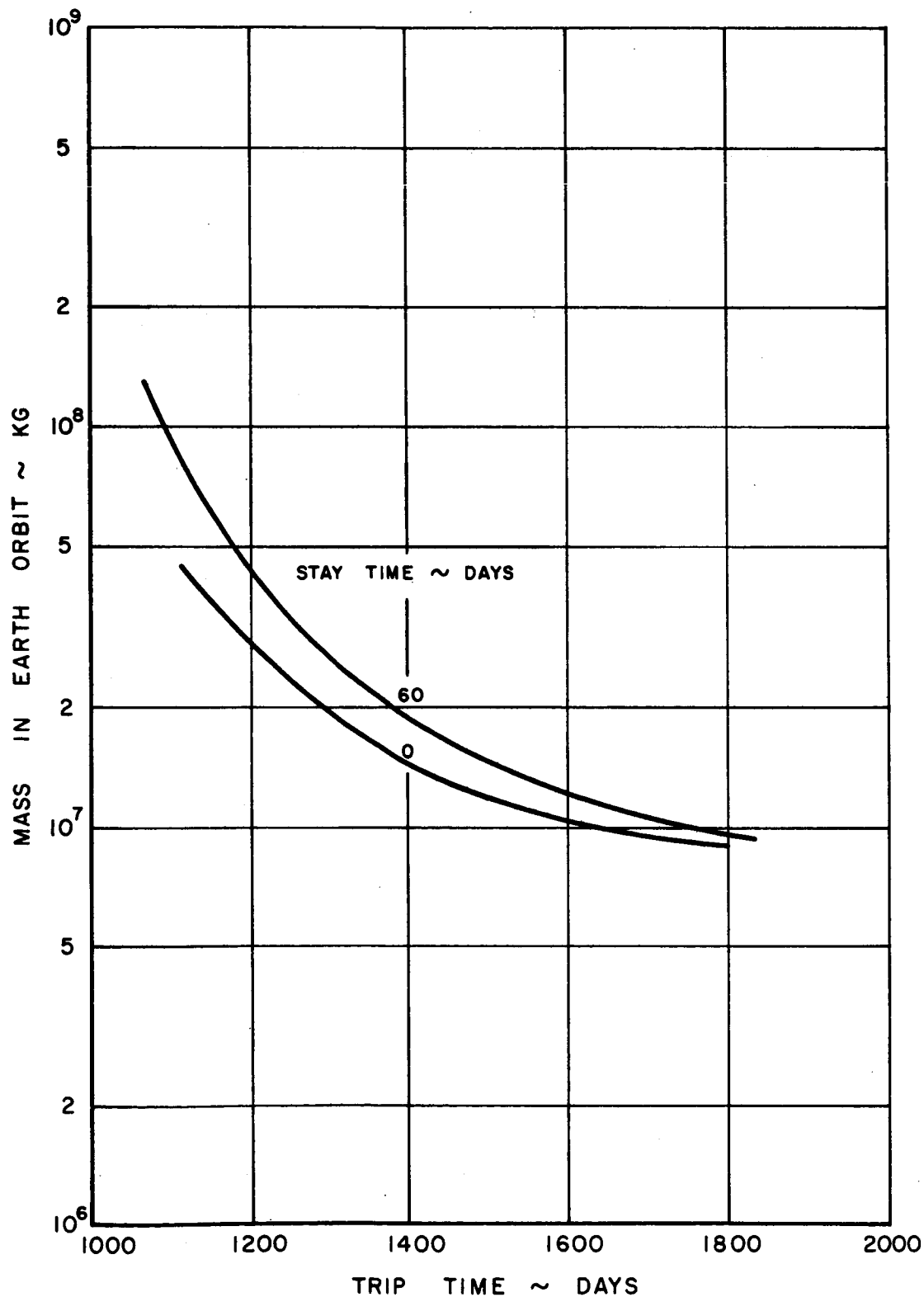
MARS MANNED ORBITAL STOPOVERS

GASEOUS-CORE NUCLEAR EARTH ESCAPE ($I_{sp}=3500$ SEC)LIGHTWEIGHT ADVANCED NUCLEAR PLANETARY STAGES ($I_{sp} = 2000$ SEC)

GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

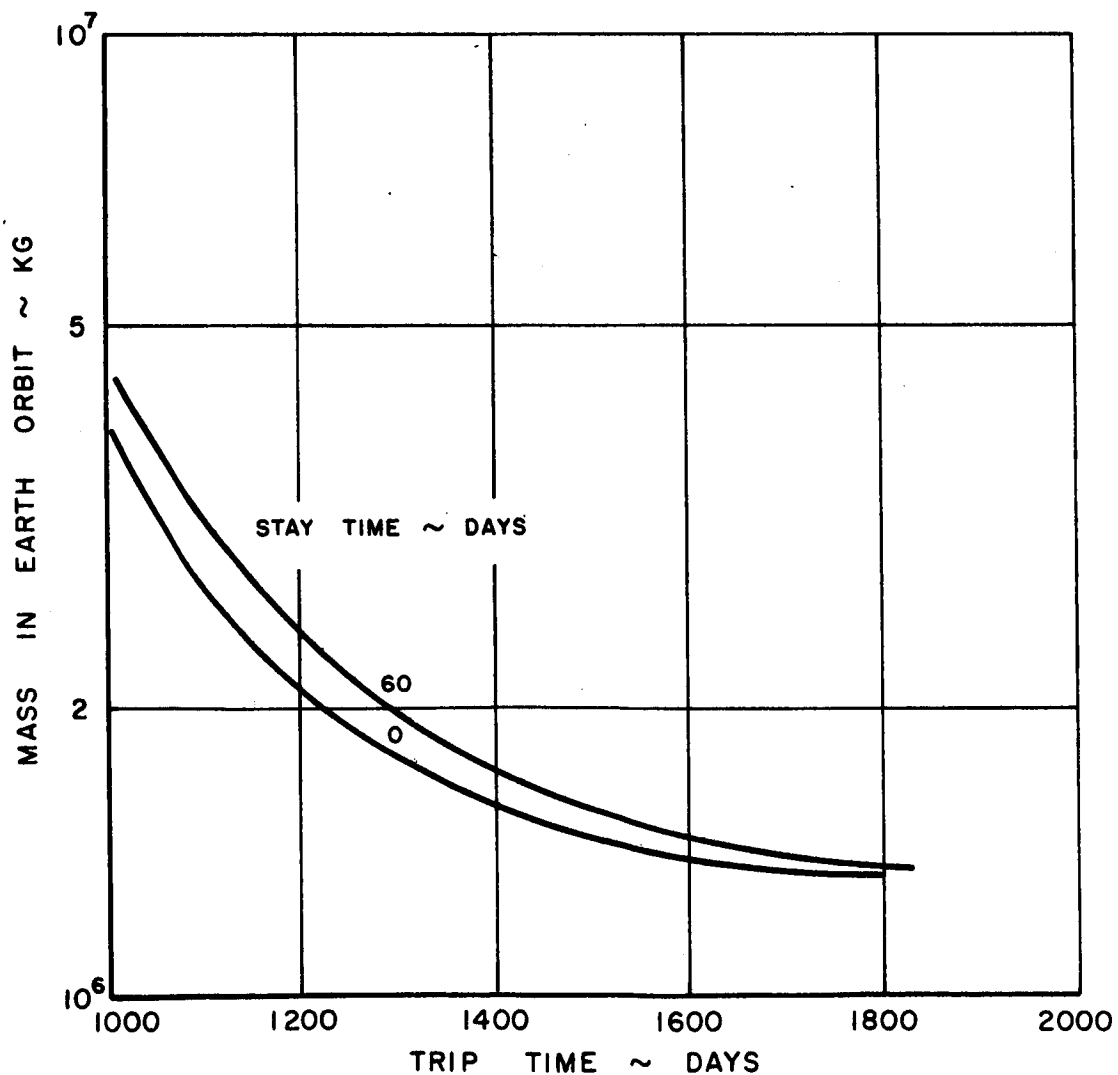
CHEMICAL EARTH ESCAPE ($I_{sp} = 430$ SEC)

SOLID-CORE NUCLEAR PLANETARY STAGES ($I_{sp} = 800$ SEC)



GANYMEDE MANNED ORBITAL STOPOVERS JUPITER III

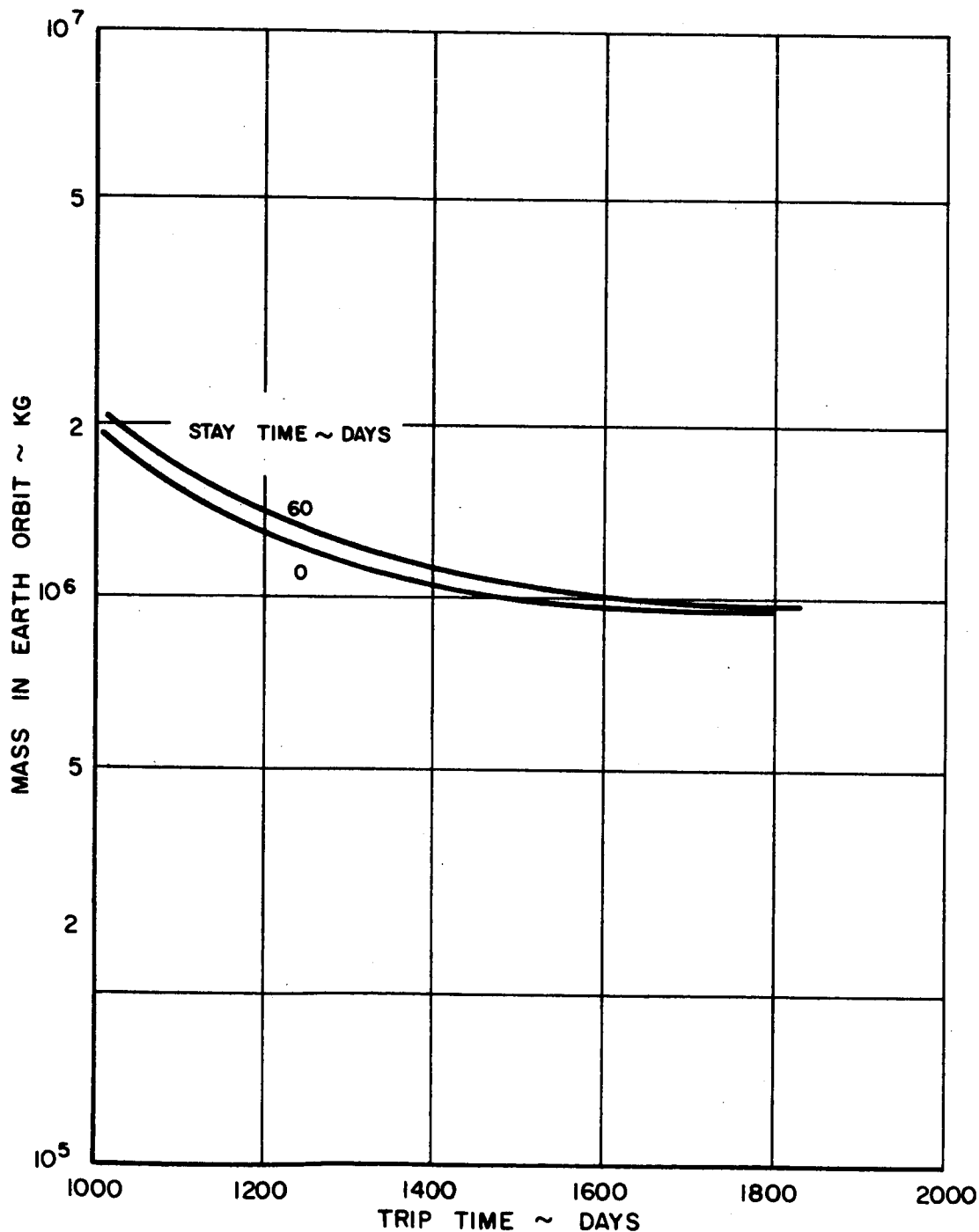
CHEMICAL EARTH ESCAPE ($I_{sp} = 430$ SEC)
LIGHTWEIGHT ADVANCED NUCLEAR PLANETARY STAGES ($I_{sp} = 2000$ SEC)



GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

CHEMICAL EARTH ESCAPE ($I_{sp} = 430$ SEC)

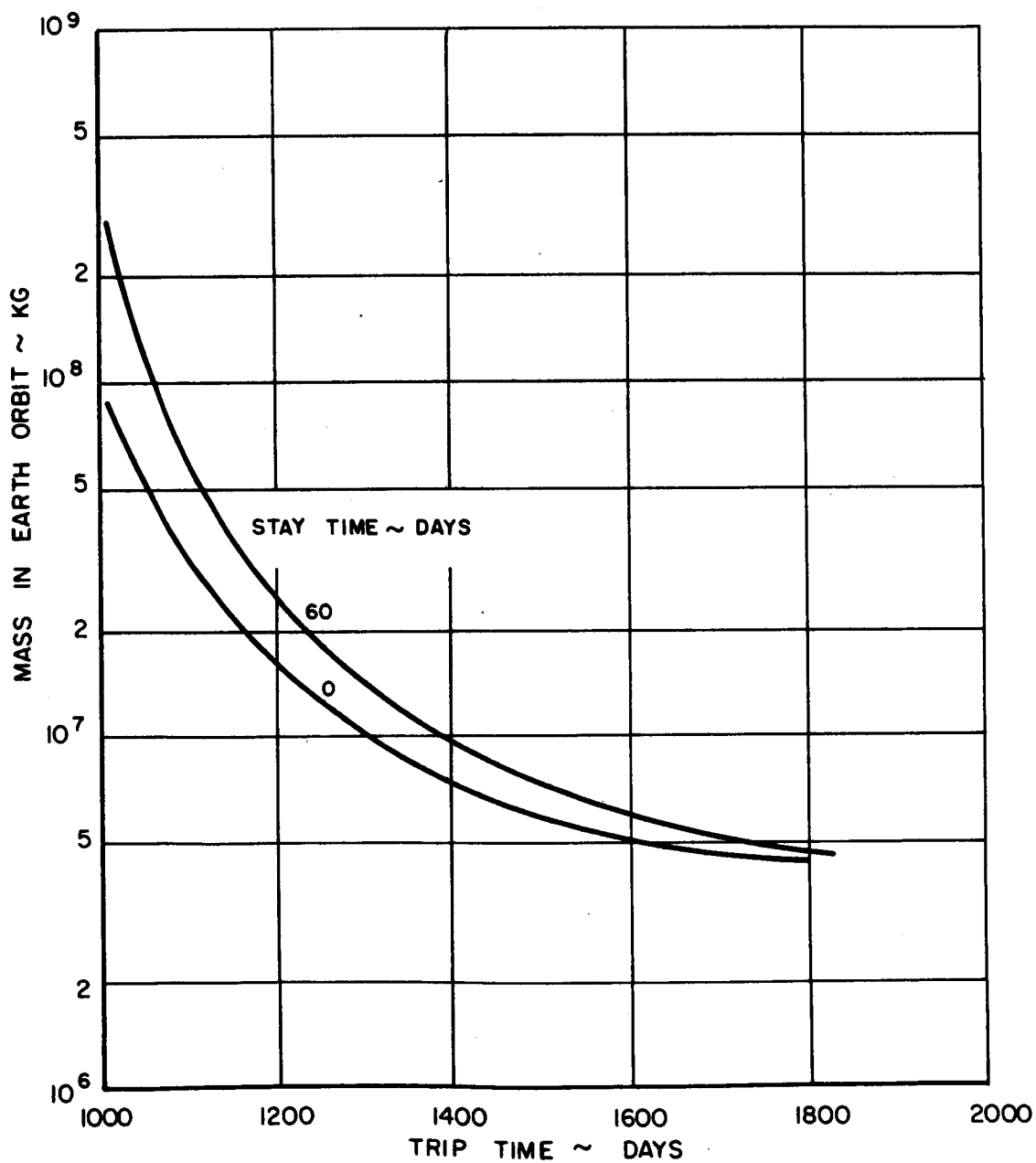
GASEOUS-CORE NUCLEAR PLANETARY STAGES ($I_{sp} = 3500$ SEC)



GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

SOLID-CORE NUCLEAR EARTH ESCAPE ($I_{sp} = 800$ SEC)

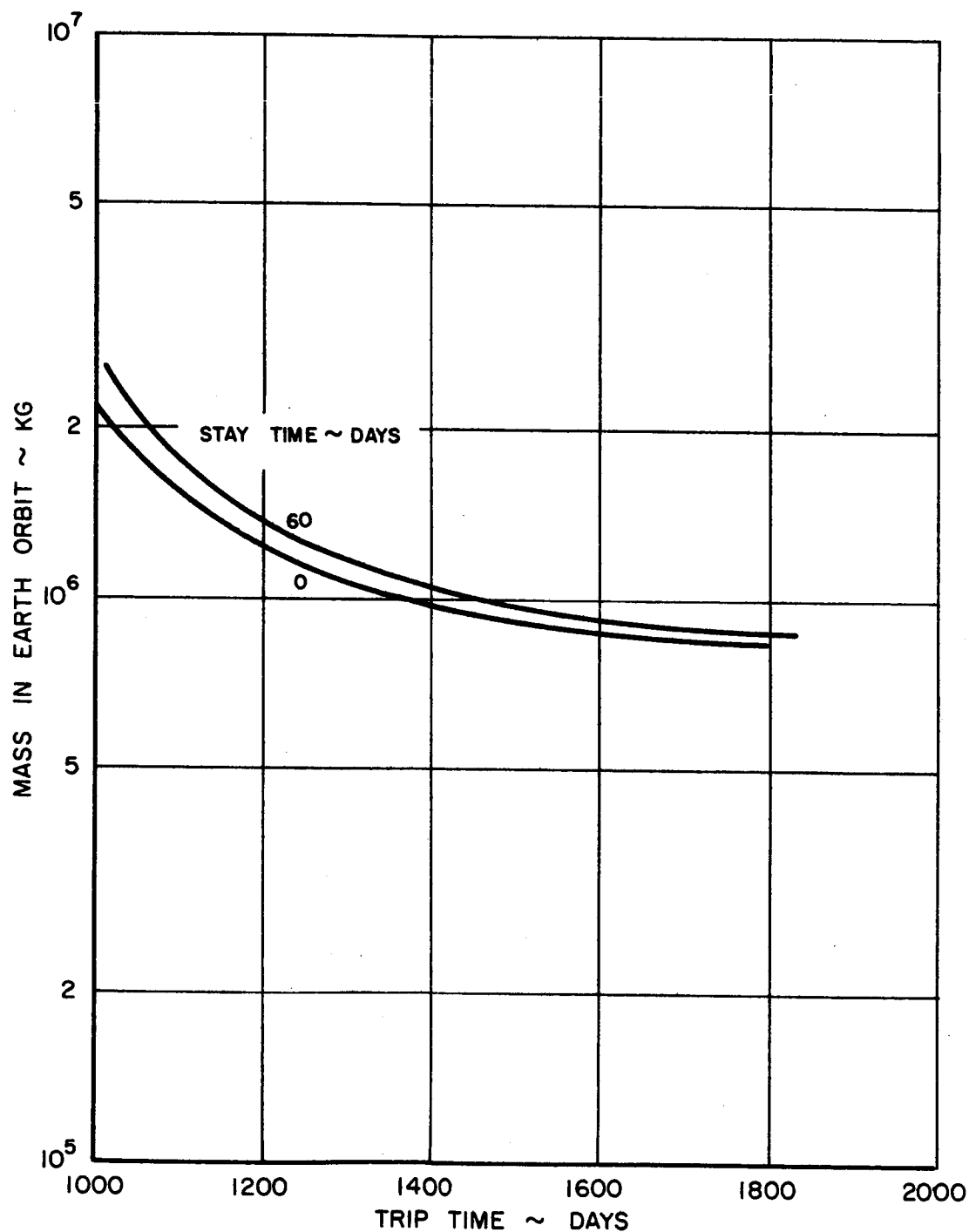
CHEMICAL PLANETARY STAGES ($I_{sp} = 430$ SEC)



GANYMEDE MANNED ORBITAL STOPOVERS JUPITER III

SOLID-CORE NUCLEAR EARTH ESCAPE ($I_{sp} = 300$ SEC)

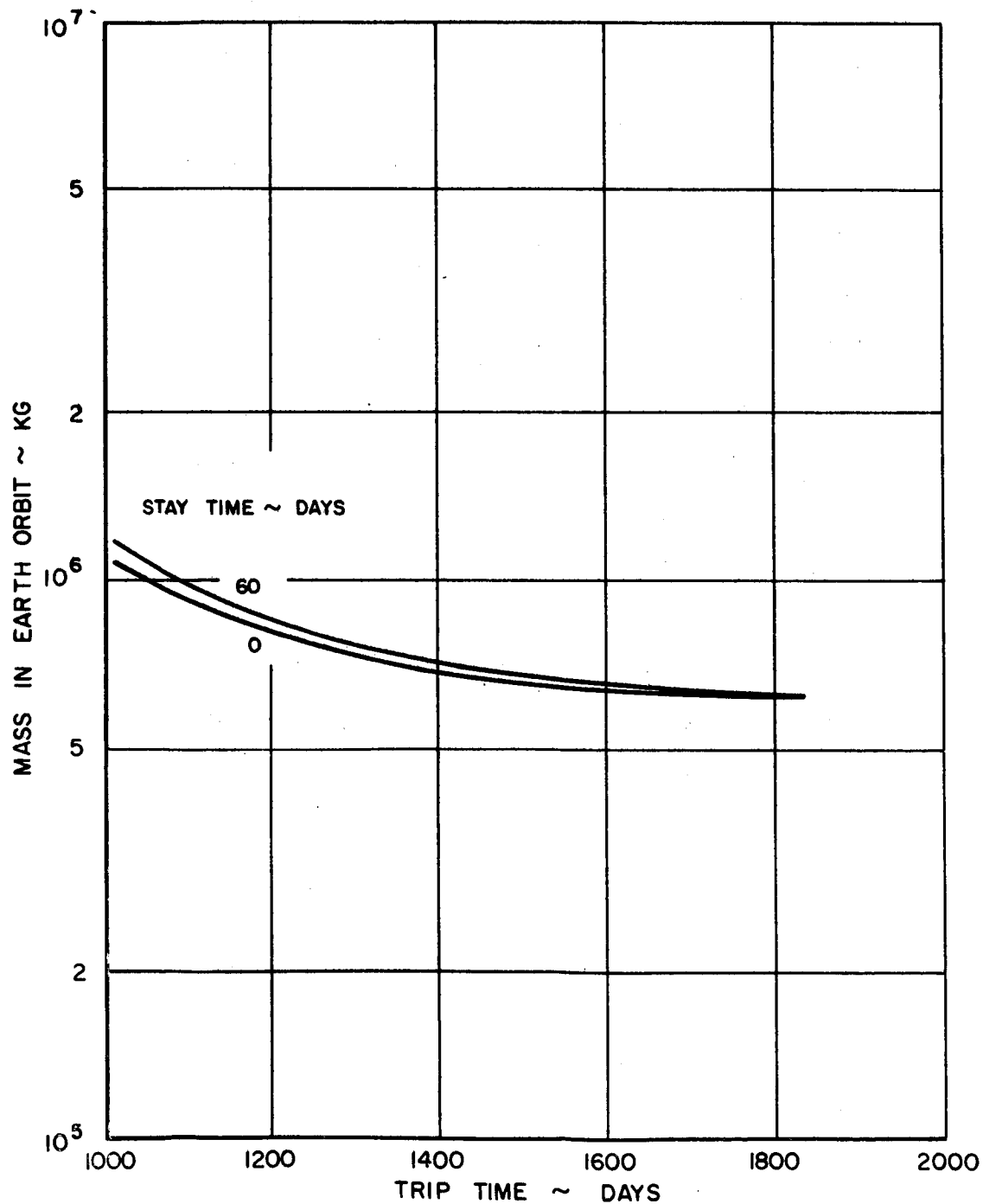
LIGHTWEIGHT ADVANCED NUCLEAR PLANETARY ($I_{sp} = 2000$ SEC)



GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

SOLID-CORE NUCLEAR EARTH ESCAPE ($I_{sp} = 800$ SEC)

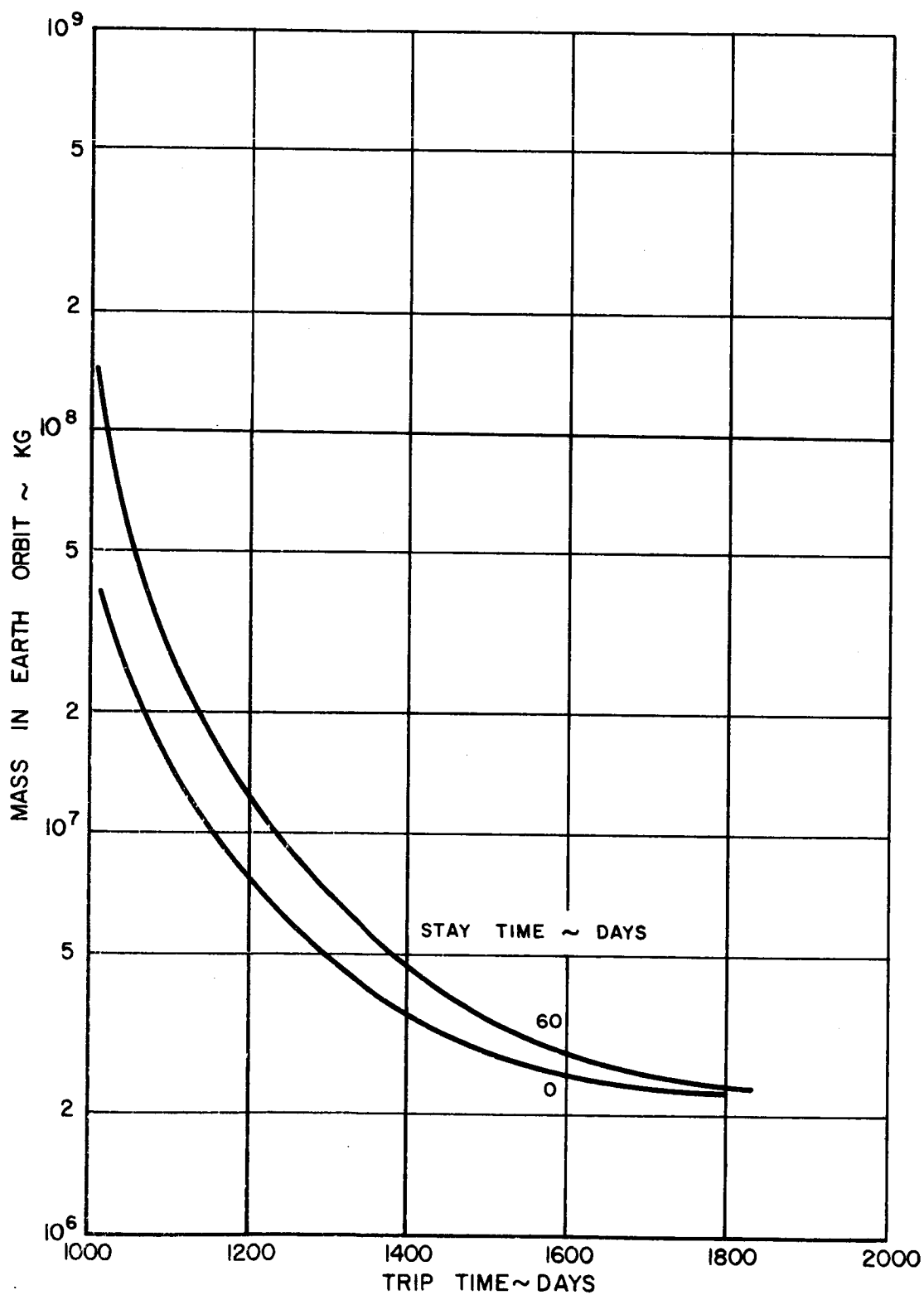
GASEOUS-CORE NUCLEAR PLANETARY STAGES ($I_{sp} = 3500$ SEC)



GANYMEDE MANNED ORBITAL STOPOVERS JUPITER III

LIGHTWEIGHT ADVANCED NUCLEAR EARTH ESCAPE ($I_{sp} = 2000$ SEC)

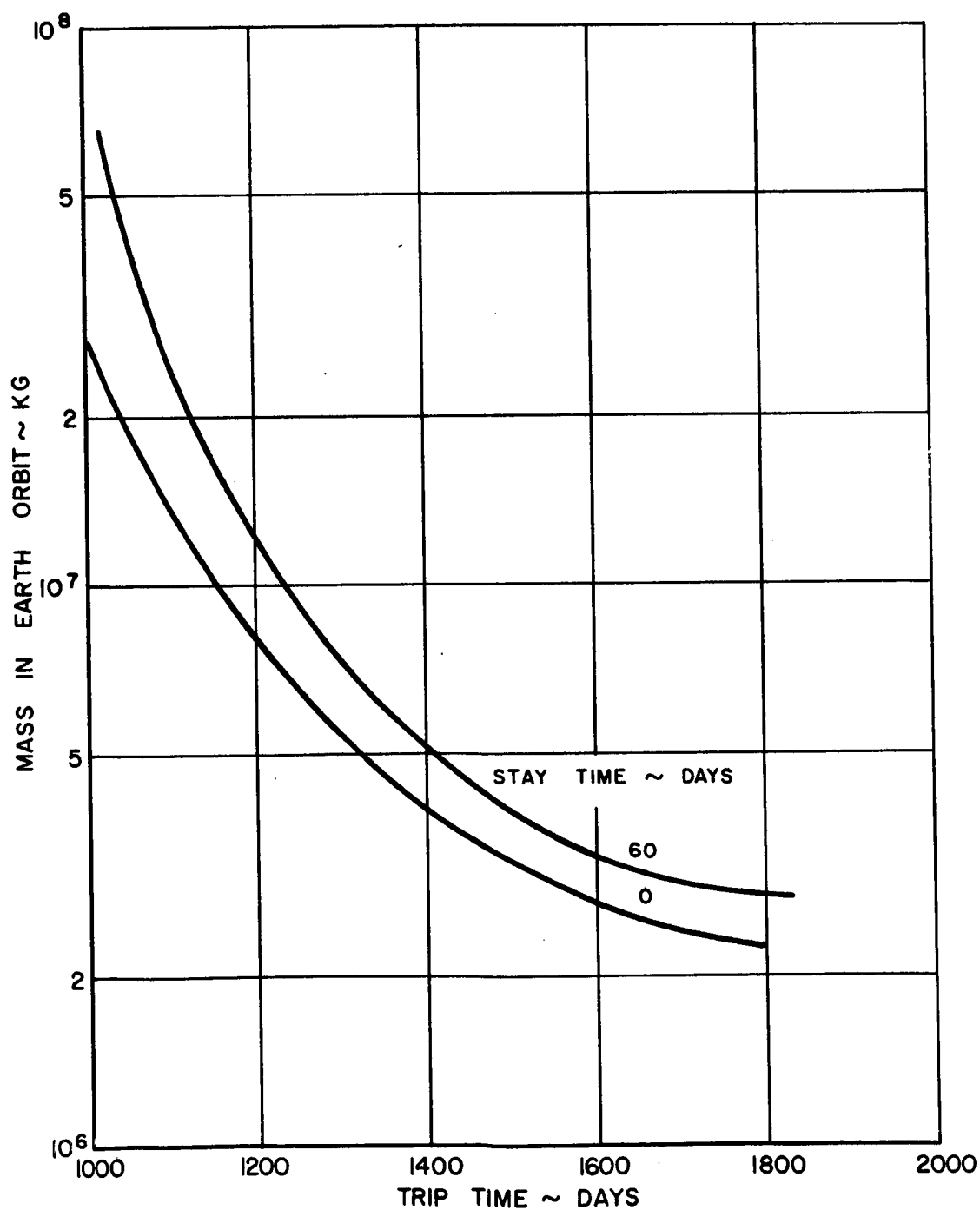
CHEMICAL PLANETARY STAGES ($I_{sp} = 430$ SEC)



GANYMEDE MANNED ORBITAL STOPOVERS JUPITER III

LIGHTWEIGHT ADVANCED NUCLEAR EARTH ESCAPE ($I_{sp} = 2000$ SEC)

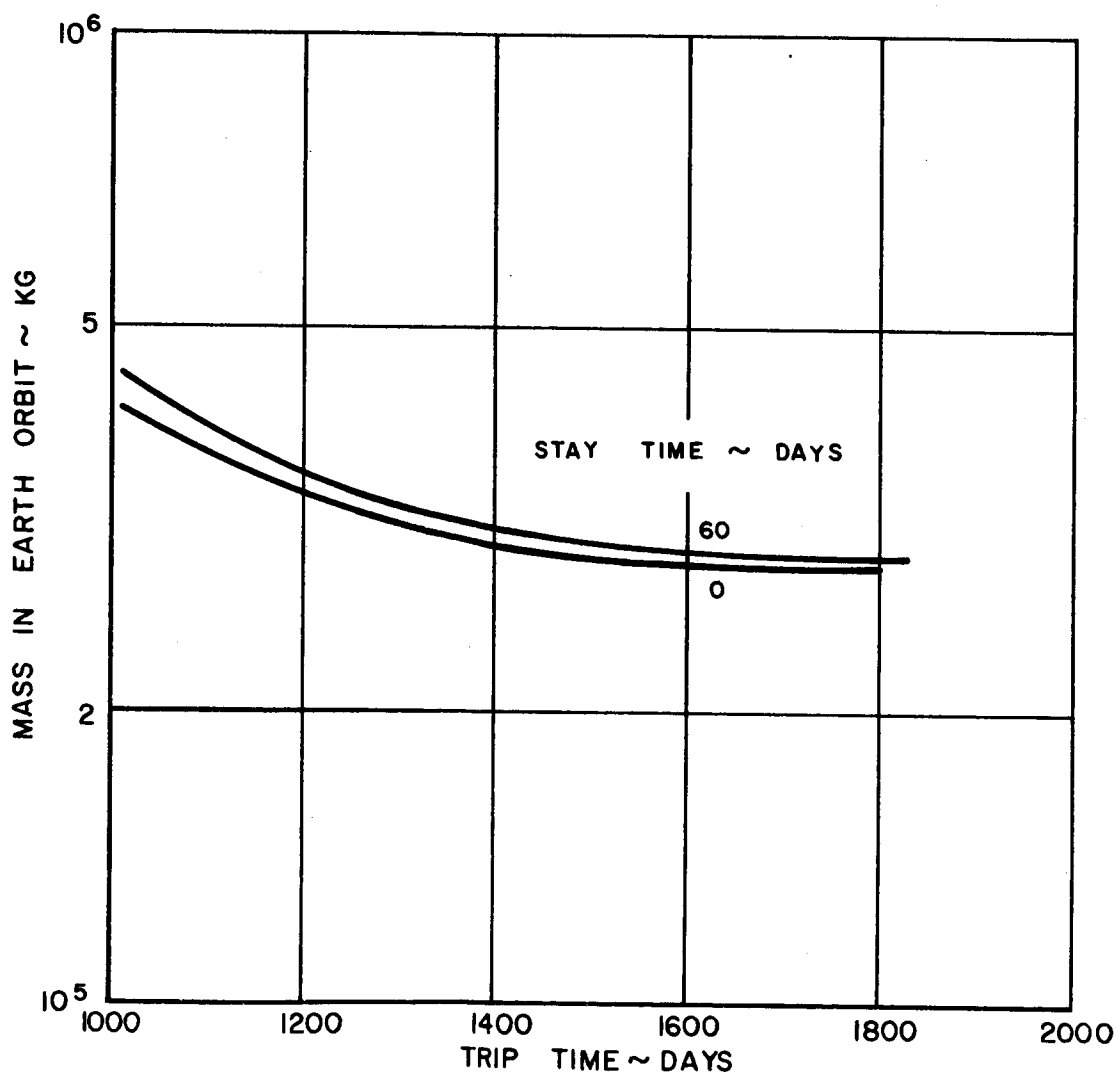
SOLID-CORE NUCLEAR PLANETARY STAGES ($I_{sp} = 800$ SEC)



GANYMEDE MANNED ORBITAL STOPOVERS JUPITER III

LIGHTWEIGHT ADVANCED NUCLEAR EARTH ESCAPE ($I_{sp} = 2000$ SEC)

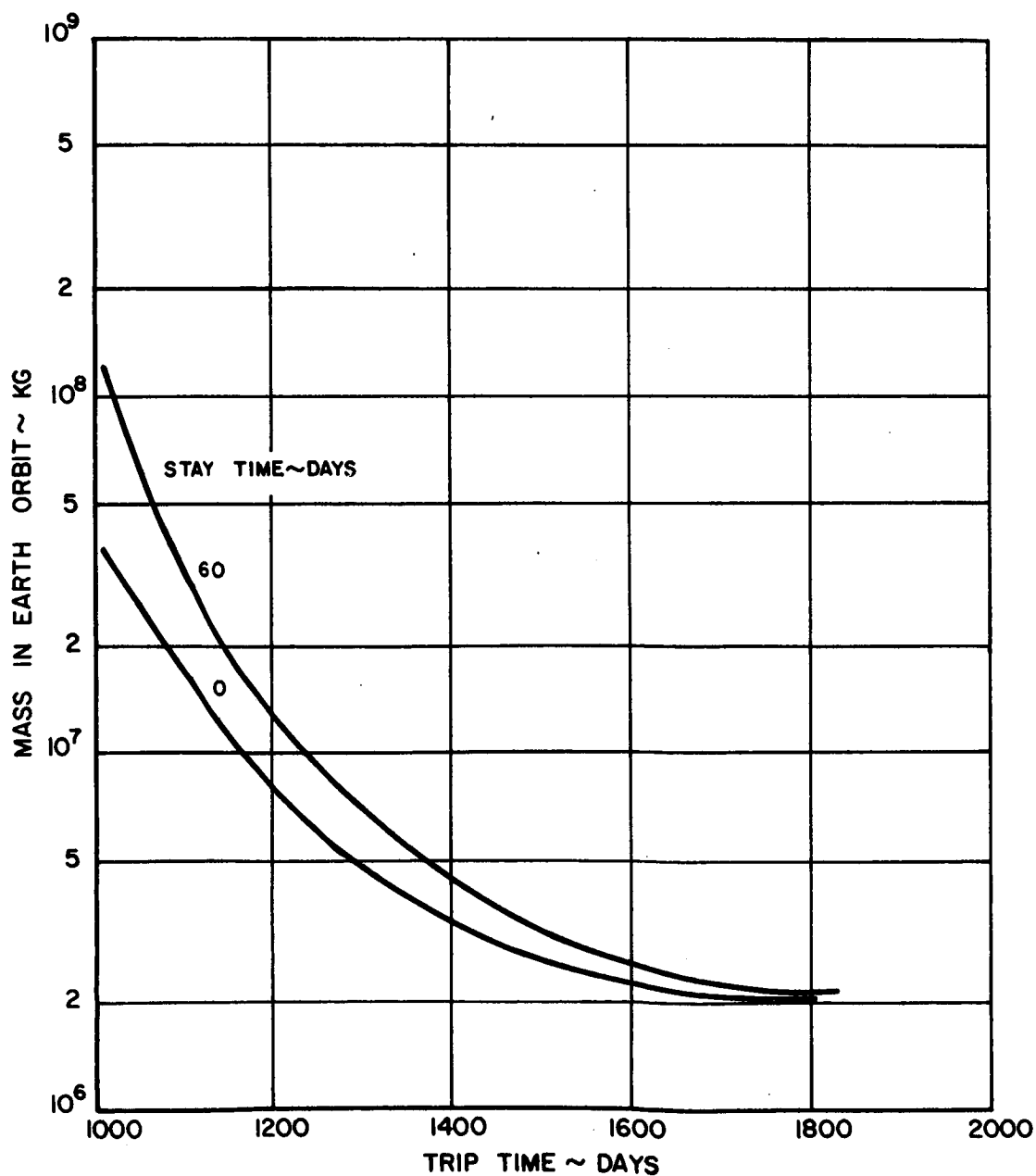
GASEOUS-CORE NUCLEAR PLANETARY STAGES ($I_{sp} = 3500$ SEC)



GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

GASEOUS-CORE NUCLEAR EARTH ESCAPE ($I_{sp} = 3500$ SEC)

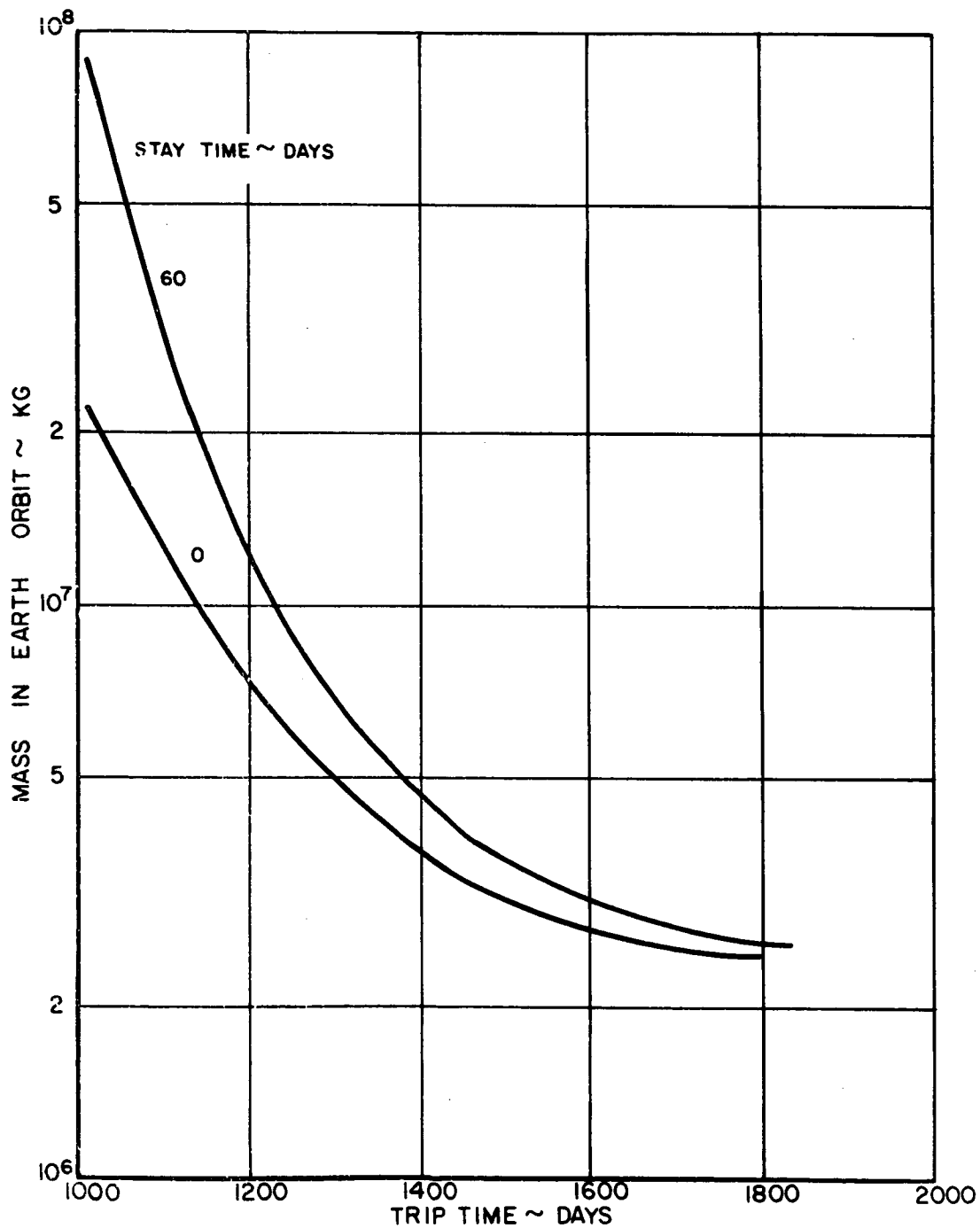
CHEMICAL PLANETARY STAGES ($I_{sp} = 430$ SEC)



GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

GASEOUS CORE NUCLEAR EARTH ESCAPE ($I_{sp} = 3500$ SEC)

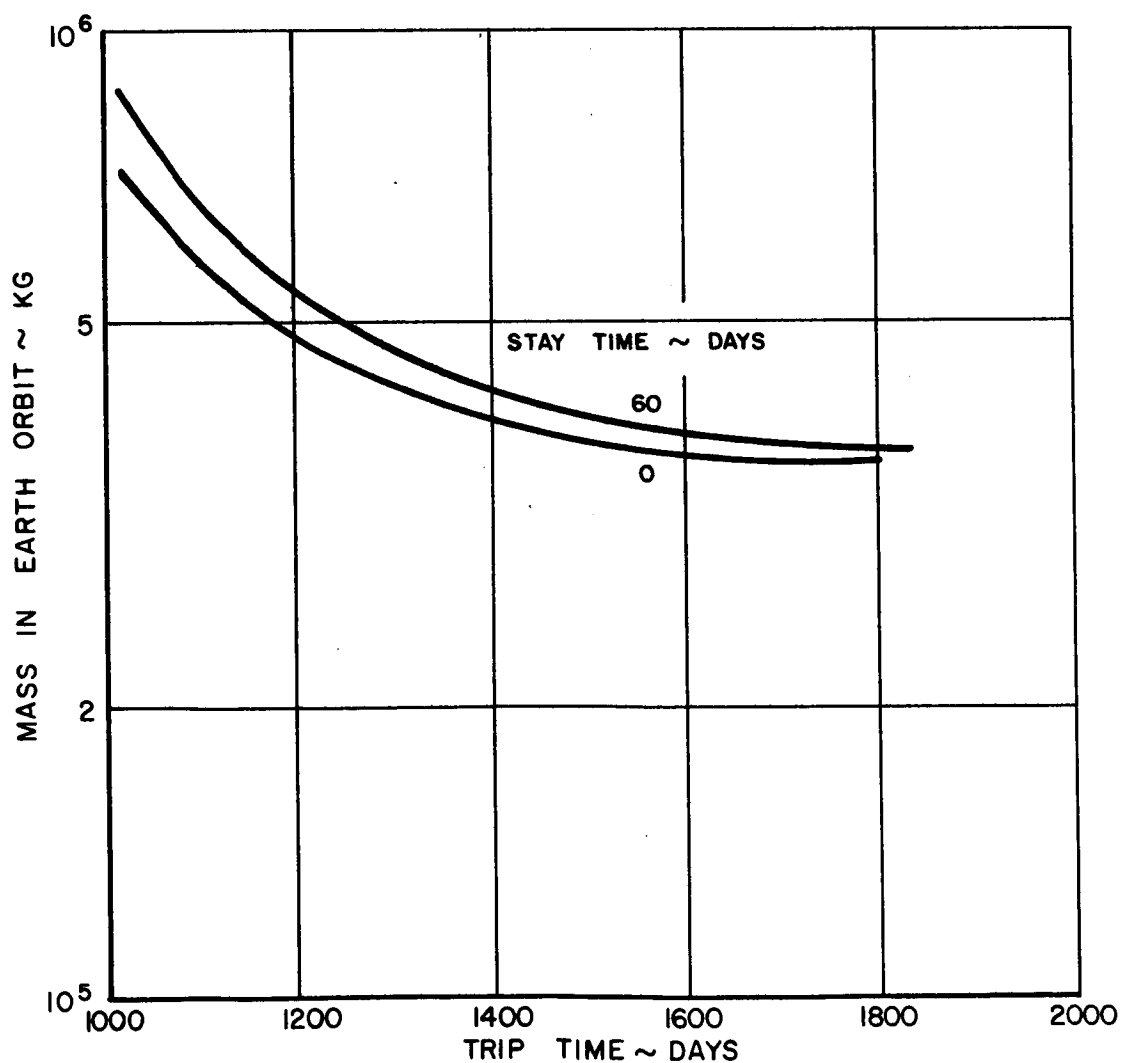
SOLID-CORE NUCLEAR PLANETARY STAGES ($I_{sp} = 800$ SEC)



GANYMEDE MANNED ORBITAL STOPOVERS JUPITER III

GASEOUS-CORE NUCLEAR EARTH ESCAPE ($I_{sp} = 3500$ SEC)

LIGHTWEIGHT ADVANCED NUCLEAR PLANETARY STAGES ($I_{sp} = 2000$ SEC)



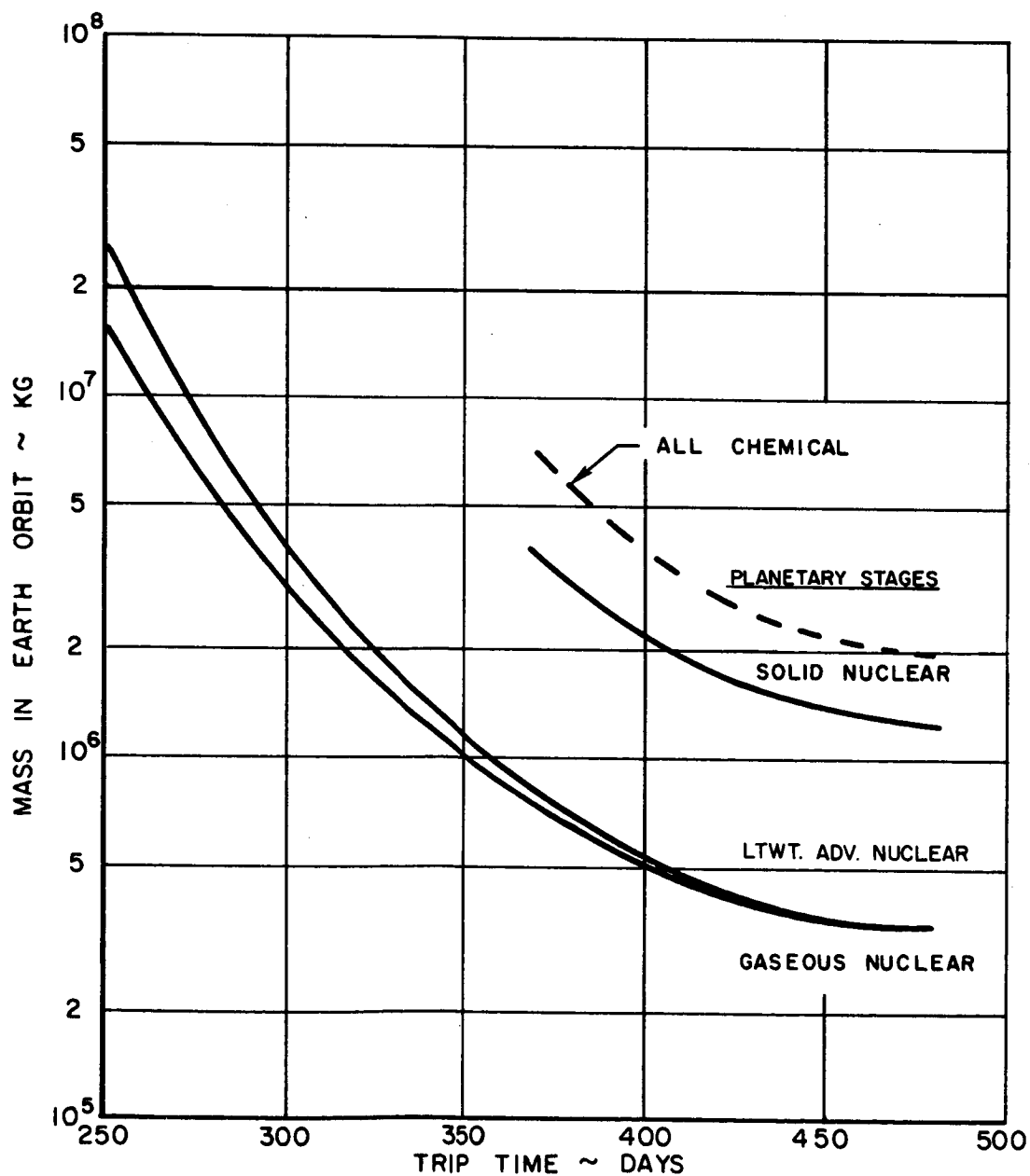
MARS MANNED ORBITAL STOPOVERS

CHEMICAL EARTH ESCAPE

20 - DAY STOPOVERS

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
LTWT. ADV. NUC.	2000	10.0	907
ADV. GAS. NUC.	3500	2.0	13600



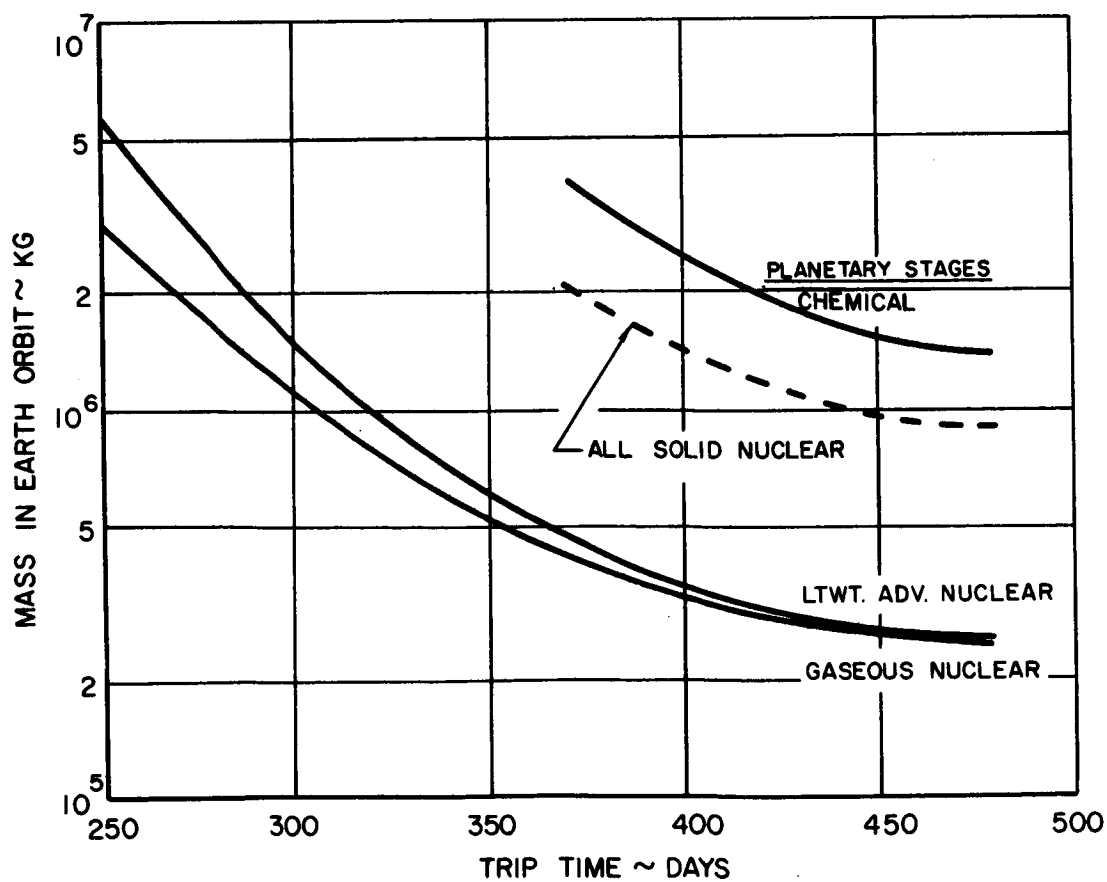
MARS MANNED ORBITAL STOPOVERS

SOLID-CORE NUCLEAR EARTH ESCAPE

20 - DAY STOPOVERS

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
LTWT. ADV. NUC.	2000	10.0	907
ADV. GAS. NUC.	3500	2.0	13600



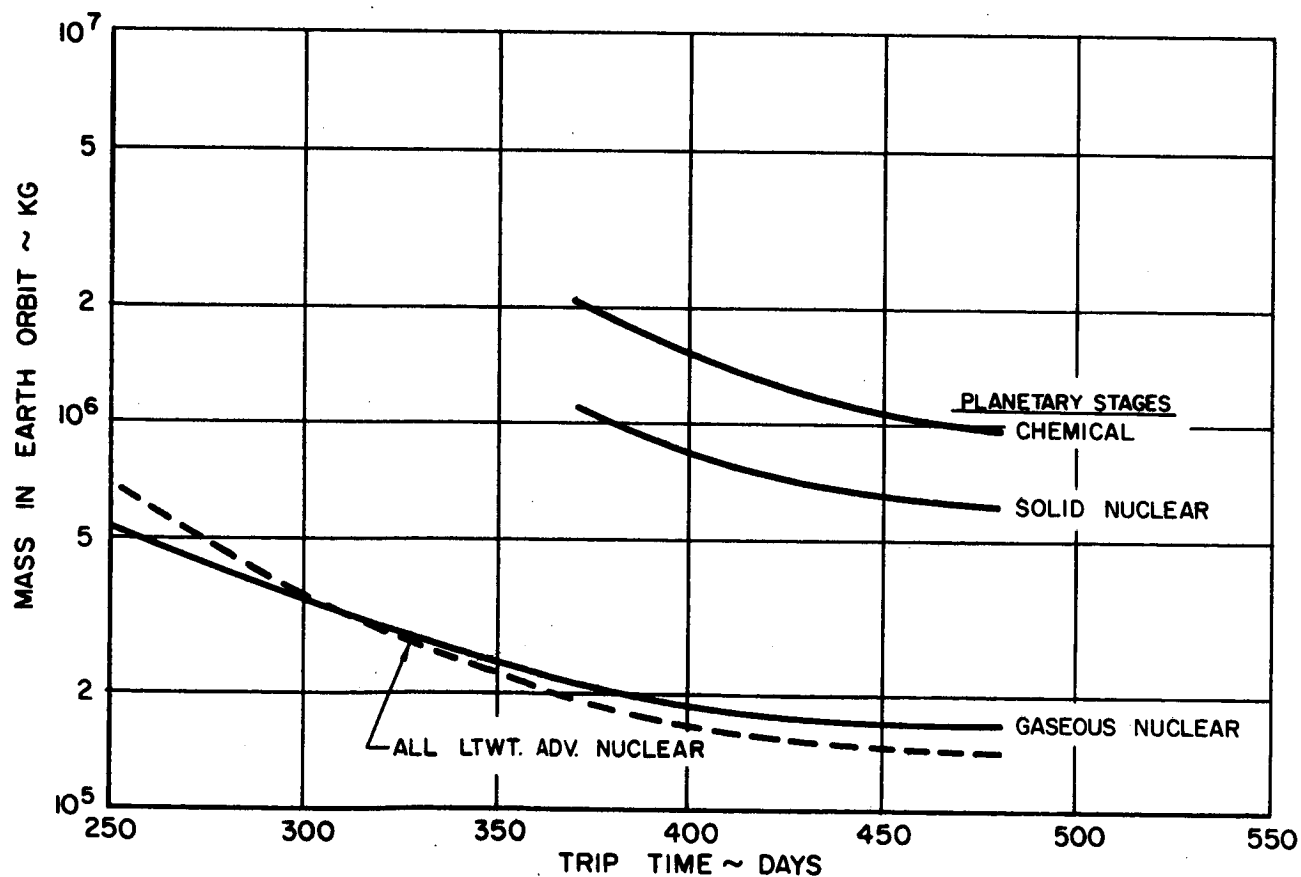
MARS MANNED ORBITAL STOPOVERS

LIQUID-CORE NUCLEAR EARTH ESCAPE

20-DAY STOPOVERS

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
LTWT. ADV. NUC.	2000	10.0	907
ADV. GAS. NUC.	3500	2.0	13600



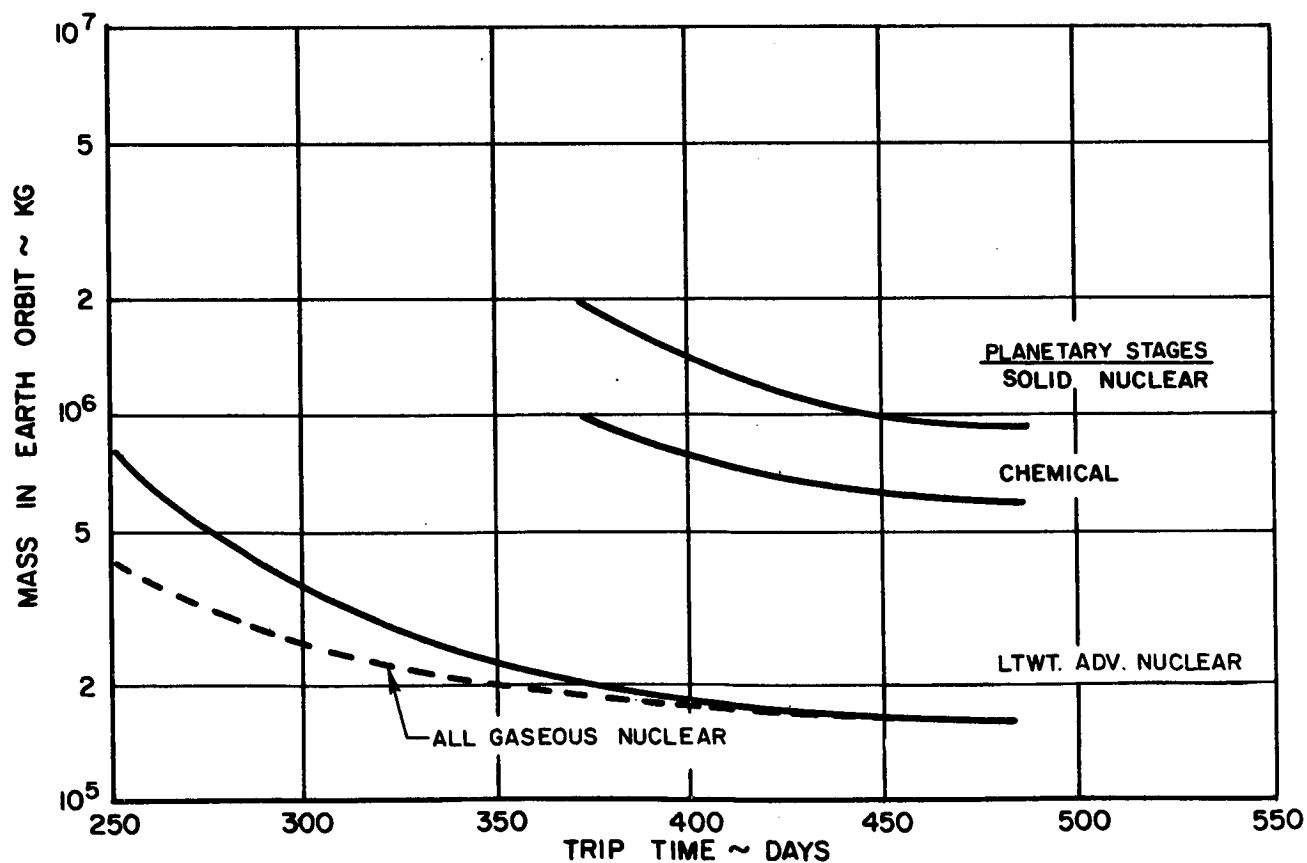
MARS MANNED ORBITAL STOPOVERS

GASEOUS - CORE NUCLEAR EARTH ESCAPE

20-DAY STOPOVERS

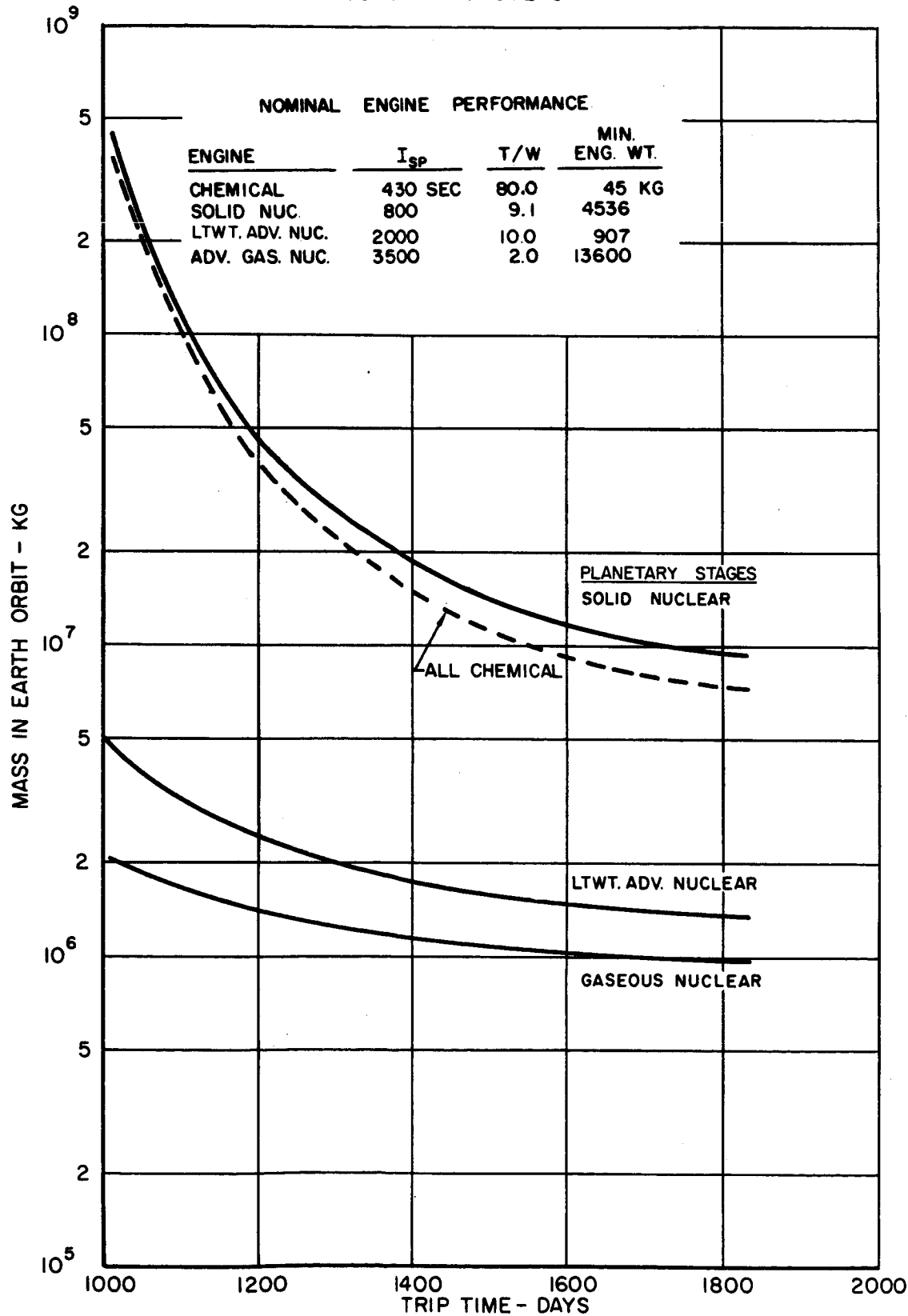
NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
LTWT. ADV. NUC.	2000	10.0	907
ADV. GAS. NUC.	3500	2.0	13600



GANYMEDE MANNED ORBITAL STOPOVERS
(JUPITER III)
CHEMICAL EARTH ESCAPE

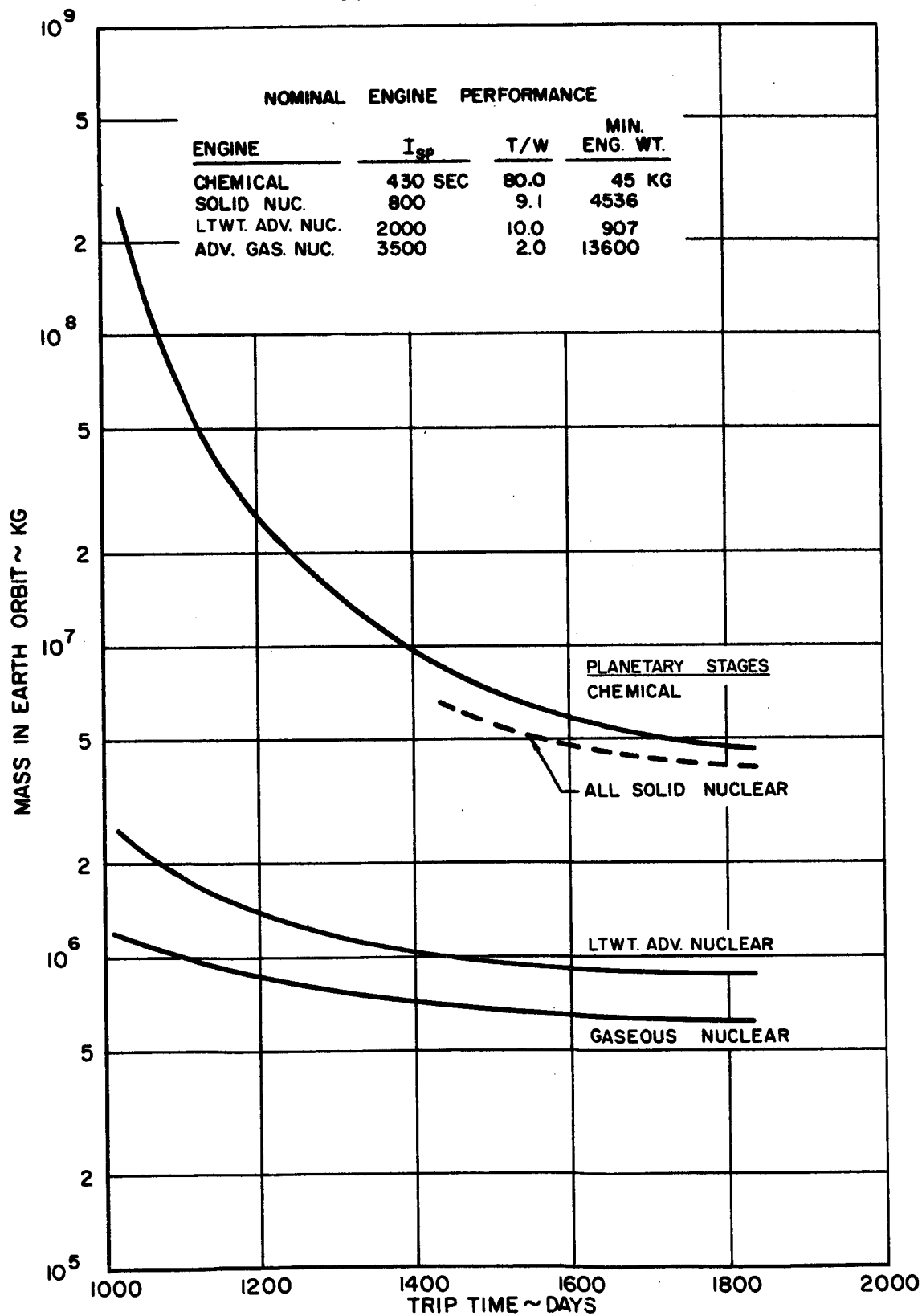
60-DAY STOPOVERS



GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

SOLID-CORE NUCLEAR EARTH ESCAPE

60-DAY STOPOVERS

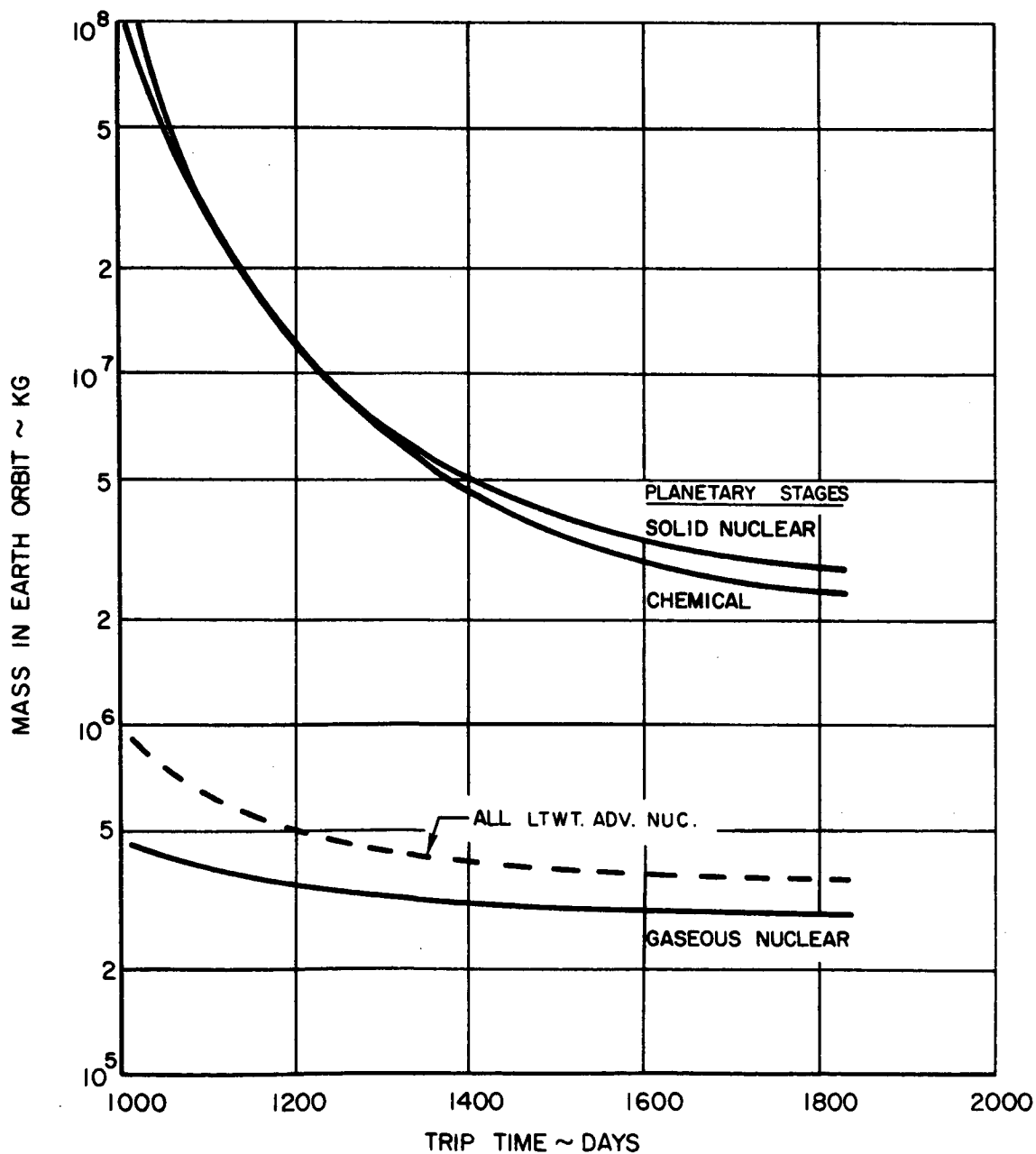


GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III) LIQUID-CORE NUCLEAR EARTH ESCAPE

60 - DAY STOPOVERS

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC	800	9.1	4536
LTWT. ADV. NUC.	2000	10.0	907
ADV. GAS. NUC.	3500	2.0	13600



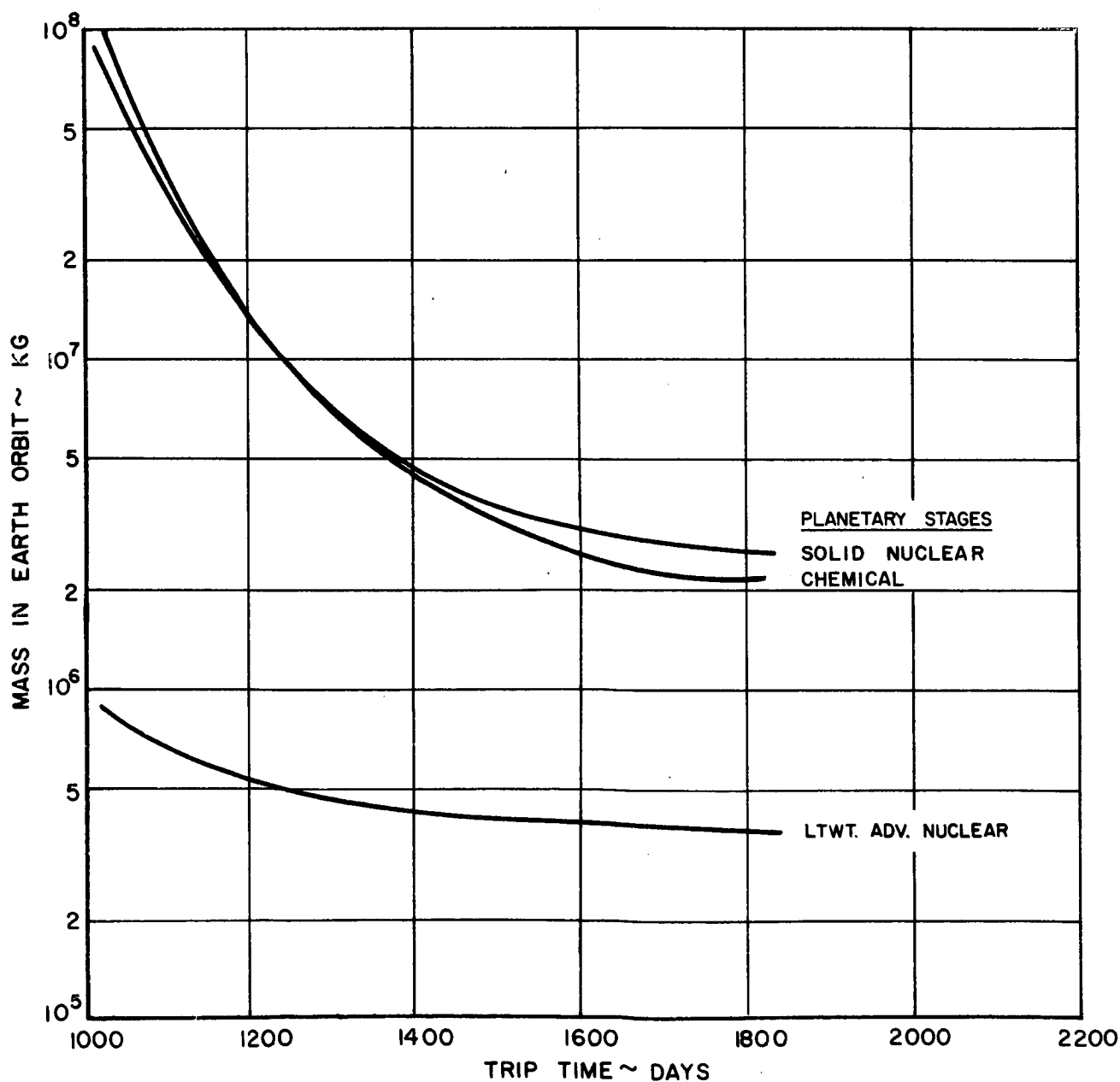
GANYMEDE MANNED ORBITAL STOPOVERS (JUPITER III)

GASEOUS-CORE NUCLEAR EARTH ESCAPE

60-DAY STOPOVER

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
LTWT. ADV. NUC.	2000	10.0	907
ADV. GAS. NUC.	3500	2.0	13600

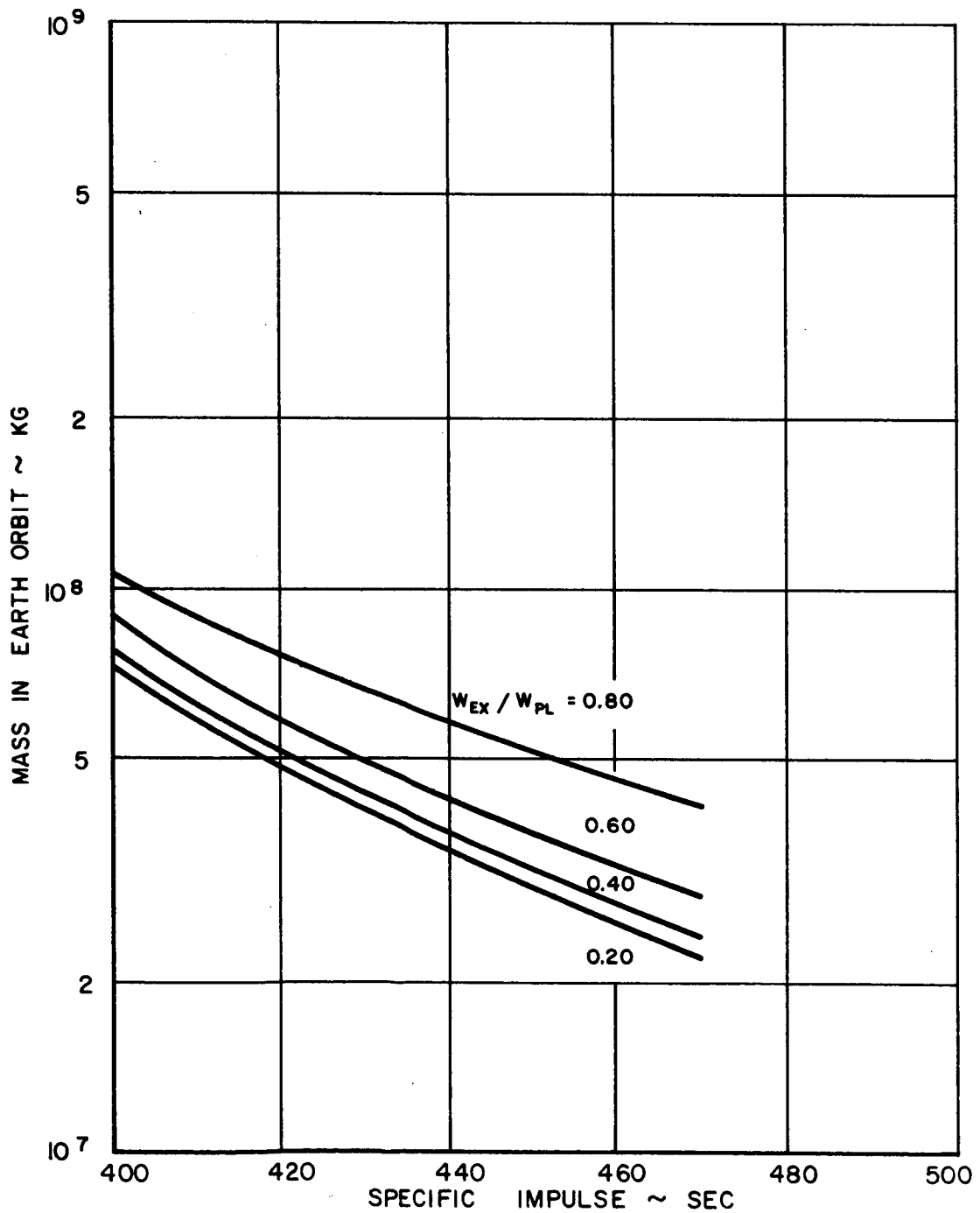


MERCURY MANNED LANDING MISSIONS

CHEMICAL

TRIP TIME = 370 DAYS

STAY TIME = 80 DAYS

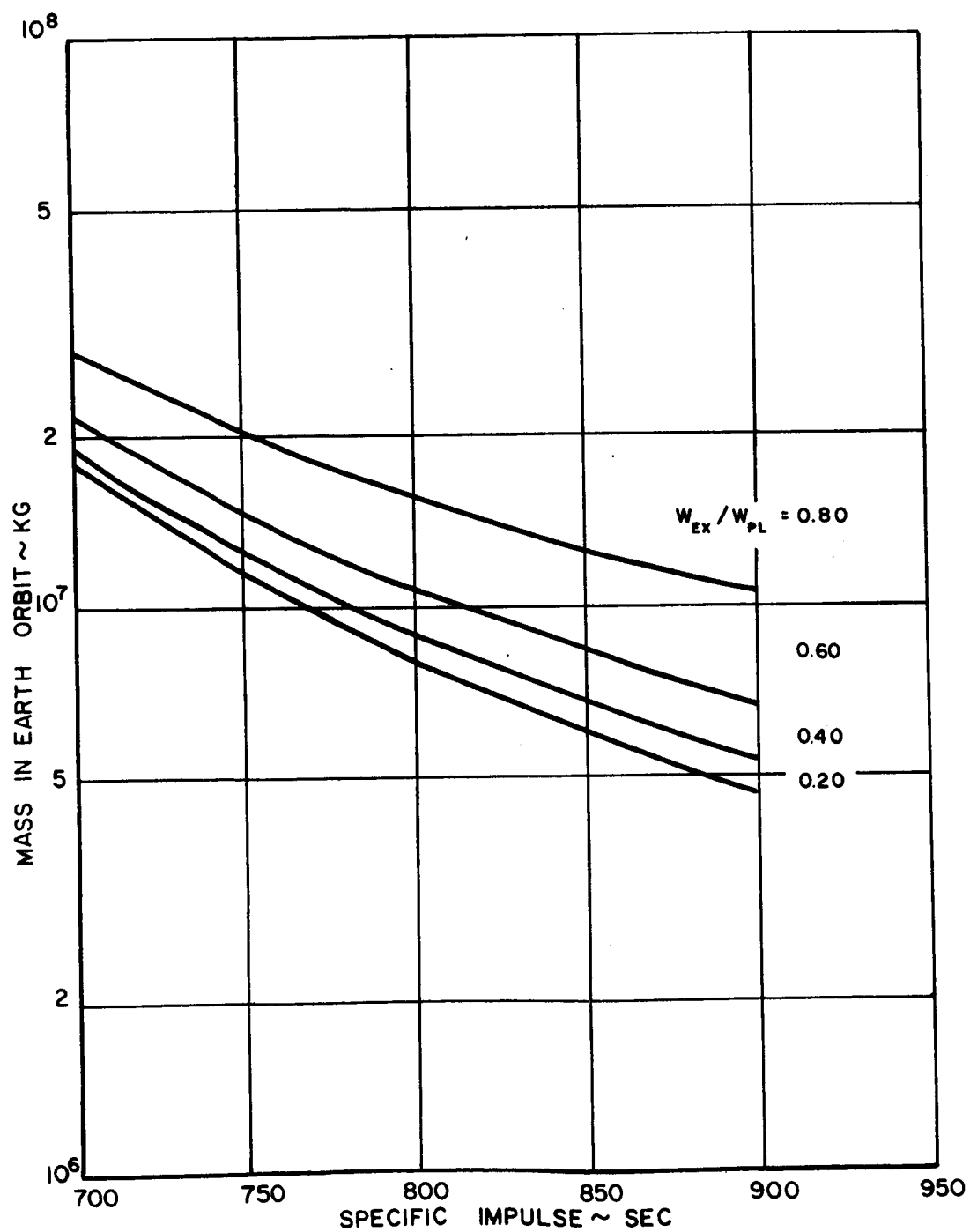


MERCURY MANNED LANDING MISSIONS

SOLID-CORE NUCLEAR

TRIP TIME = 370 DAYS

STAY TIME = 80 DAYS

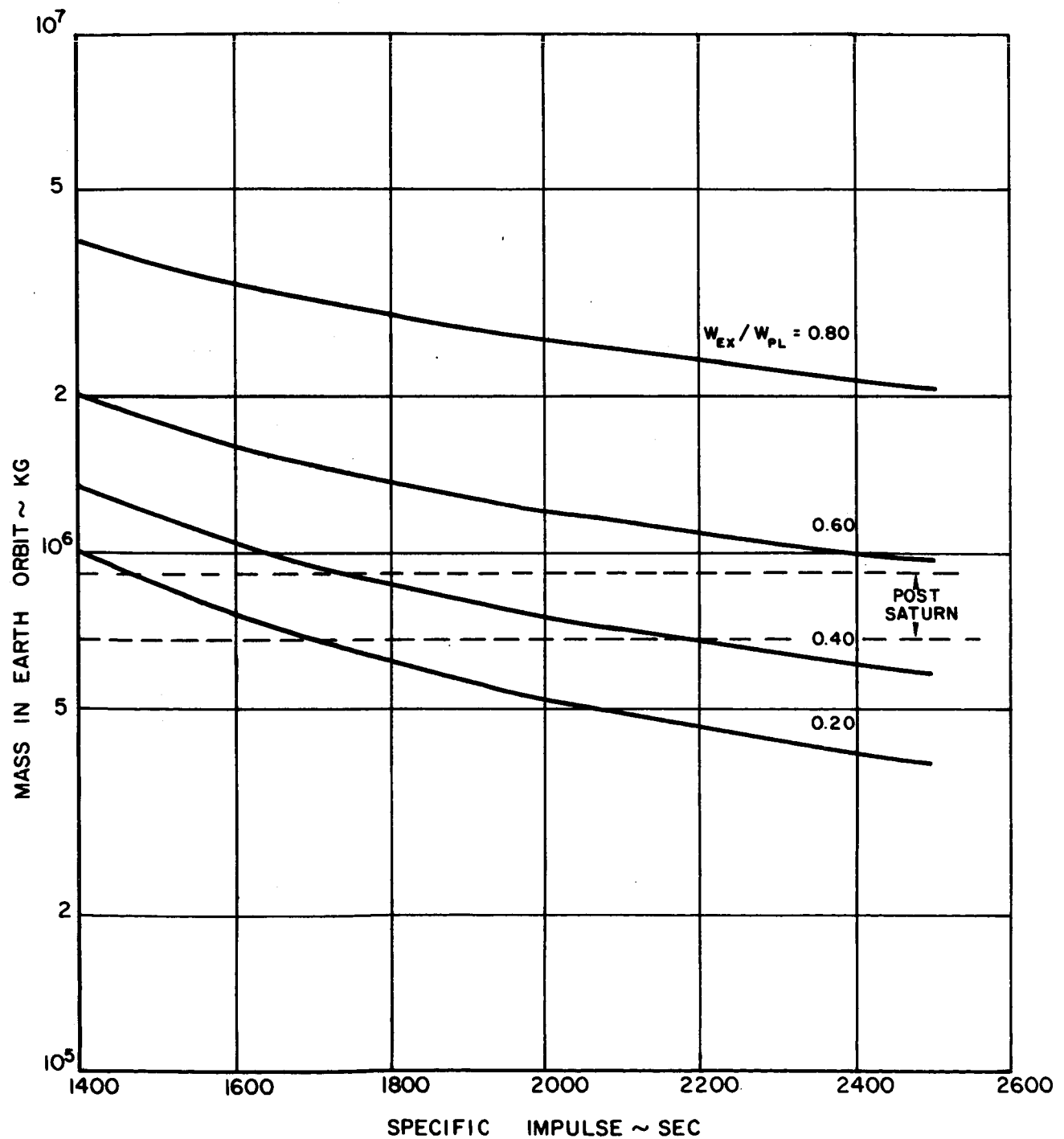


MERCURY MANNED LANDING MISSIONS

LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 370 DAYS

STAY TIME = 80 DAYS

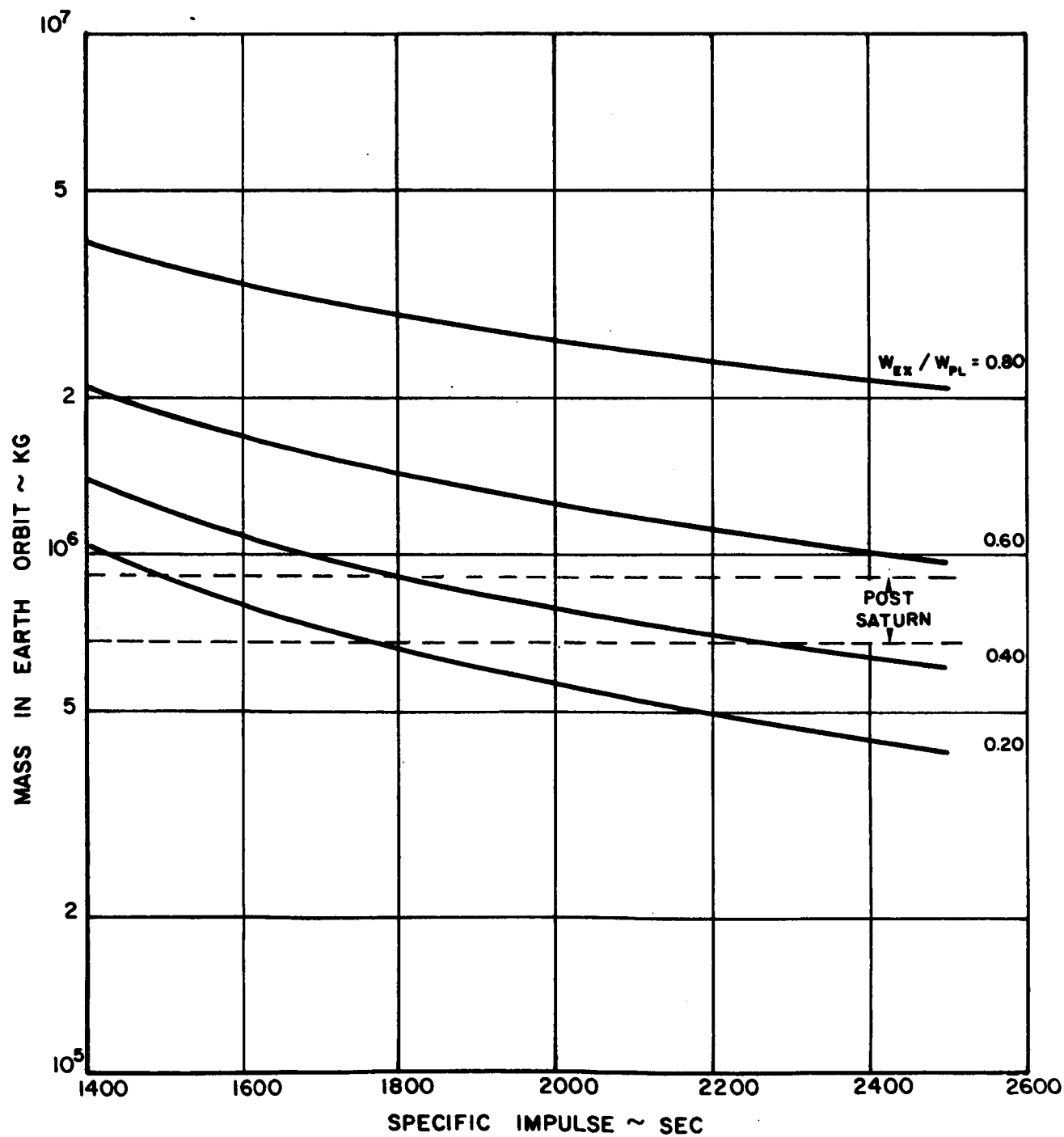


MERCURY MANNED LANDING MISSIONS

GASEOUS-CORE NUCLEAR

TRIP TIME = 370 DAYS

STAY TIME = 80 DAYS

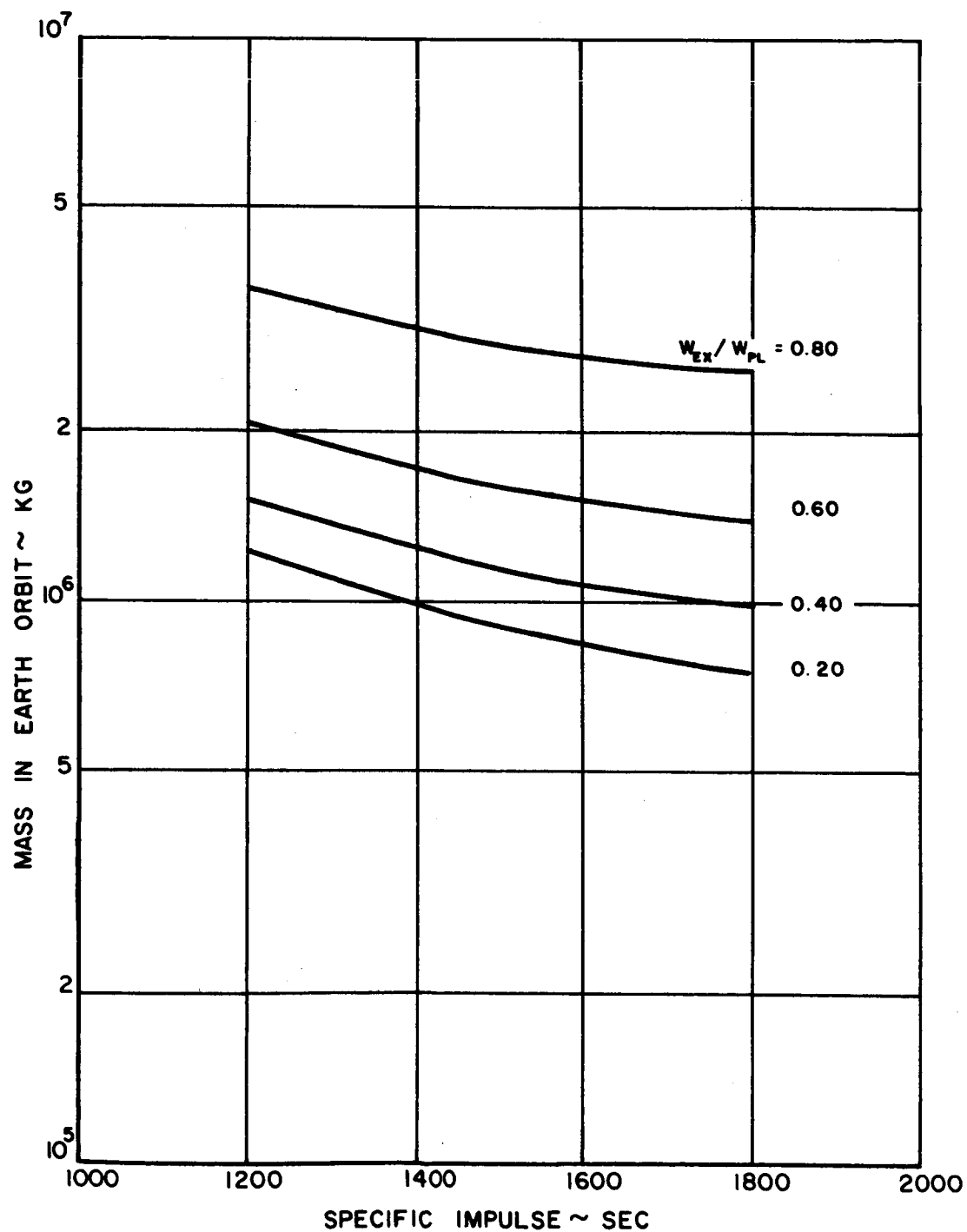


MERCURY MANNED LANDING MISSIONS

PULSED NUCLEAR

TRIP TIME = 370 DAYS

STAY TIME = 80 DAYS

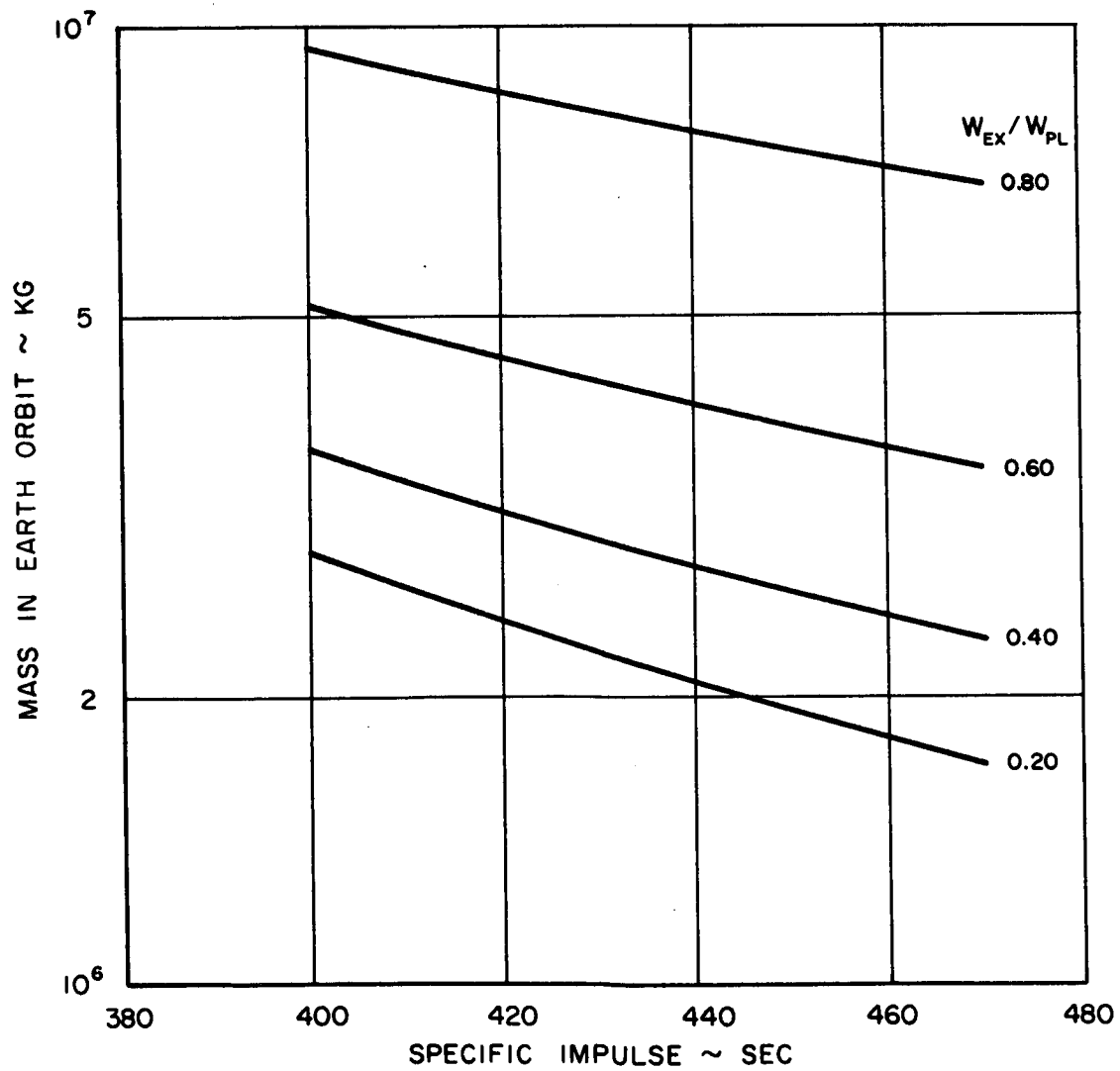


MARS MANNED LANDING MISSIONS

CHEMICAL

TRIP TIME = 470 DAYS

STAY TIME = 0

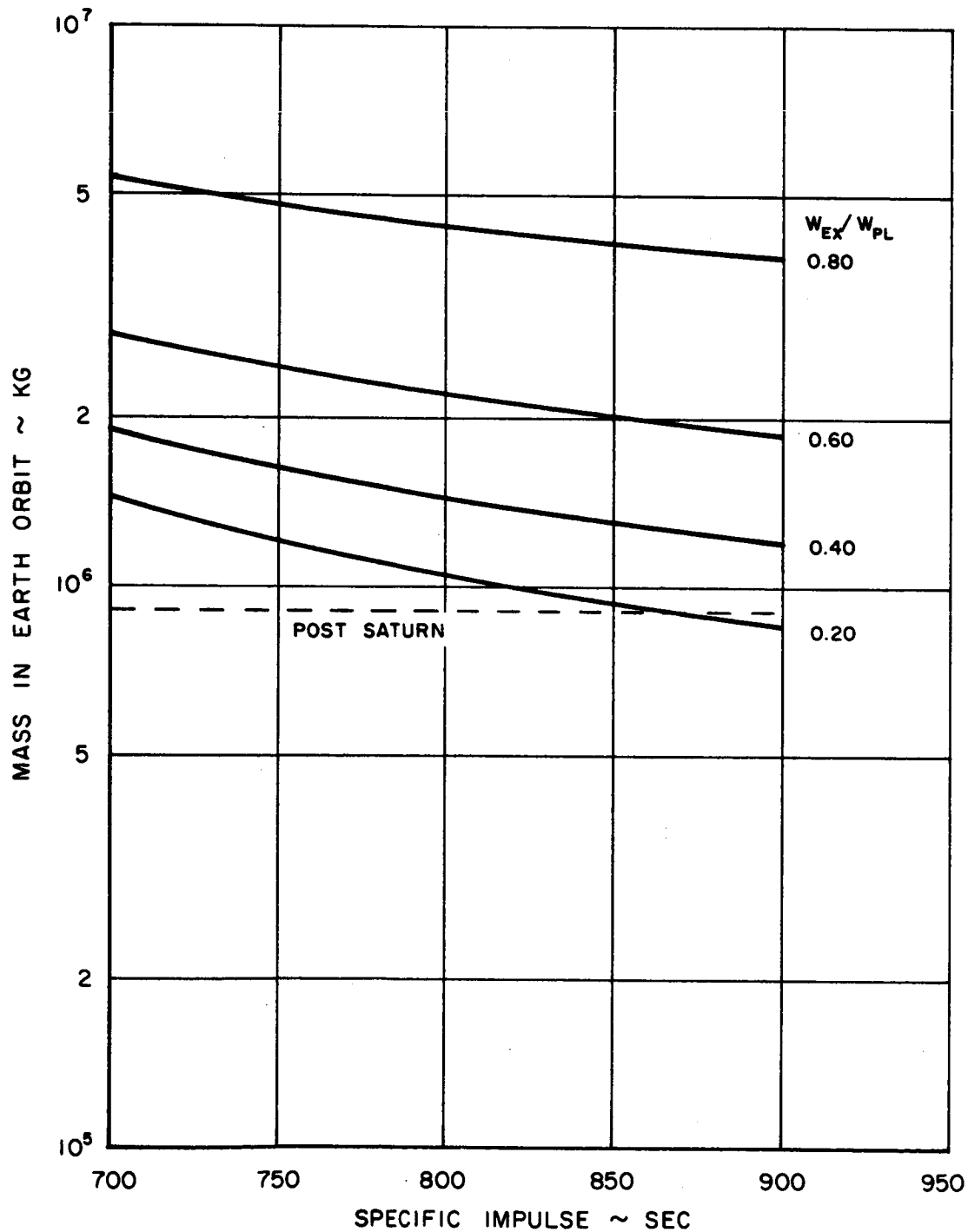


MARS MANNED LANDING MISSIONS

SOLID-CORE NUCLEAR

TRIP TIME = 470 DAYS

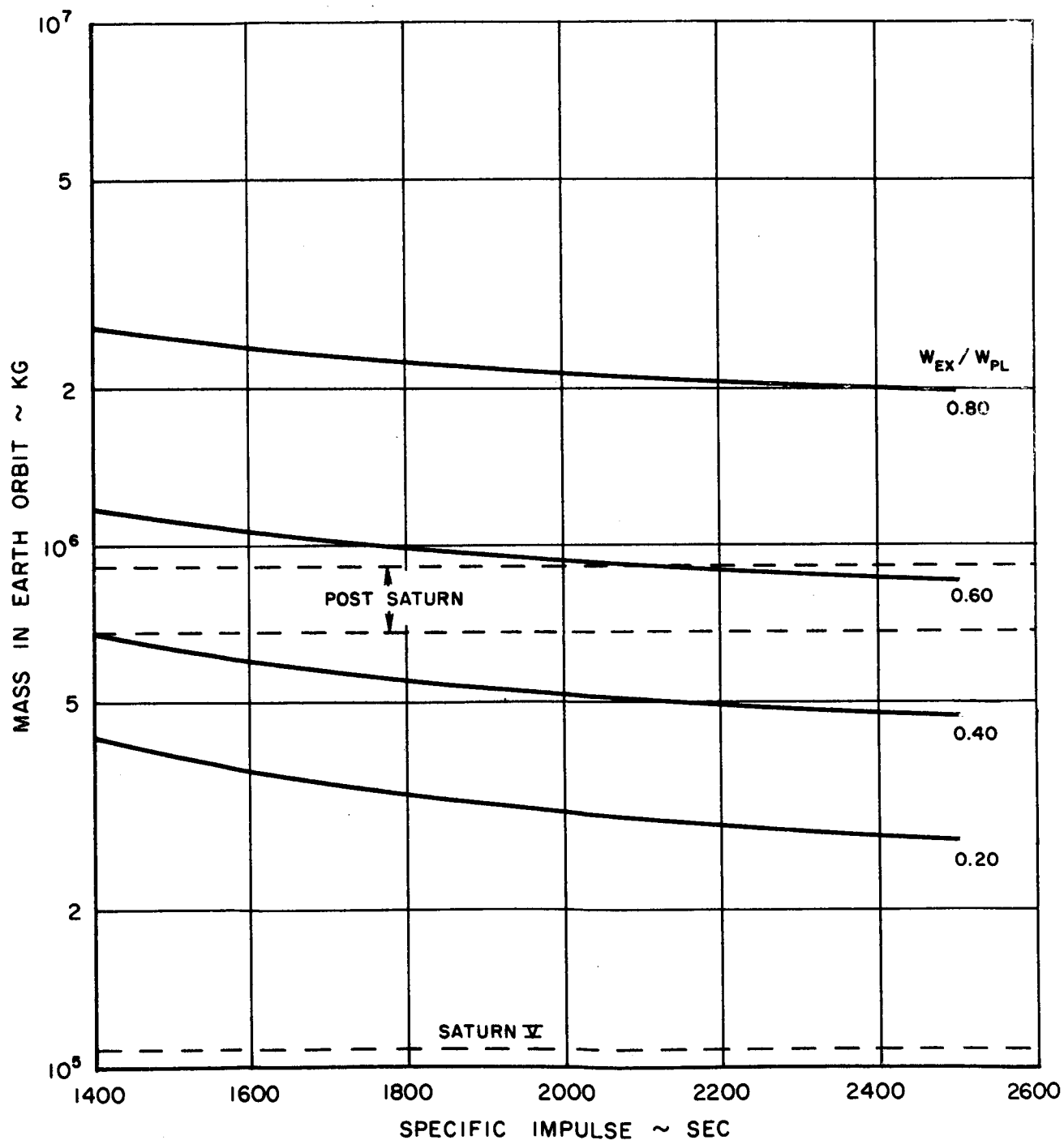
STAY TIME = 0



MARS MANNED LANDING MISSIONS

LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 470 DAYS
STAY TIME = 0

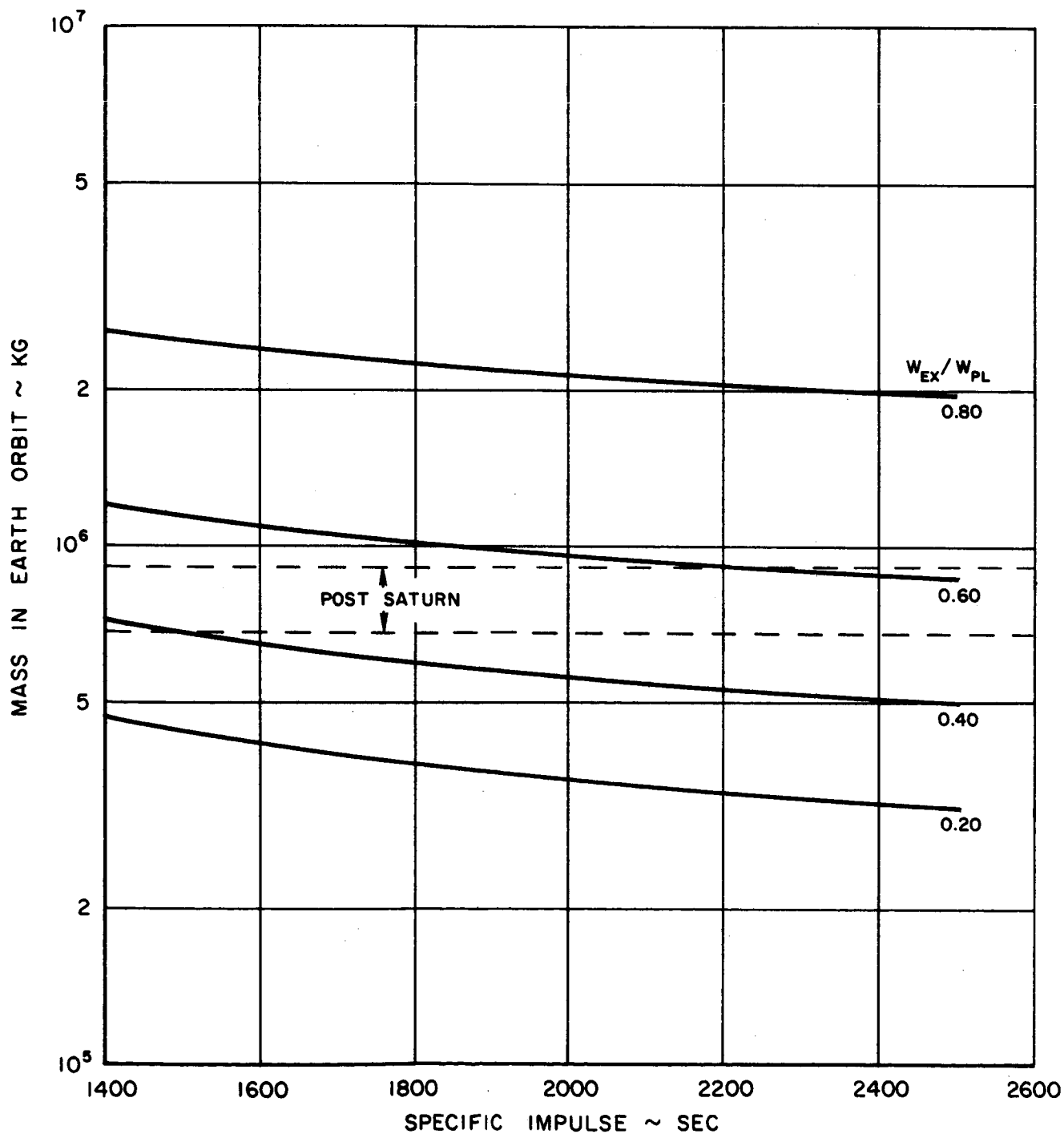


MARS MANNED LANDING MISSIONS

GASEOUS-CORE NUCLEAR

TRIP TIME = 470 DAYS

STAY TIME = 0

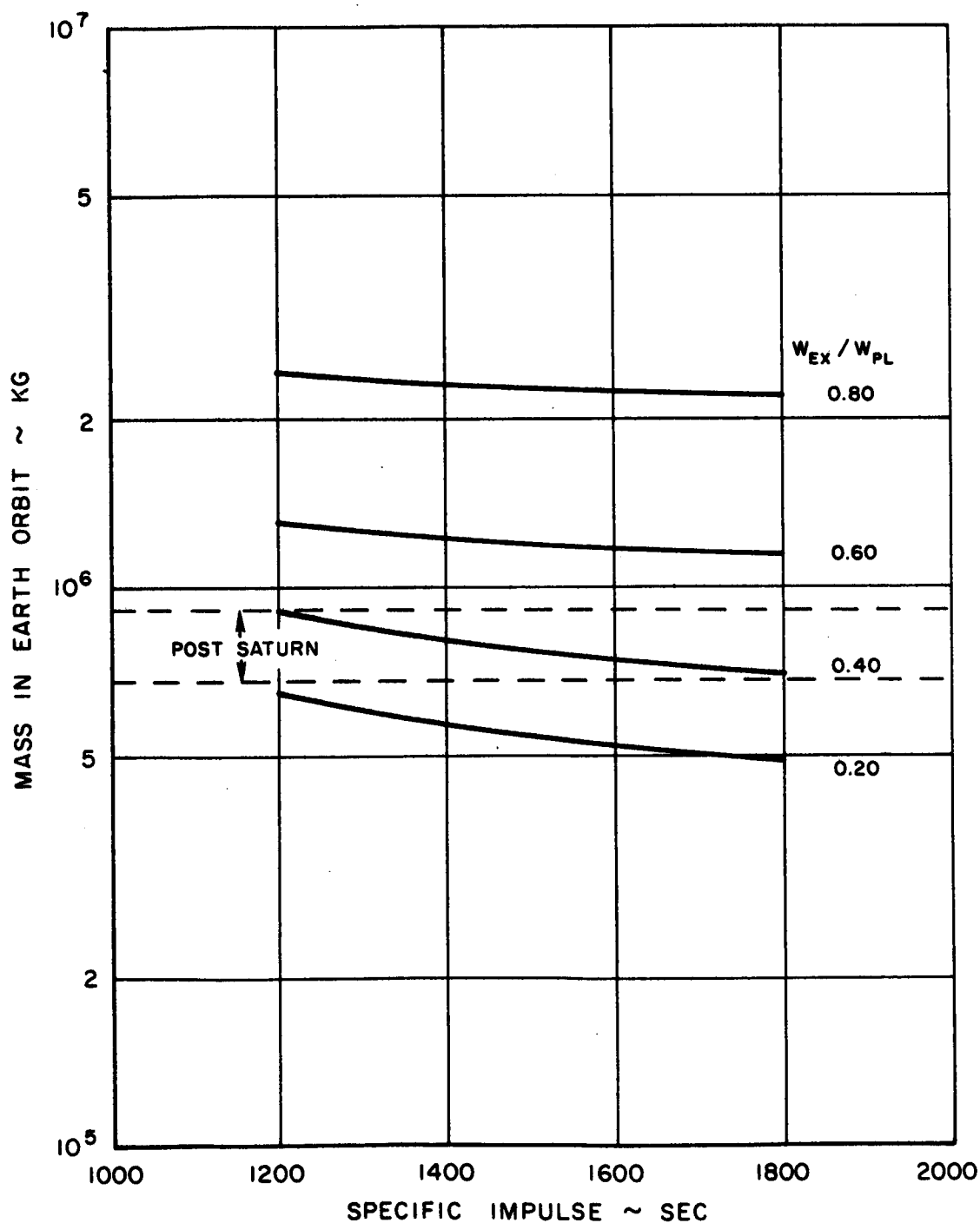


MARS MANNED LANDING MISSIONS

PULSED NUCLEAR

TRIP TIME = 470 DAYS

STAY TIME = 0

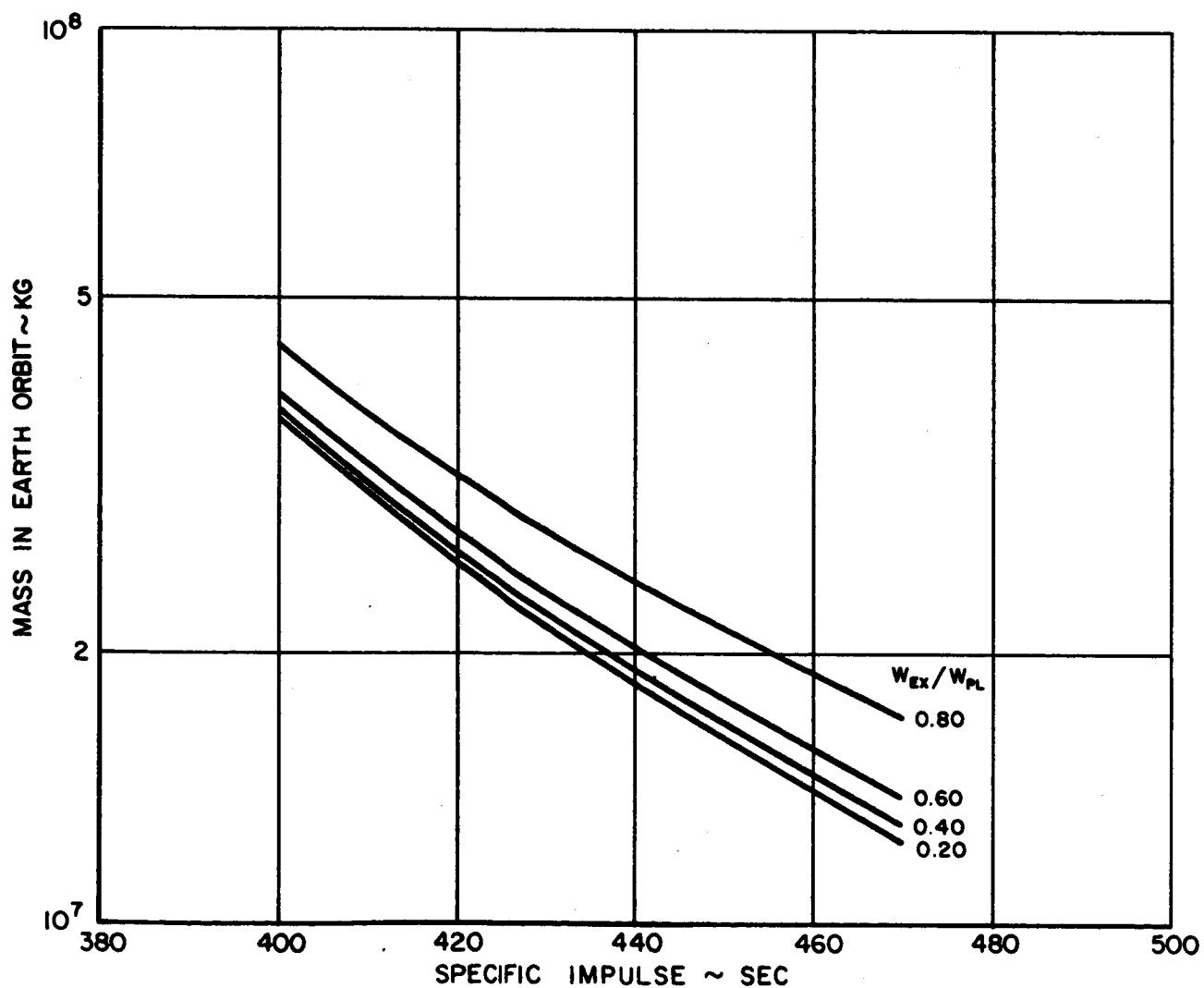


CERES MANNED LANDING MISSION

CHEMICAL

TRIP TIME = 780 DAYS

STAY TIME = 0

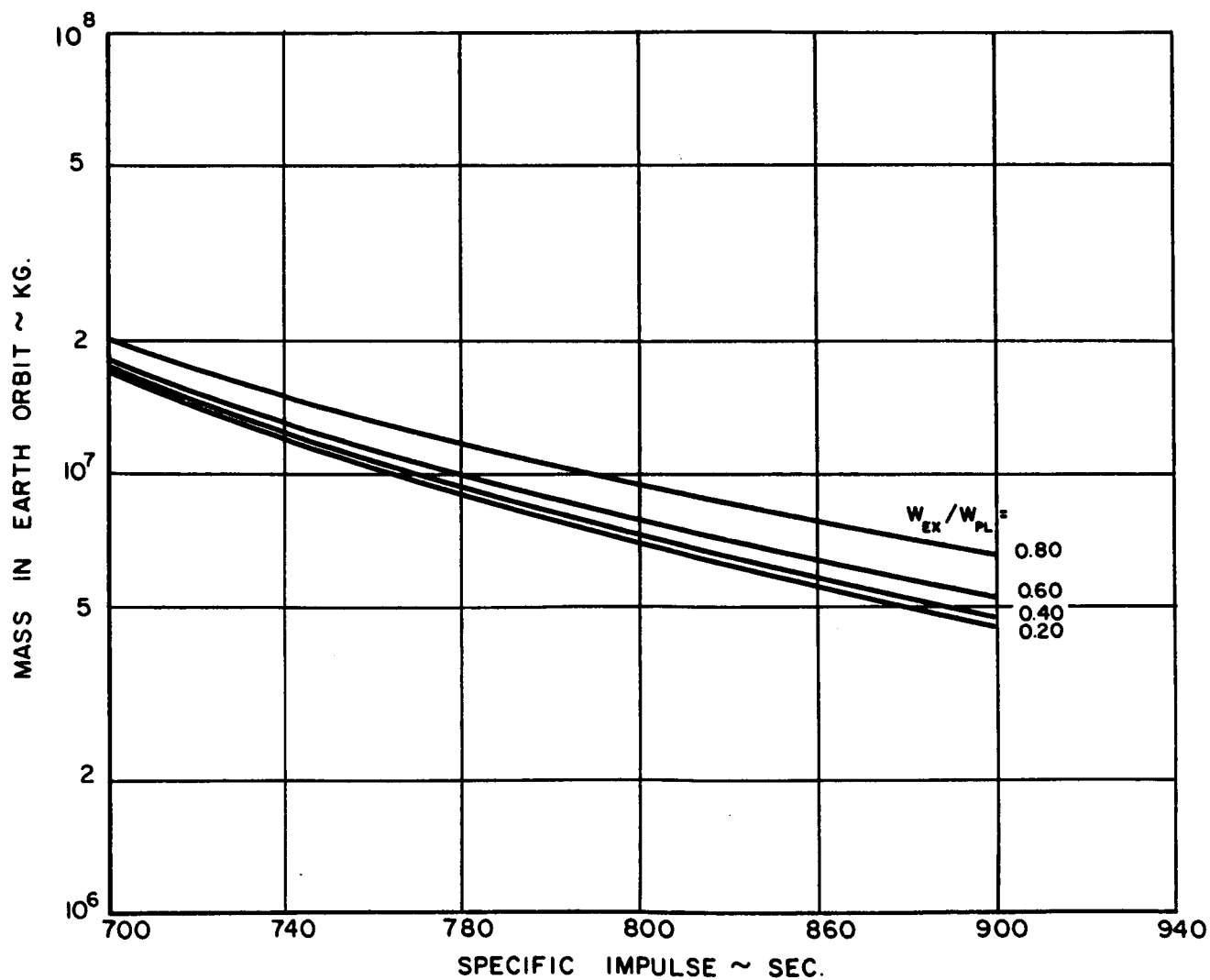


CERES MANNED LANDING MISSIONS

SOLID-CORE NUCLEAR

TRIP TIME = 780 DAYS

STAY TIME = 0

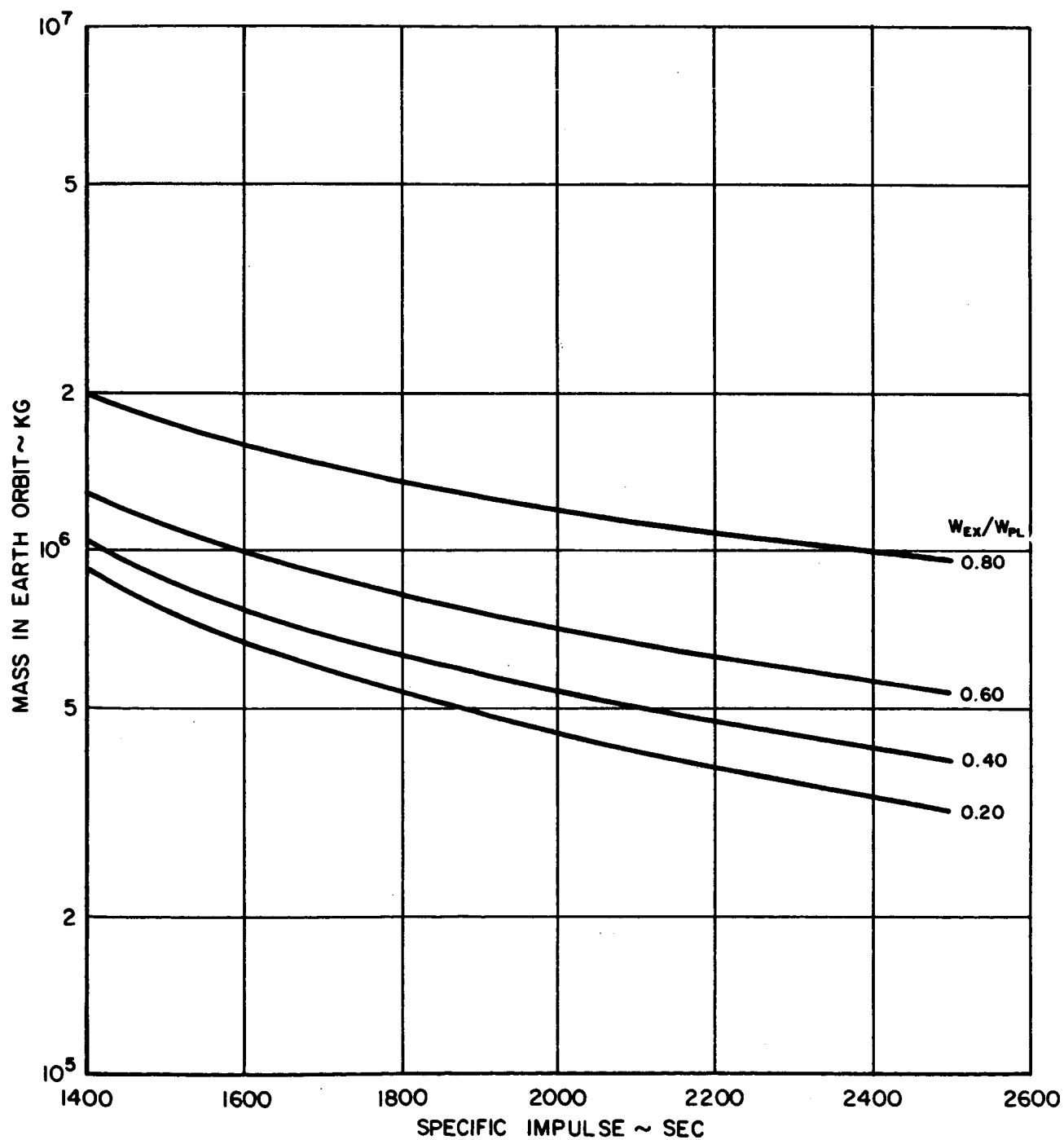


CERES MANNED LANDING MISSION

LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 780 DAYS

STAY TIME = 0

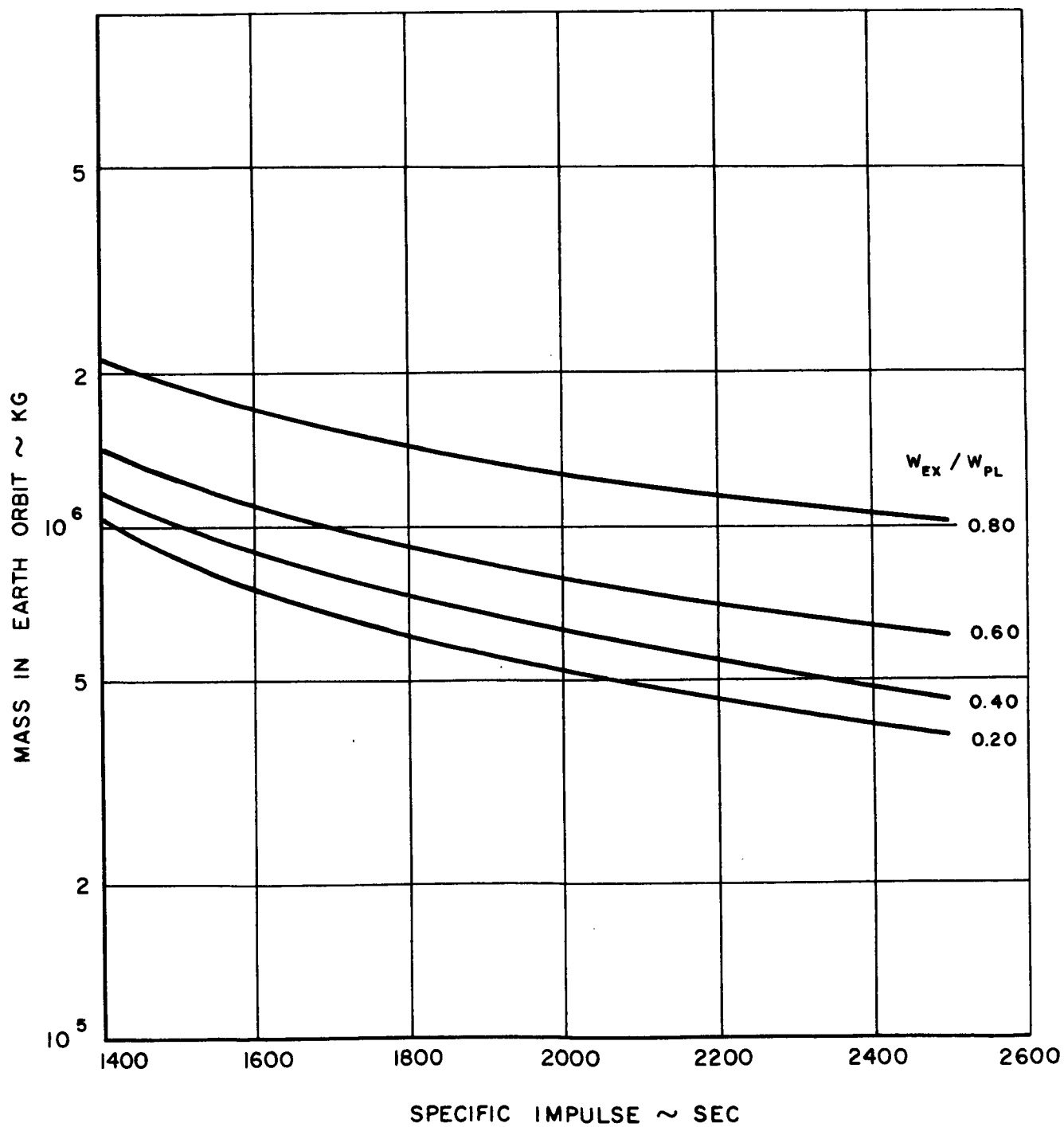


CERES MANNED LANDING MISSION

GASEOUS-CORE NUCLEAR

TRIP TIME = 780 DAYS

STAY TIME = 0

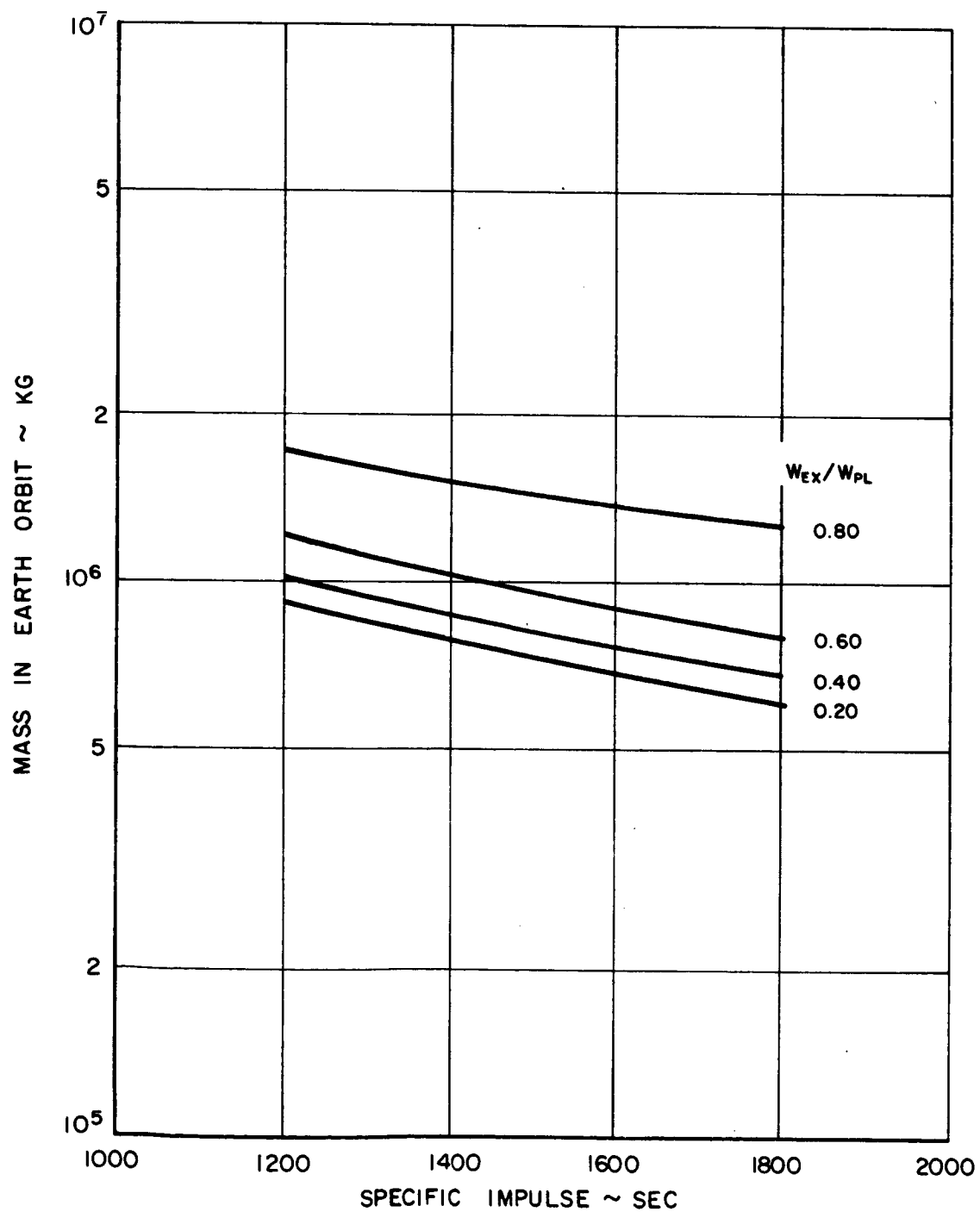


CERES MANNED LANDING MISSION

PULSED NUCLEAR

TRIP TIME = 780 DAYS

STAY TIME = 0

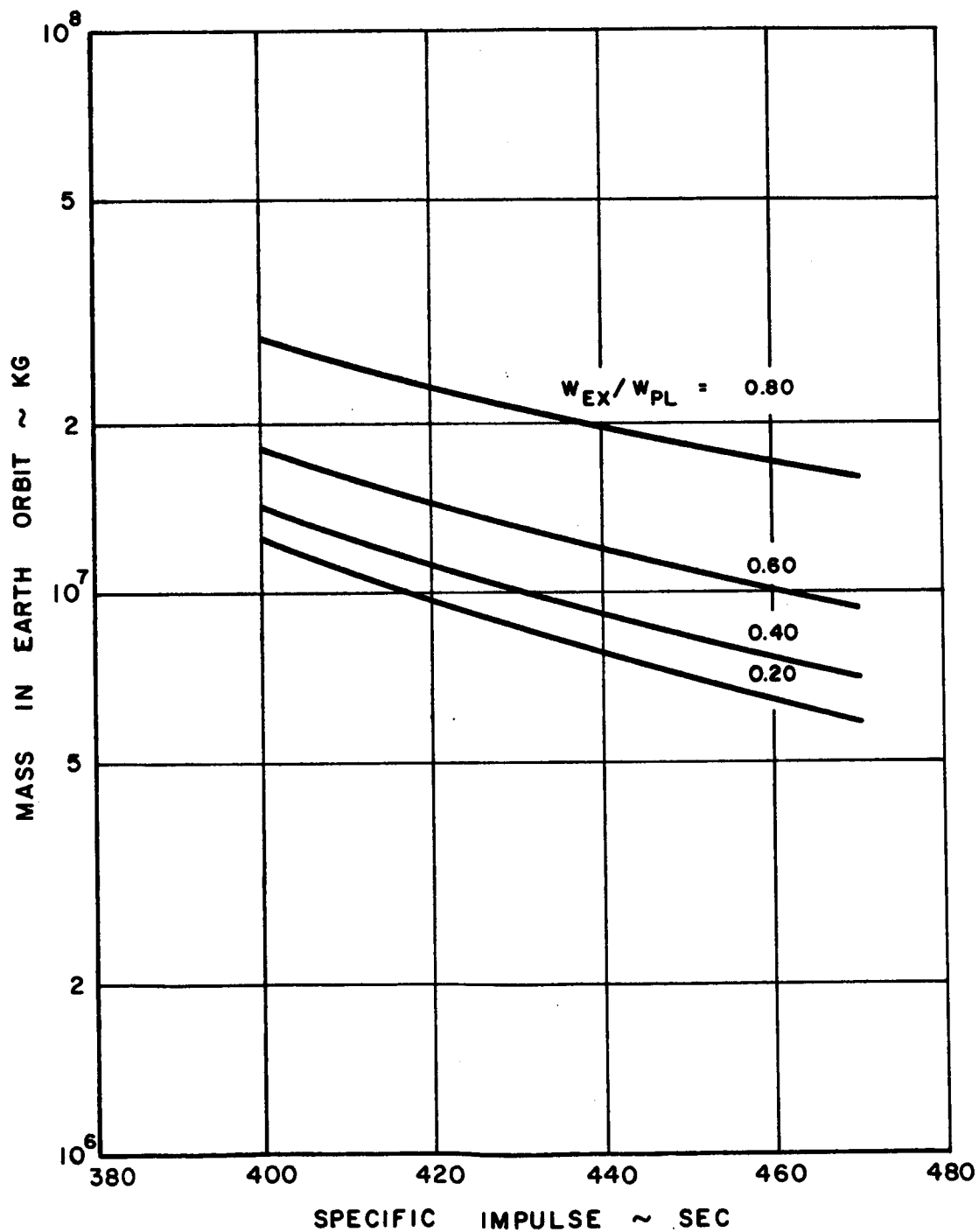


GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

CHEMICAL

TRIP TIME = 1800 DAYS

STAY TIME = 0

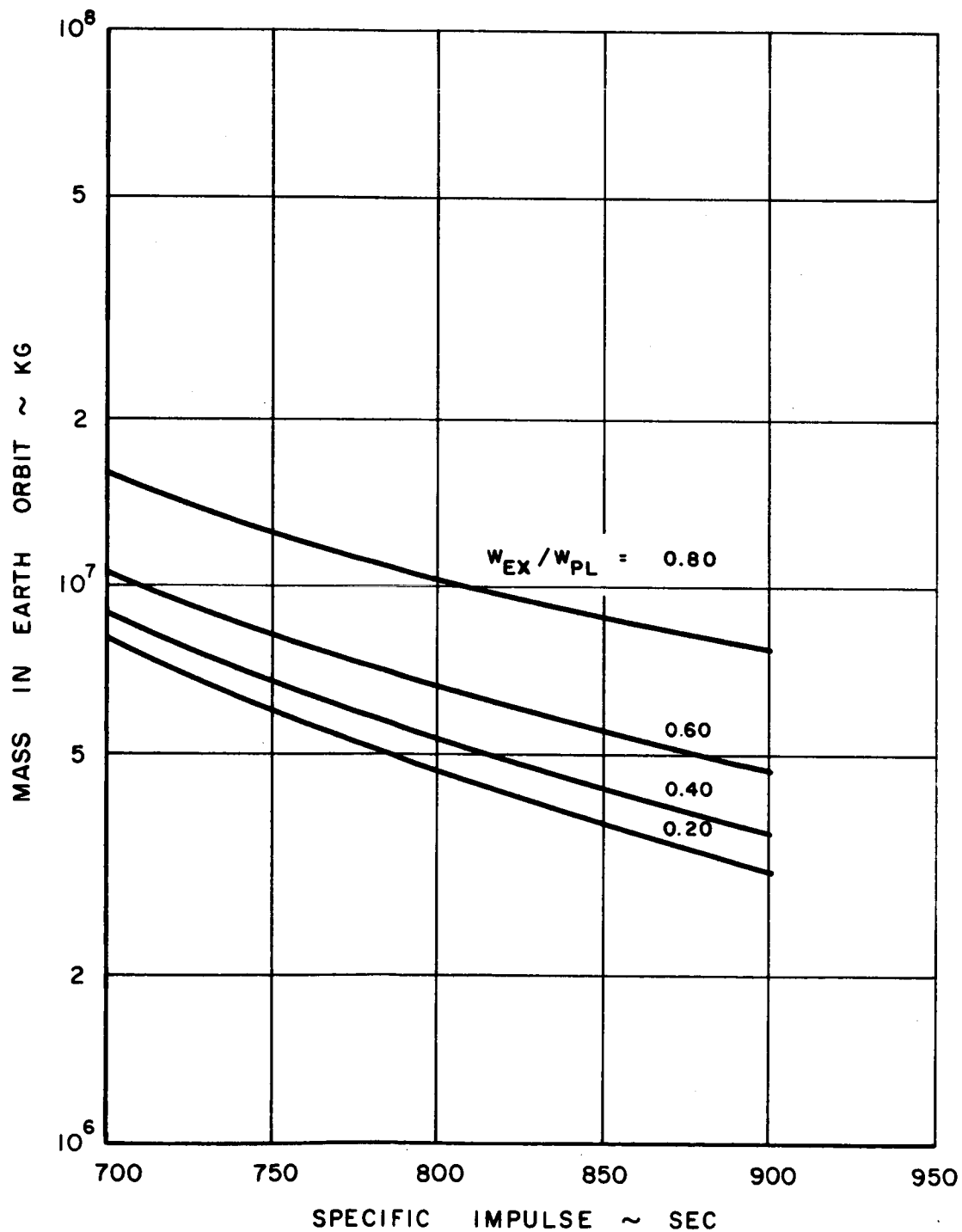


GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

SOLID - CORE NUCLEAR

TRIP TIME = 1800 DAYS

STAY TIME = 0

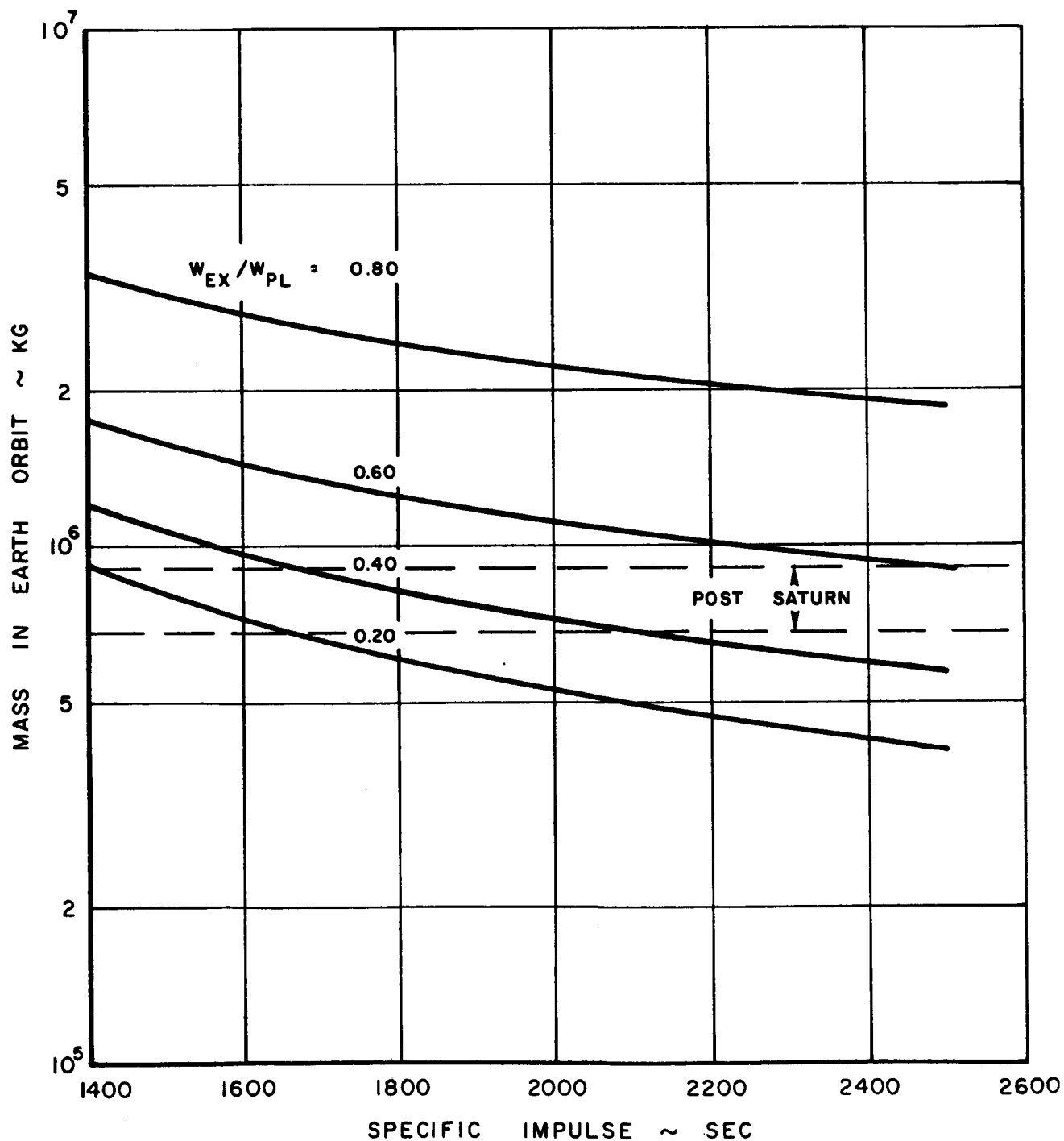


GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 1800 DAYS

STAY TIME = 0

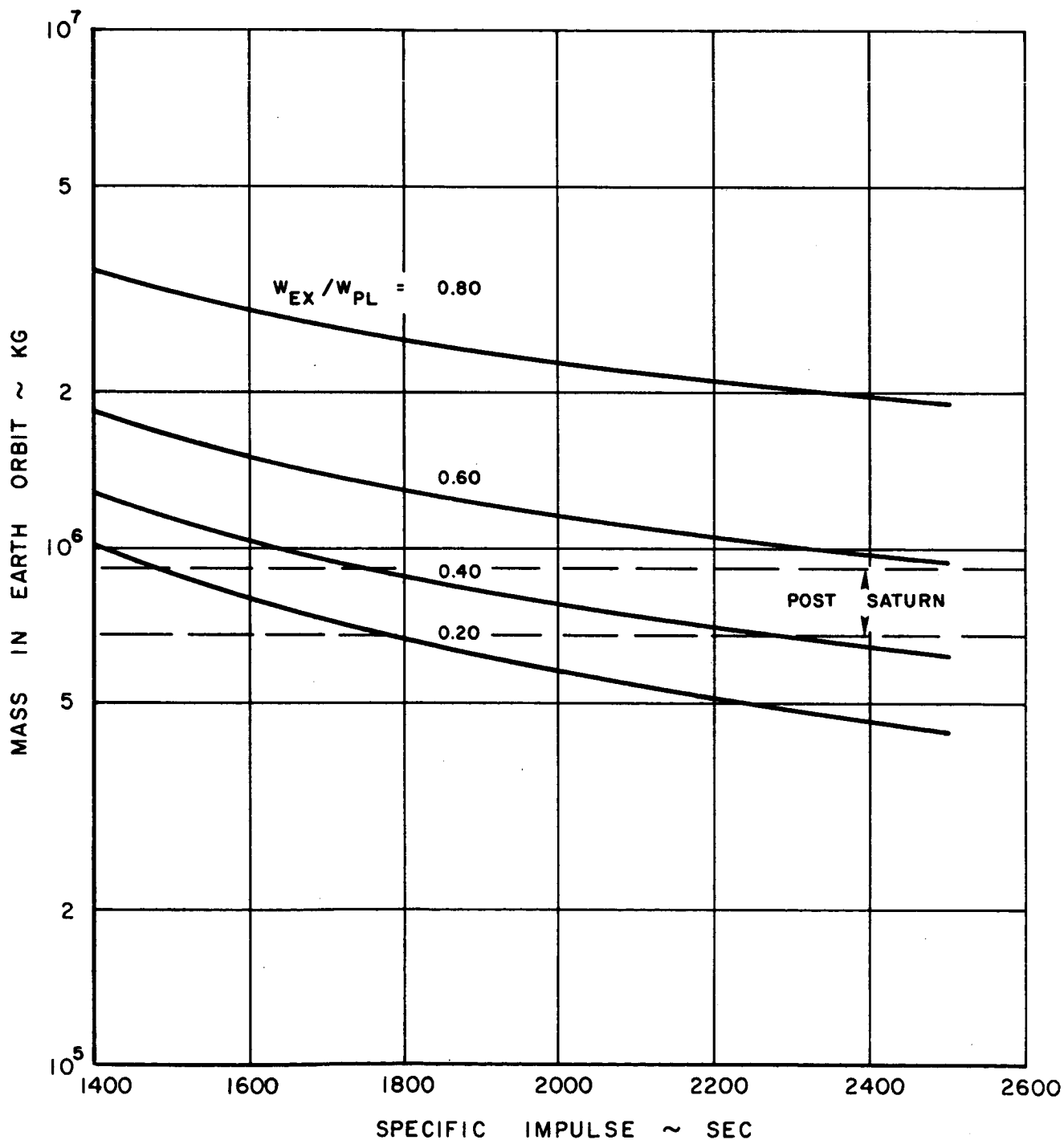


GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

GASEOUS - CORE NUCLEAR

TRIP TIME = 1800 DAYS

STAY TIME = 0

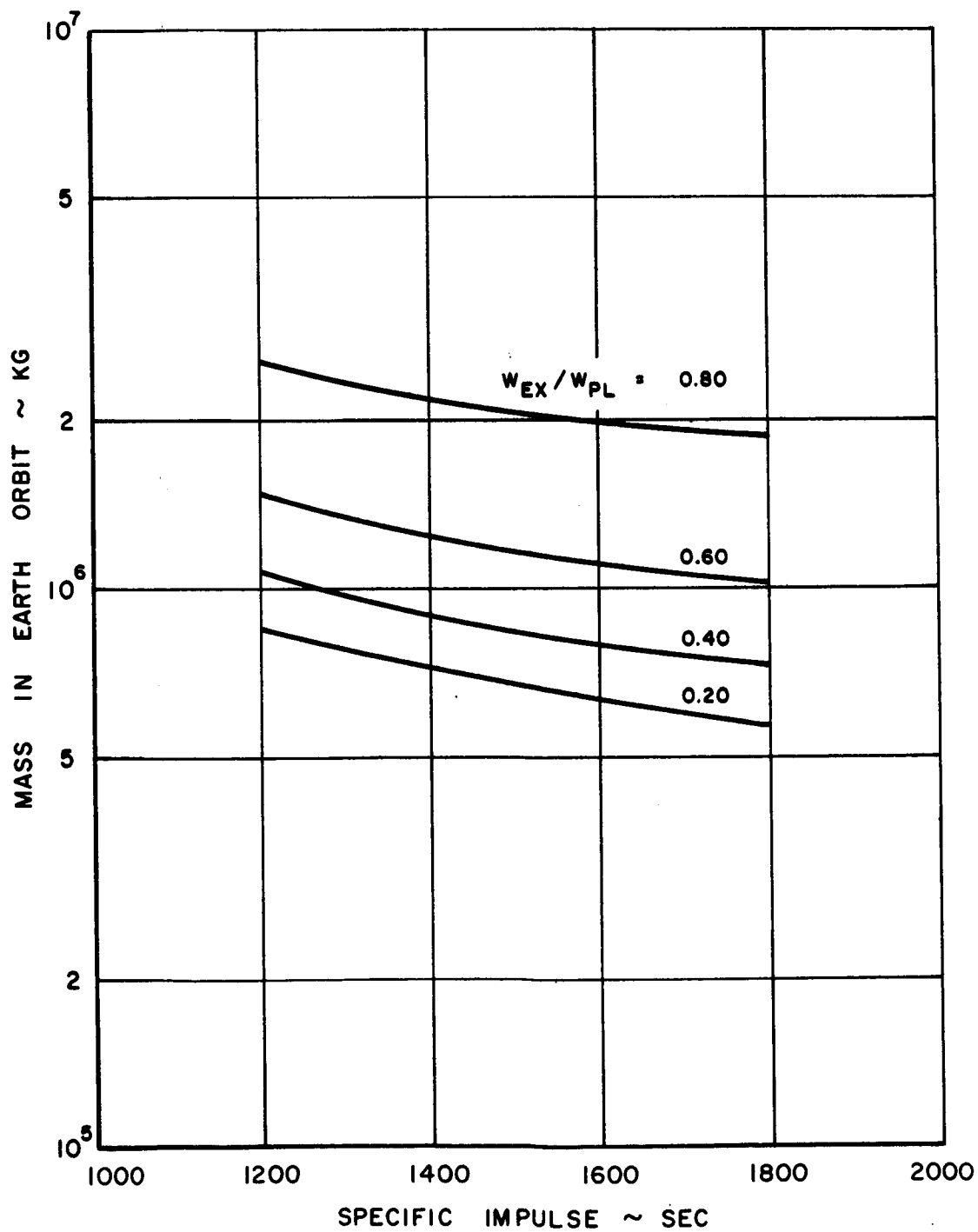


GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

PULSED NUCLEAR

TRIP TIME = 1800 DAYS

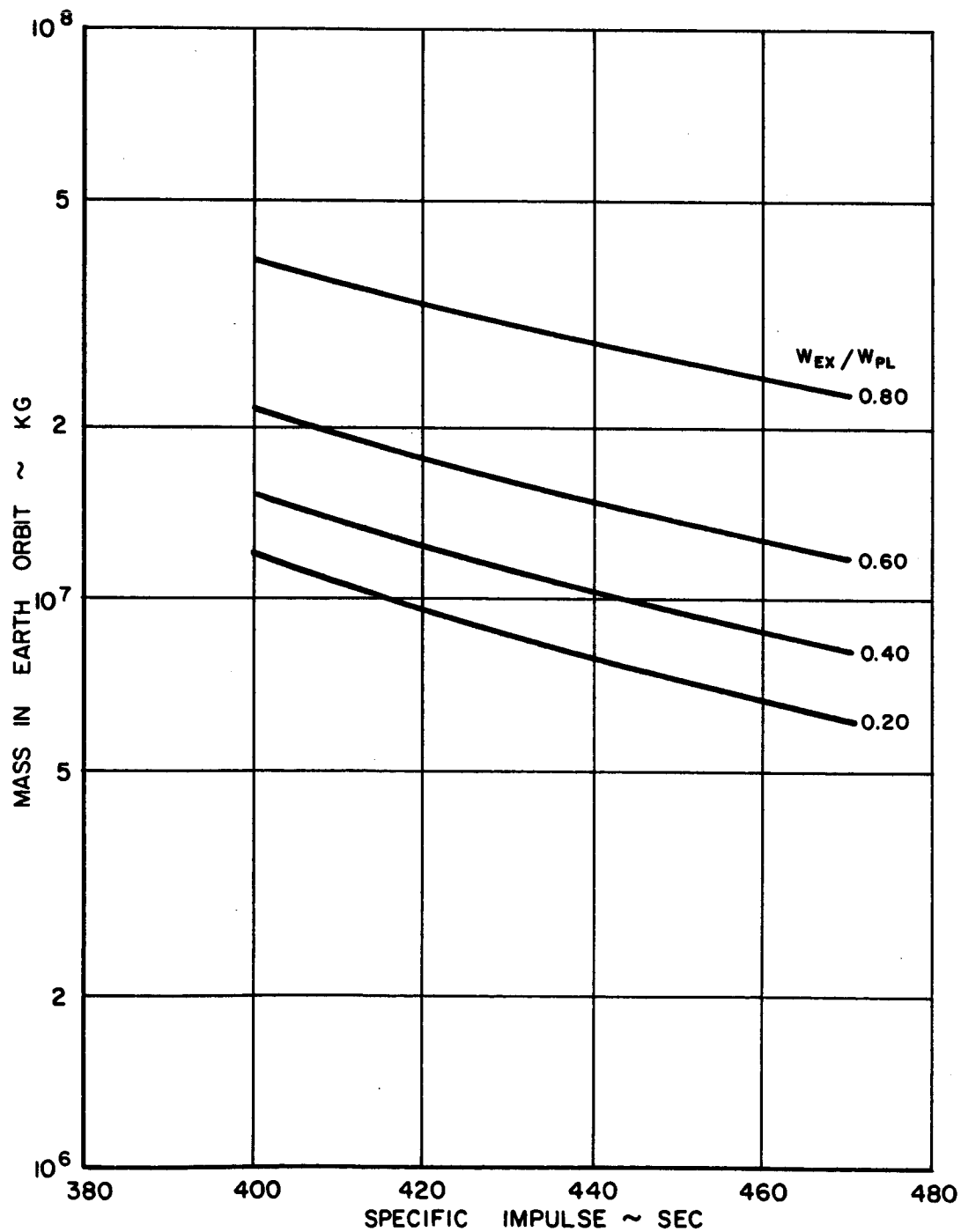
STAY TIME = 0



TITAN MANNED LANDING MISSIONS
(SATURN VI)
CHEMICAL

TRIP TIME = 4000 DAYS

STAY TIME = 0 DAYS

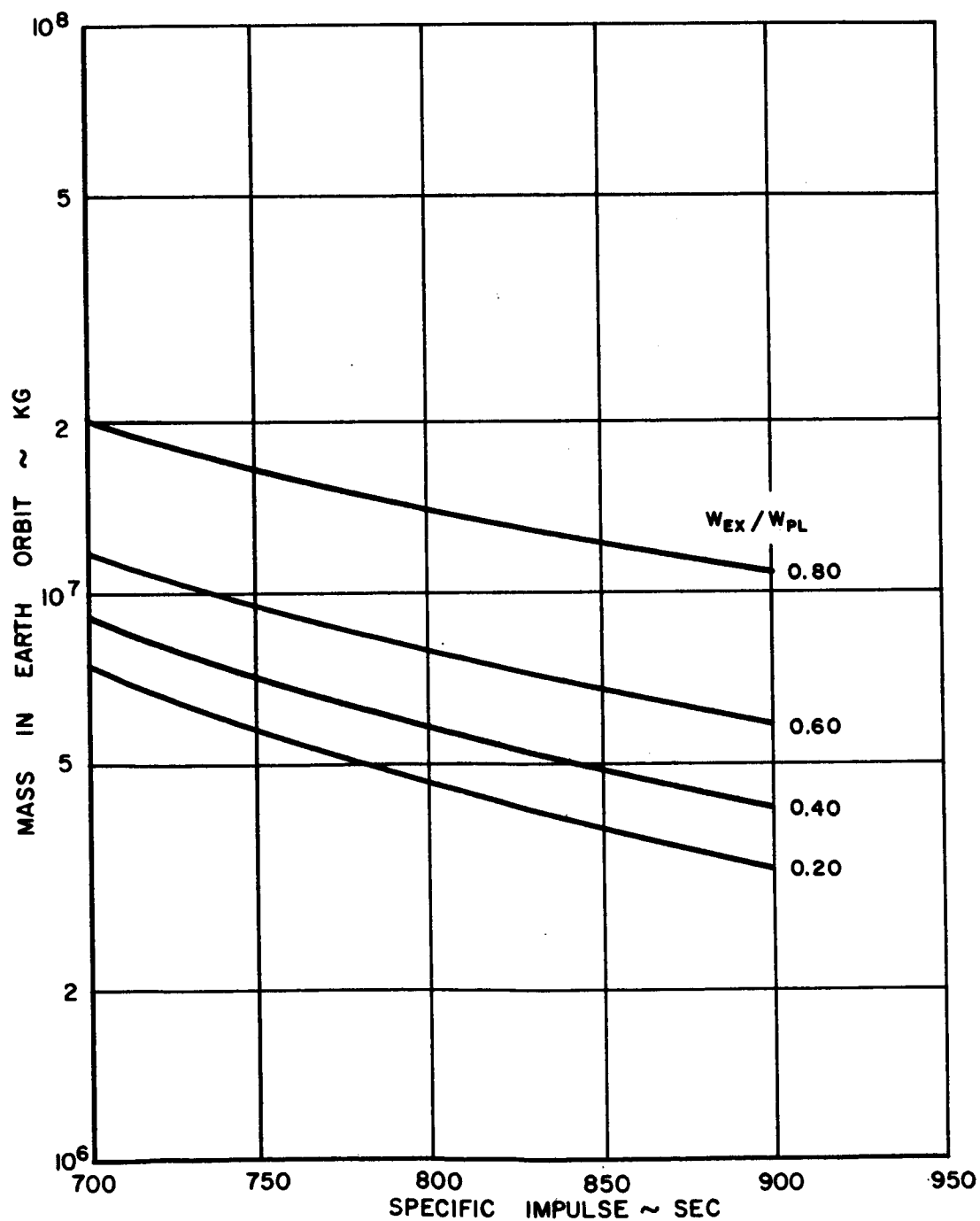


TITAN MANNED LANDING MISSIONS (SATURN VI)

SOLID-CORE NUCLEAR

TRIP TIME = 4000 DAYS

STAY TIME = 0 DAYS

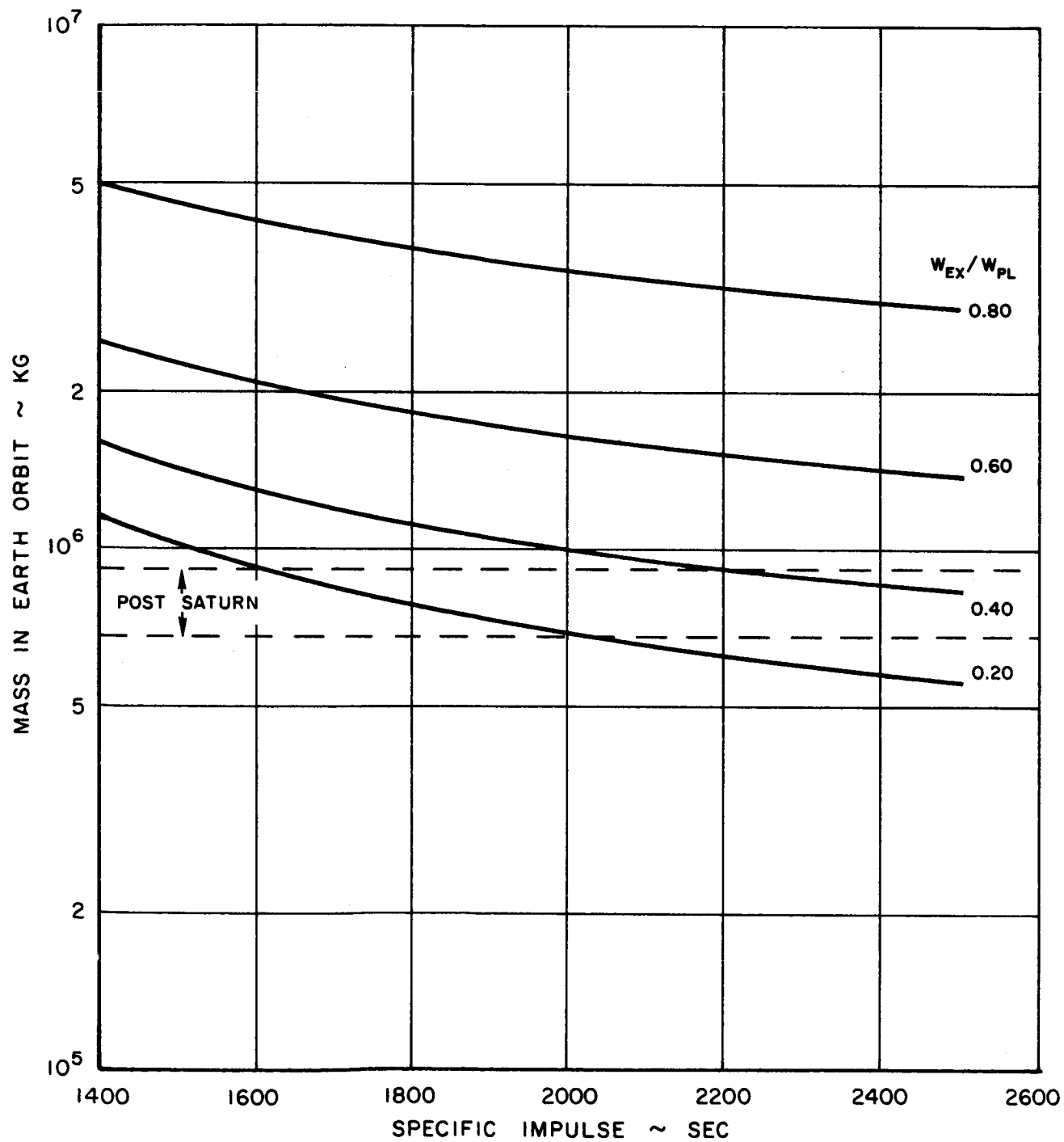


TITAN MANNED LANDING MISSIONS (SATURN VI)

LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 4000 DAYS

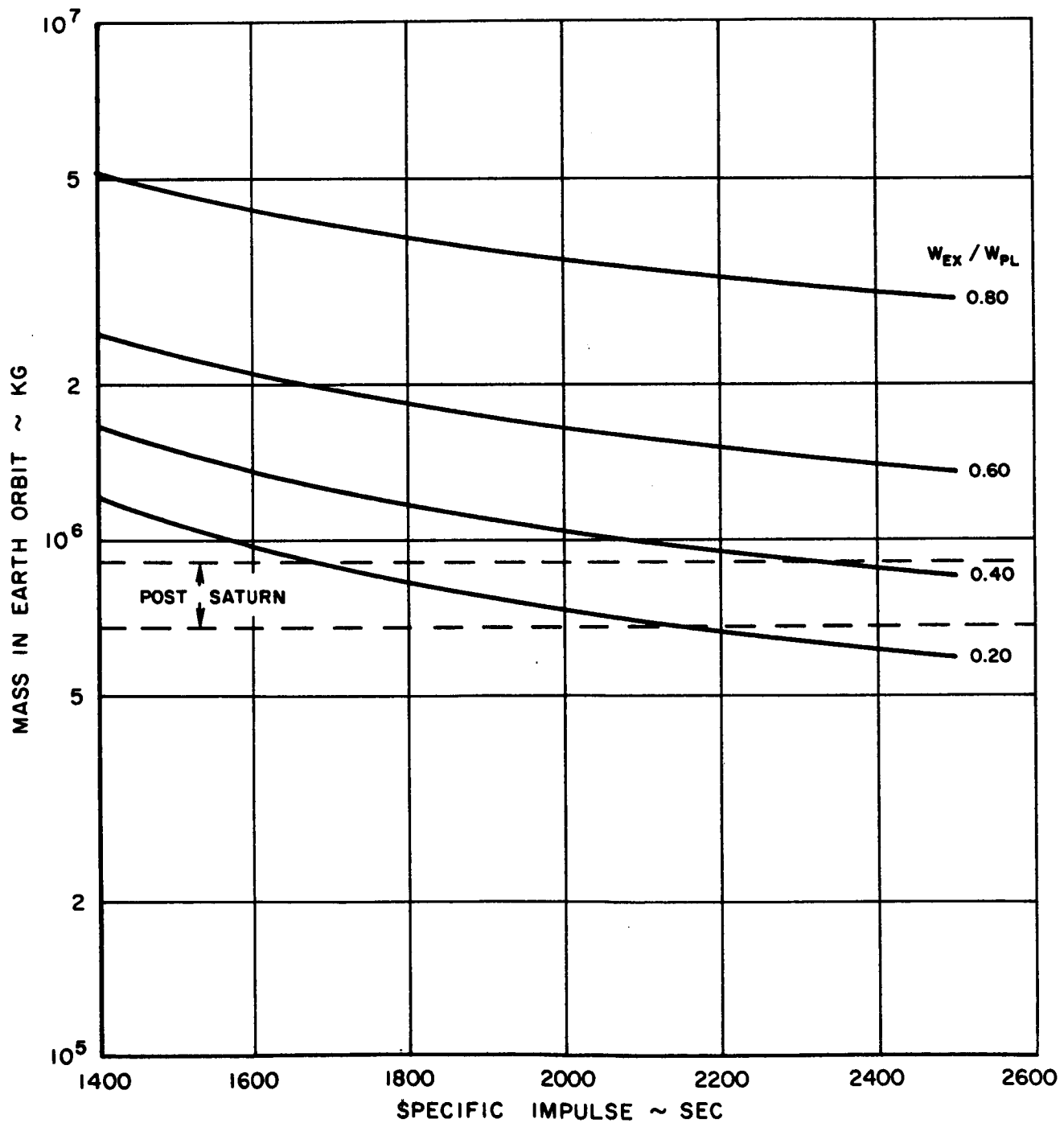
STAY TIME = 0



TITAN MANNED LANDING MISSIONS
(SATURN VI)
GASEOUS-CORE NUCLEAR

TRIP TIME = 4000 DAYS

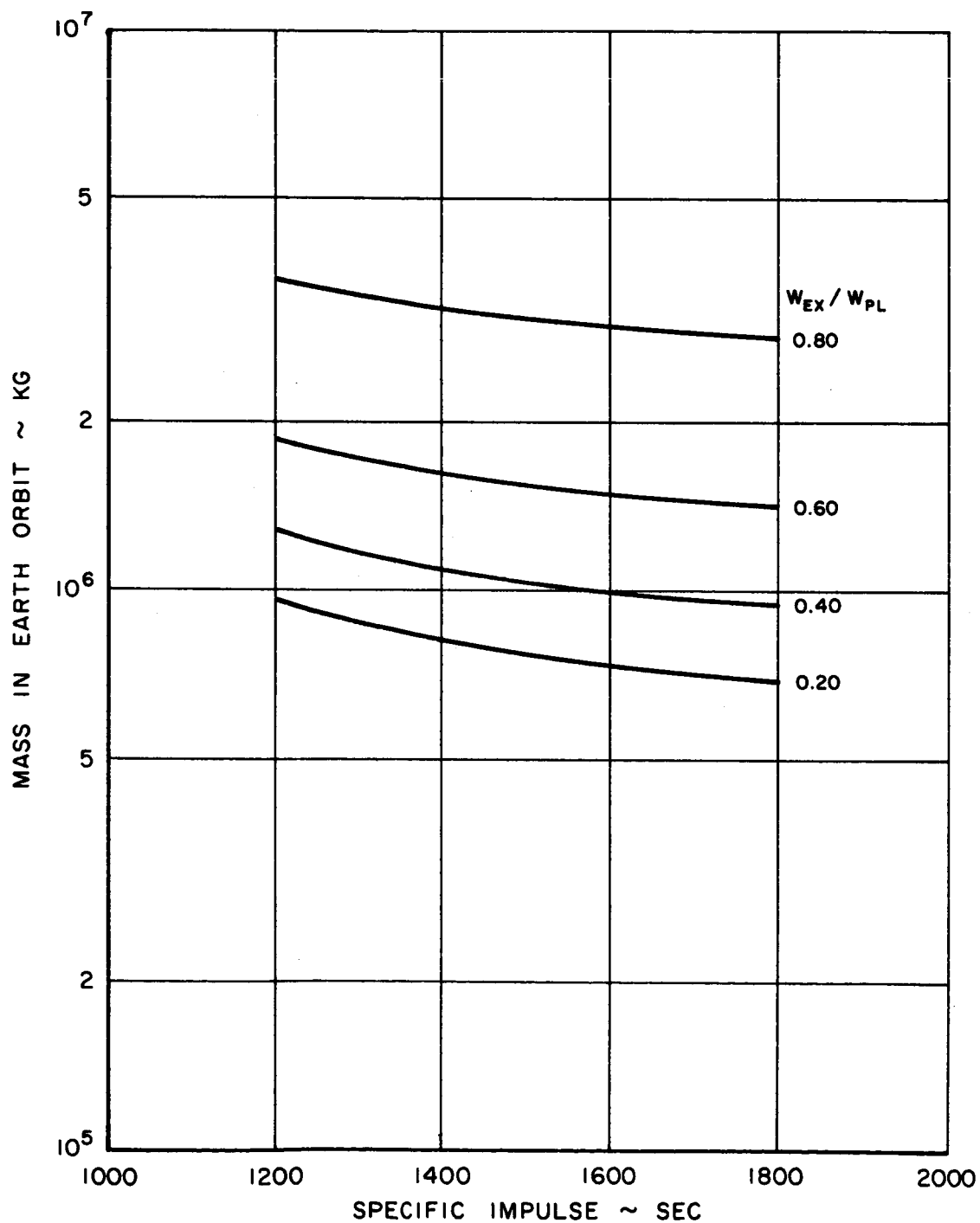
STAY TIME = 0



TITAN MANNED LANDING MISSIONS
(SATURN VI)
PULSED NUCLEAR

TRIP TIME = 4000 DAYS

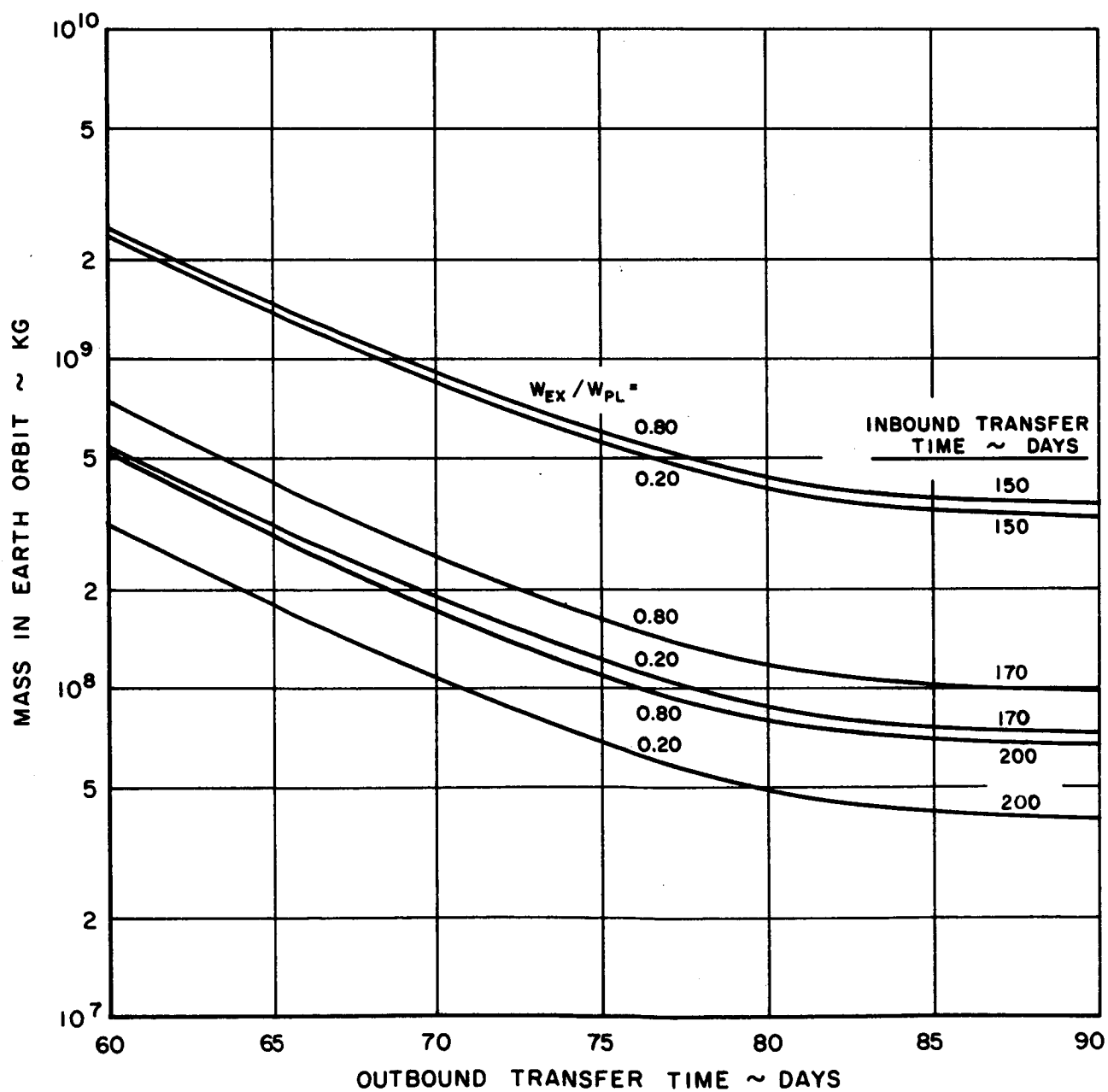
STAY TIME = 0



MERCURY MANNED LANDING MISSIONS

CHEMICAL

$$I_{SP} = 430 \text{ SEC}$$

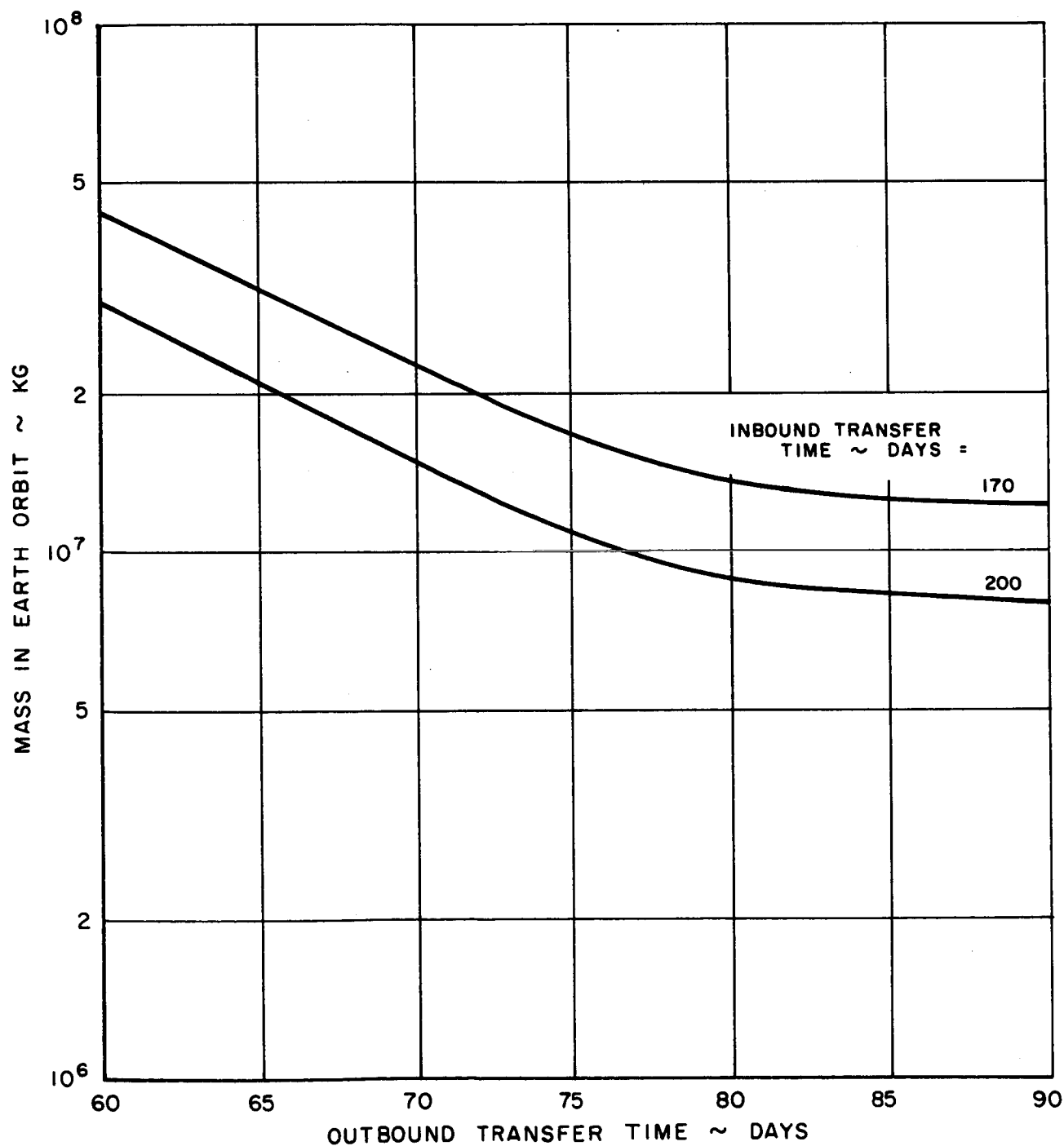


MERCURY MANNED LANDING MISSIONS

SOLID-CORE NUCLEAR

$$W_{EX}/W_{PL} = 0.20$$

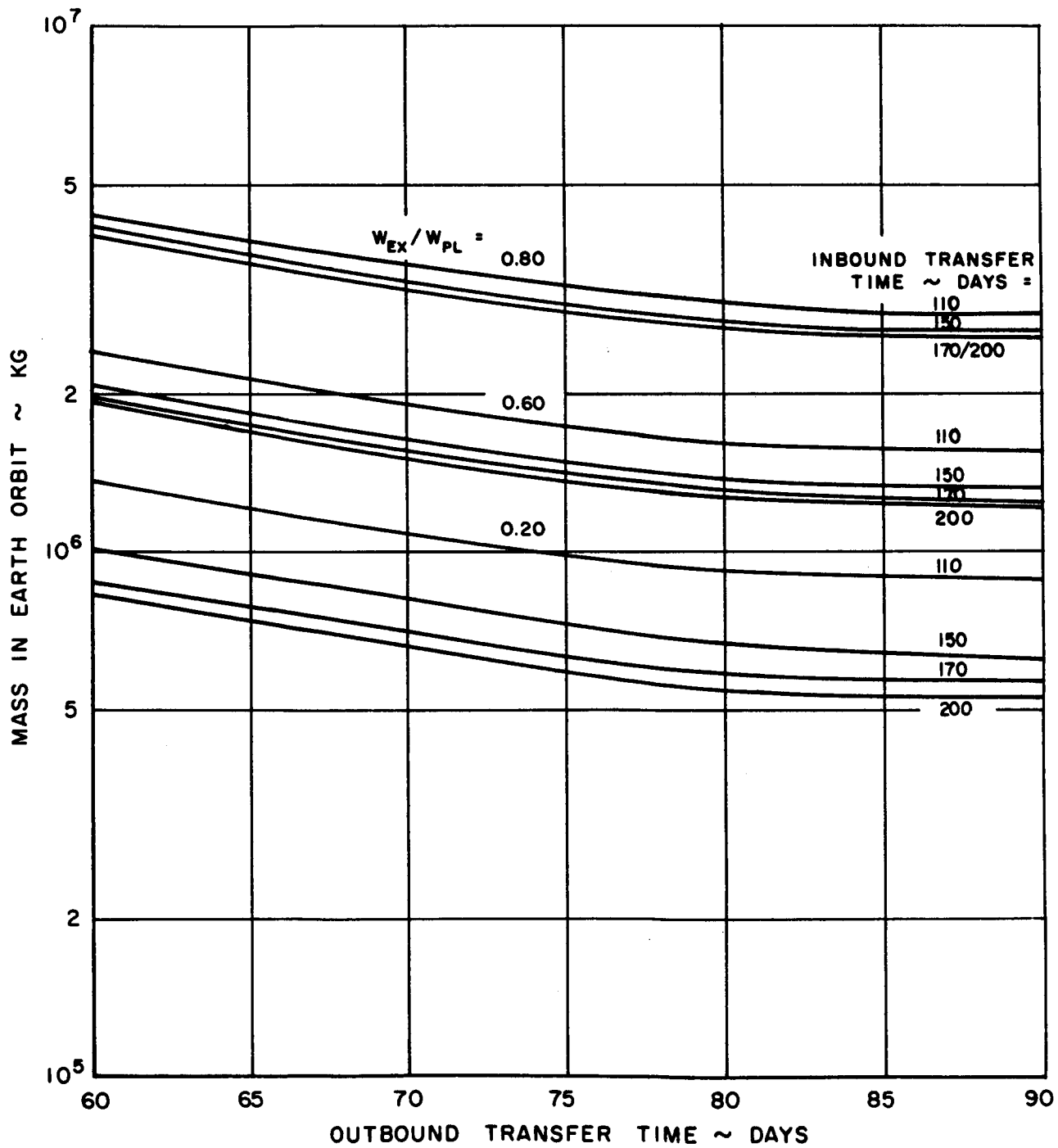
$$I_{SP} = 800 \text{ SEC}$$



MERCURY MANNED LANDING MISSIONS

LIGHTWEIGHT ADVANCED NUCLEAR

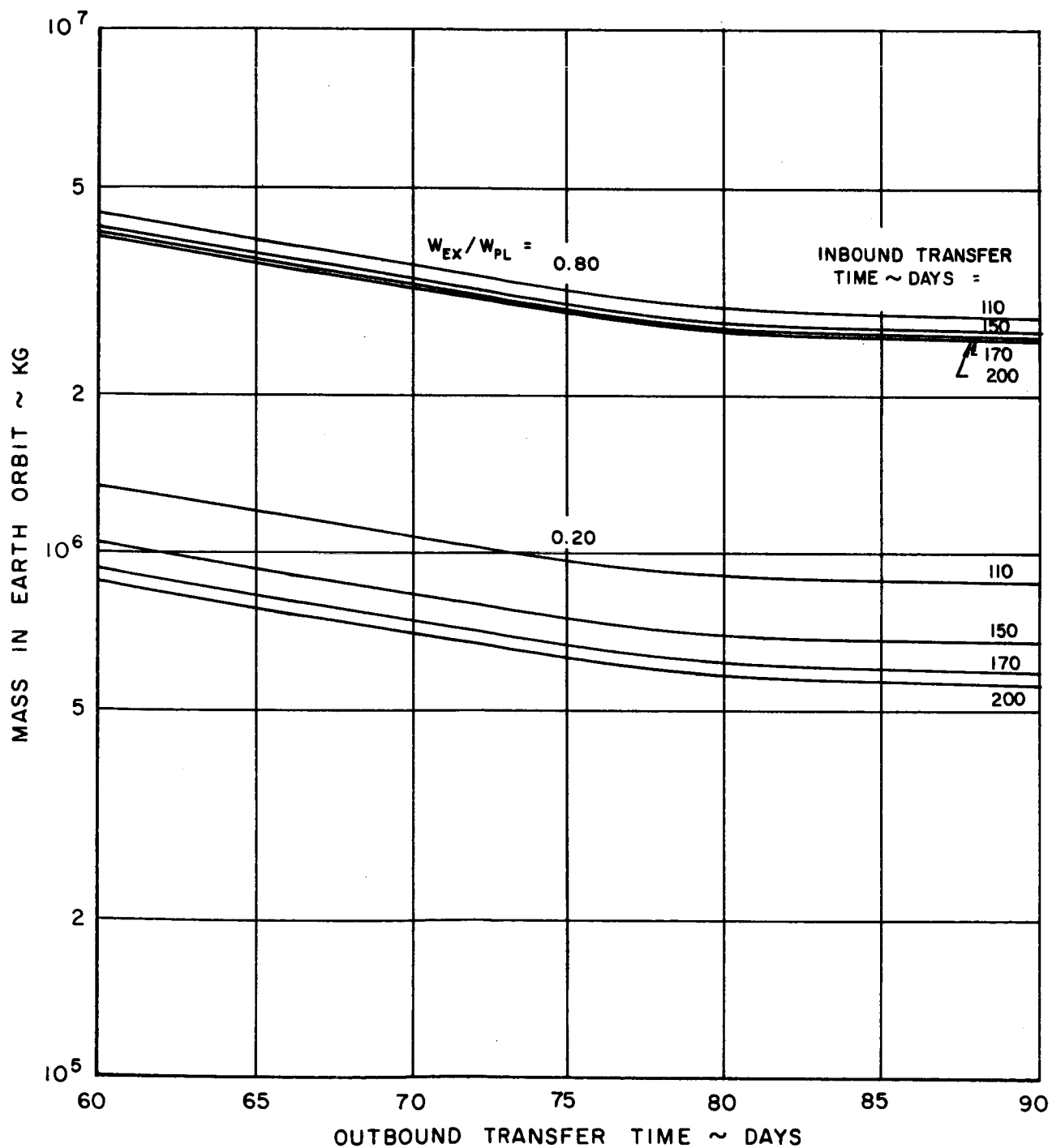
$$I_{SP} = 2000 \text{ SEC}$$



MERCURY MANNED LANDING MISSIONS

GASEOUS-CORE NUCLEAR

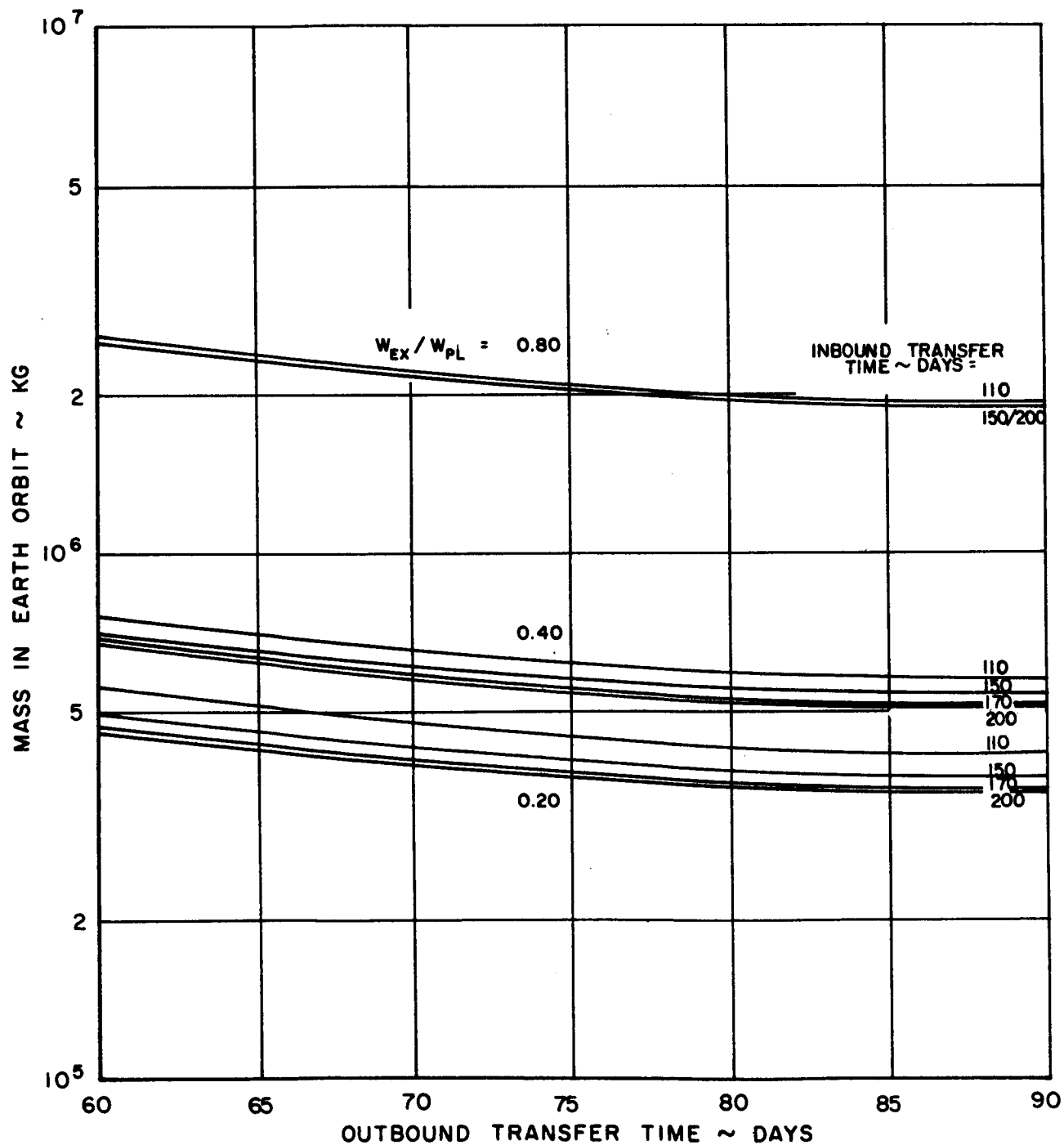
$$I_{SP} = 2000 \text{ SEC}$$



MERCURY MANNED LANDING MISSIONS

GASEOUS-CORE NUCLEAR

$$I_{SP} = 3500 \text{ SEC}$$



MERCURY MANNED LANDING MISSIONS

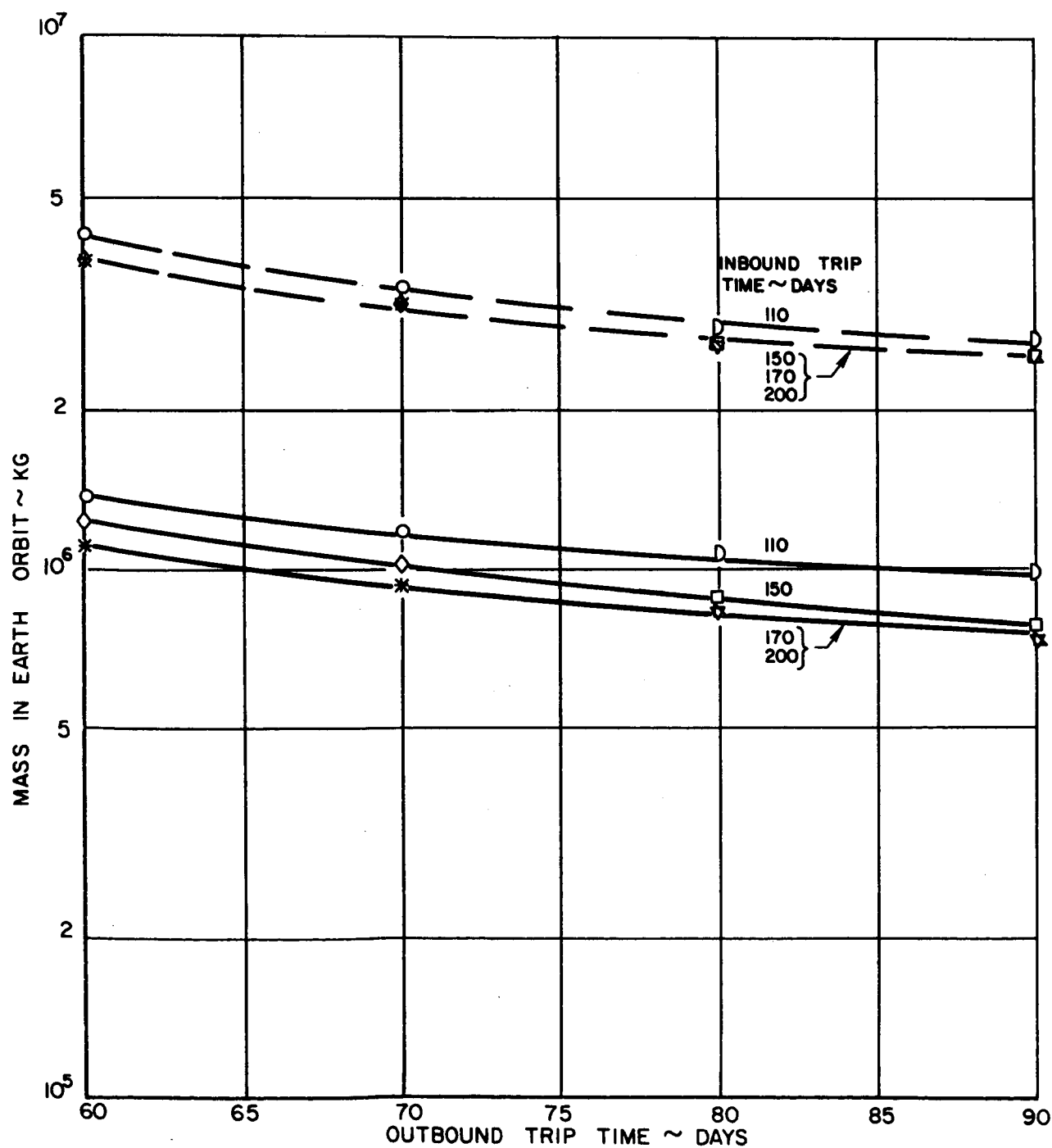
PULSED NUCLEAR

$$I_{SP} = 1800 \text{ SEC}$$

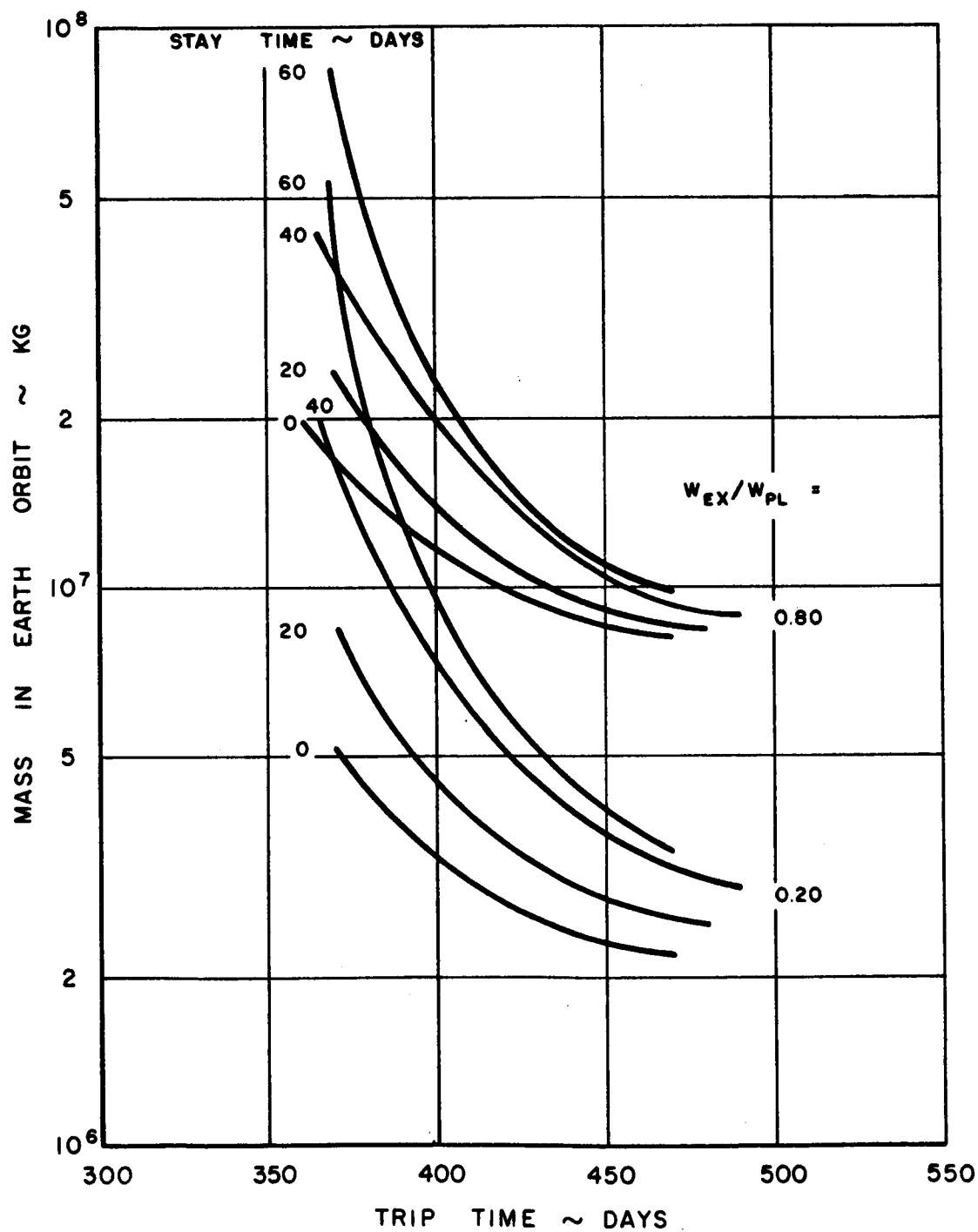
$$\text{---} \frac{W_{EX}}{W_{PL}} = 0.20$$

$$\text{---} \frac{W_{EX}}{W_{PL}} = 0.80$$

SYMBOL	D	O	□	◇	▽	X	△	+
STOPOVER TIME	50	55	60	65	70	75	80	85



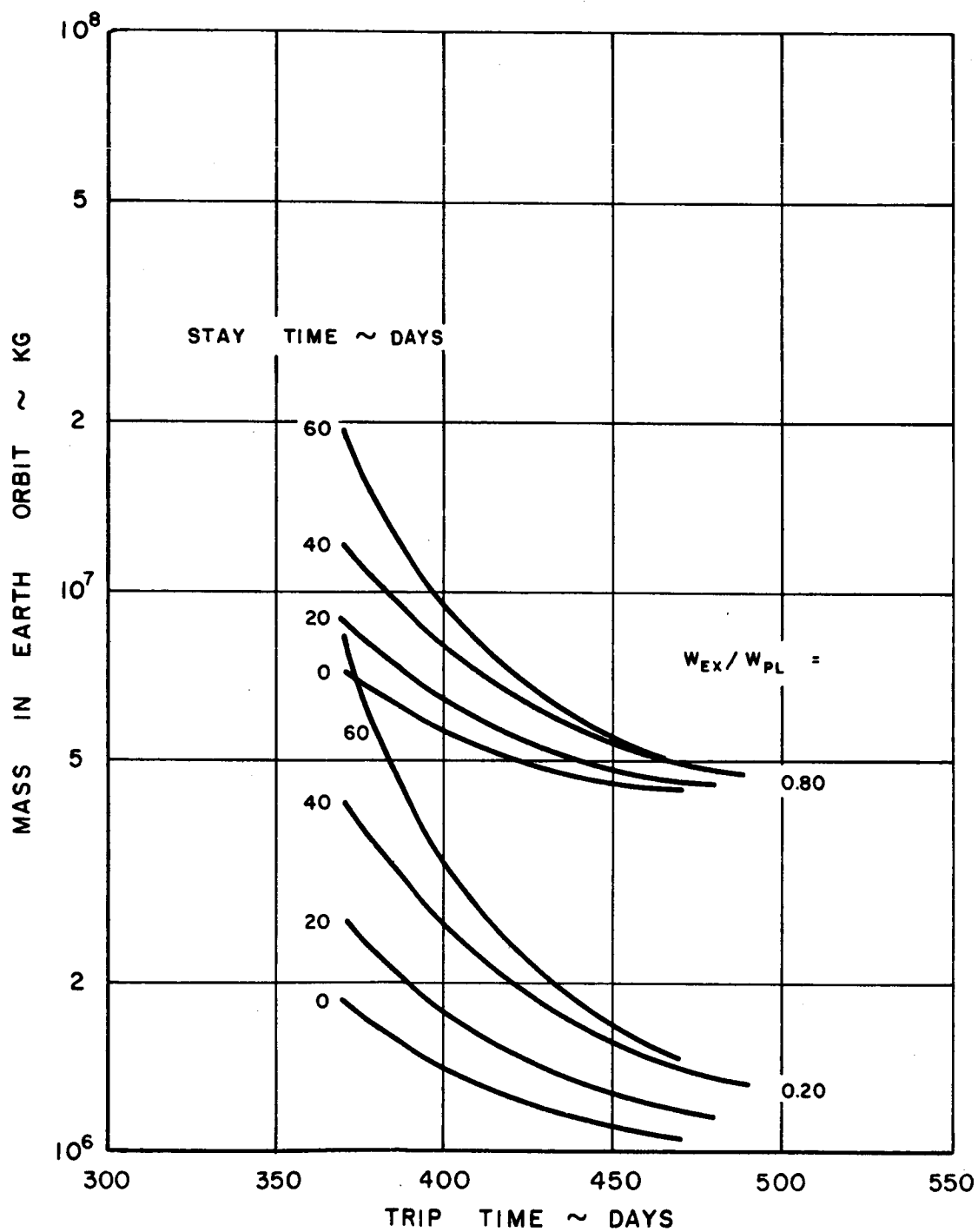
MARS MANNED LANDING MISSIONS CHEMICAL

 $I_{sp} = 430 \text{ SEC}$ 

MARS MANNED LANDING MISSIONS

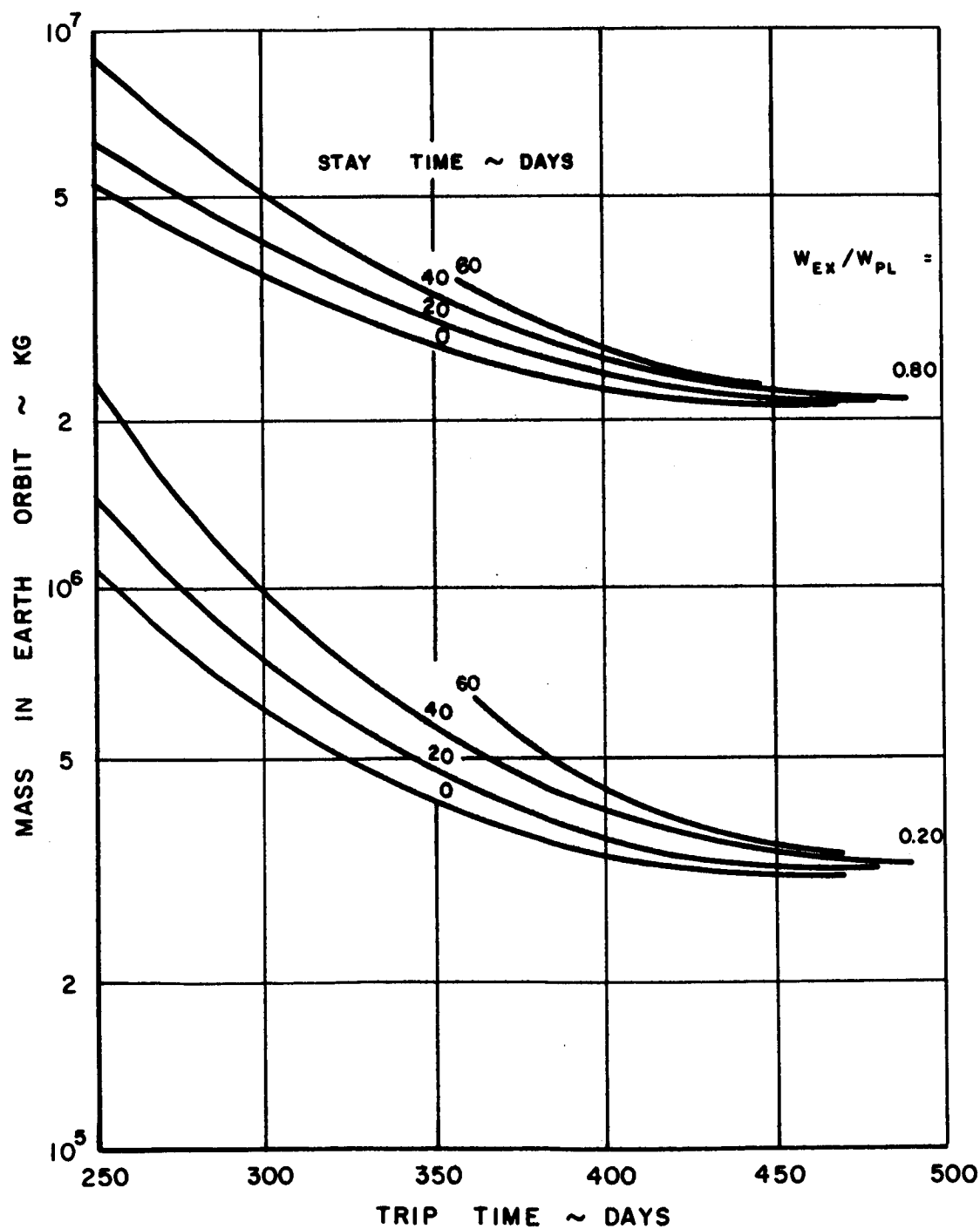
SOLID-CORE NUCLEAR

$$I_{SP} = 800 \text{ SEC}$$



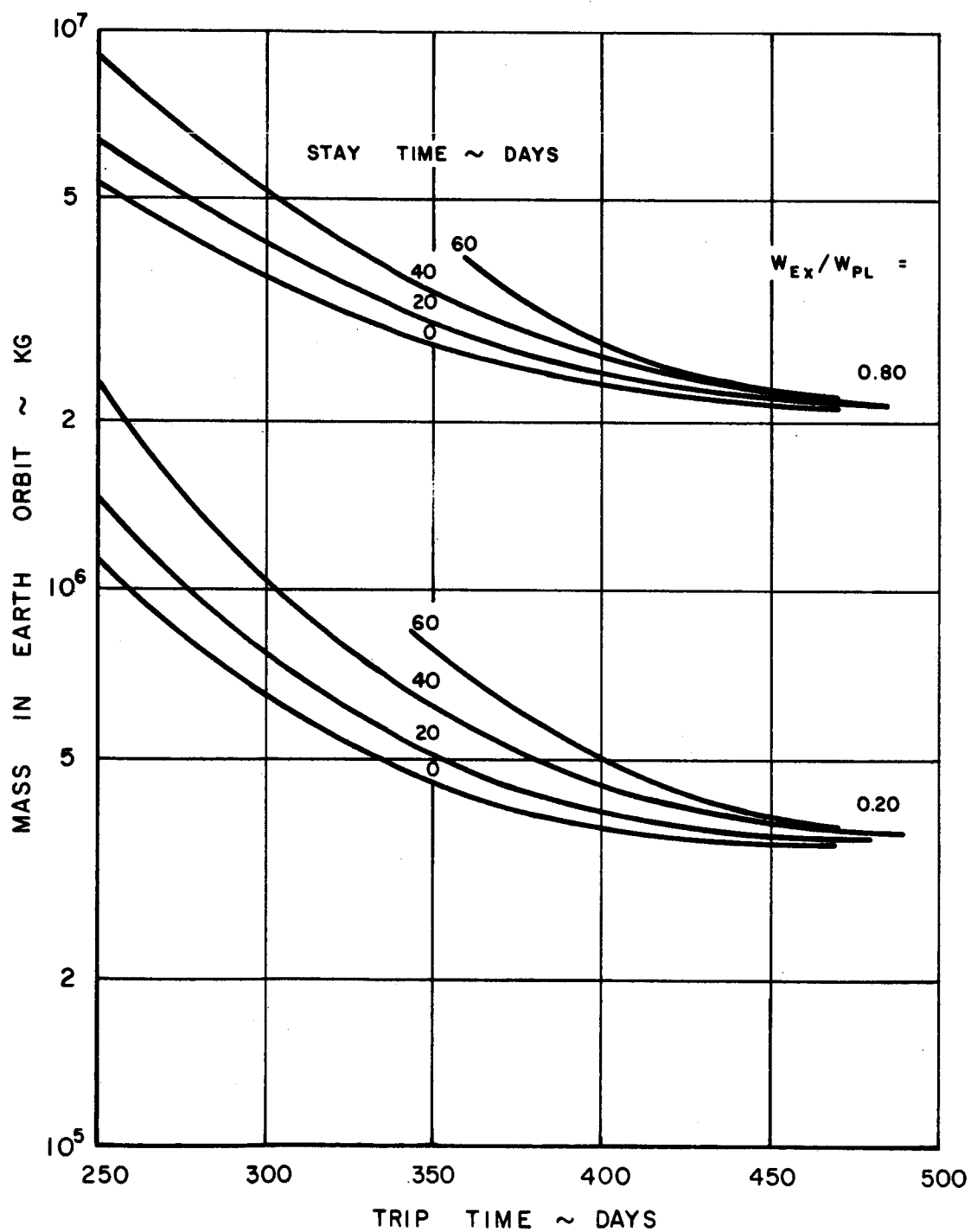
MARS MANNED LANDING MISSIONS LIGHTWEIGHT ADVANCED NUCLEAR

$$I_{SP} = 2000 \text{ SEC}$$



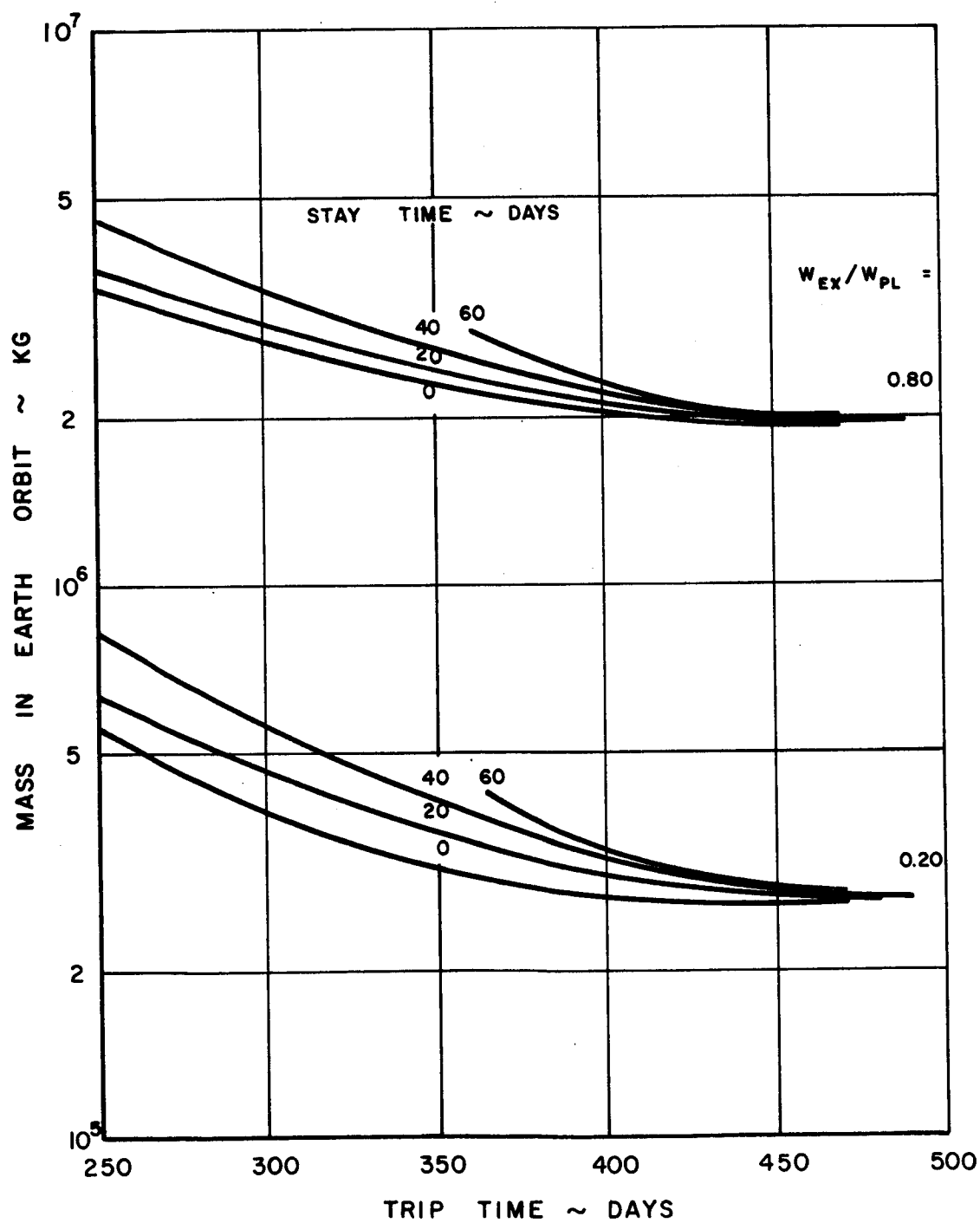
MARS MANNED LANDING MISSIONS

GASEOUS - CORE NUCLEAR

 $I_{SP} = 2000 \text{ SEC}$ 

MARS MANNED LANDING MISSIONS

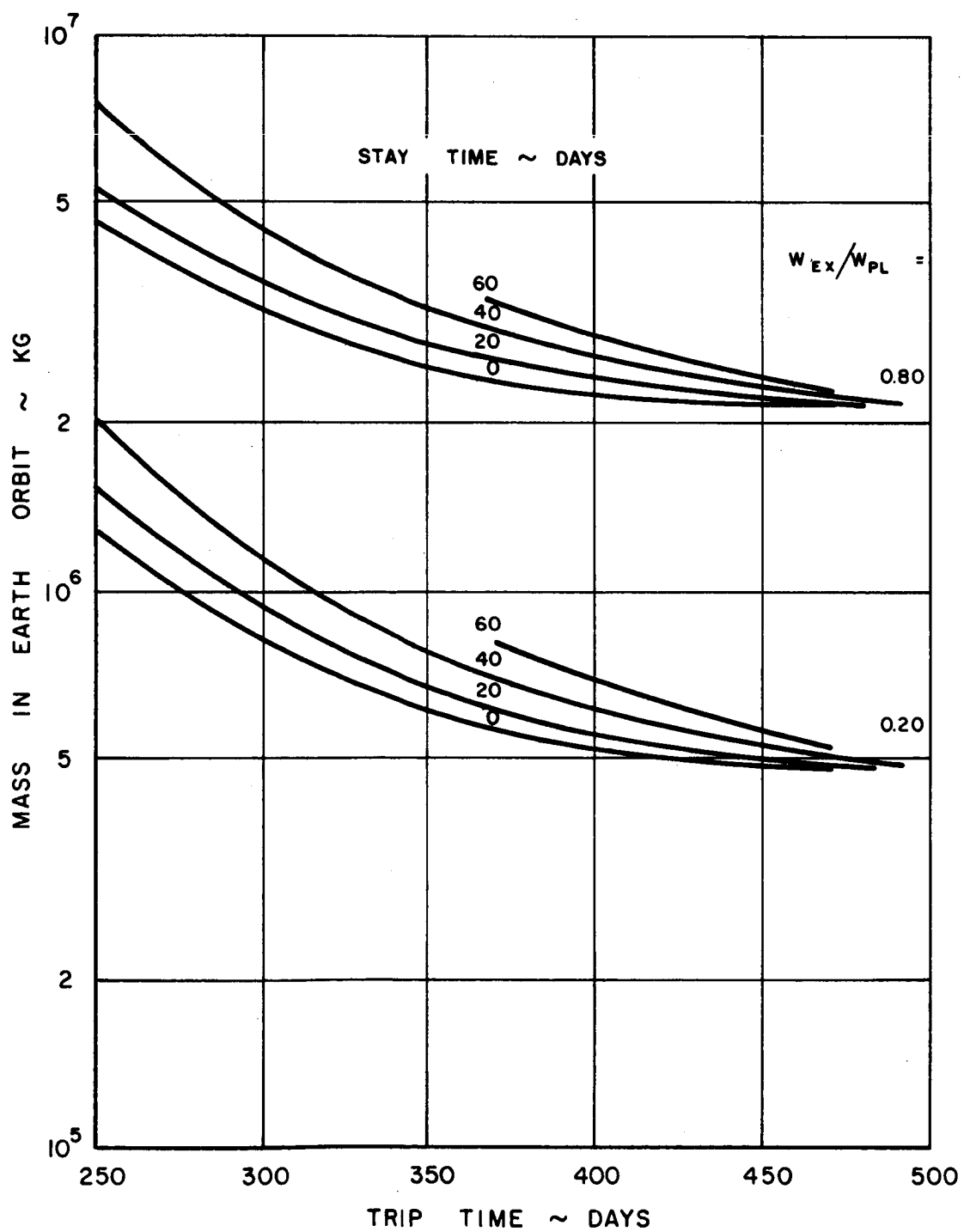
GASEOUS - CORE NUCLEAR

 $I_{SP} = 3500 \text{ SEC}$ 

MARS MANNED LANDING MISSIONS

PULSED NUCLEAR

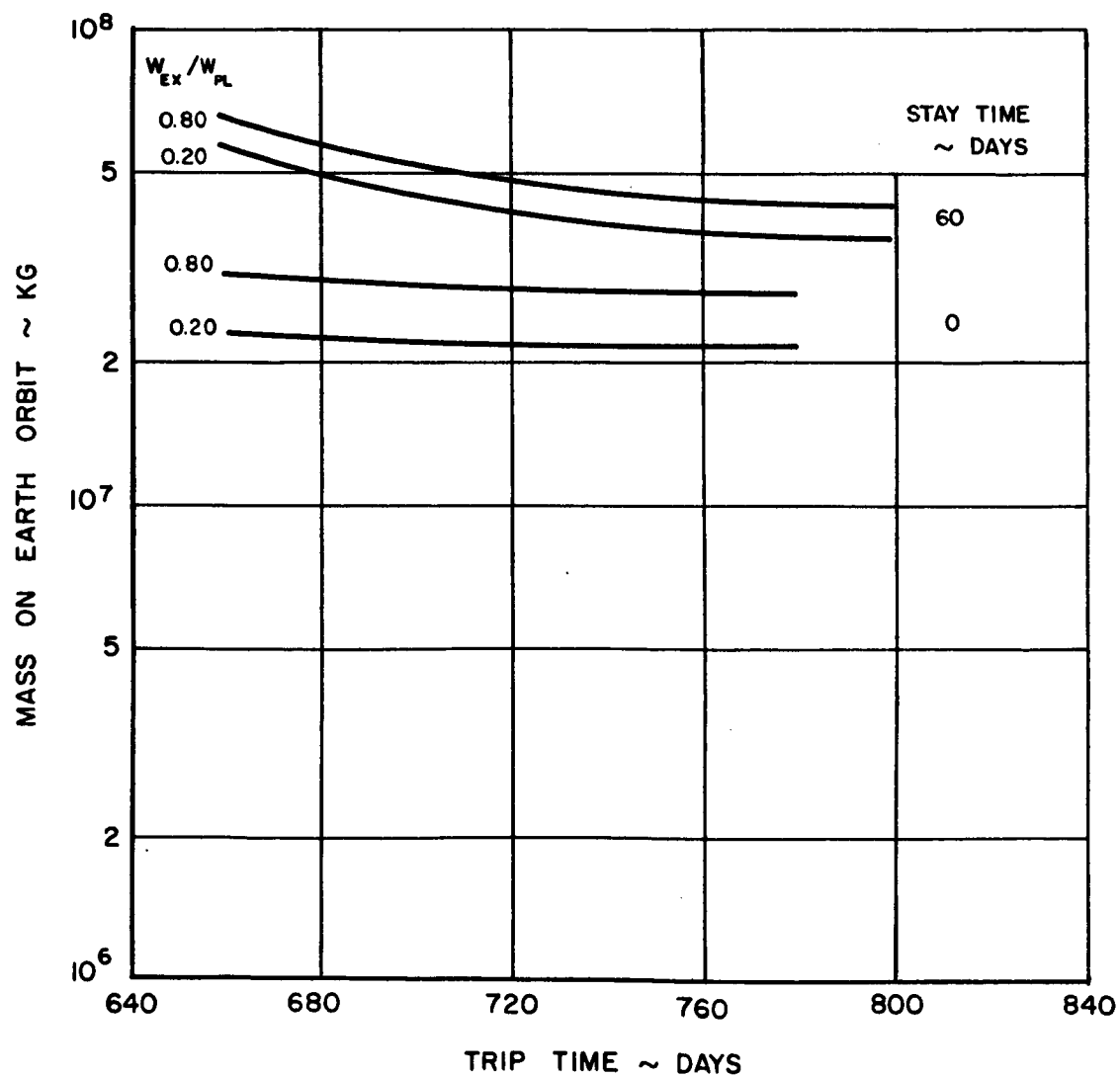
$$I_{sp} = 1800 \text{ SEC}$$



CERES MANNED LANDING MISSION

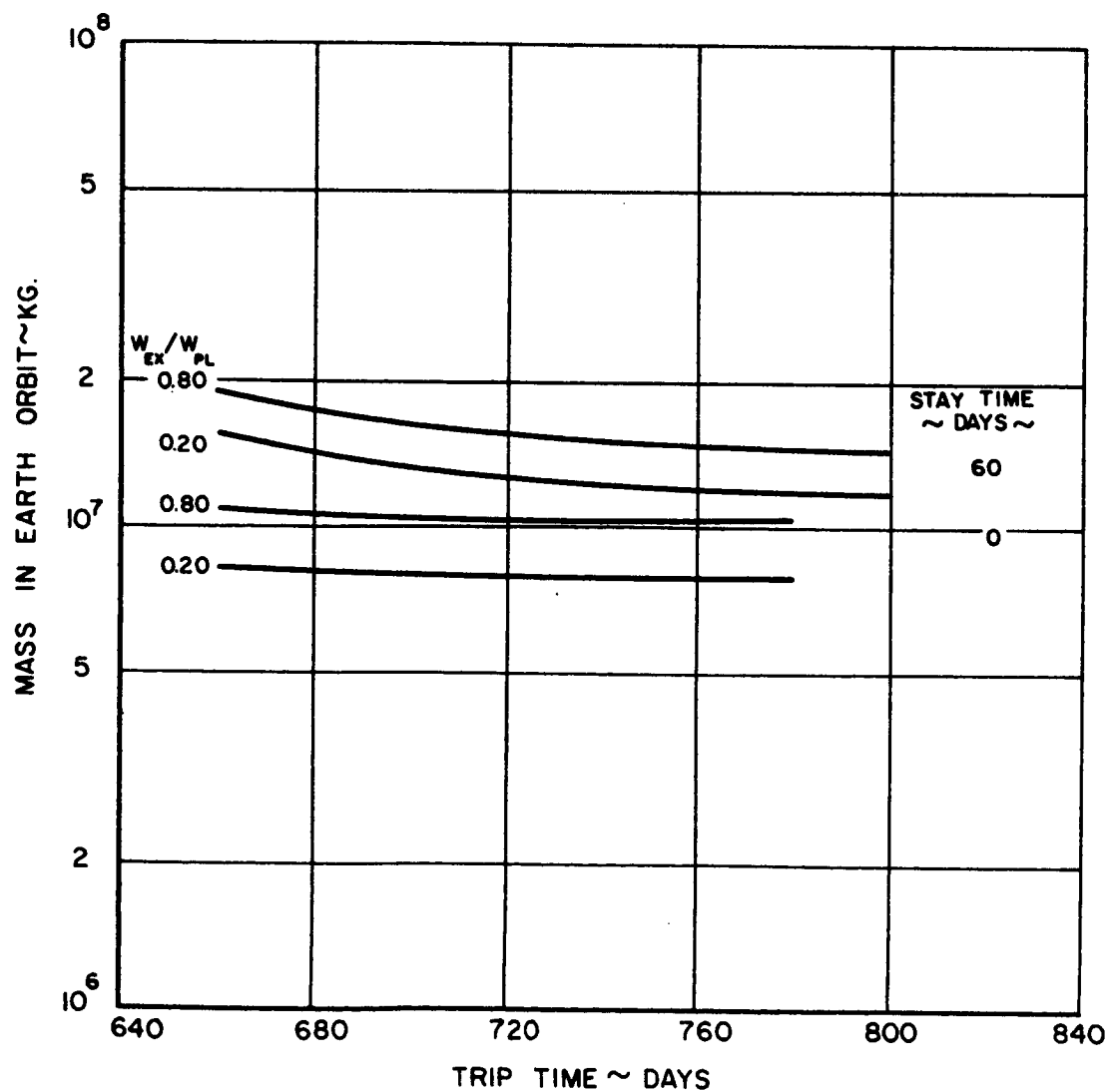
CHEMICAL

$$I_{SP} = 430 \text{ SEC}$$



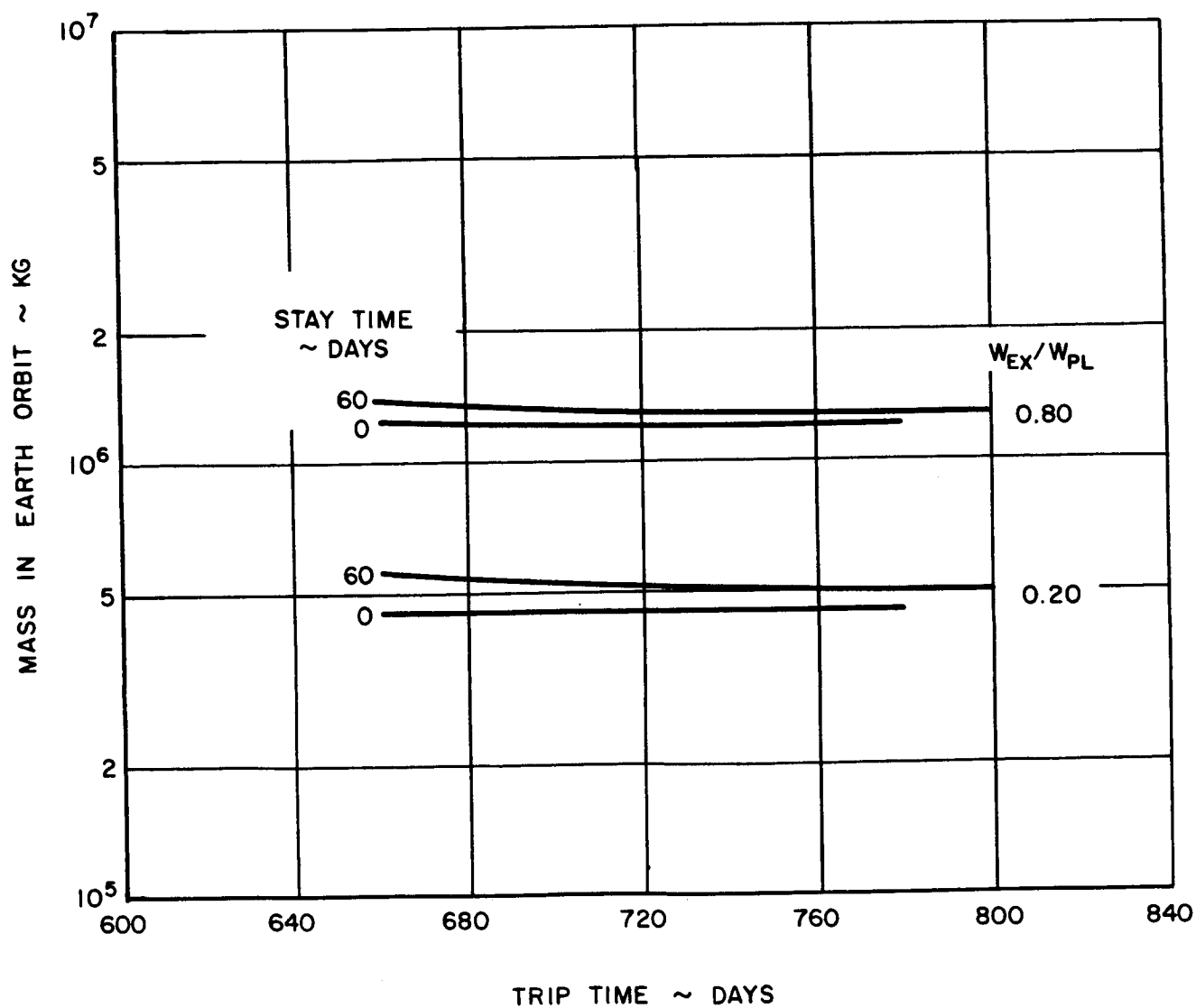
CERES MANNED LANDING MISSION
SOLID-CORE NUCLEAR

$$I_{SP} = 800 \text{ SEC.}$$



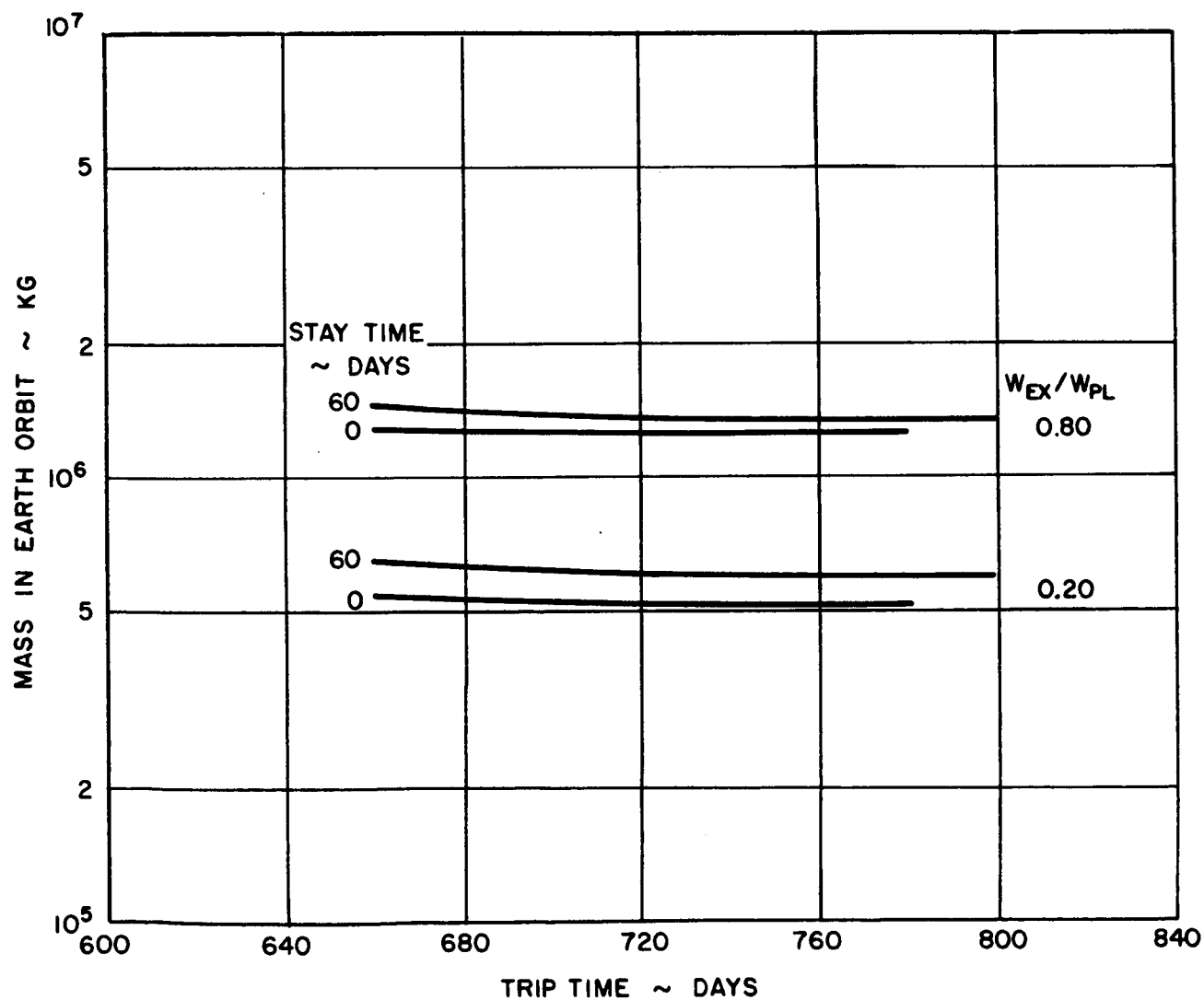
CERES MANNED LANDING MISSION

LIGHTWEIGHT ADVANCED NUCLEAR

 $I_{SP} = 2000 \text{ SEC.}$ 

CERES MANNED LANDING MISSION

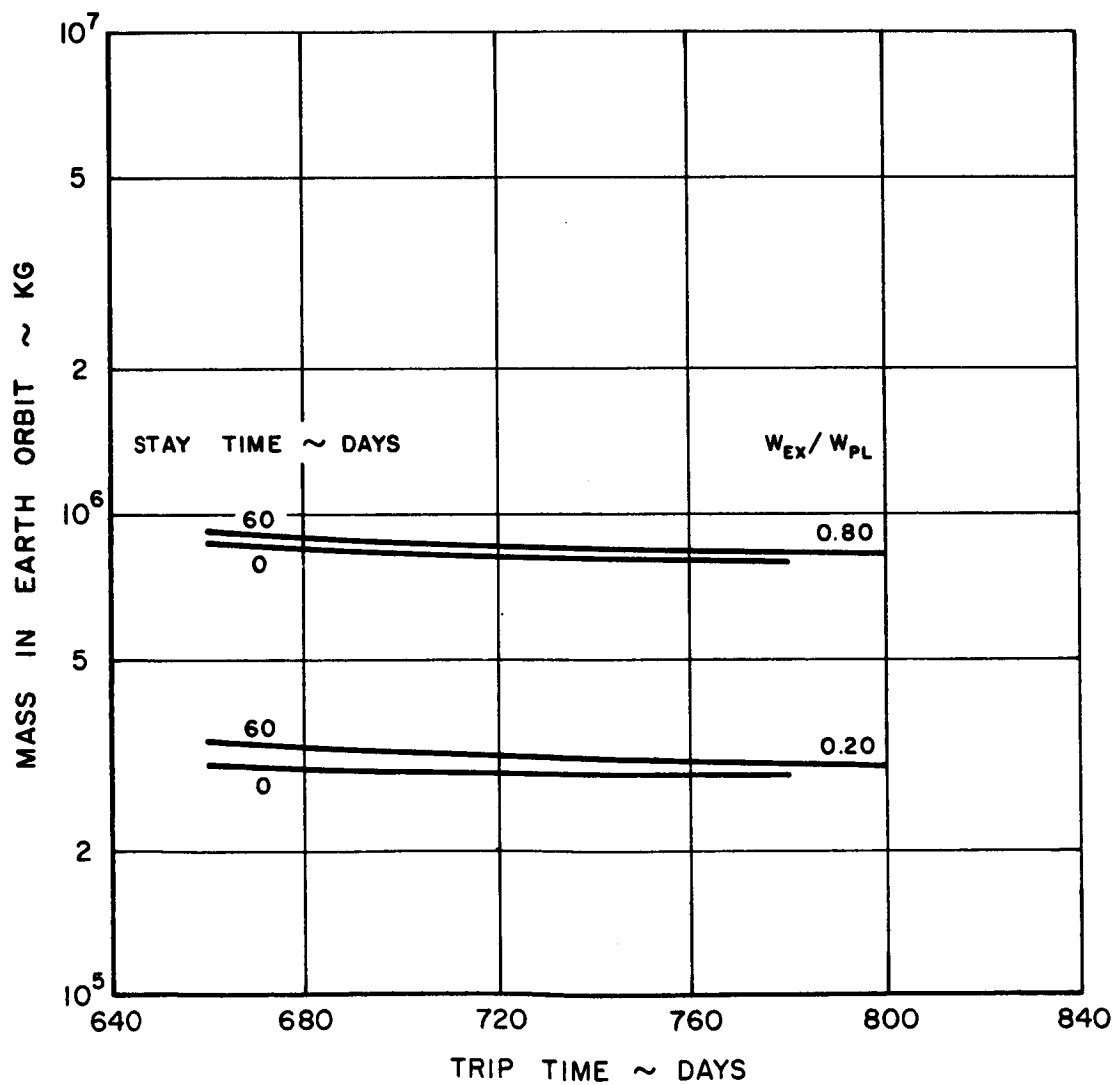
GASEOUS-CORE NUCLEAR

 $I_{SP} = 2000 \text{ SEC.}$ 

CERES MANNED LANDING MISSIONS

GASEOUS-CORE NUCLEAR

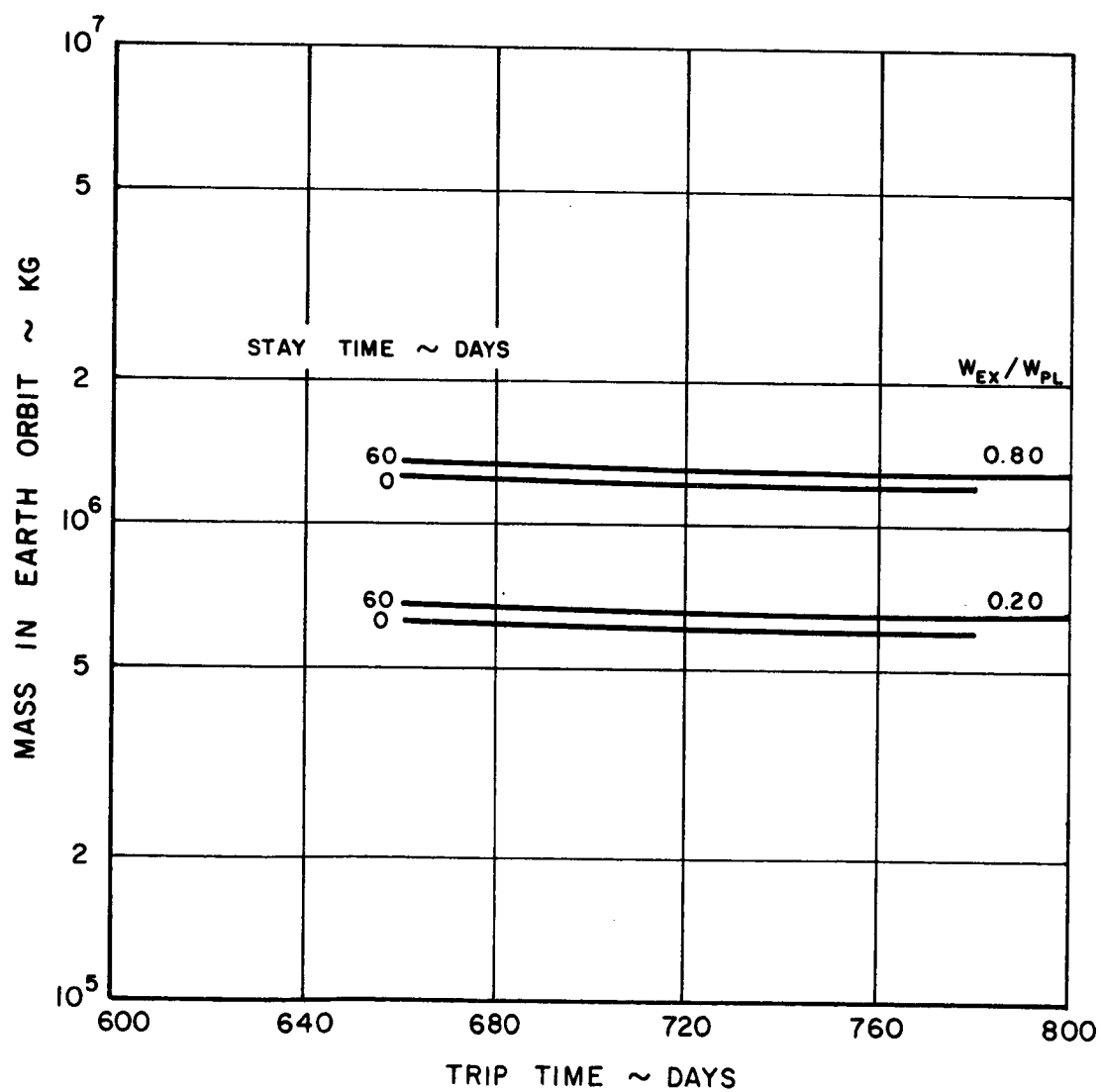
$$I_{sp} = 3500 \text{ SEC}$$



CERES MANNED LANDING MISSIONS

PULSED NUCLEAR

$$I_{sp} = 1800 \text{ SEC}$$



GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

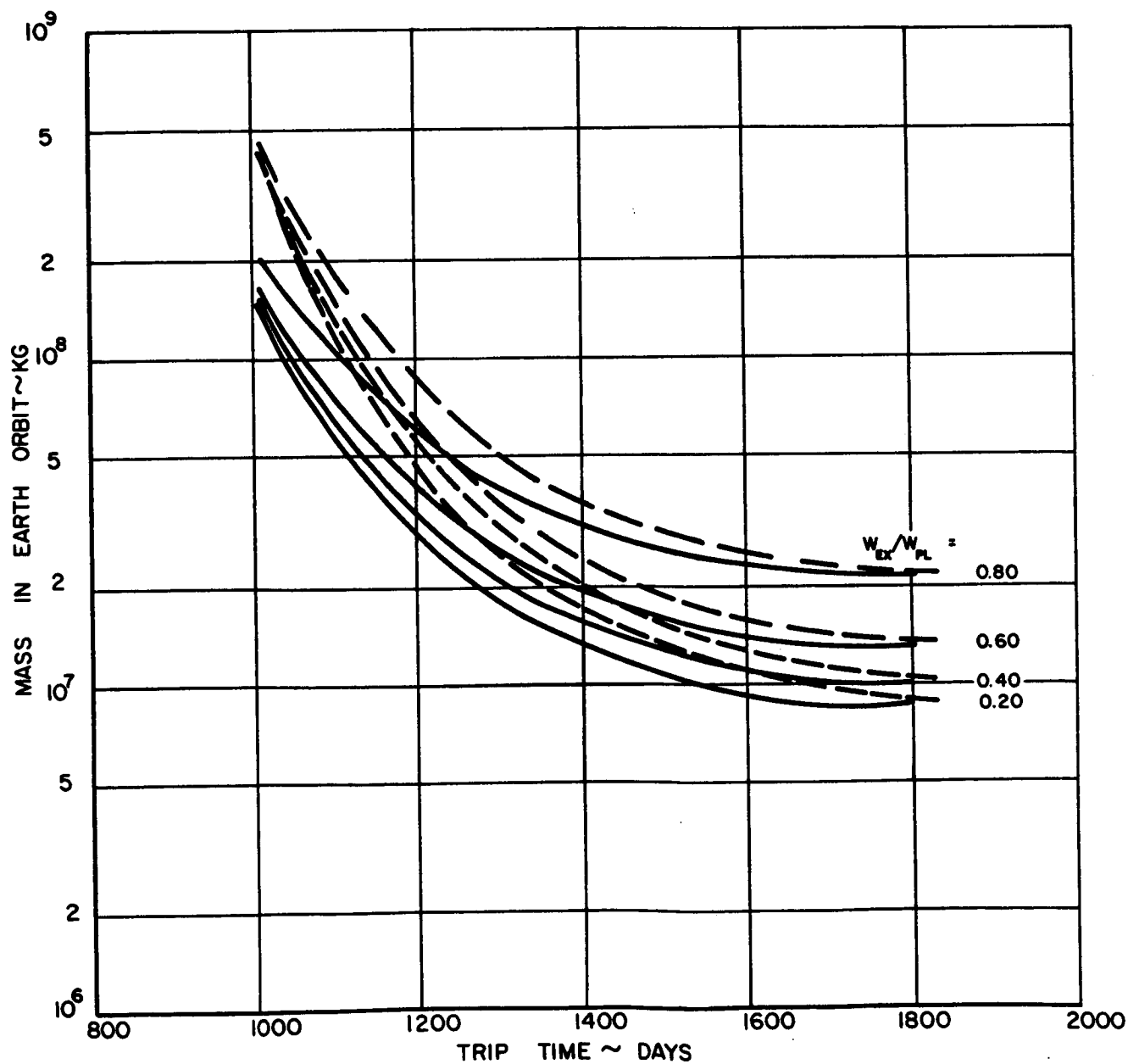
CHEMICAL

$I_{sp} = 430 \text{ SEC}$

NOMINAL

--- 60-DAY STAY

— ZERO STAY



GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

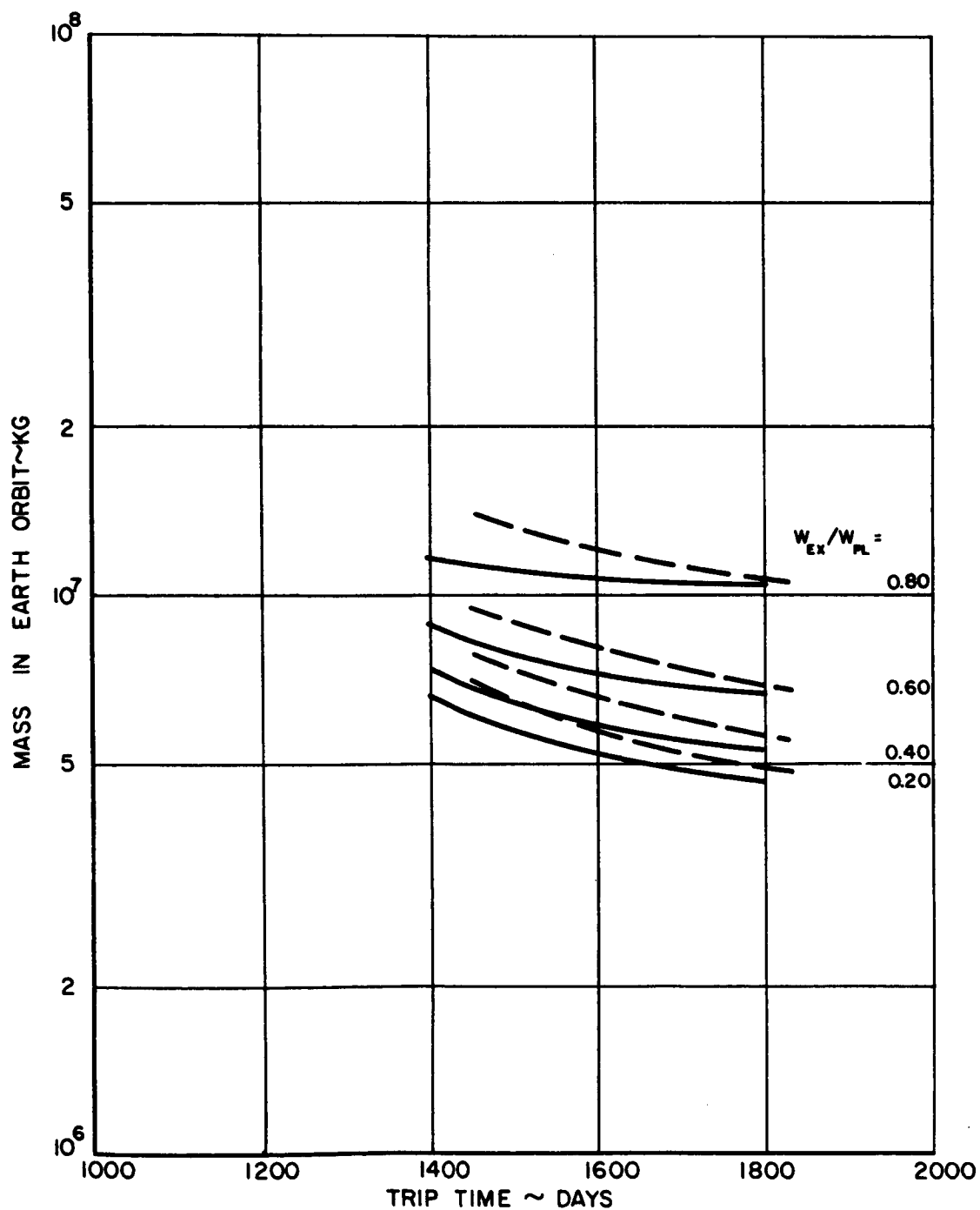
SOLID-CORE NUCLEAR

$I_{sp} = 800 \text{ SEC}$

NOMINAL

--- 60 - DAY STAY

— ZERO STAY



GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

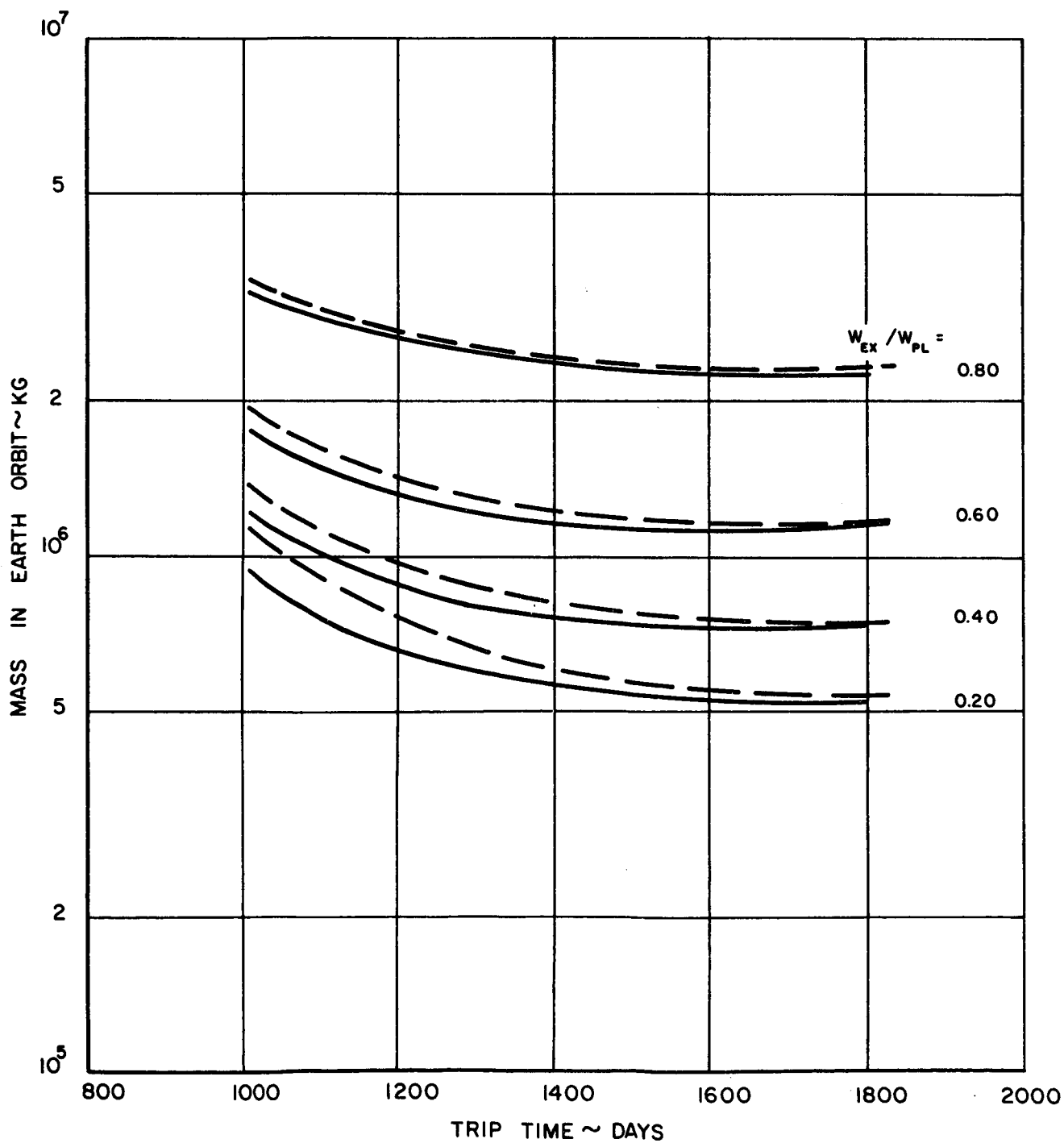
LIGHTWEIGHT ADVANCED NUCLEAR

$I_{sp} = 2000 \text{ SEC}$

NOMINAL

--- 60-DAY STAY

— ZERO STAY



GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

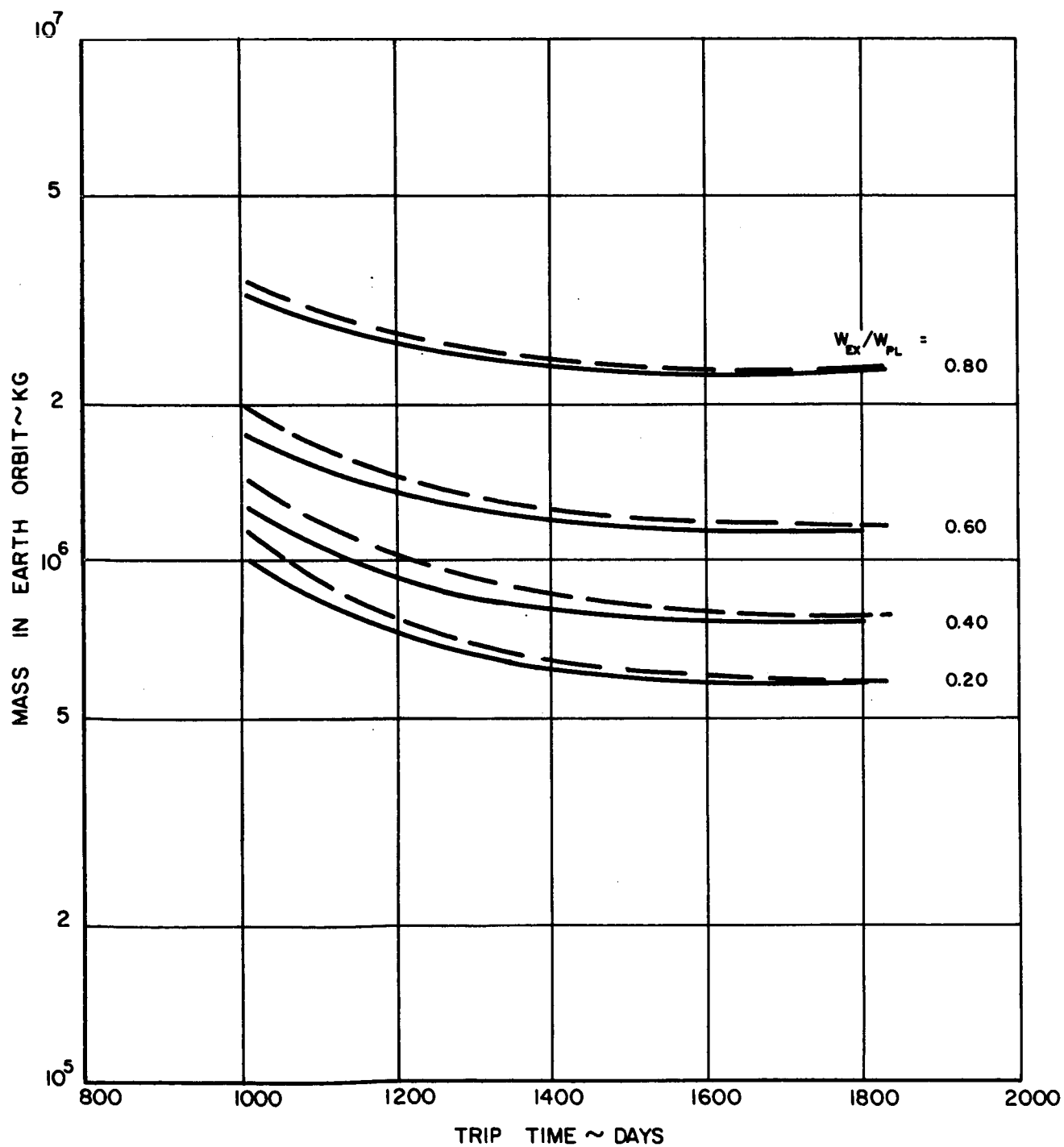
GASEOUS-CORE NUCLEAR

$I_{sp} = 2000 \text{ SEC}$

NOMINAL

--- 60-DAY STAY

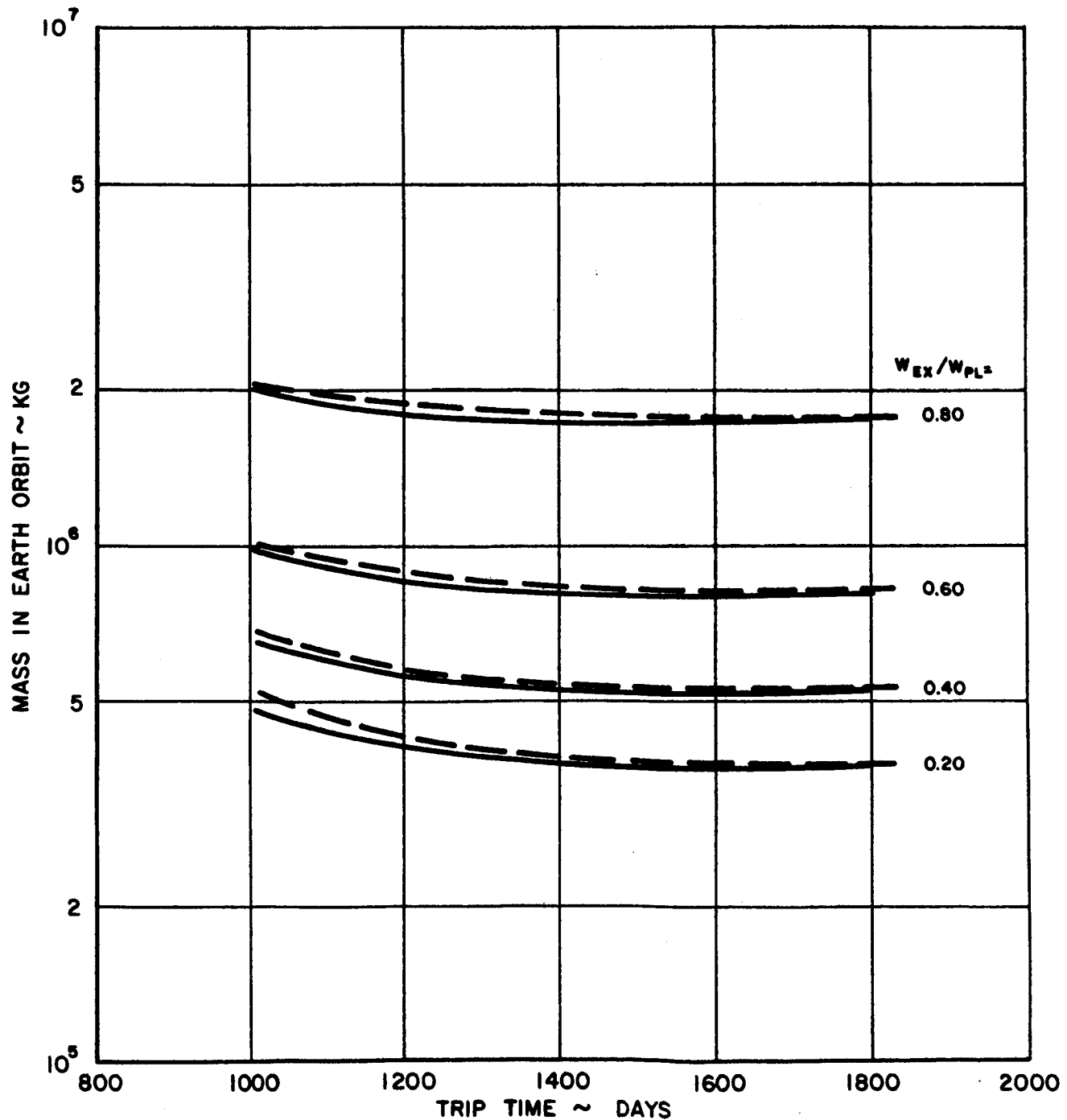
— ZERO STAY



GANYMEDE MANNED LANDING MISSIONS
(JUPITER III)
GASEOUS-CORE NUCLEAR

$I_{sp} = 3500 \text{ SEC}$

--- 60-DAY STAY
— ZERO STAY

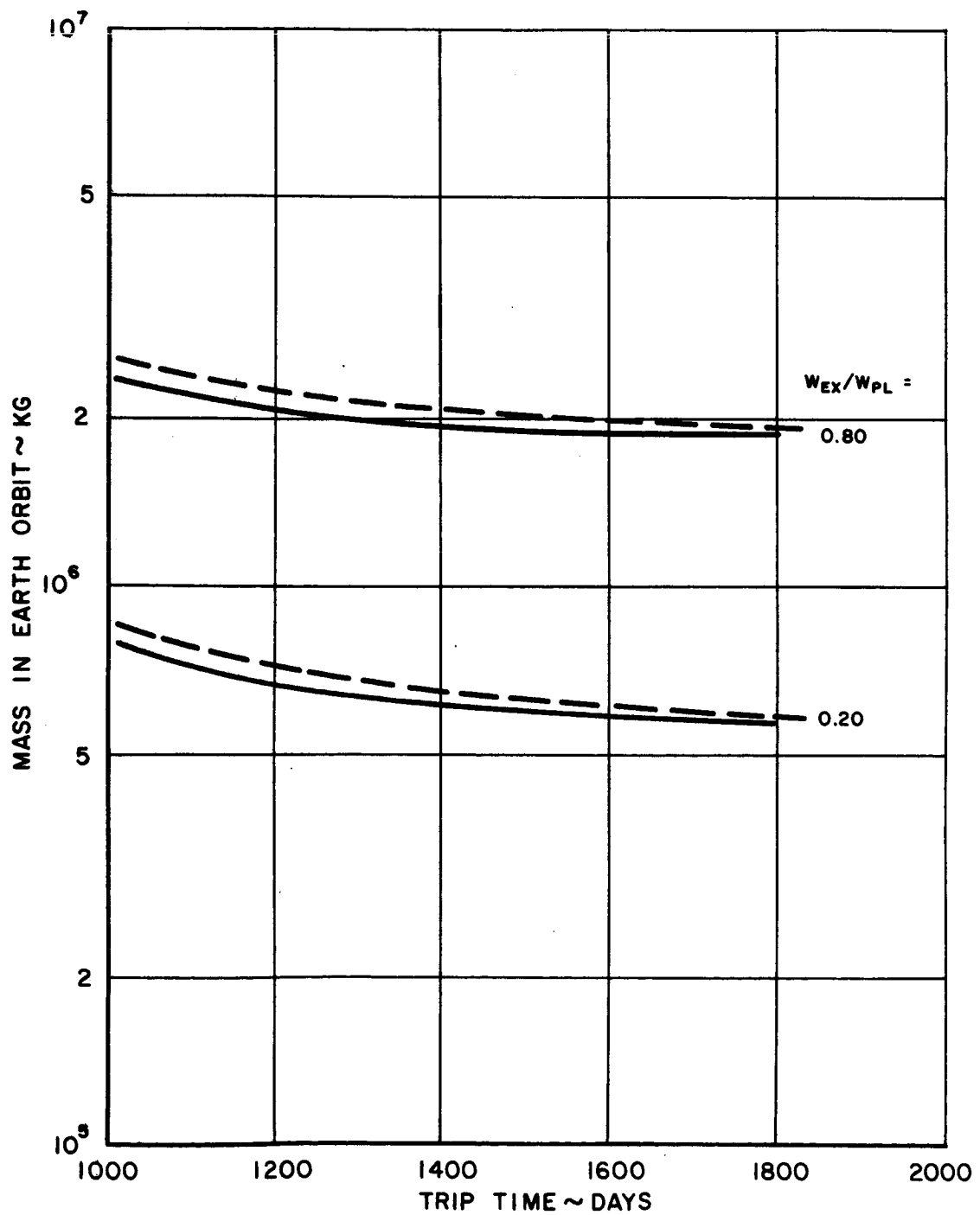


GANYMEDE MANNED LANDING MISSIONS
(JUPITER III)
PULSED NUCLEAR

$I_{sp} = 1800 \text{ SEC}$

--- 60-DAY STAY

— ZERO STAY



TITAN MANNED LANDING MISSIONS
(SATURN VI)

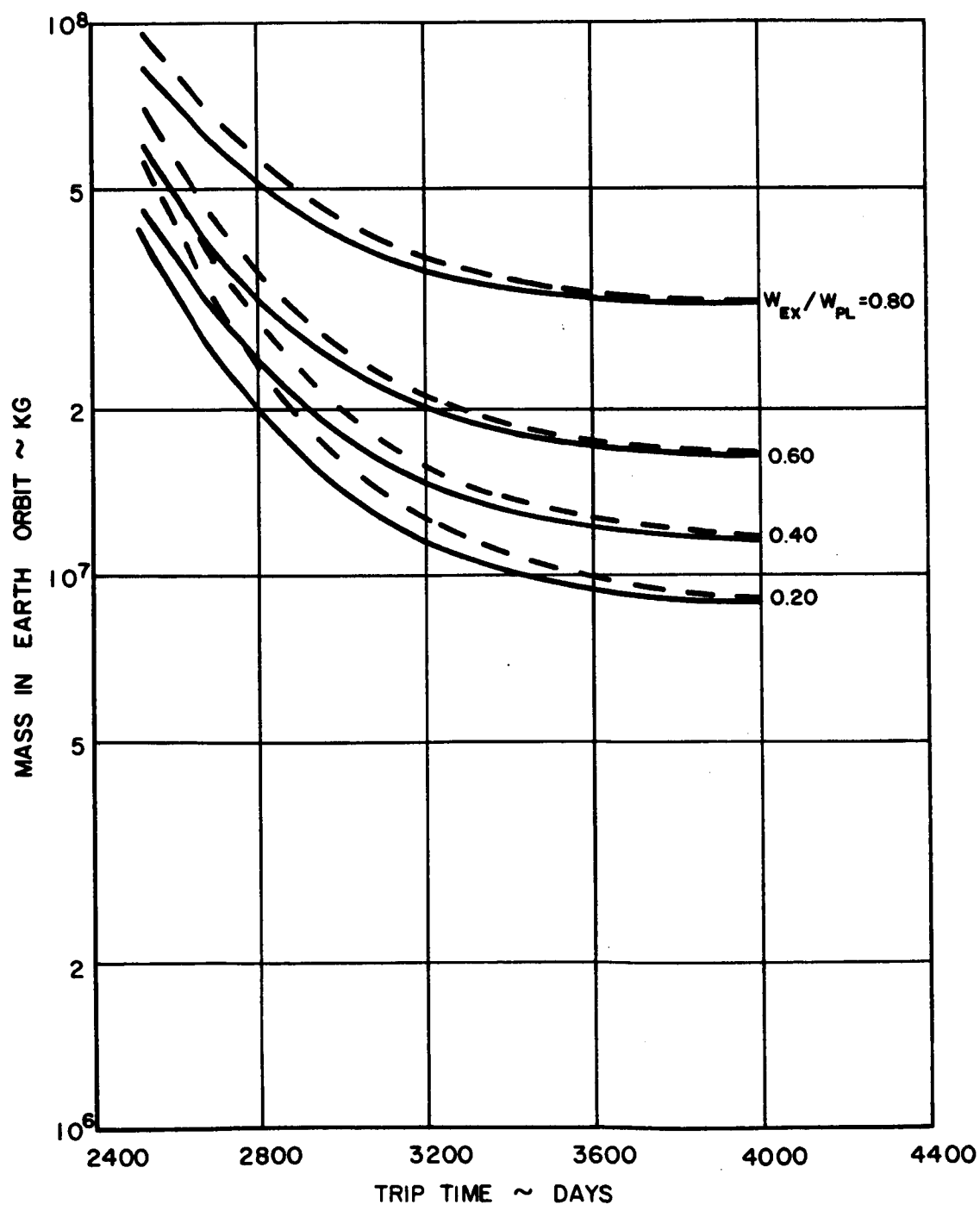
CHEMICAL

Isp = 430 SEC

NOMINAL

--- 60 - DAY STAY

— ZERO STAY



TITAN MANNED LANDING MISSIONS (SATURN VI)

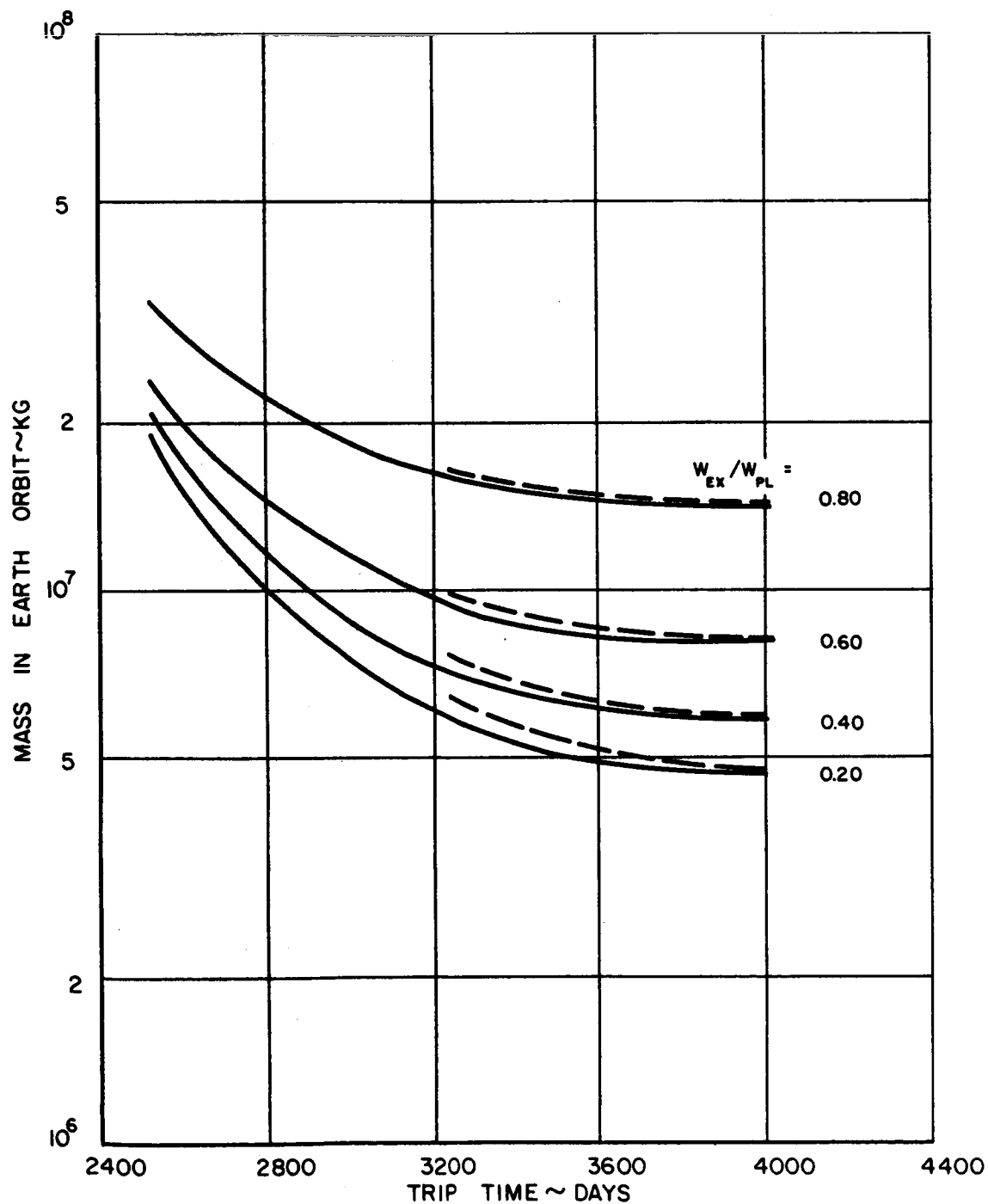
SOLID-CORE NUCLEAR

$I_{sp} = 800 \text{ SEC}$

NOMINAL

--- 60-DAY STAY

— ZERO STAY



TITAN MANNED LANDING MISSIONS (SATURN VI)

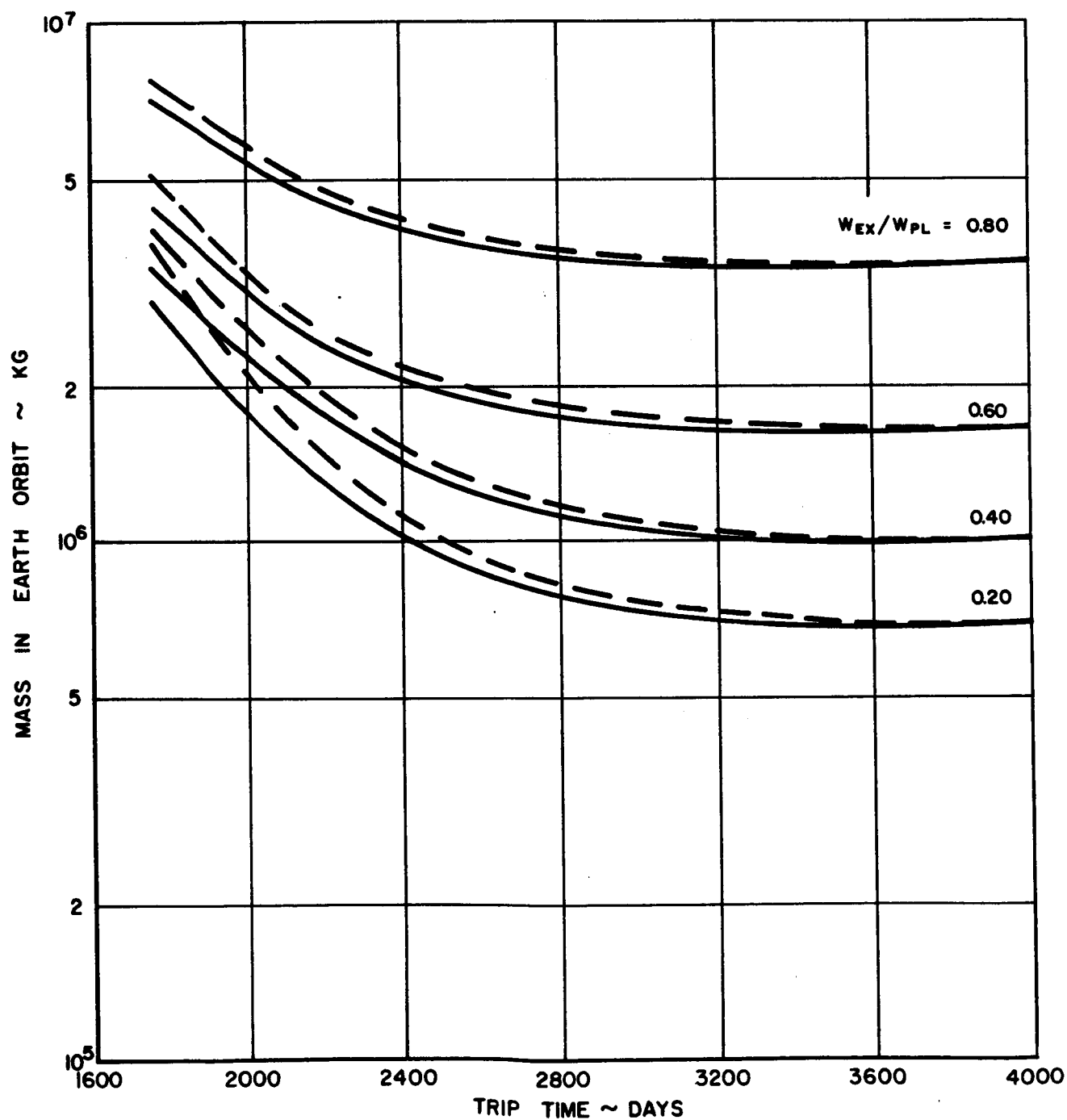
LIGHTWEIGHT ADVANCED NUCLEAR

Isp = 2000 SEC

NOMINAL

--- 60 - DAY STAY

— ZERO STAY



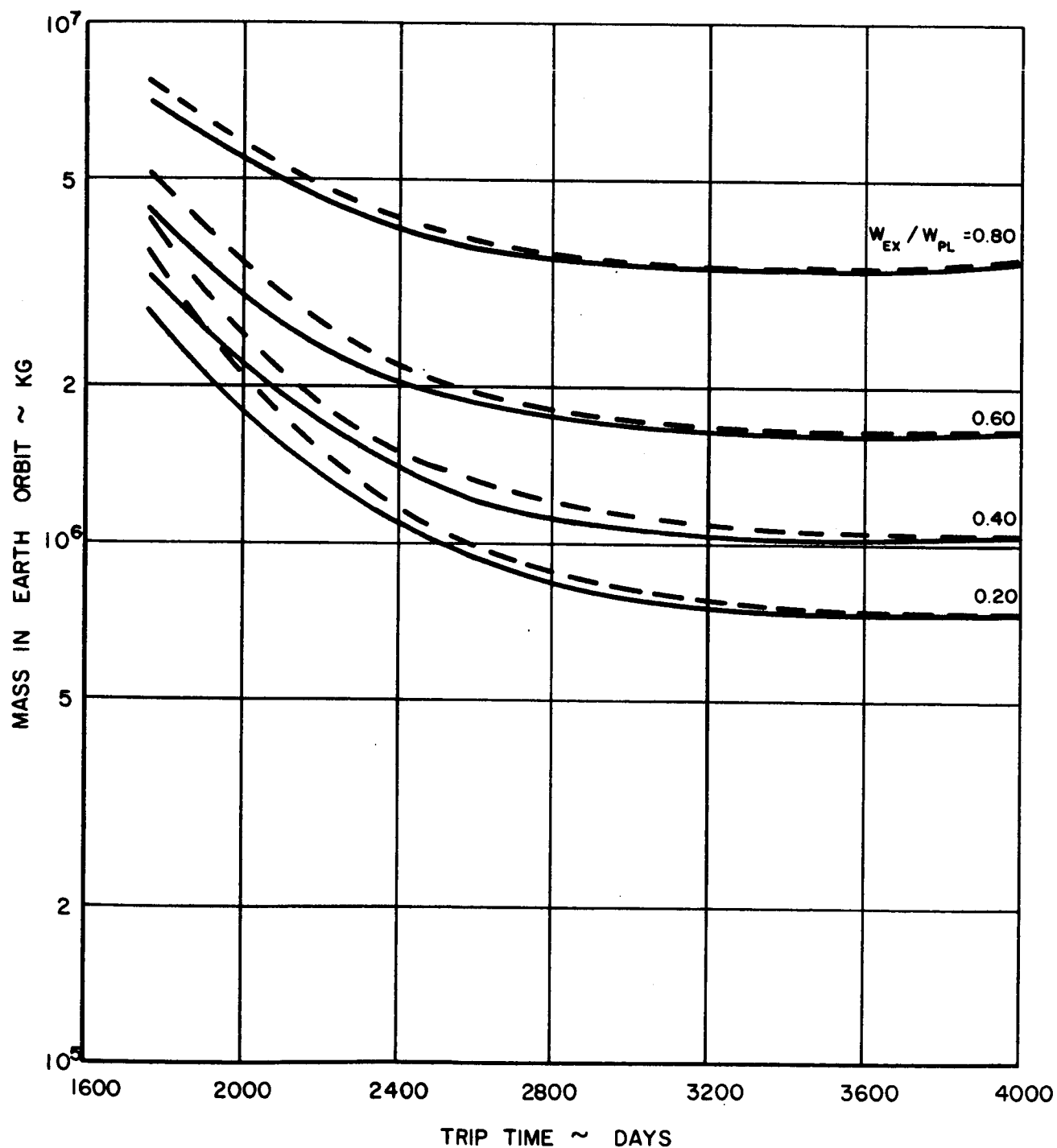
TITAN MANNED LANDING MISSIONS
(SATURN VI)
GASEOUS-CORE NUCLEAR

$I_{sp} = 2000 \text{ SEC}$

NOMINAL

--- 60-DAY STAY

— ZERO STAY

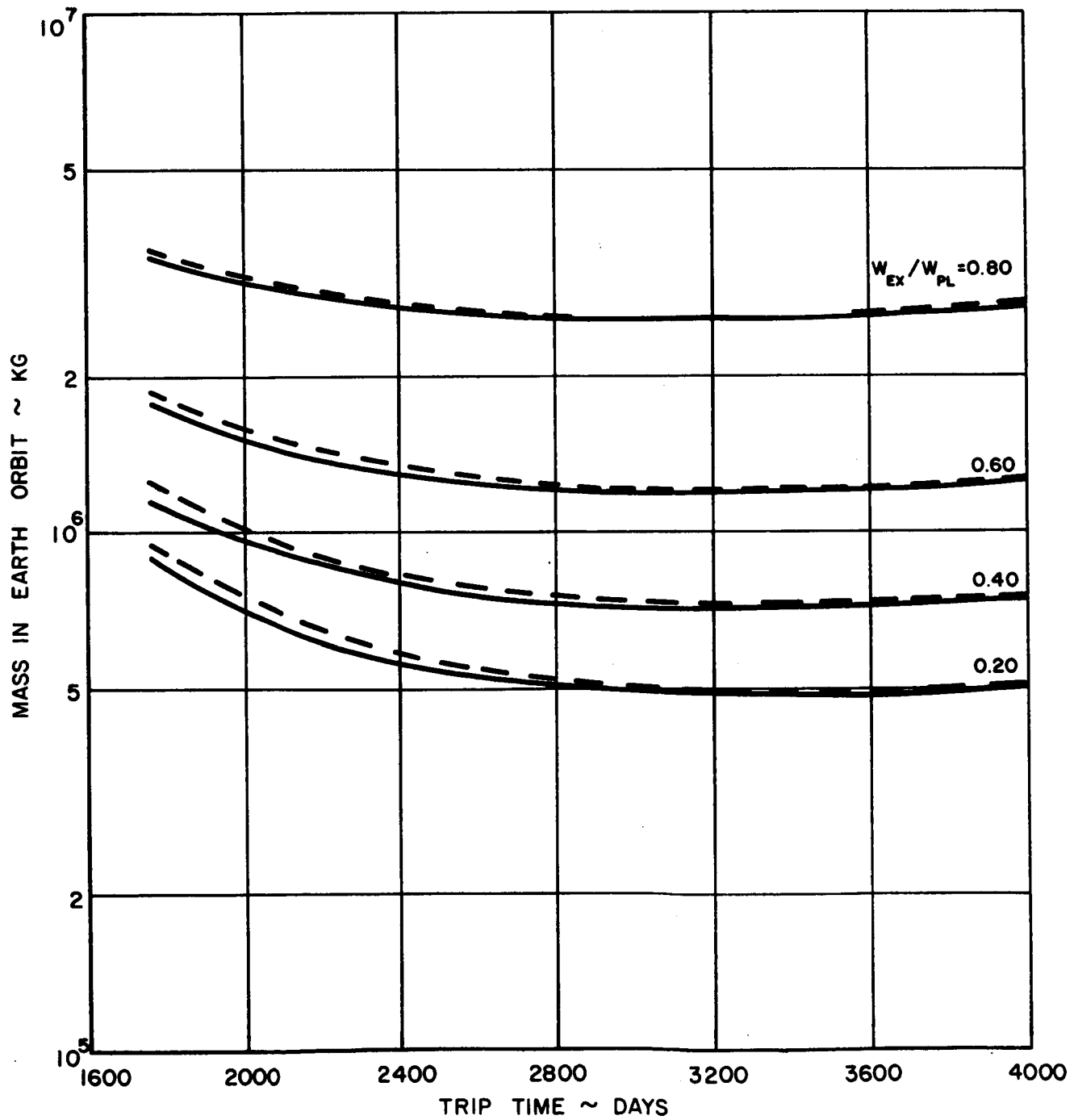


TITAN MANNED LANDING MISSIONS
(SATURN VI)
GASEOUS - CORE NUCLEAR

$I_{sp} = 3500 \text{ SEC}$

--- 60-DAY STAY

— ZERO STAY

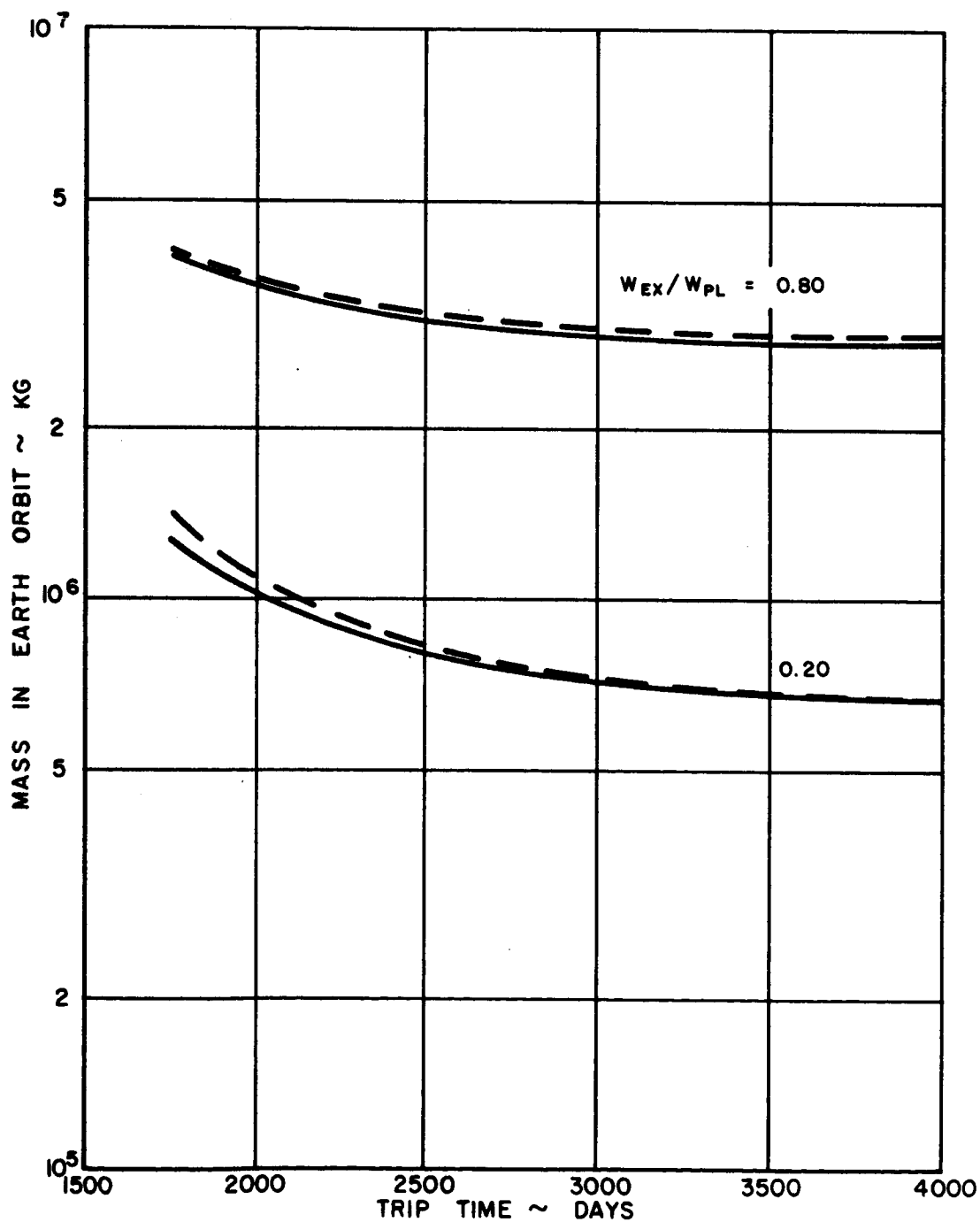


TITAN MANNED LANDING MISSIONS
(SATURN VI)
PULSED NUCLEAR

$I_{sp} = 1800 \text{ SEC}$

--- 60 - DAY STAY

— ZERO STAY



MERCURY MANNED LANDING MISSIONS

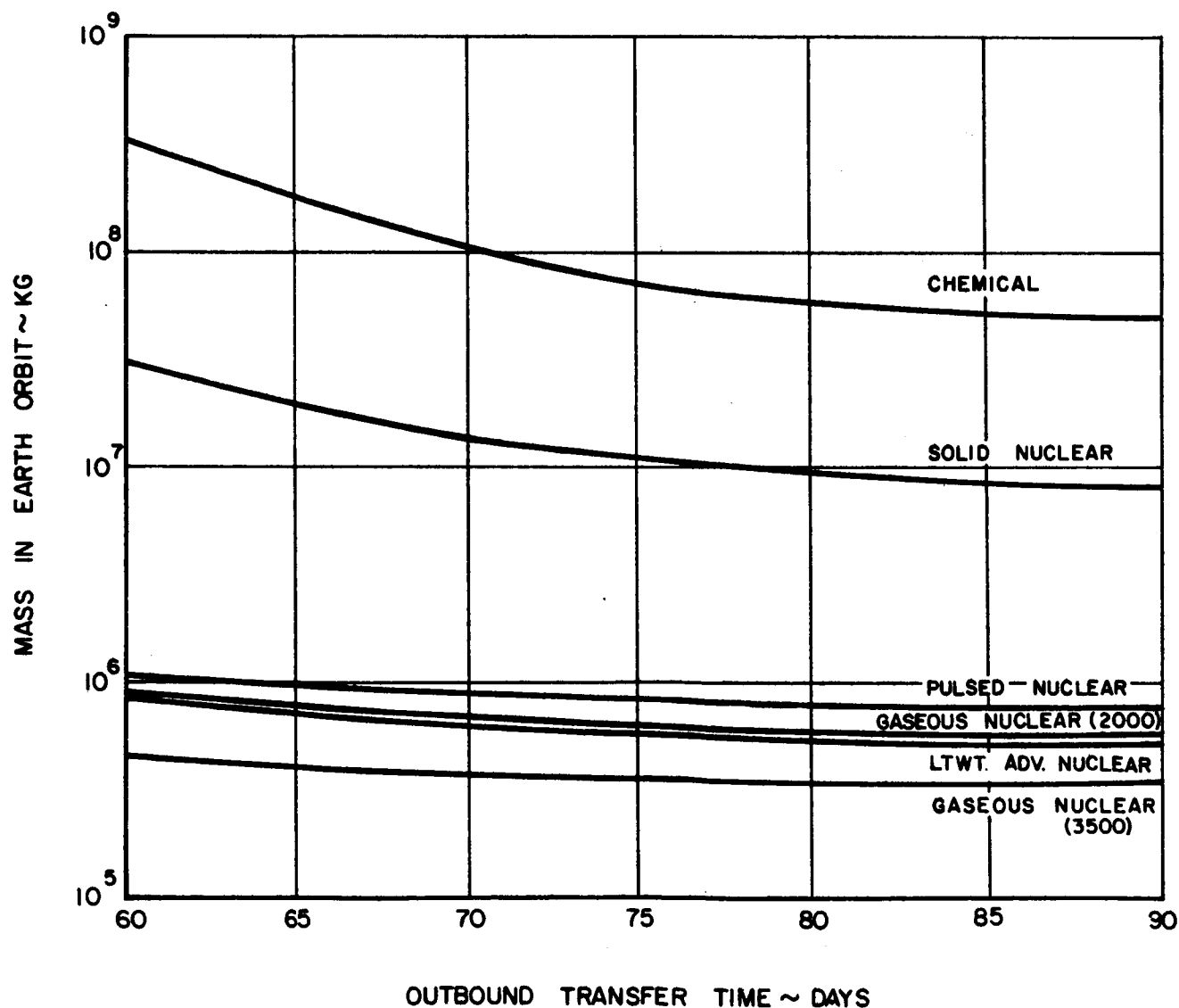
INBOUND LEG DURATION = 200 DAYS

STAY TIME = 80-85 DAYS

$W_{EX} / W_{PL} = 0.20$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600



MARS MANNED LANDING MISSIONS

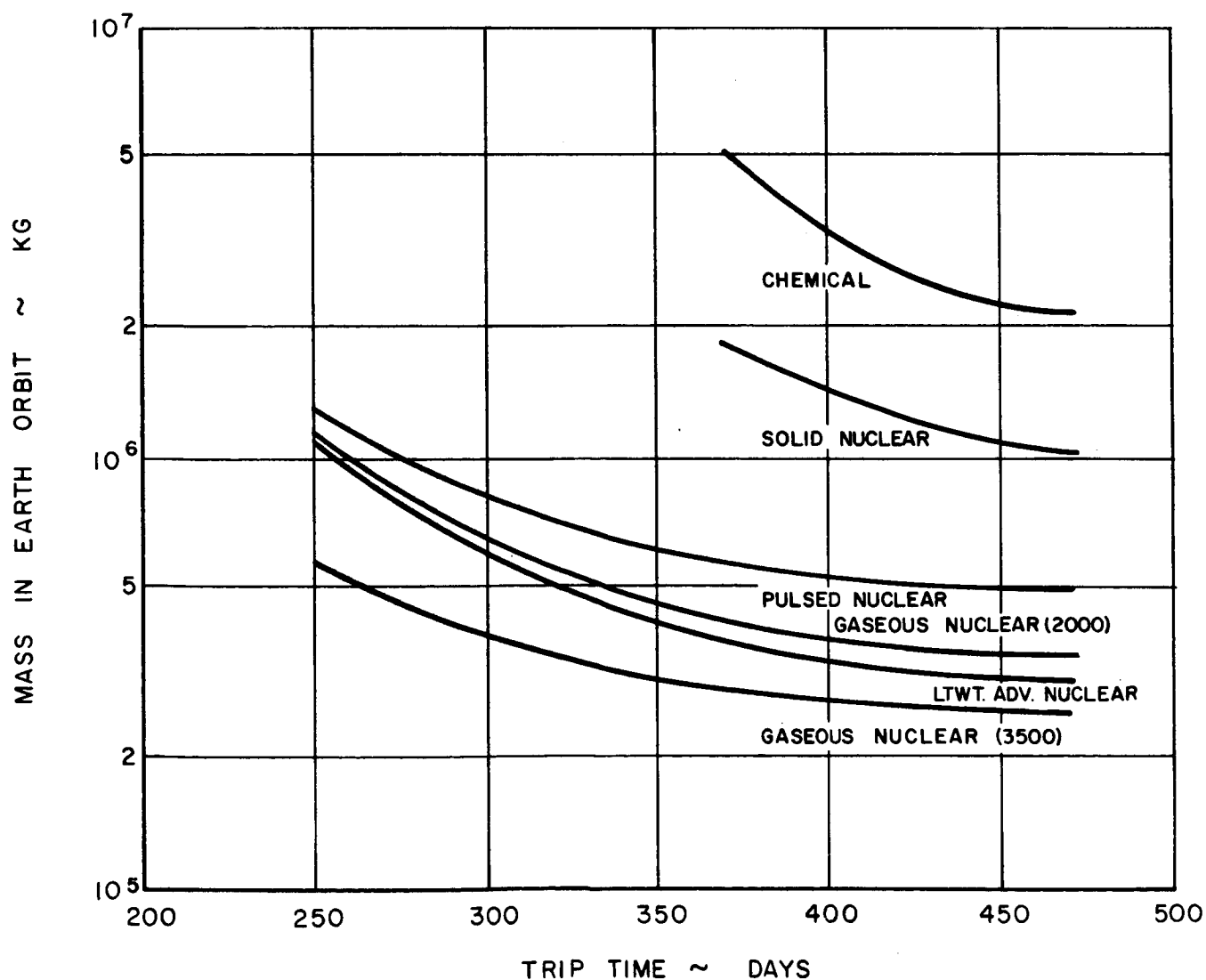
NOMINAL

STAY TIME = 0

 $W_{EX}/W_{PL} = 0.20$

NOMINAL ENGINE PERFORMANCE

ENGINE	I_{sp}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	60.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600



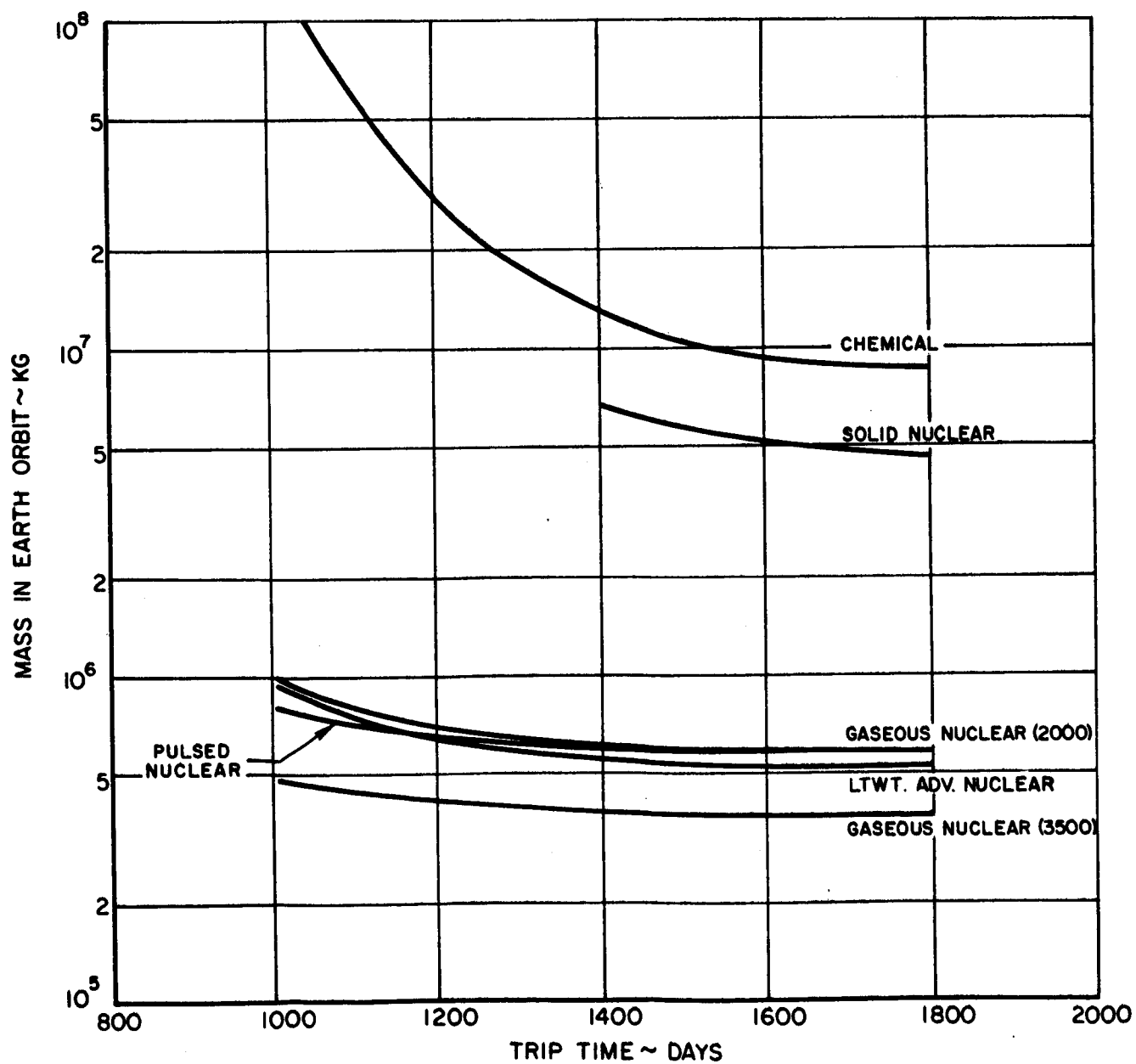
GANYMEDE MANNED LANDING MISSIONS (JUPITER II)

STAY TIME=0

$$W_{EX}/W_{PL} = 0.20$$

NOMINAL ENGINE PERFORMANCE

ENGINE	Isp	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600



TITAN MANNED LANDING MISSIONS

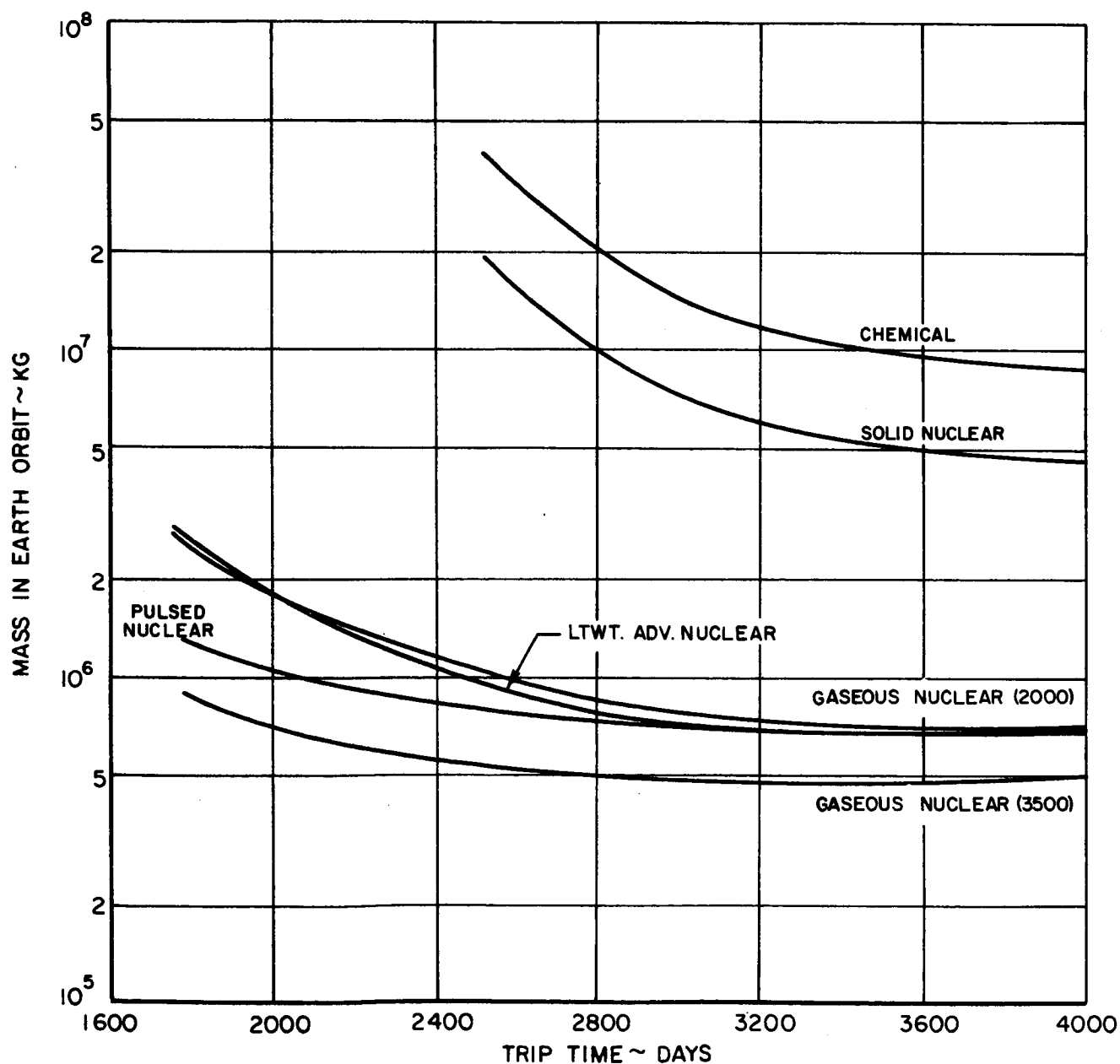
SATURN VI

STAY TIME=0

$$W_{EX}/W_{PL} = 0.20$$

NOMINAL ENGINE PERFORMANCE

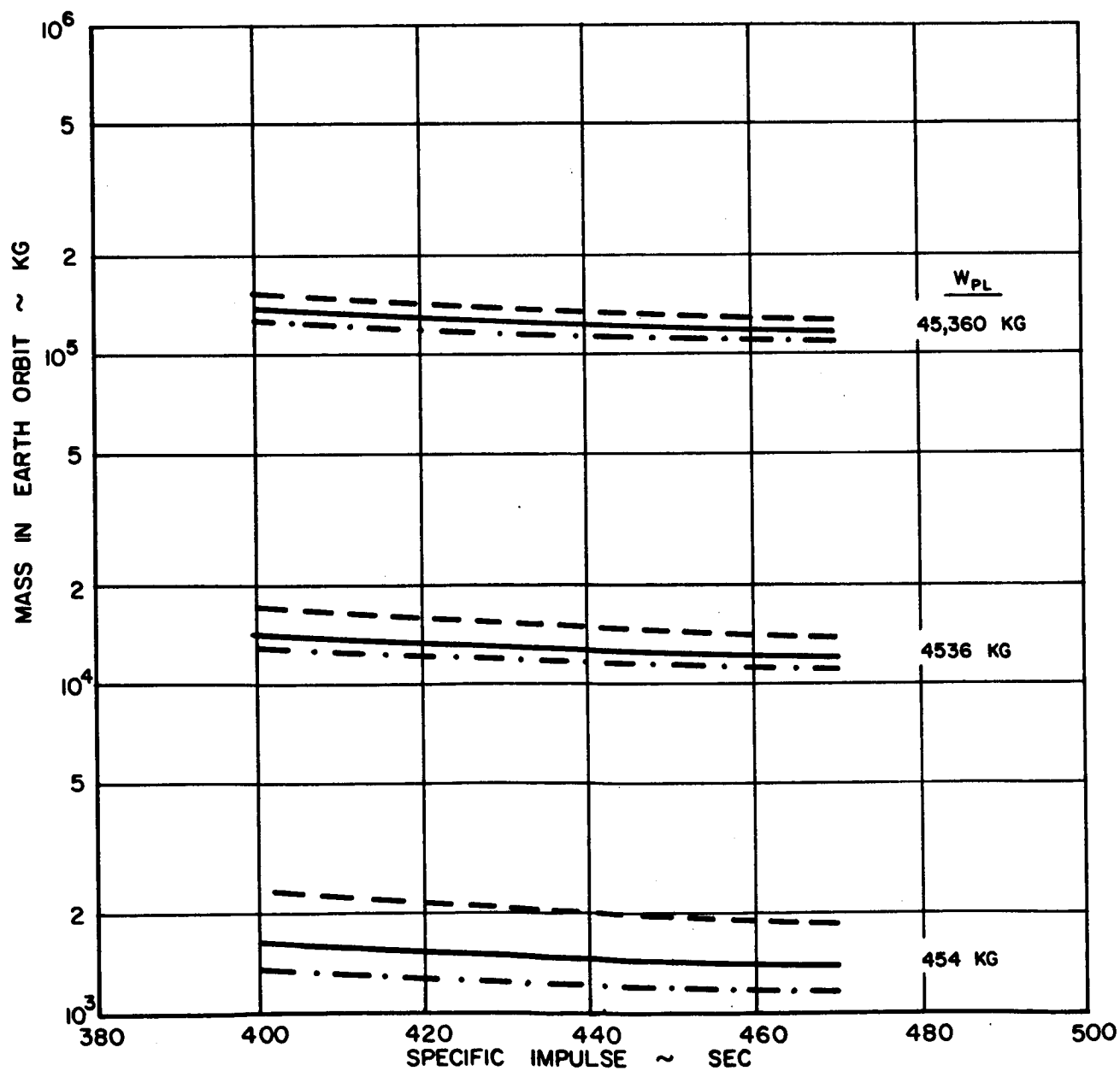
ENGINE	I_{SP}	T/W	MIN. ENG. WT.
CHEMICAL	430 SEC	80.0	45 KG
SOLID NUC.	800	9.1	4536
PULSED NUC.	1800	3.0	—
LTWT. ADV. NUC.	2000	10.0	907
GASEOUS NUC.	2000	10.0	13600
ADV. GAS. NUC.	3500	2.0	13600



MARS FLYBYS

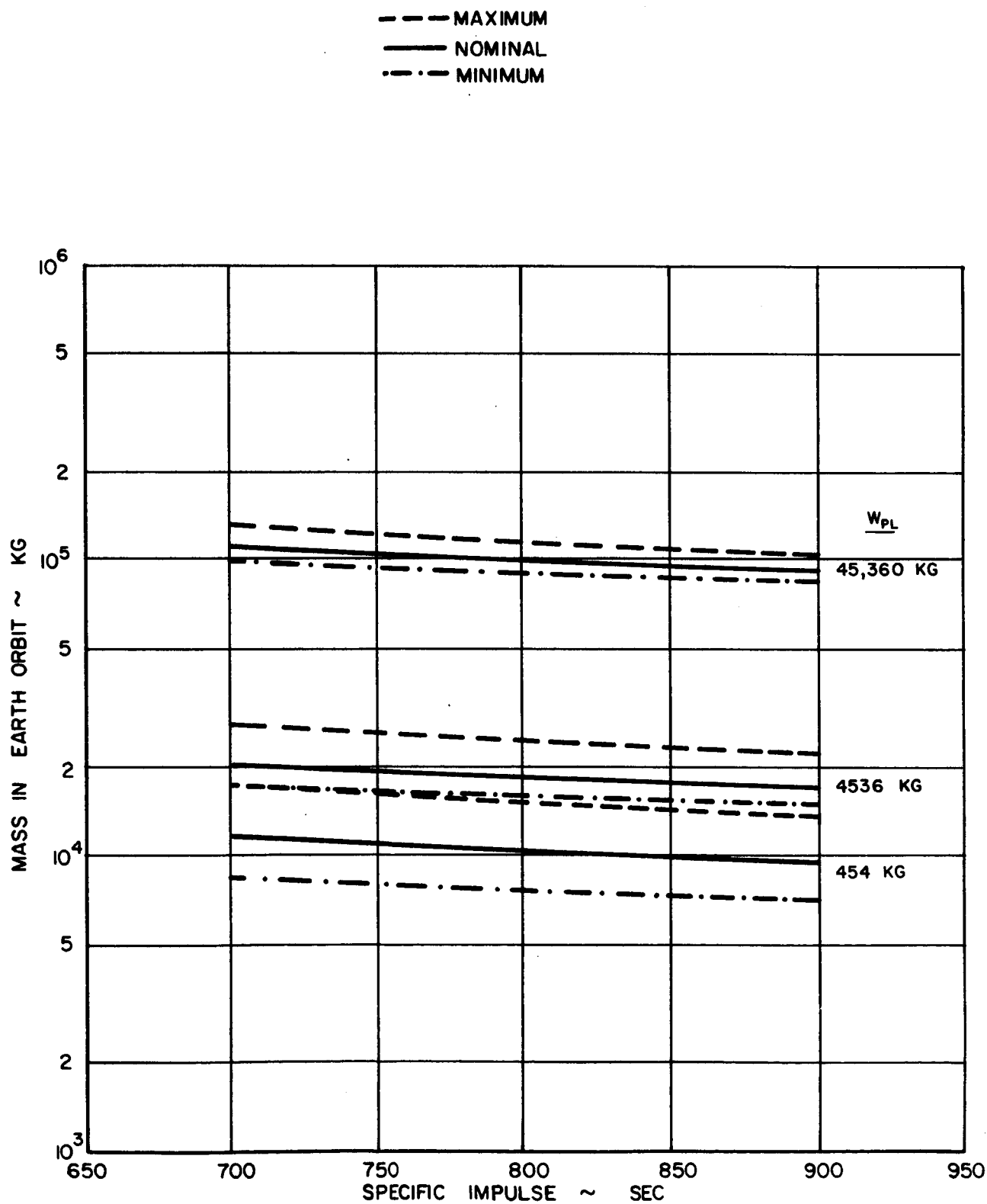
CHEMICAL

--- MAXIMUM
— NOMINAL
- - - MINIMUM



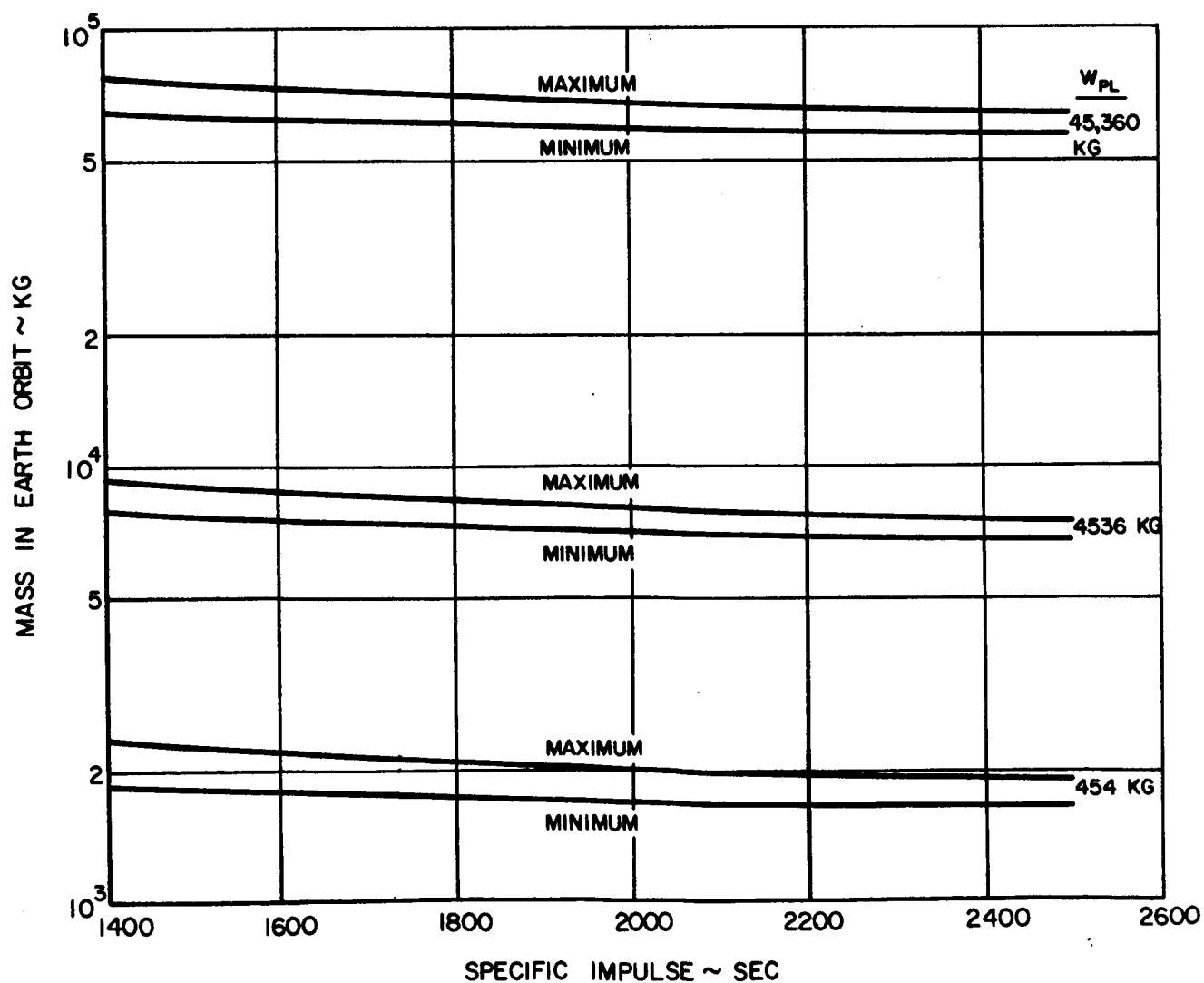
MARS FLYBYS

SOLID-CORE NUCLEAR



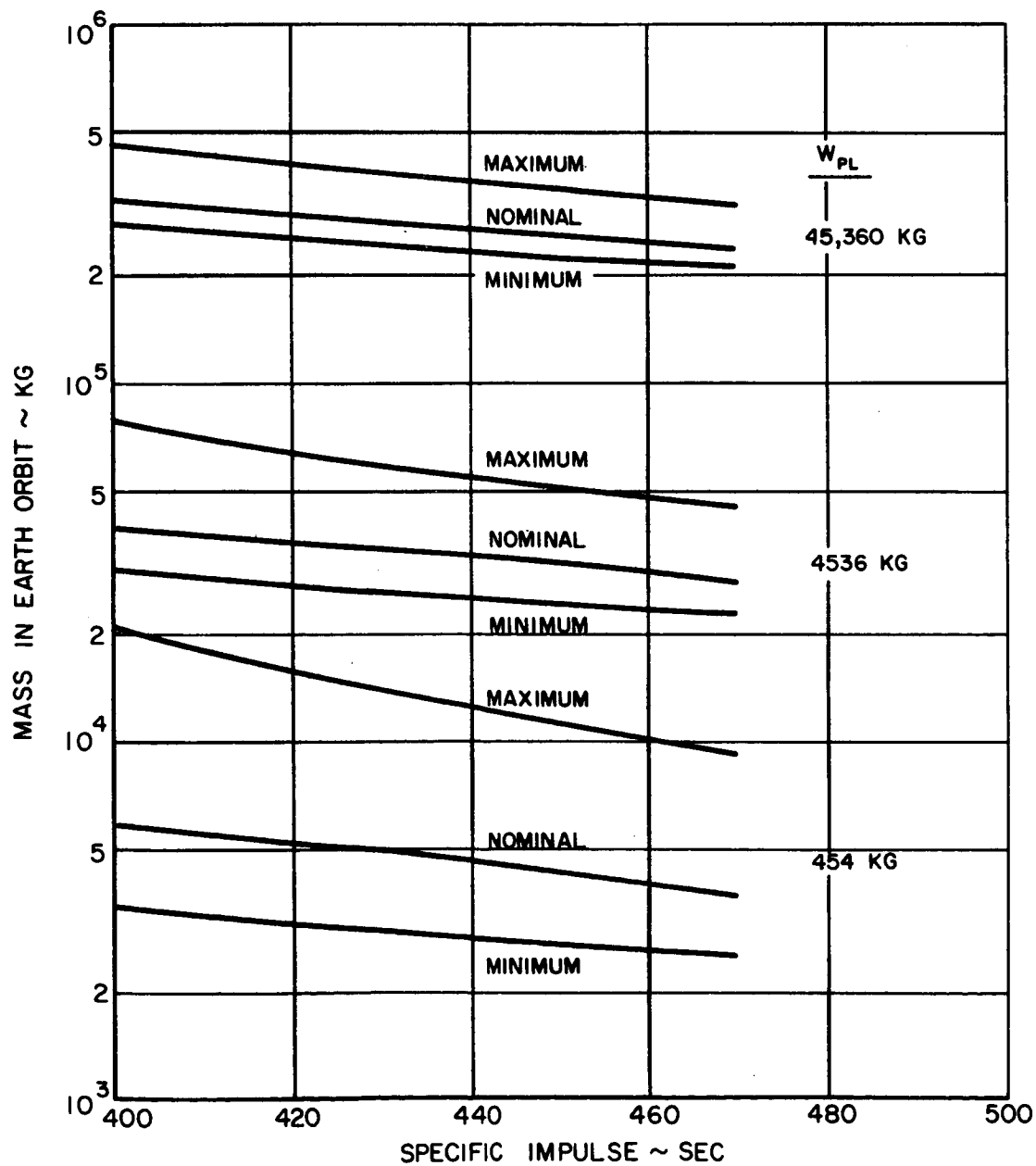
MARS FLYBYS

LIGHTWEIGHT ADVANCED NUCLEAR

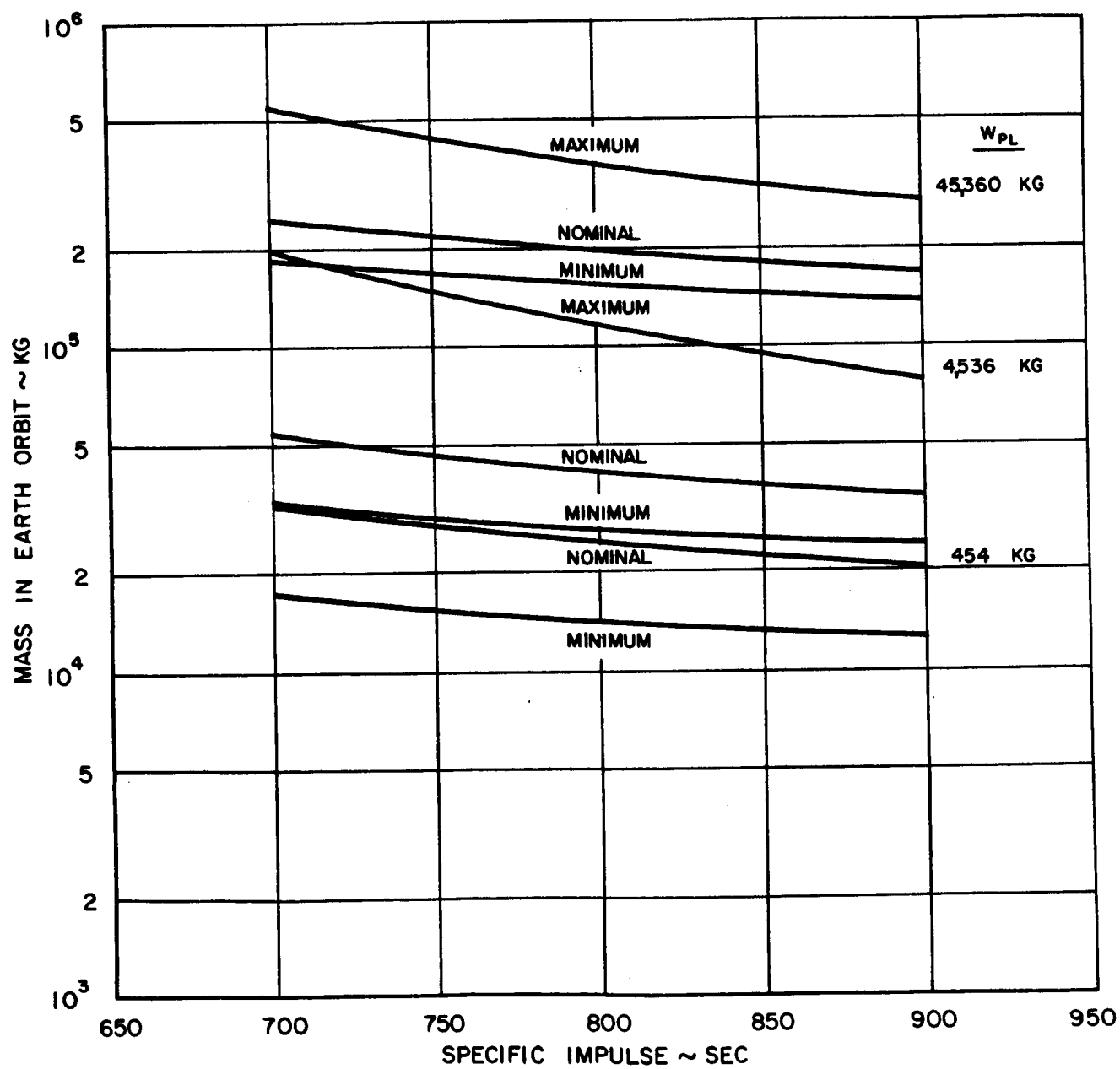


JUPITER FLYBYS

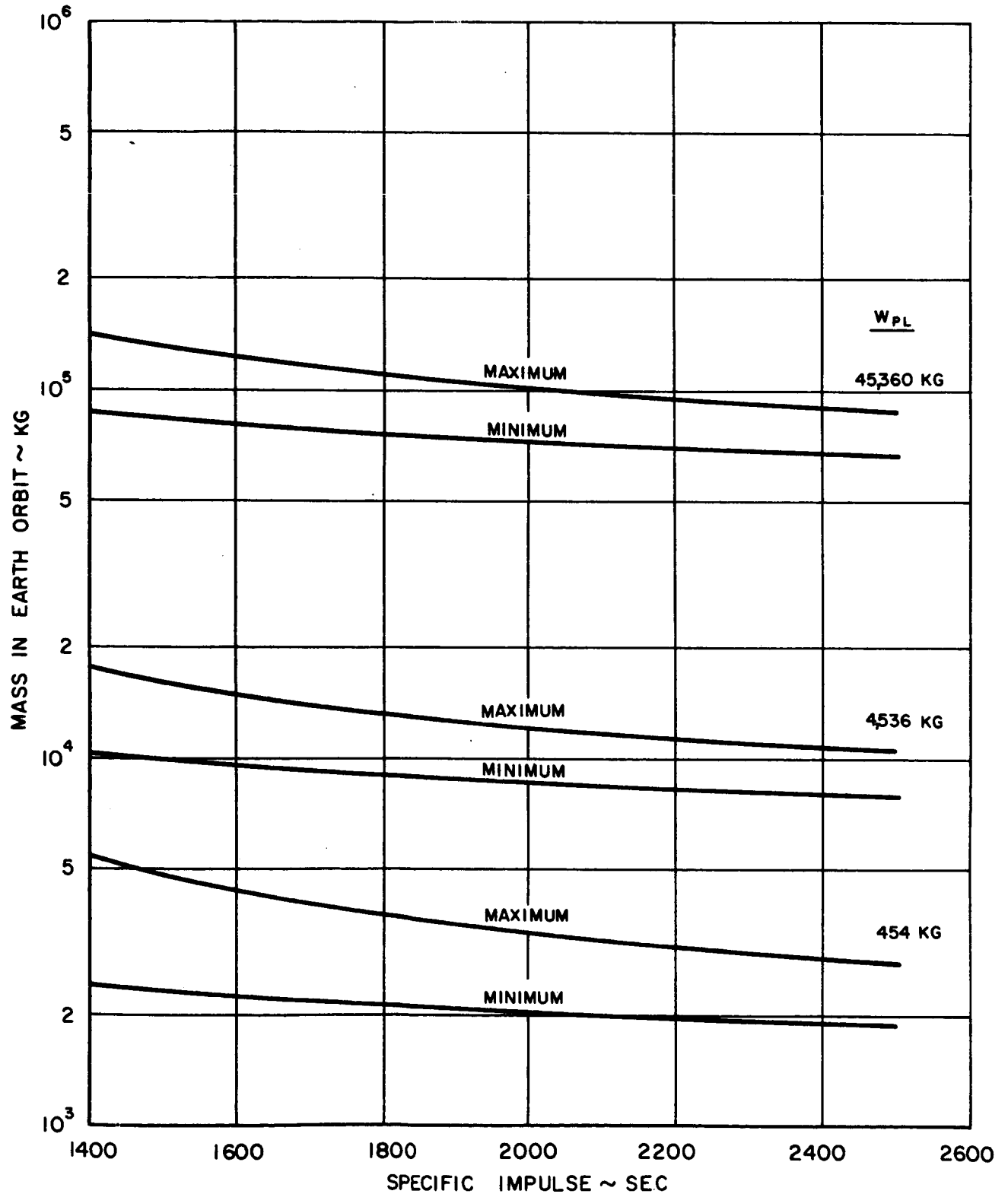
CHEMICAL



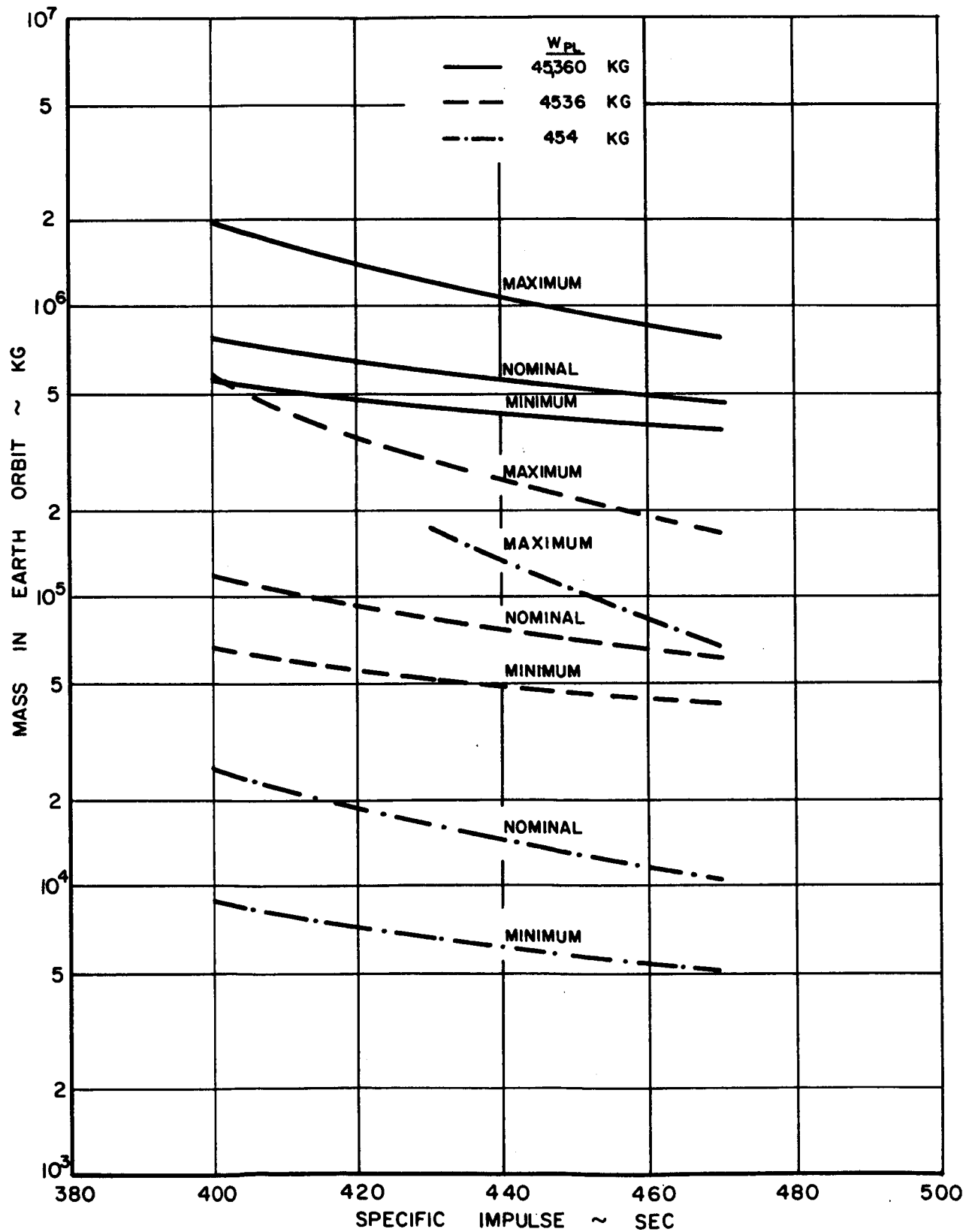
JUPITER FLYBYS SOLID-CORE NUCLEAR



JUPITER FLYBYS LIGHTWEIGHT ADVANCED NUCLEAR

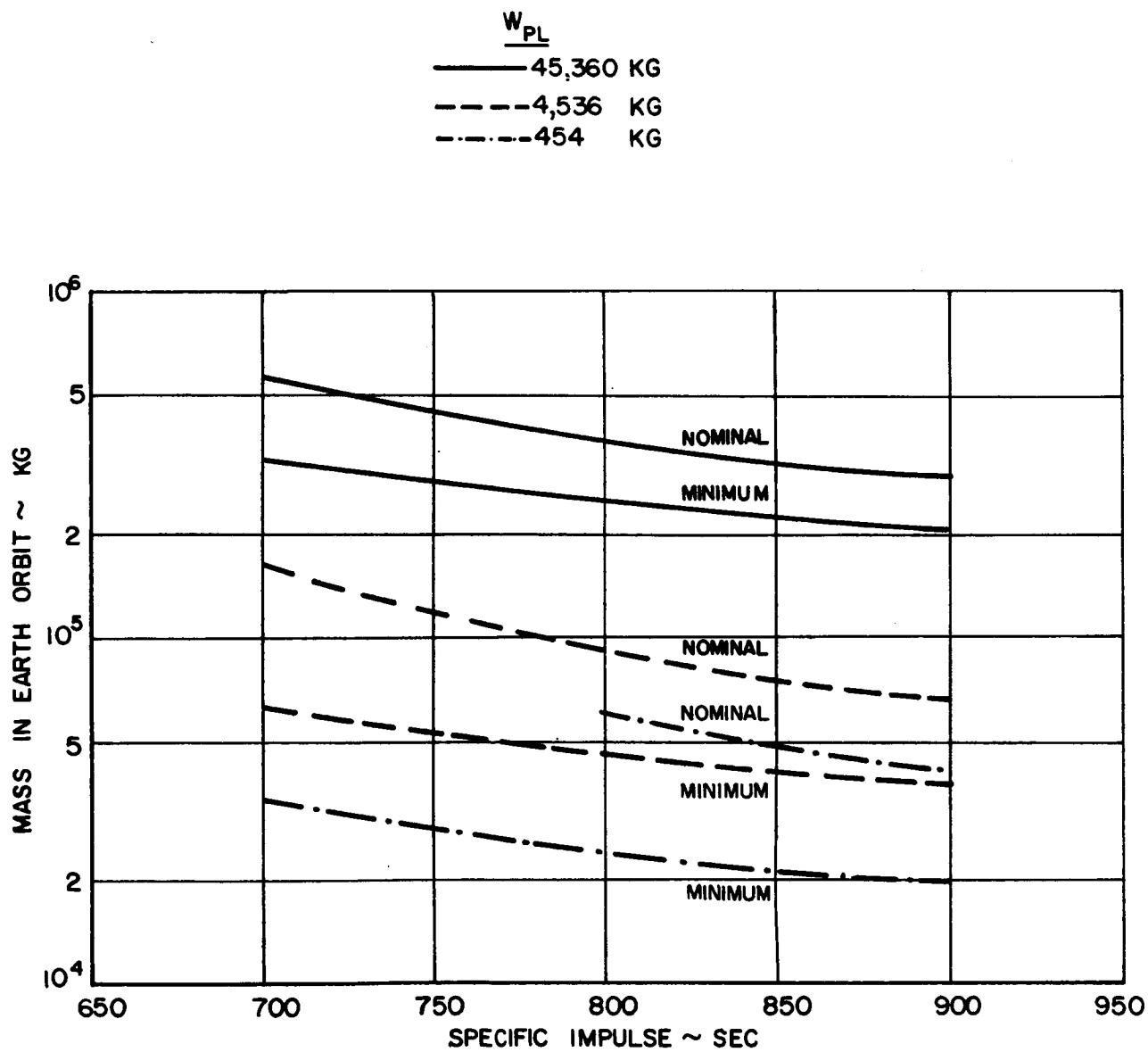


NEPTUNE FLYBYS
CHEMICAL

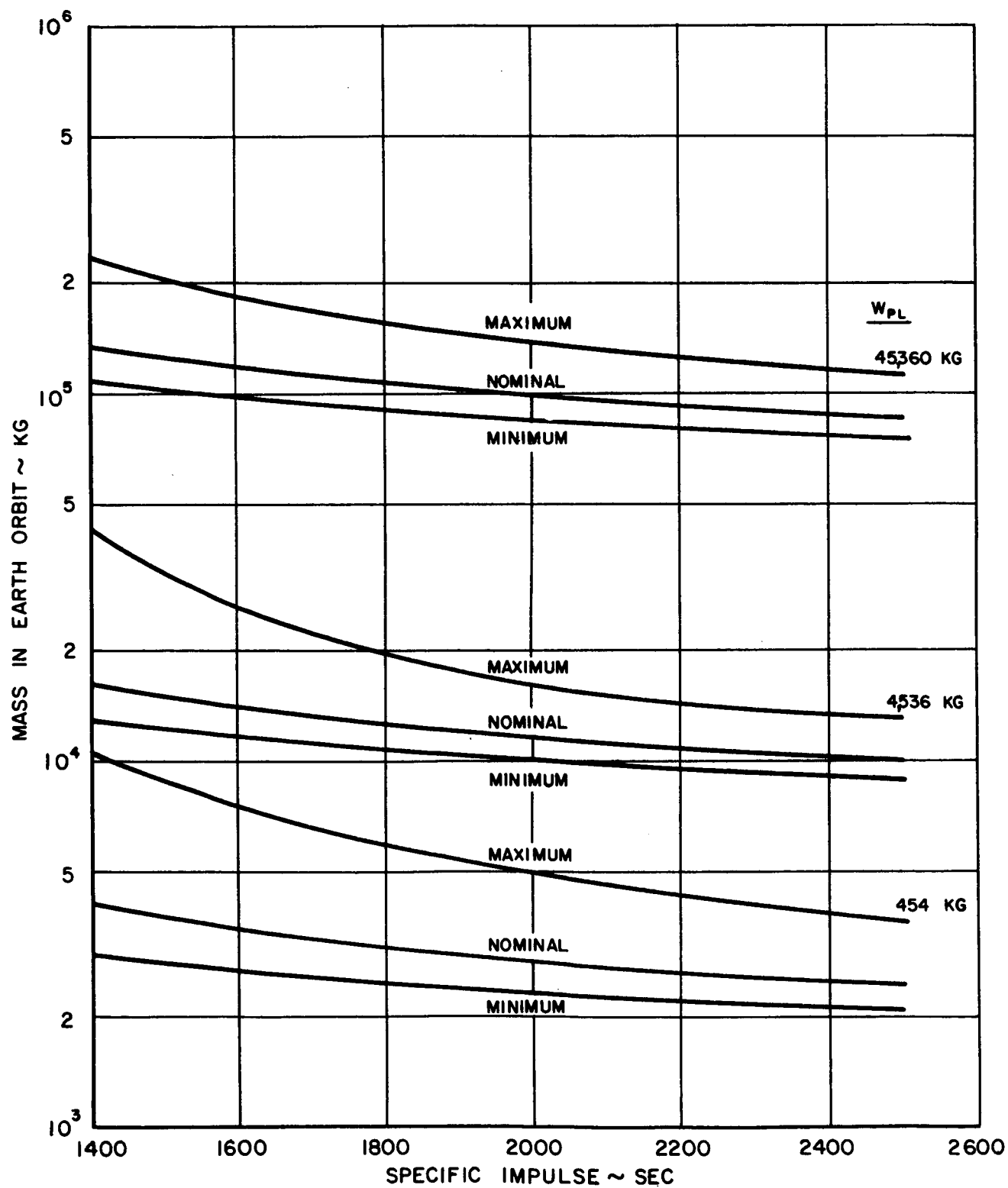


NEPTUNE FLYBYS

SOLID-CORE NUCLEAR

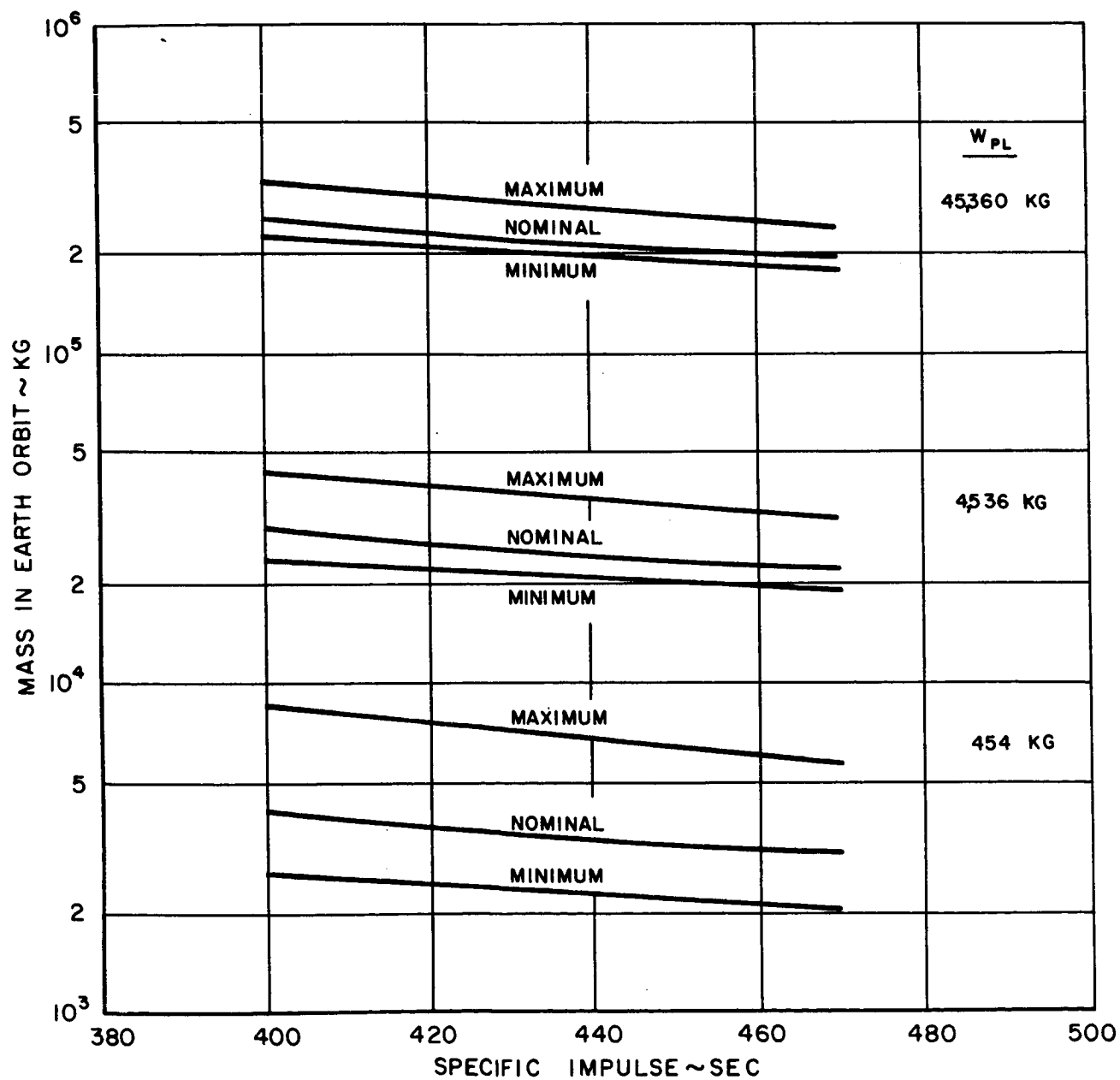


NEPTUNE FLYBYS LIGHTWEIGHT ADVANCED NUCLEAR



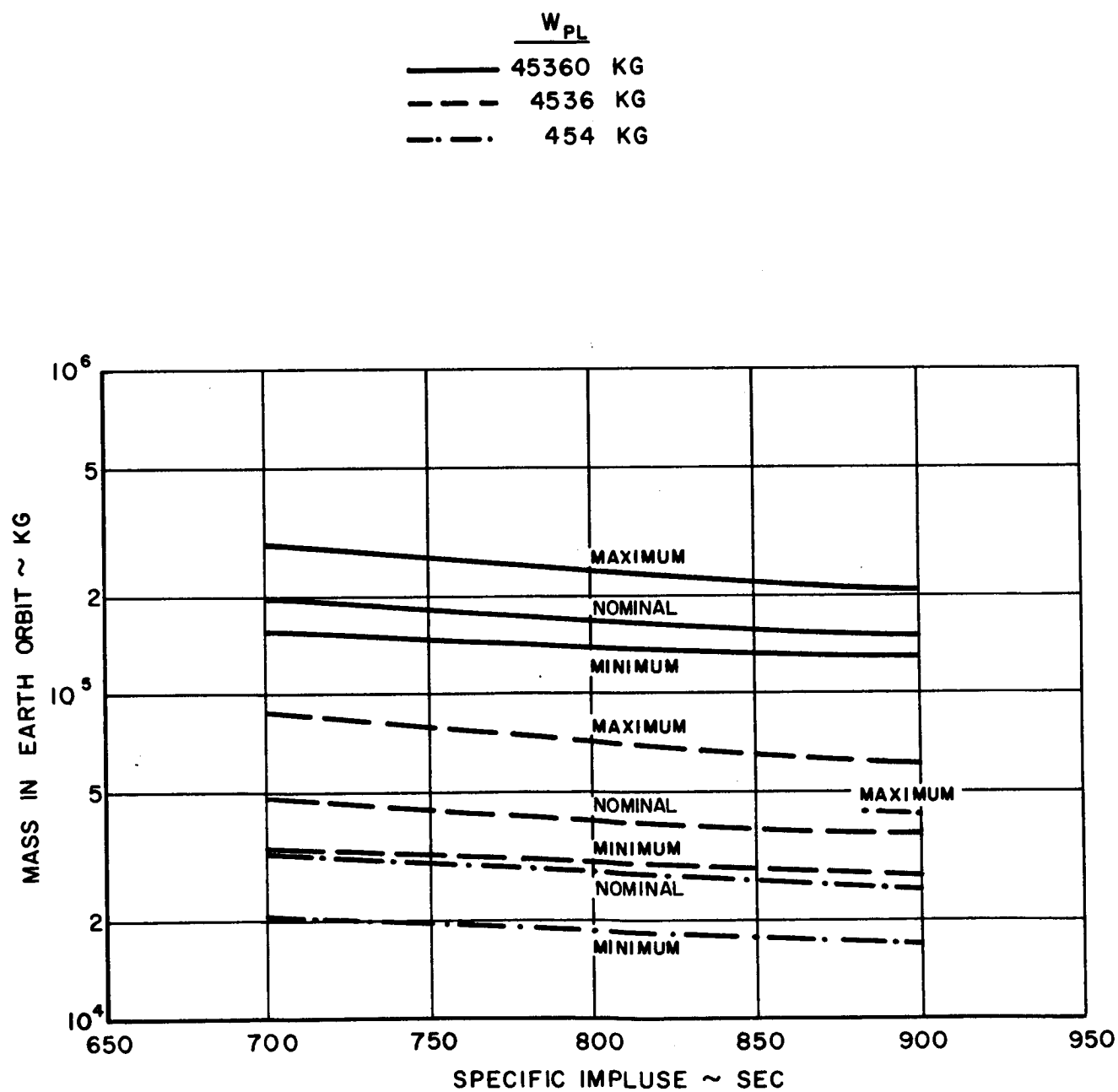
MARS ORBITAL STOPOVERS

CHEMICAL

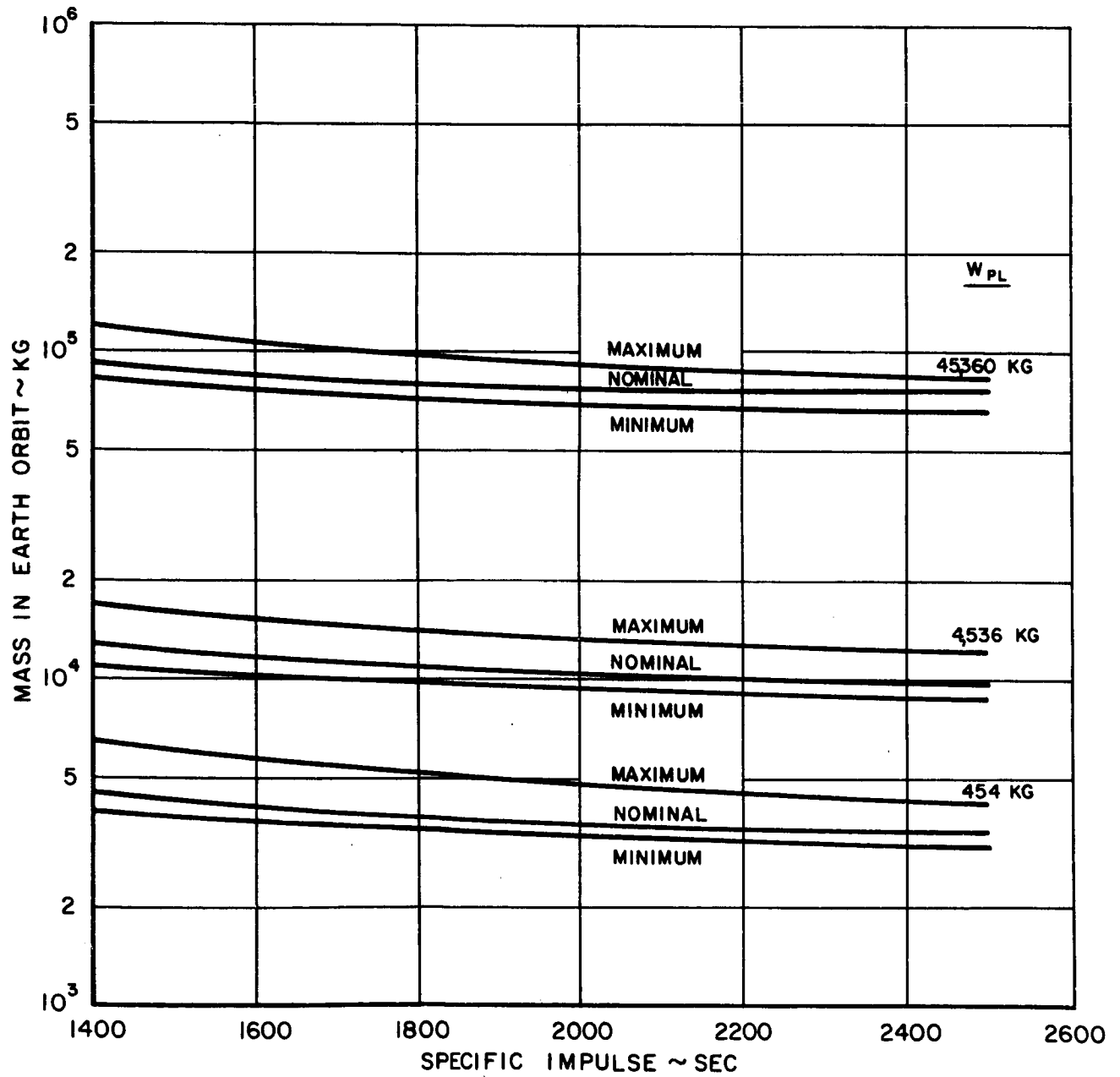


MARS ORBITAL STOPOVERS

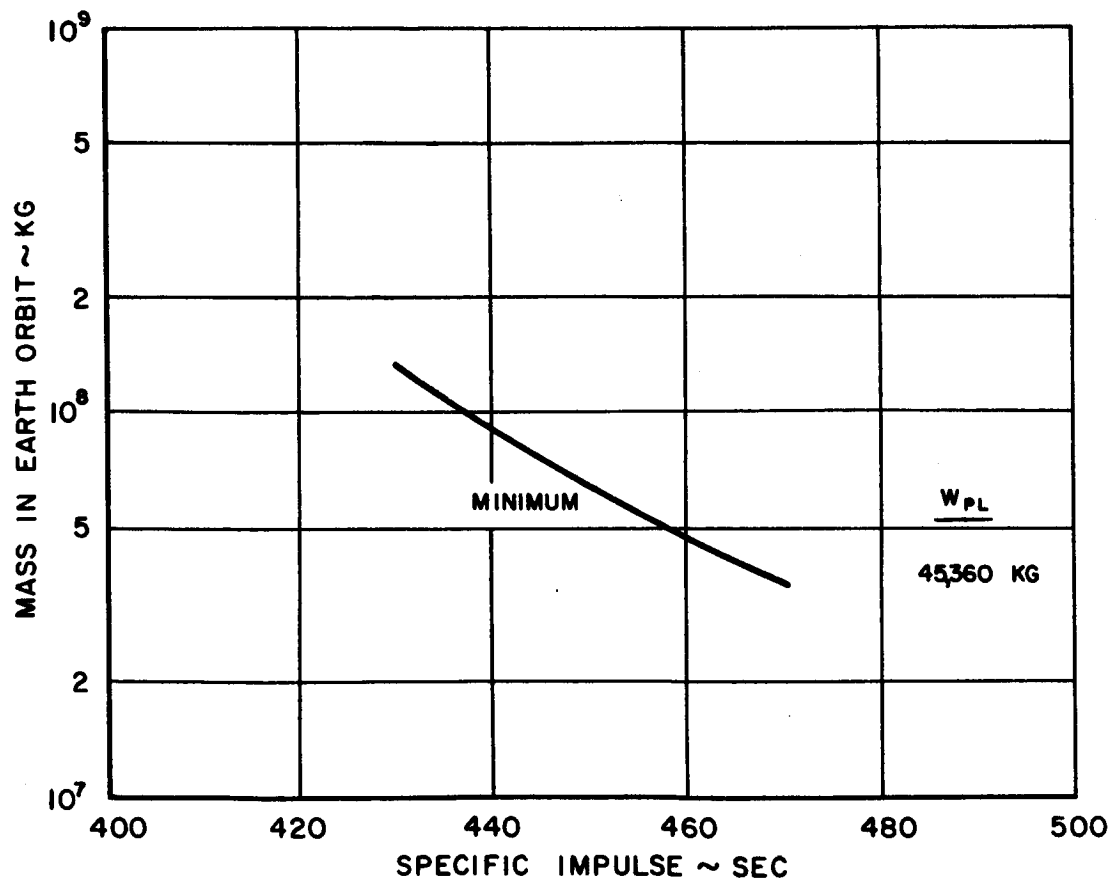
SOLID-CORE NUCLEAR



MARS ORBITAL STOPOVERS
LIGHTWEIGHT ADVANCED NUCLEAR

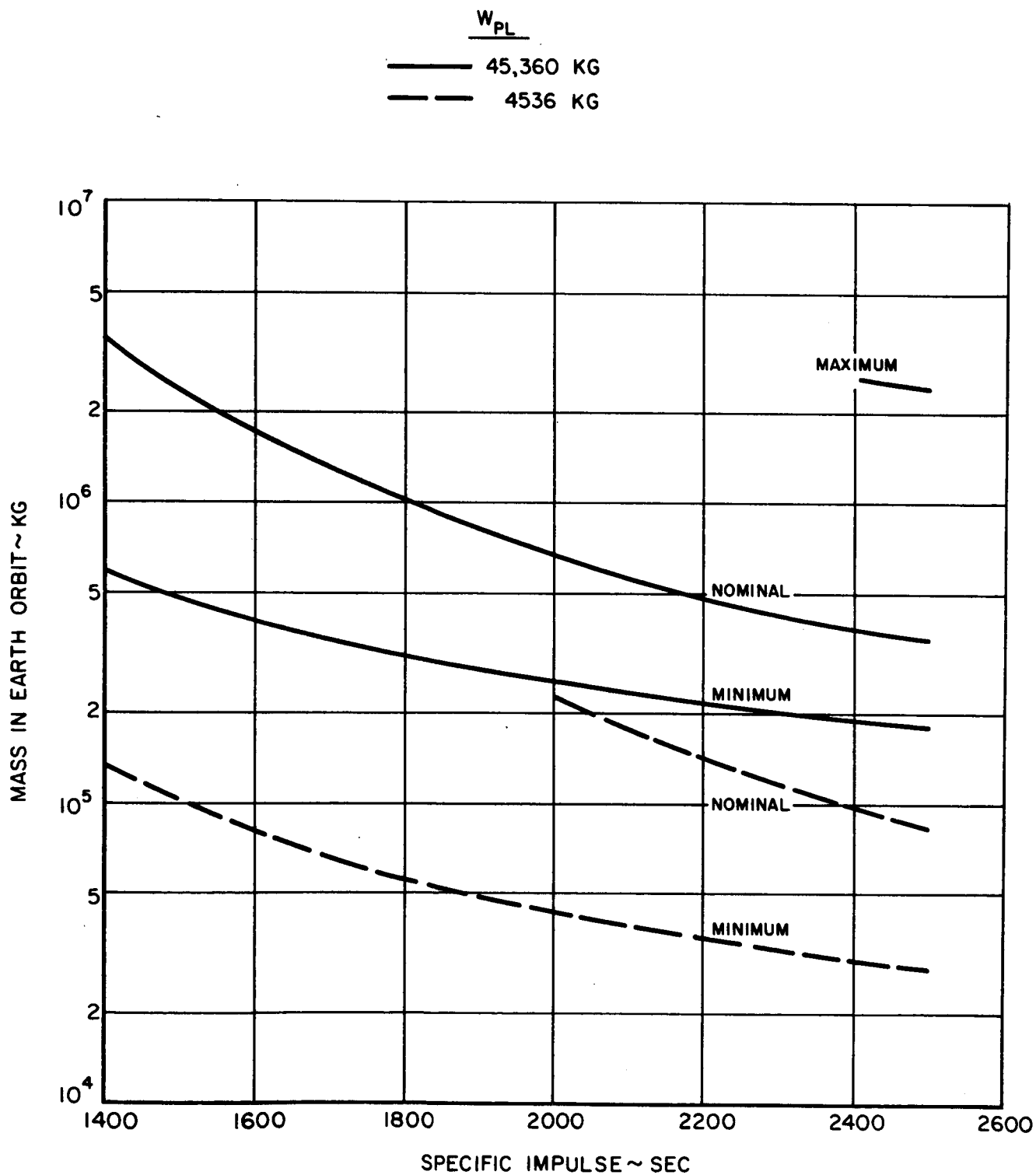


JUPITER ORBITAL STOPOVERS CHEMICAL



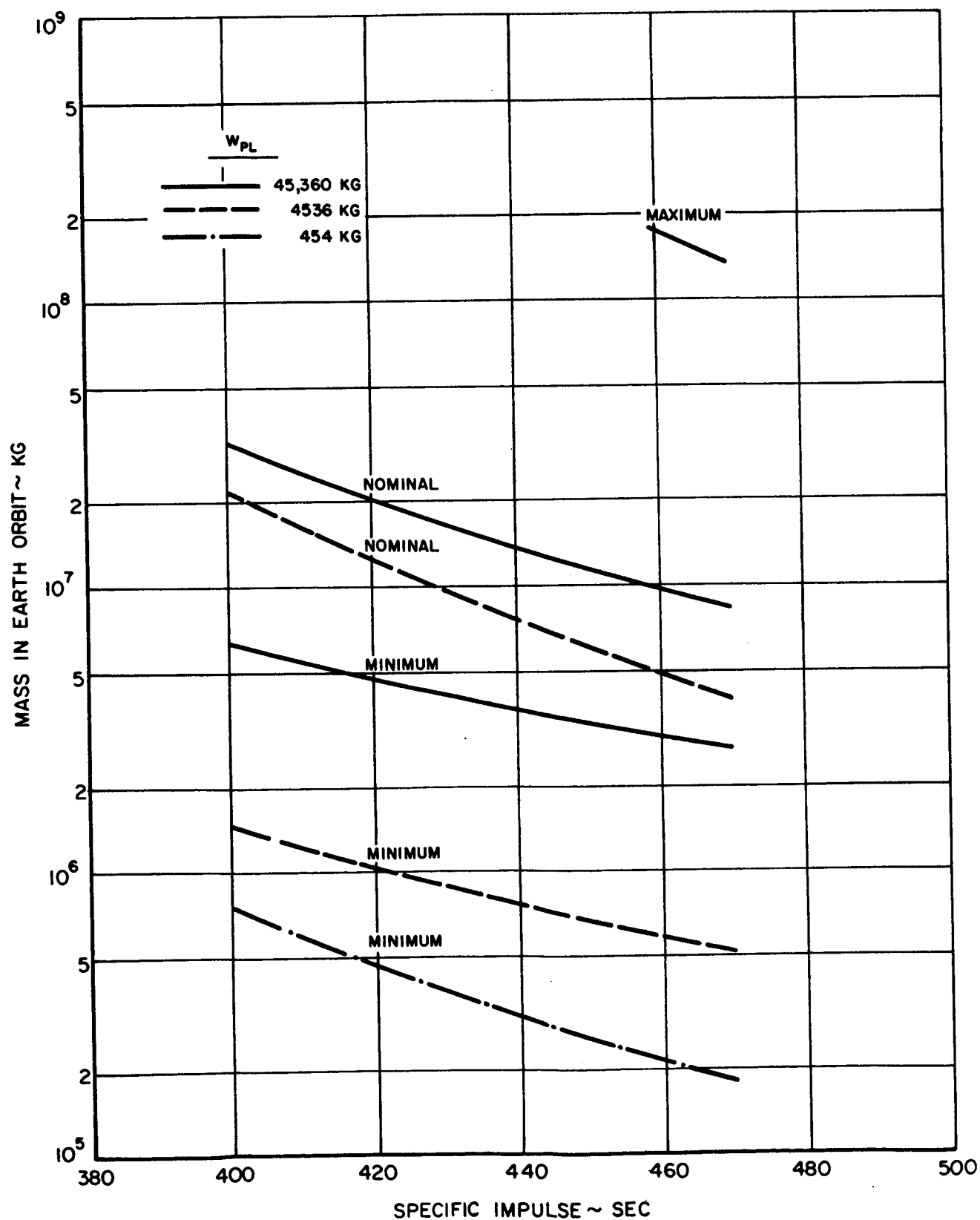
JUPITER ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR

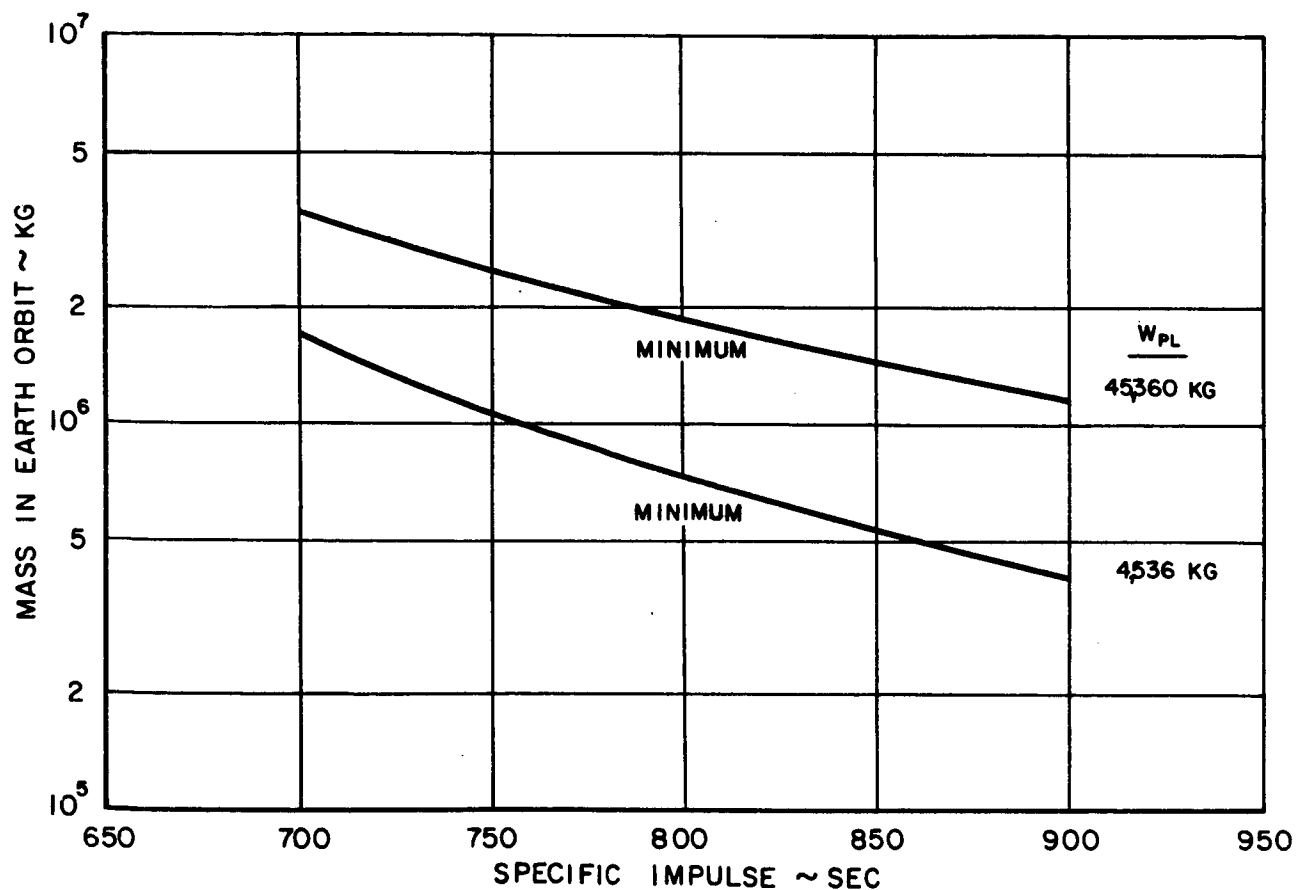


NEPTUNE ORBITAL STOPOVERS

CHEMICAL

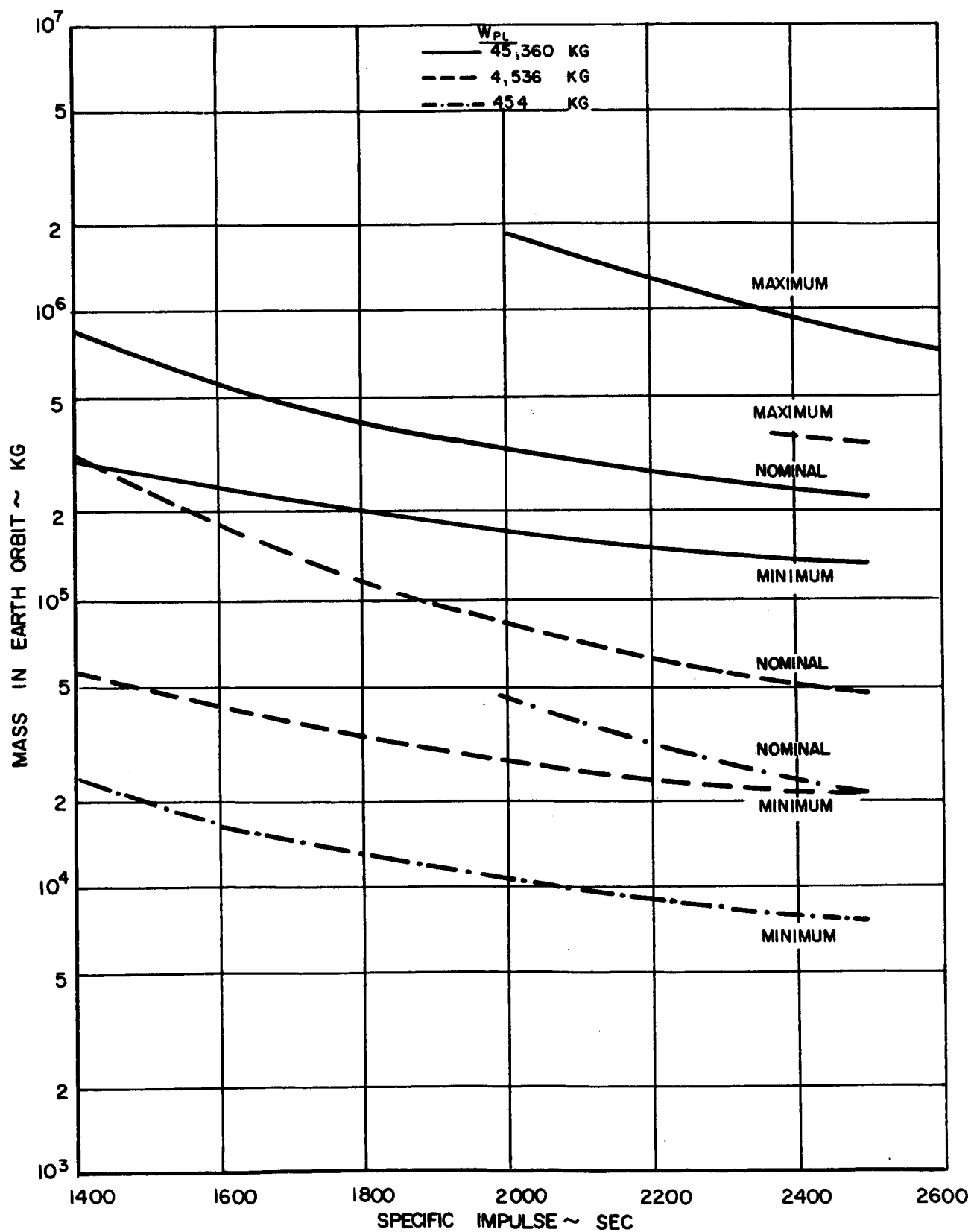


NEPTUNE ORBITAL STOPOVERS SOLID-CORE NUCLEAR



NEPTUNE ORBITAL STOPOVERS

LIGHTWEIGHT ADVANCED NUCLEAR

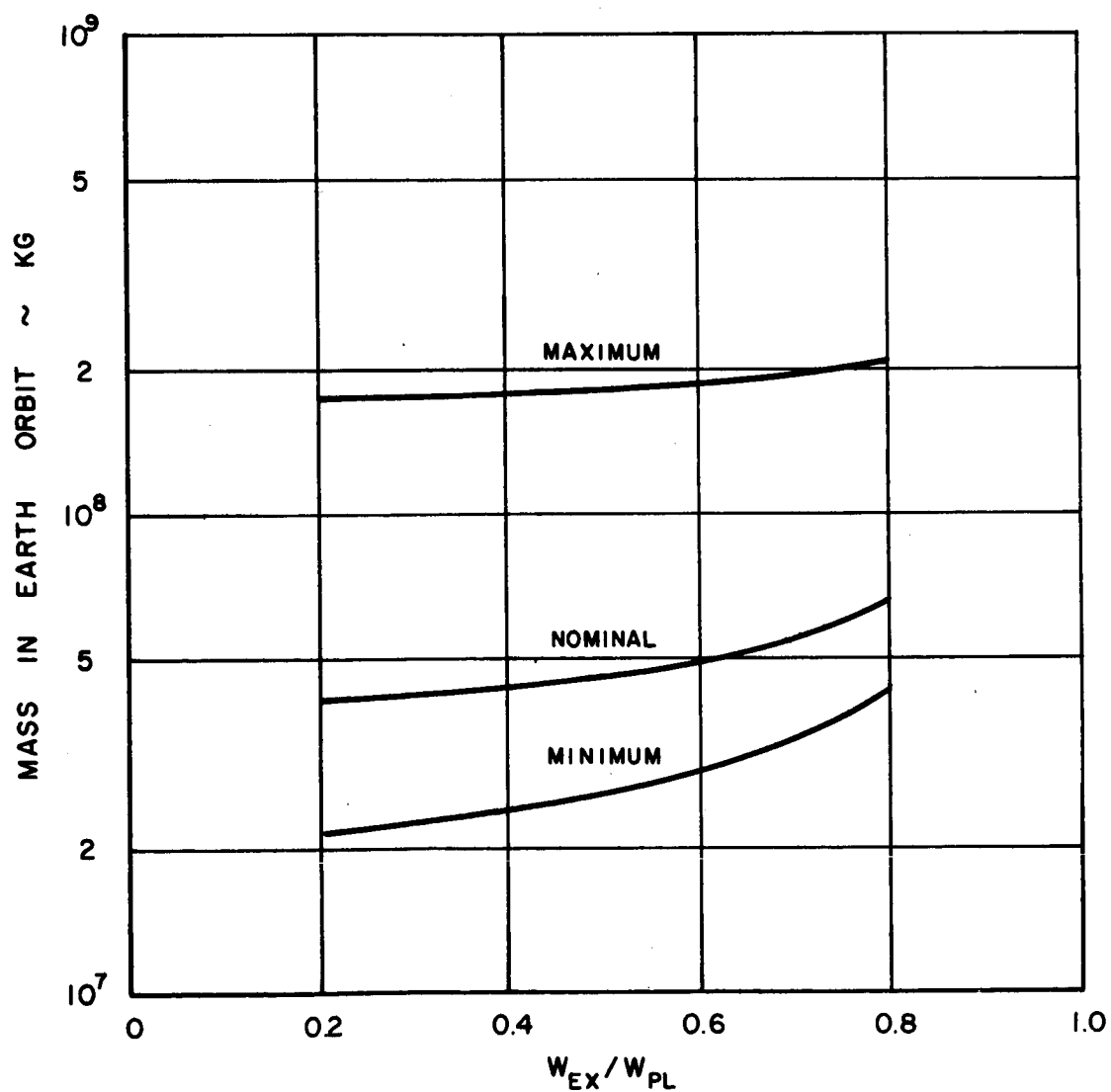


MERCURY MANNED LANDING MISSIONS

CHEMICAL

TRIP TIME = 370 DAYS

STAY TIME = 80 DAYS

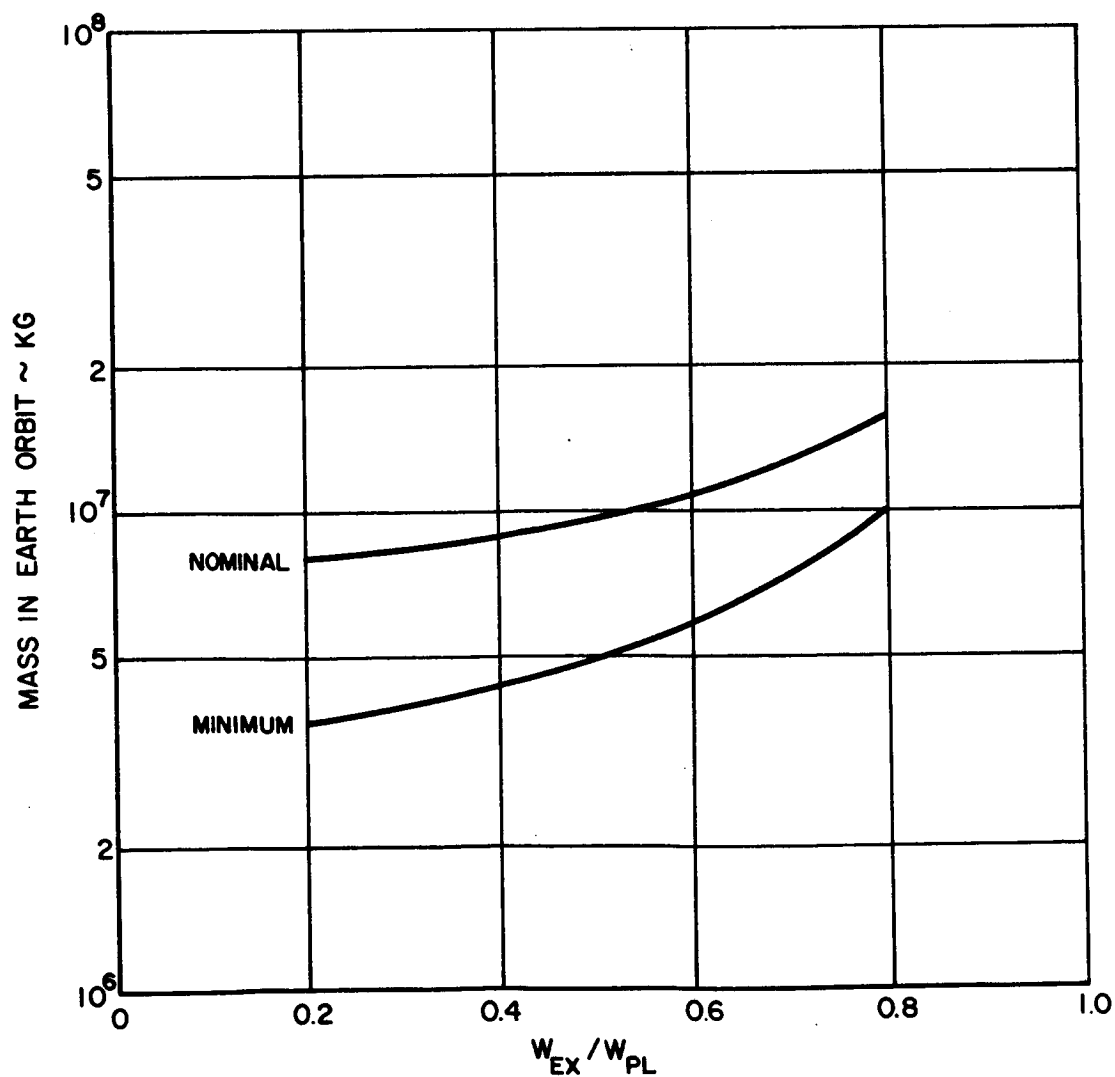
 $I_{SP} = 430$ SEC

MERCURY MANNED LANDING MISSION

SOLID-CORE NUCLEAR

TRIP TIME = 370 DAYS

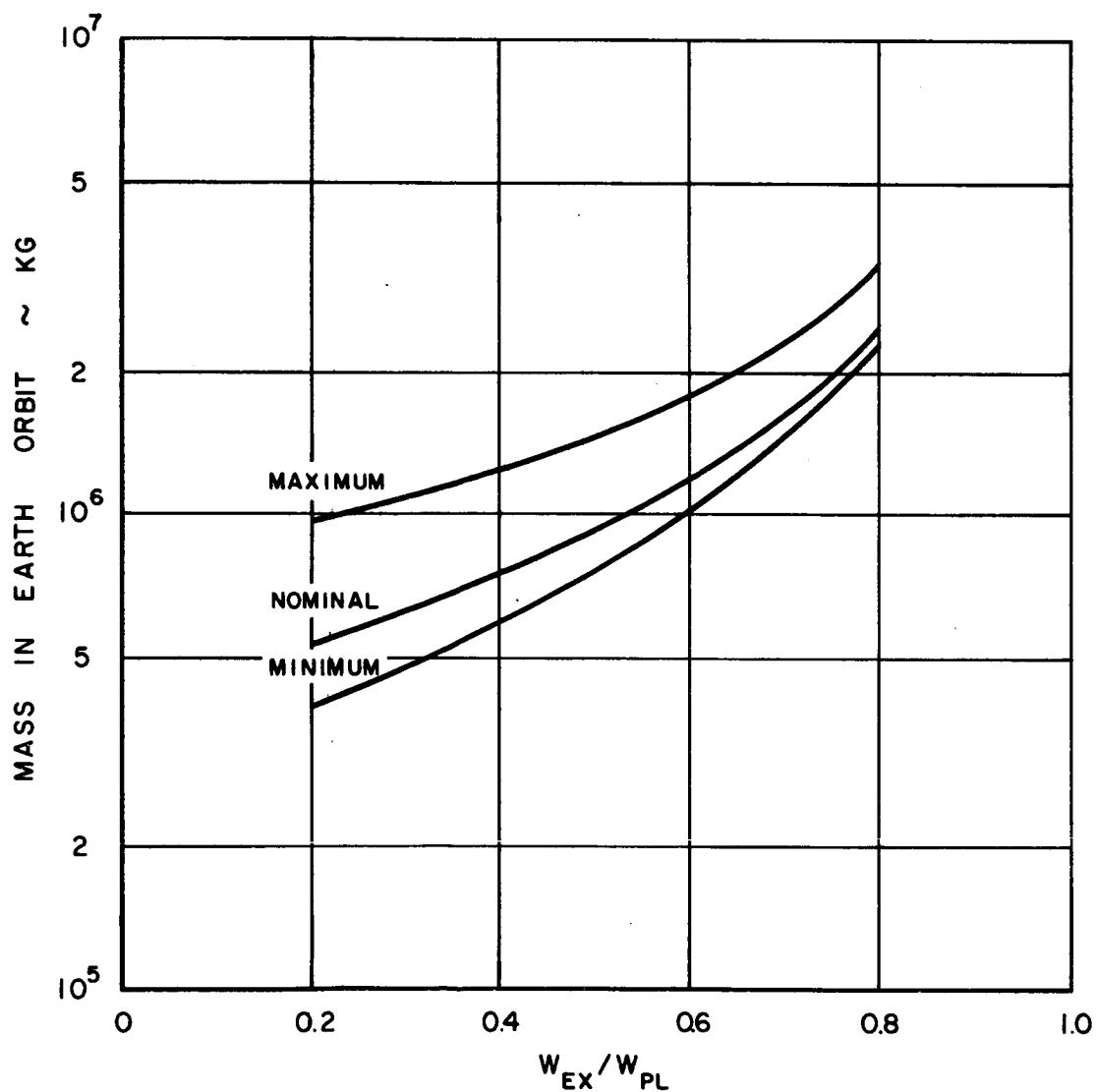
STAY TIME = 80 DAYS

 $I_{SP} = 800 \text{ SEC}$ 

MERCURY MANNED LANDING MISSIONS

LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 370 DAYS
STAY TIME = 80 DAYS
 $I_{SP} = 2000$ SEC

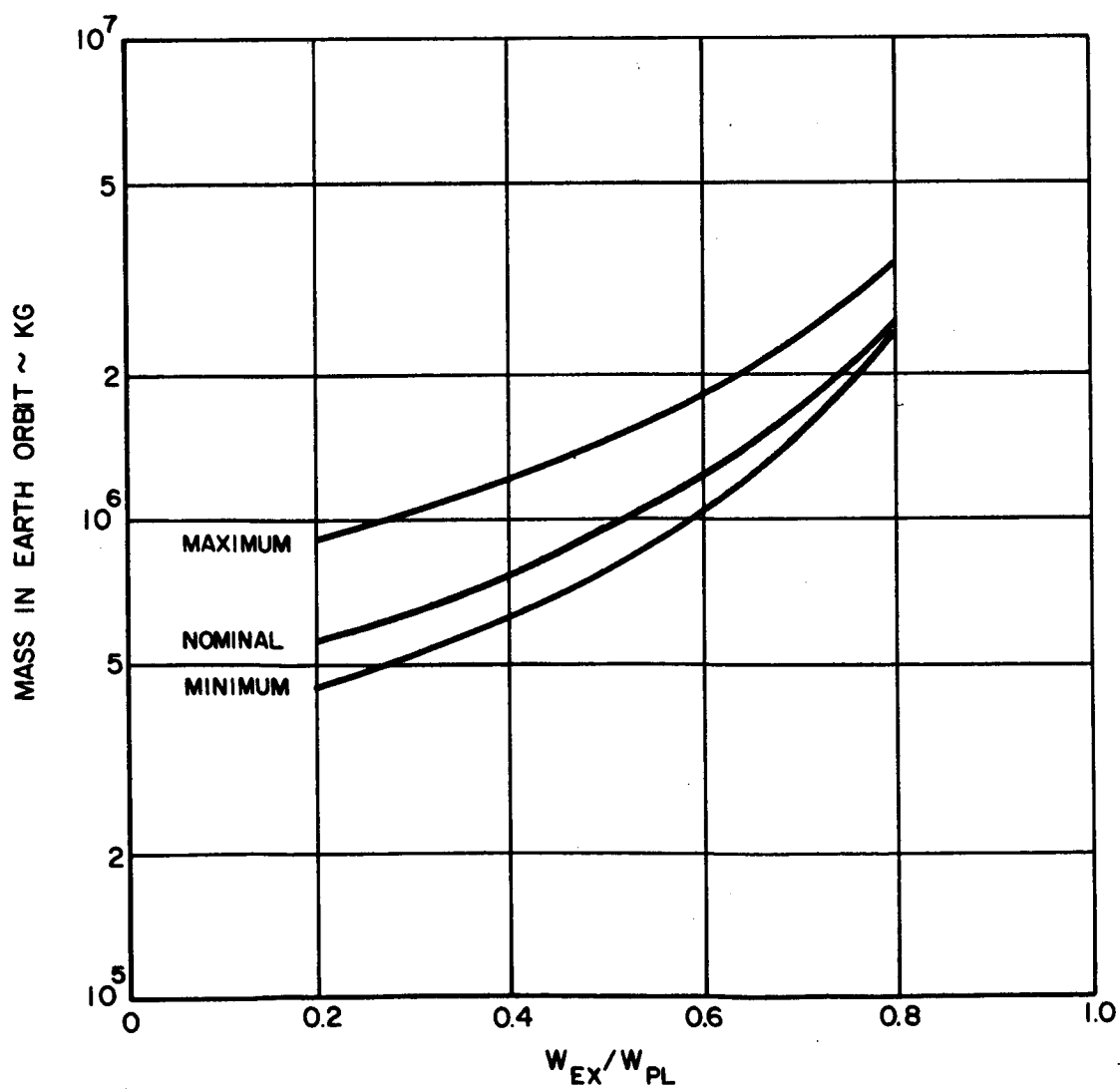


MERCURY MANNED LANDING MISSIONS

GASEOUS-CORE NUCLEAR

TRIP TIME = 370 DAYS

STAY TIME = 80 DAYS

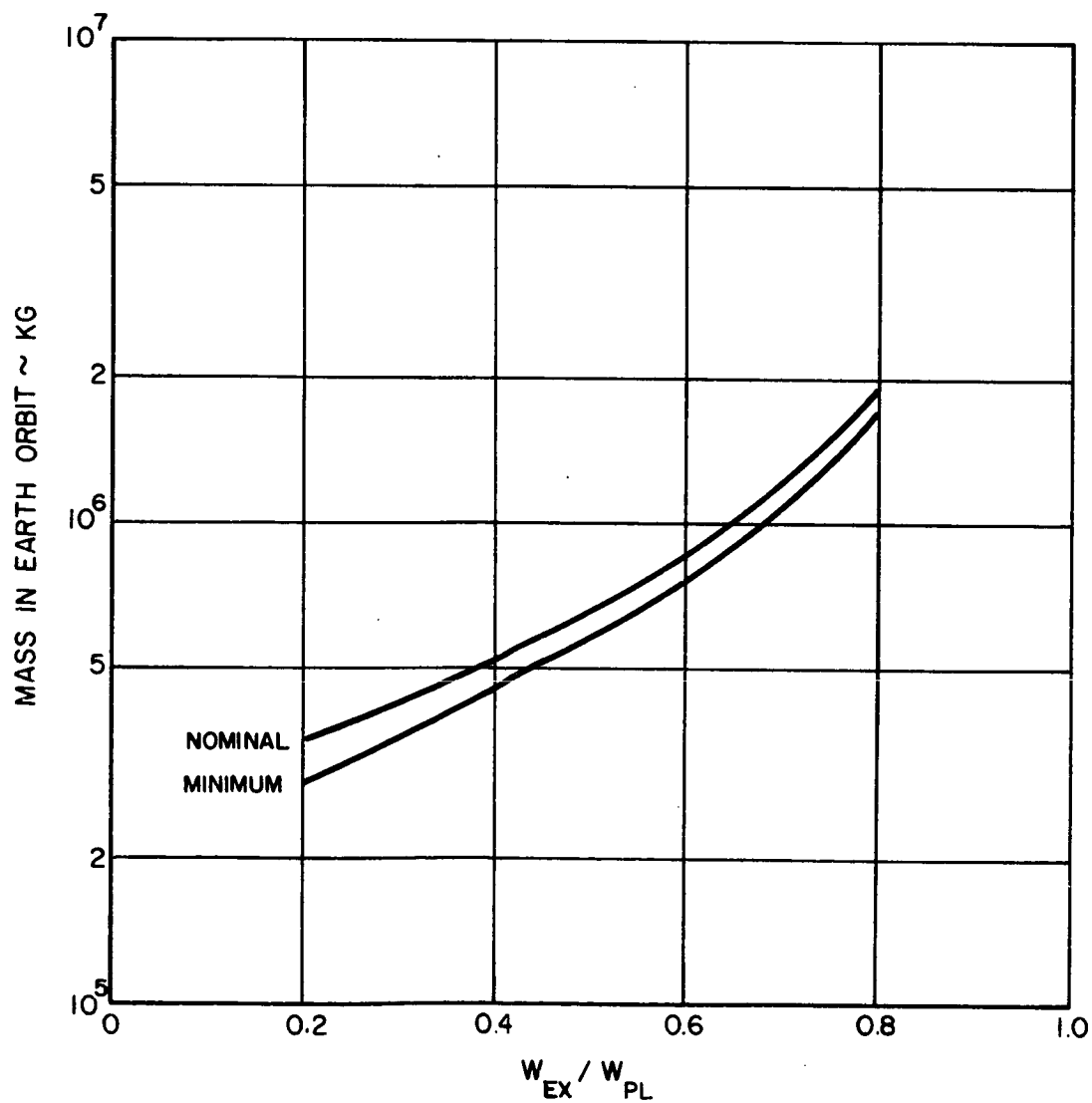
 $I_{SP} = 2000 \text{ SEC}$ 

MERCURY MANNED LANDING MISSIONS

GASEOUS-CORE NUCLEAR

TRIP TIME = 370 DAYS

STAY TIME = 80 DAYS

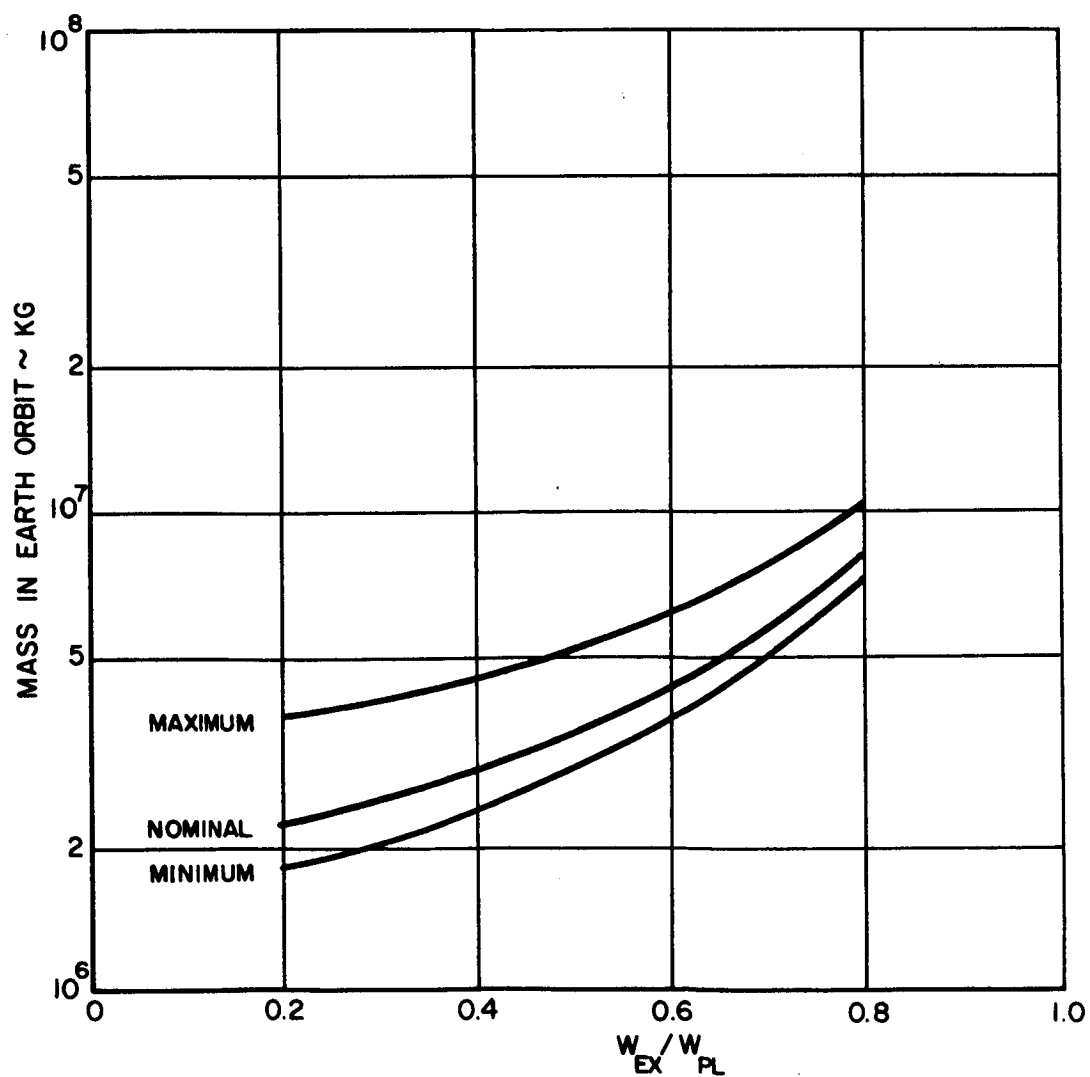
 $I_{sp} = 3500 \text{ SEC}$ 

MARS MANNED LANDING MISSIONS

CHEMICAL

TRIP TIME = 470 DAYS

STAY TIME = 0

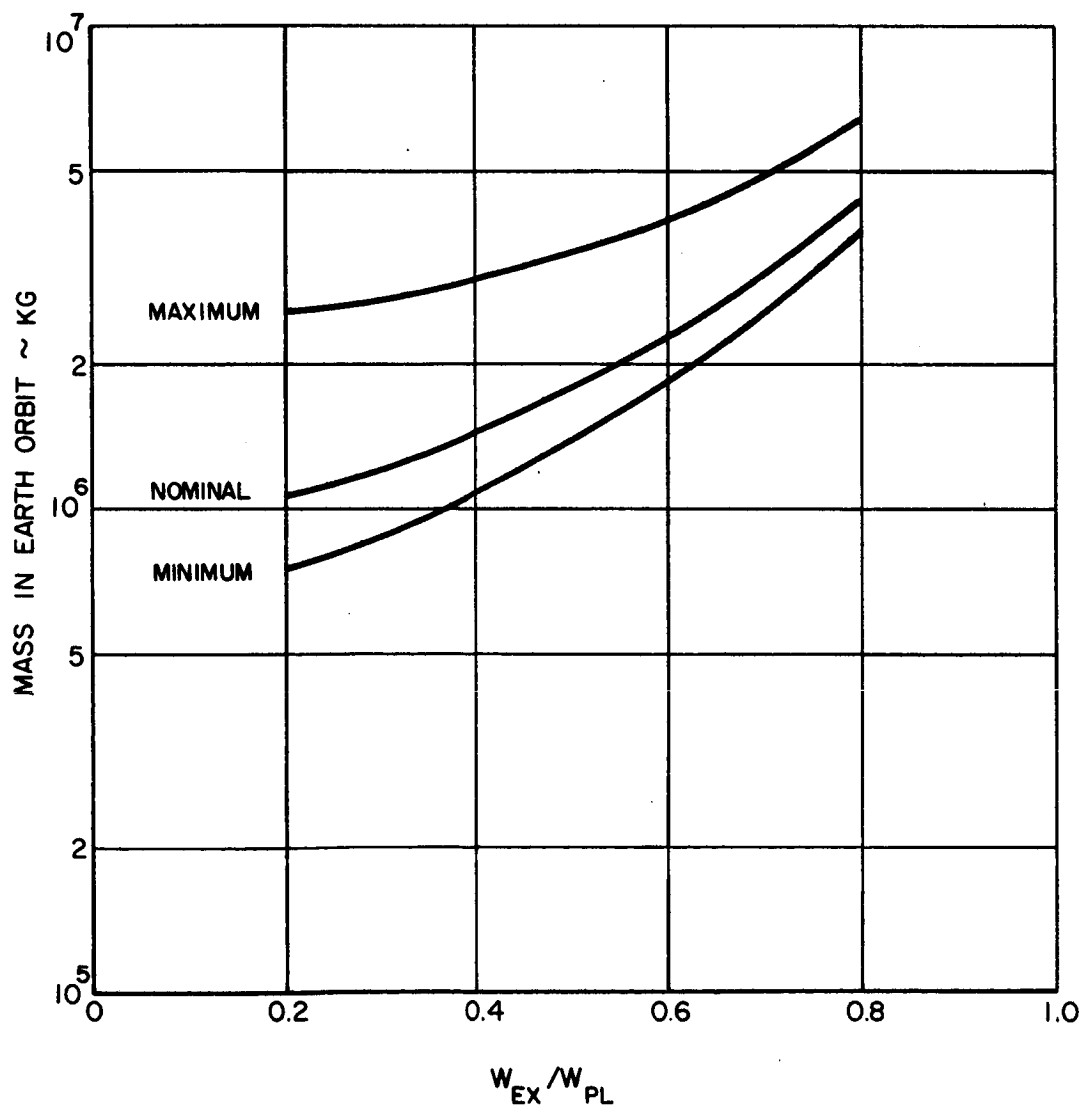
 $I_{sp} = 430$ SEC

MARS MANNED LANDING MISSION

SOLID-CORE NUCLEAR

TRIP TIME = 470 DAYS

STAY TIME = 0

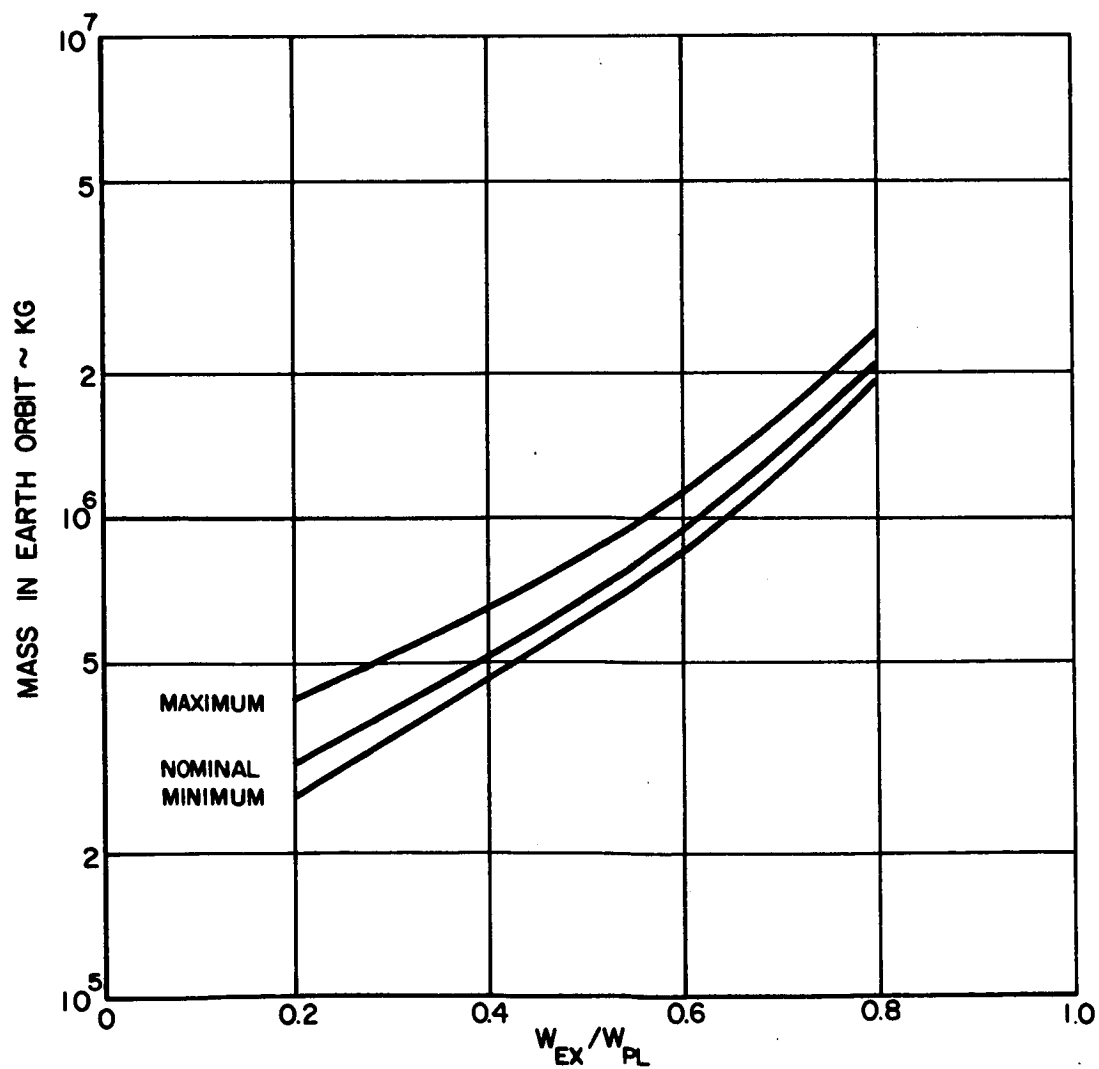
 $I_{SP} = 800 \text{ SEC}$ 

MARS MANNED LANDING MISSION
LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 470 DAYS

STAY TIME = 0

$I_{sp} = 2000$ SEC

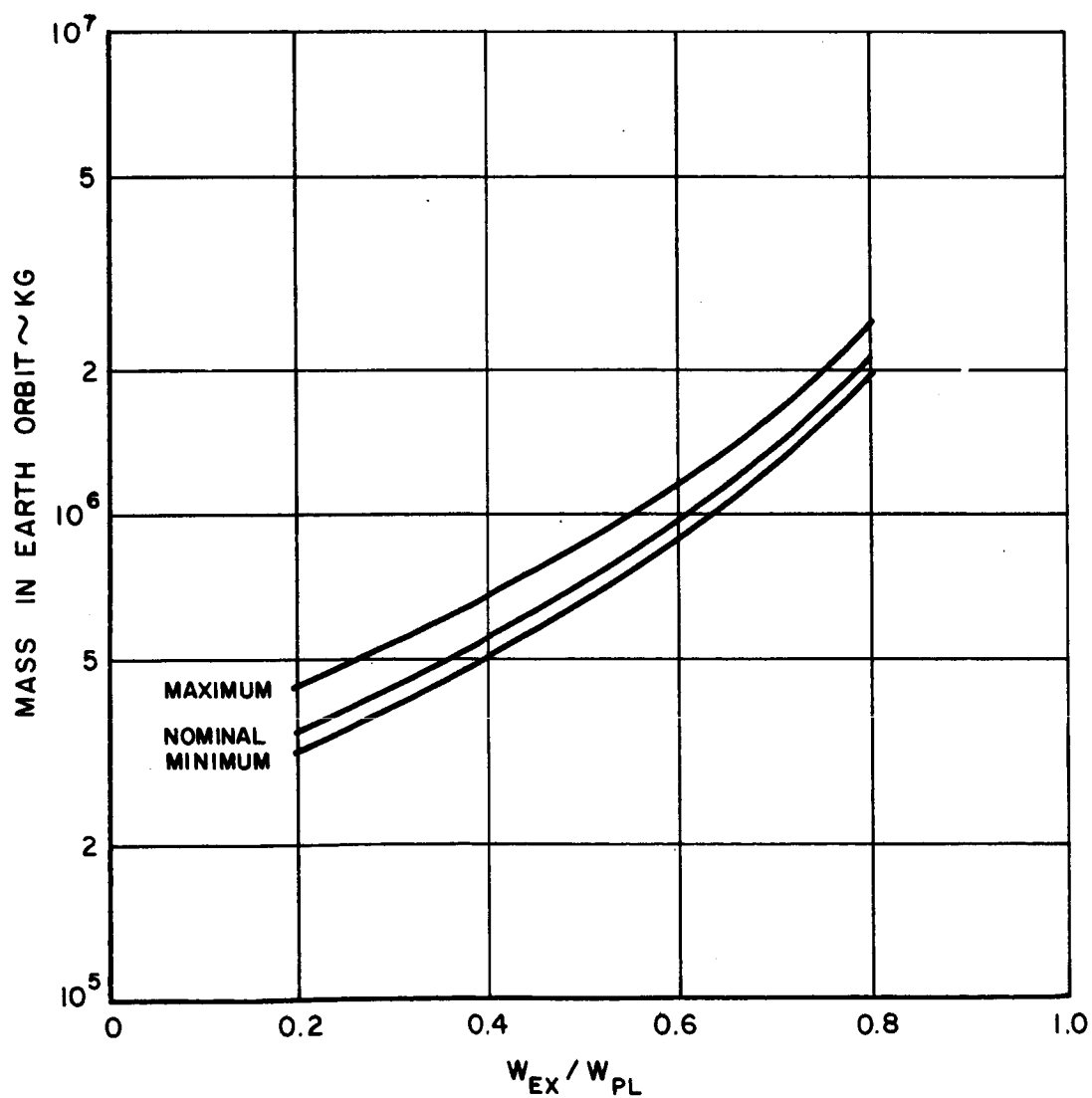


MARS MANNED LANDING MISSIONS

GASEOUS-CORE NUCLEAR

TRIP TIME = 470 DAYS

STAY TIME = 0

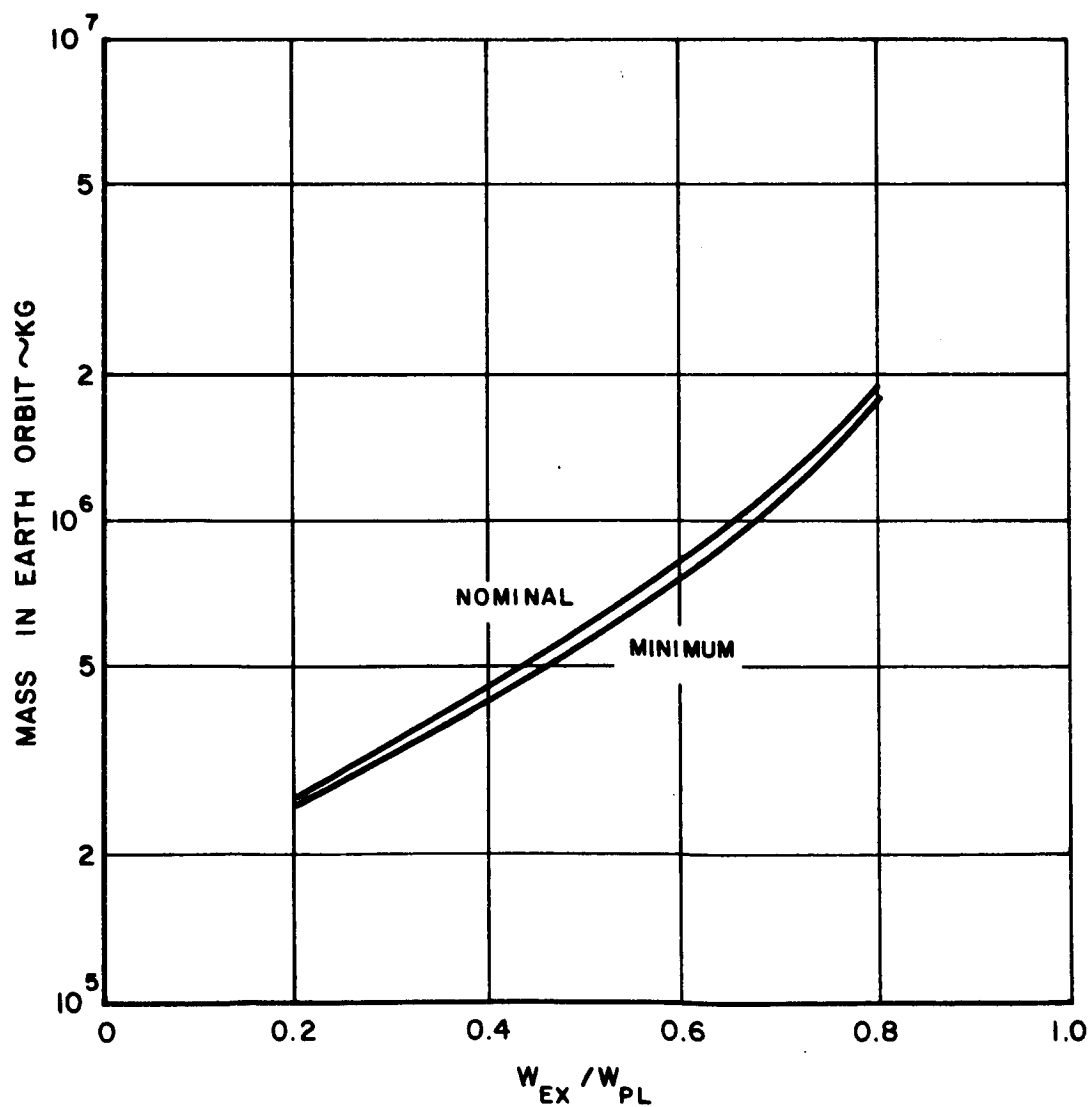
 $I_{SP} = 2000$ SEC

MARS MANNED LANDING MISSIONS

GASEOUS-CORE NUCLEAR

TRIP TIME = 470 DAYS

STAY TIME = 0

 $I_{sp} = 3500$ SEC

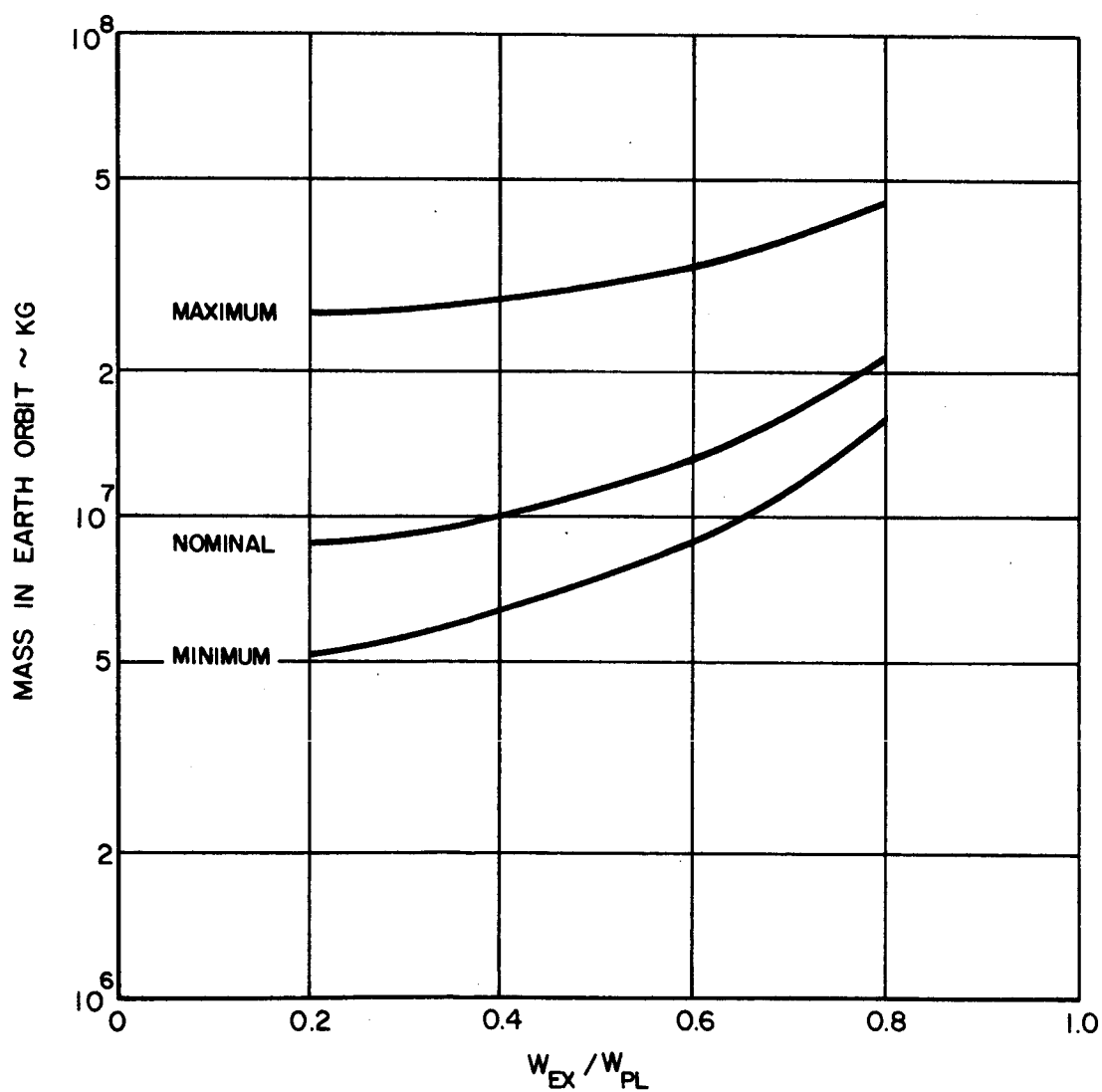
GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

CHEMICAL

TRIP TIME = 1800 DAYS

STAY TIME = 0

$I_{SP} = 430 \text{ SEC}$



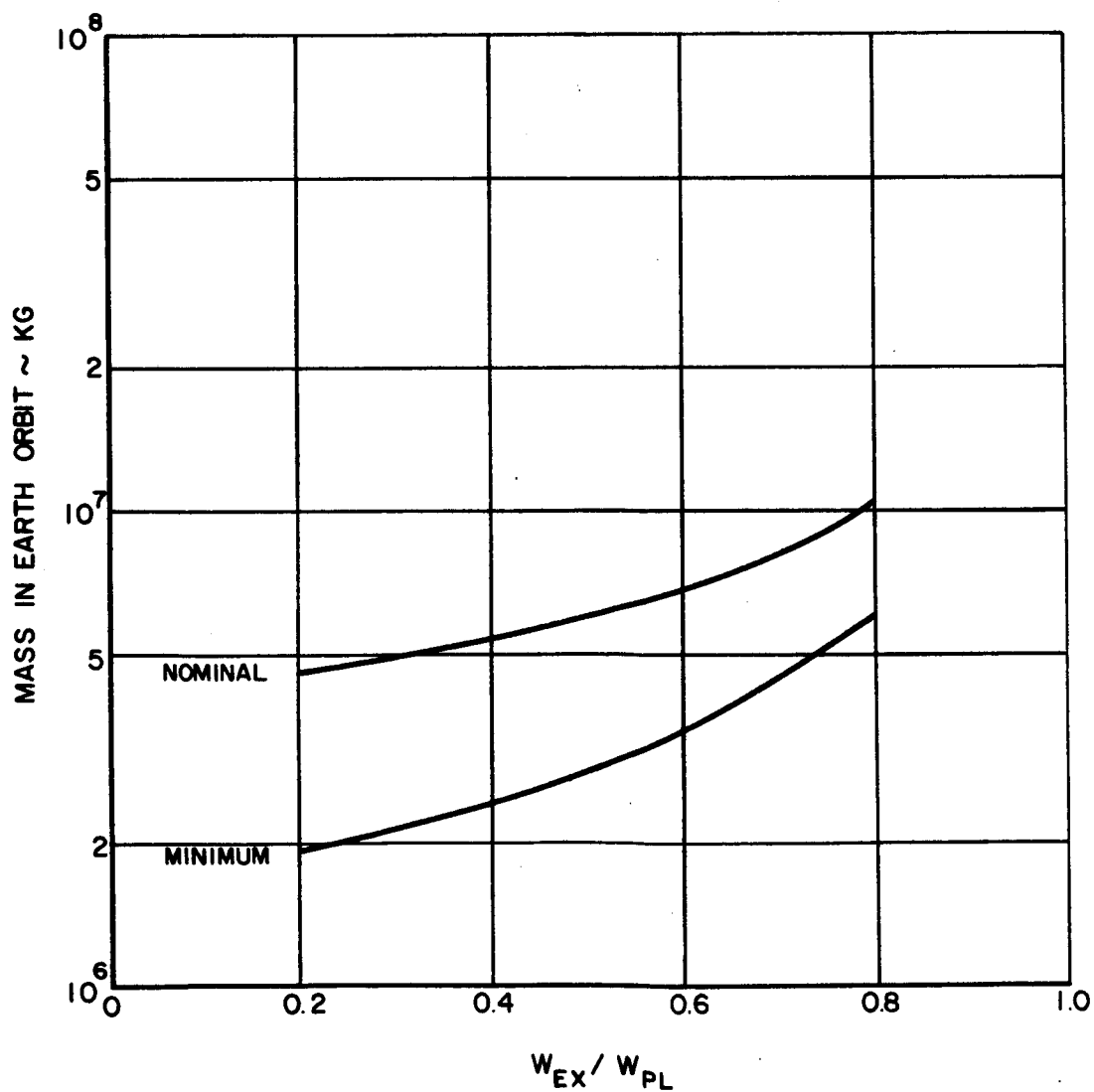
GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

SOLID-CORE NUCLEAR

TRIP TIME = 1800 DAYS

STAY TIME = 0

$I_{sp} = 800$ SEC



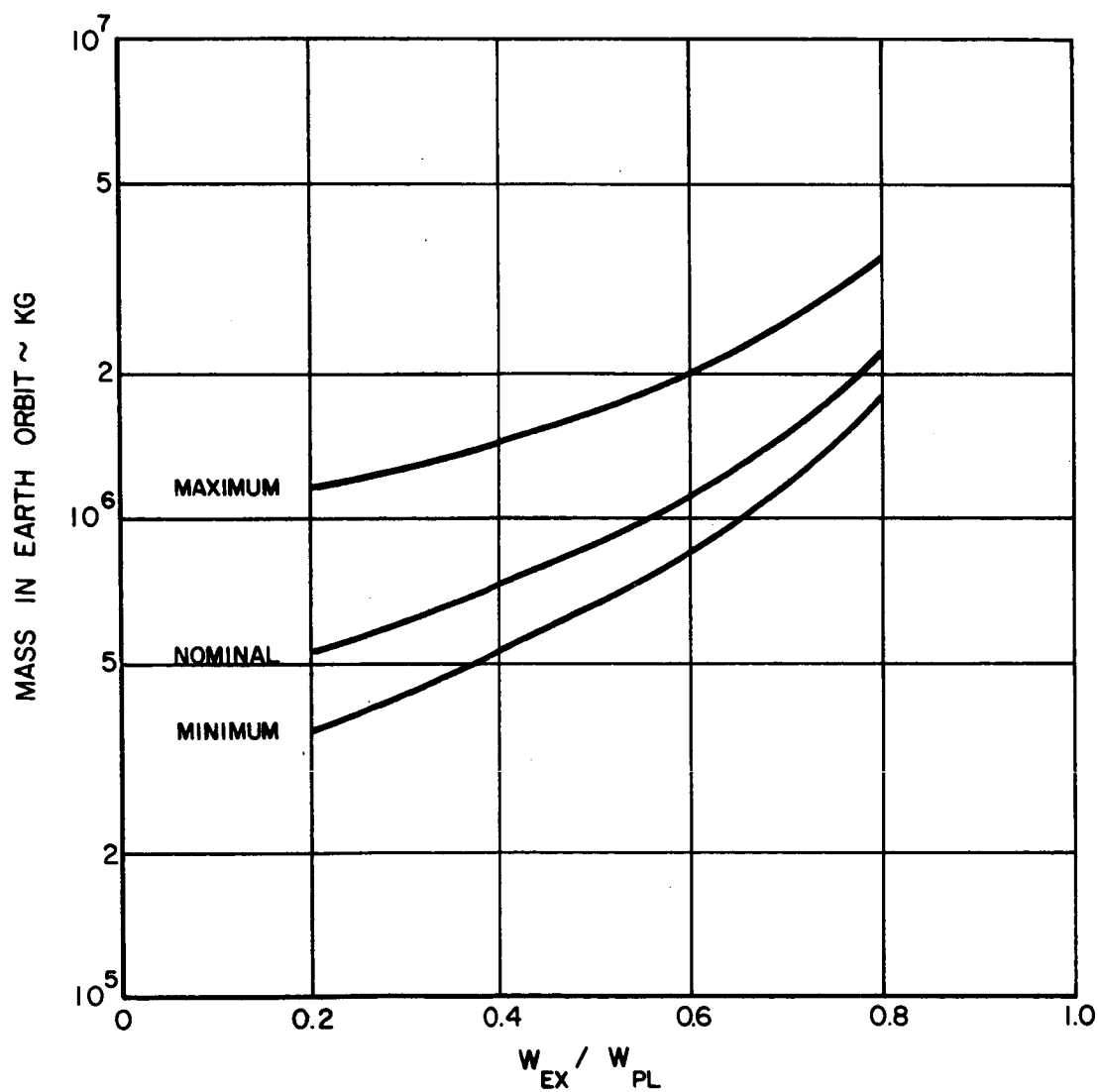
GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 1800 DAYS

STAY TIME = 0

$I_{SP} = 2000 \text{ SEC}$



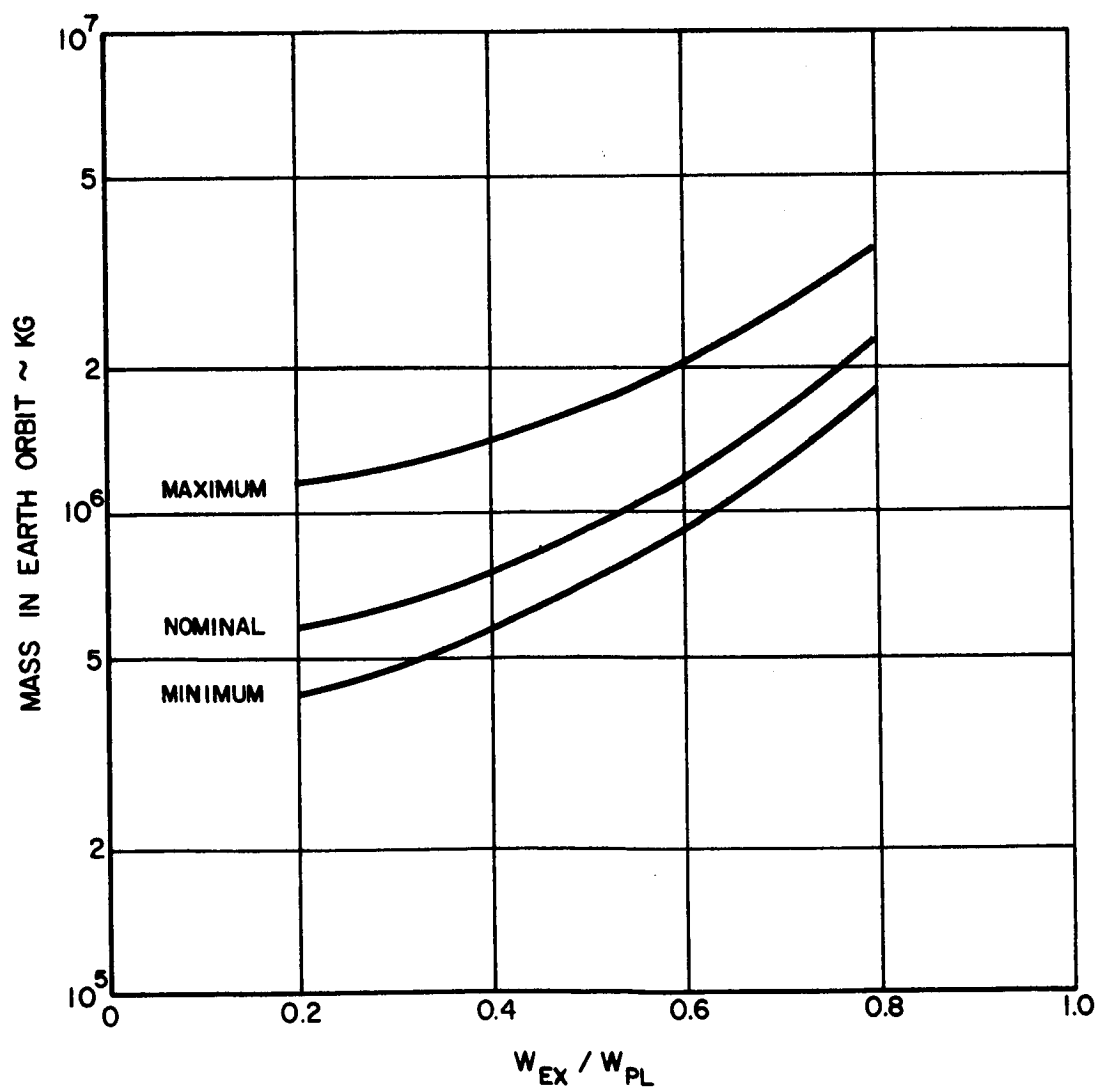
GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

GASEOUS-CORE NUCLEAR

TRIP TIME = 1800 DAYS

STAY TIME = 0

$I_{SP} = 2000 \text{ SEC}$



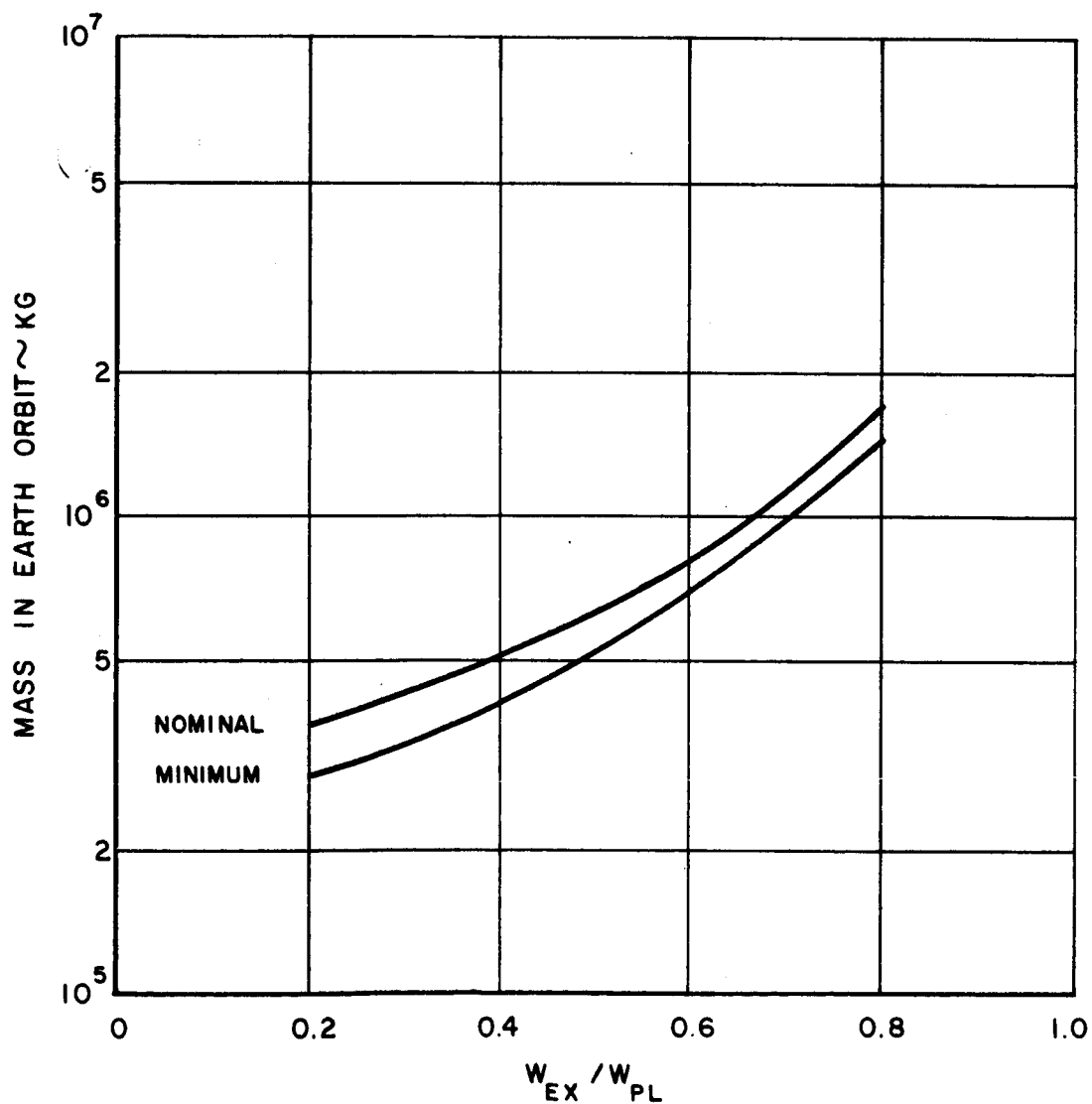
GANYMEDE MANNED LANDING MISSIONS (JUPITER III)

GASEOUS - CORE NUCLEAR

TRIP TIME = 1800 DAYS

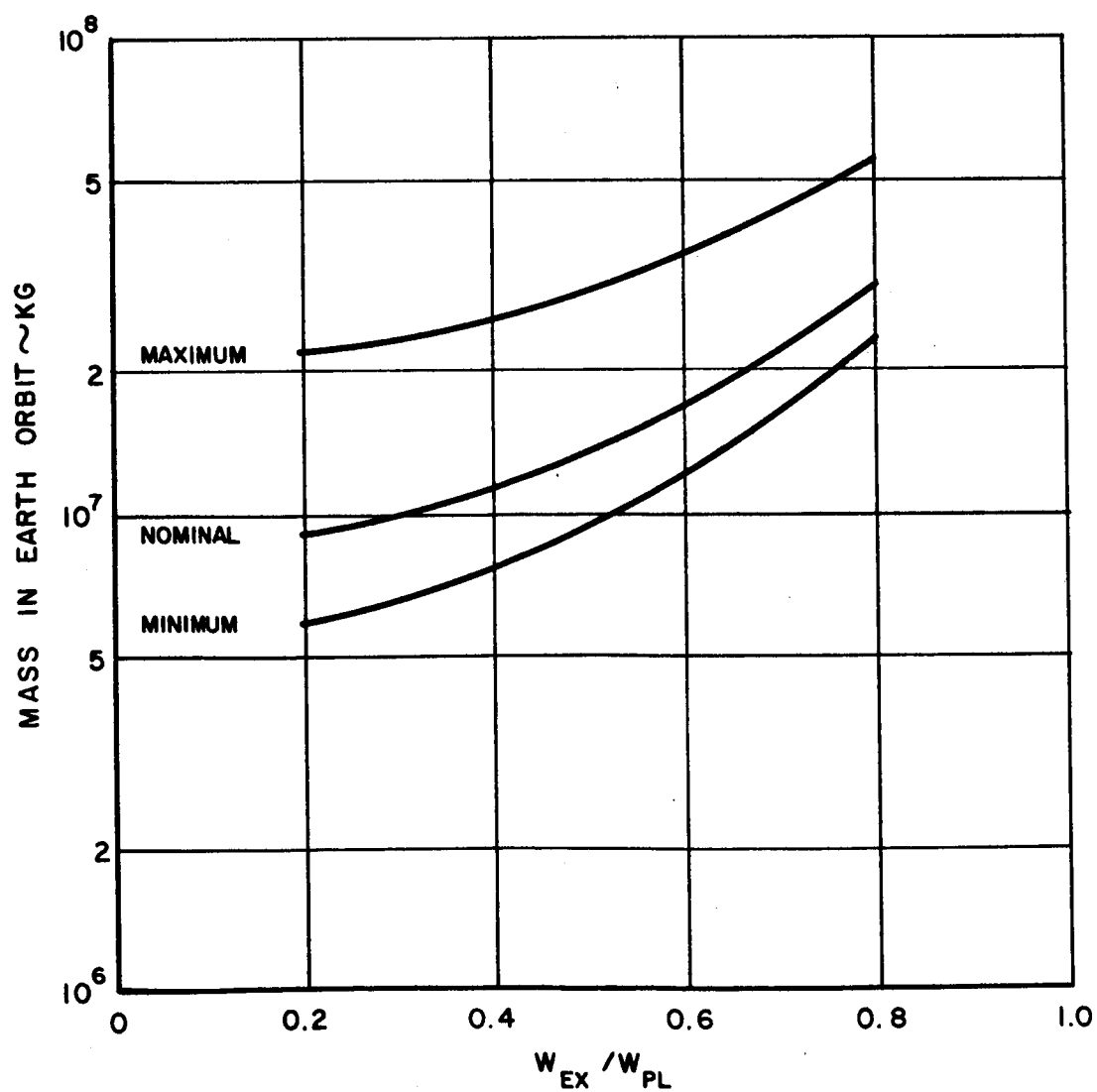
STAY TIME = 0

$I_{sp} = 3500$ SEC



TITAN MANNED LANDING MISSIONS
(SATURN VI)
CHEMICAL

TRIP TIME = 4000
STAY TIME = 0
 $I_{sp} = 430$ SEC

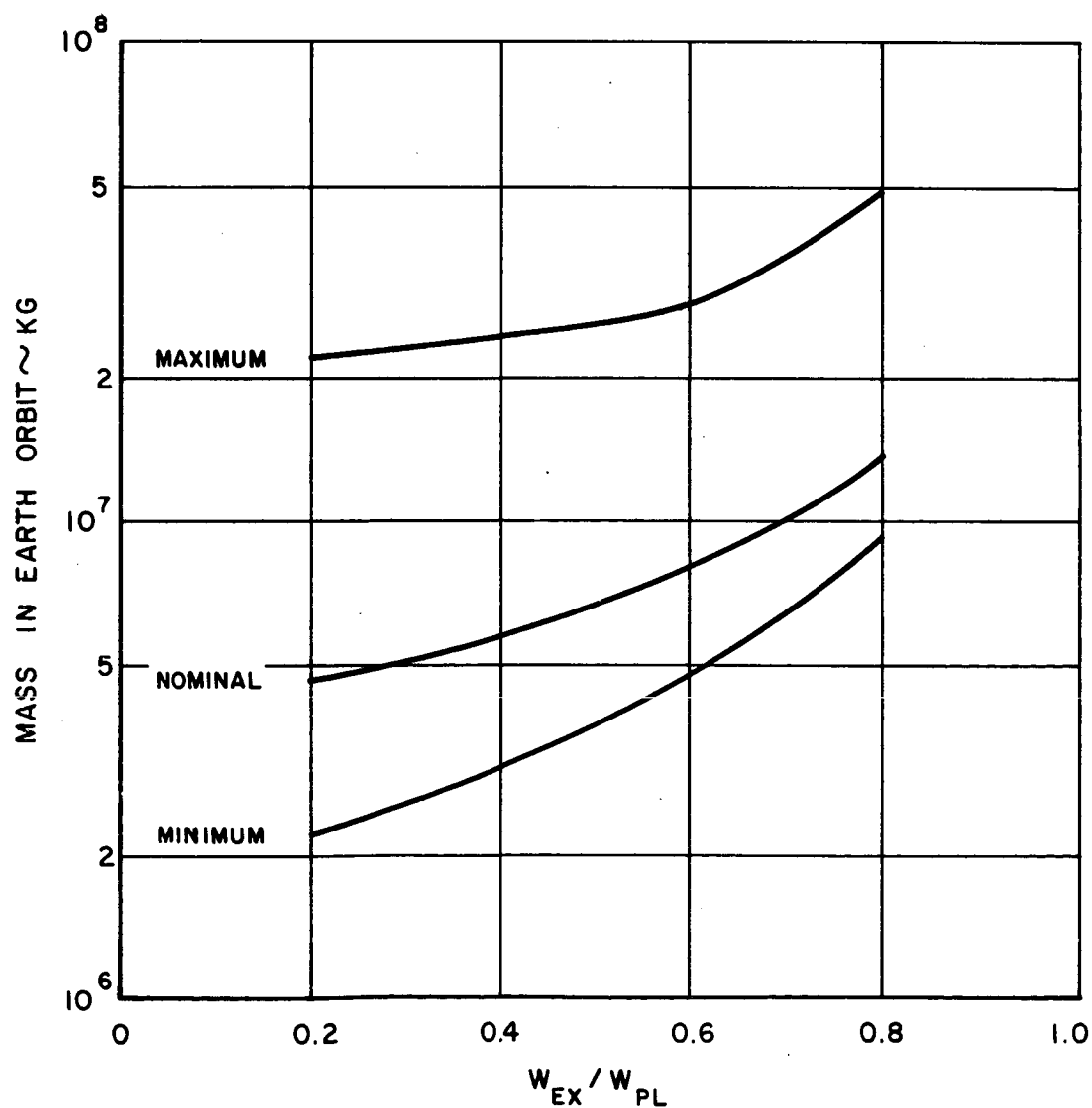


TITAN MANNED LANDING MISSIONS
(SATURN VI)
SOLID - CORE NUCLEAR

TRIP TIME = 4000 DAYS

STAY TIME = 0

$I_{SP} = 800 \text{ SEC}$



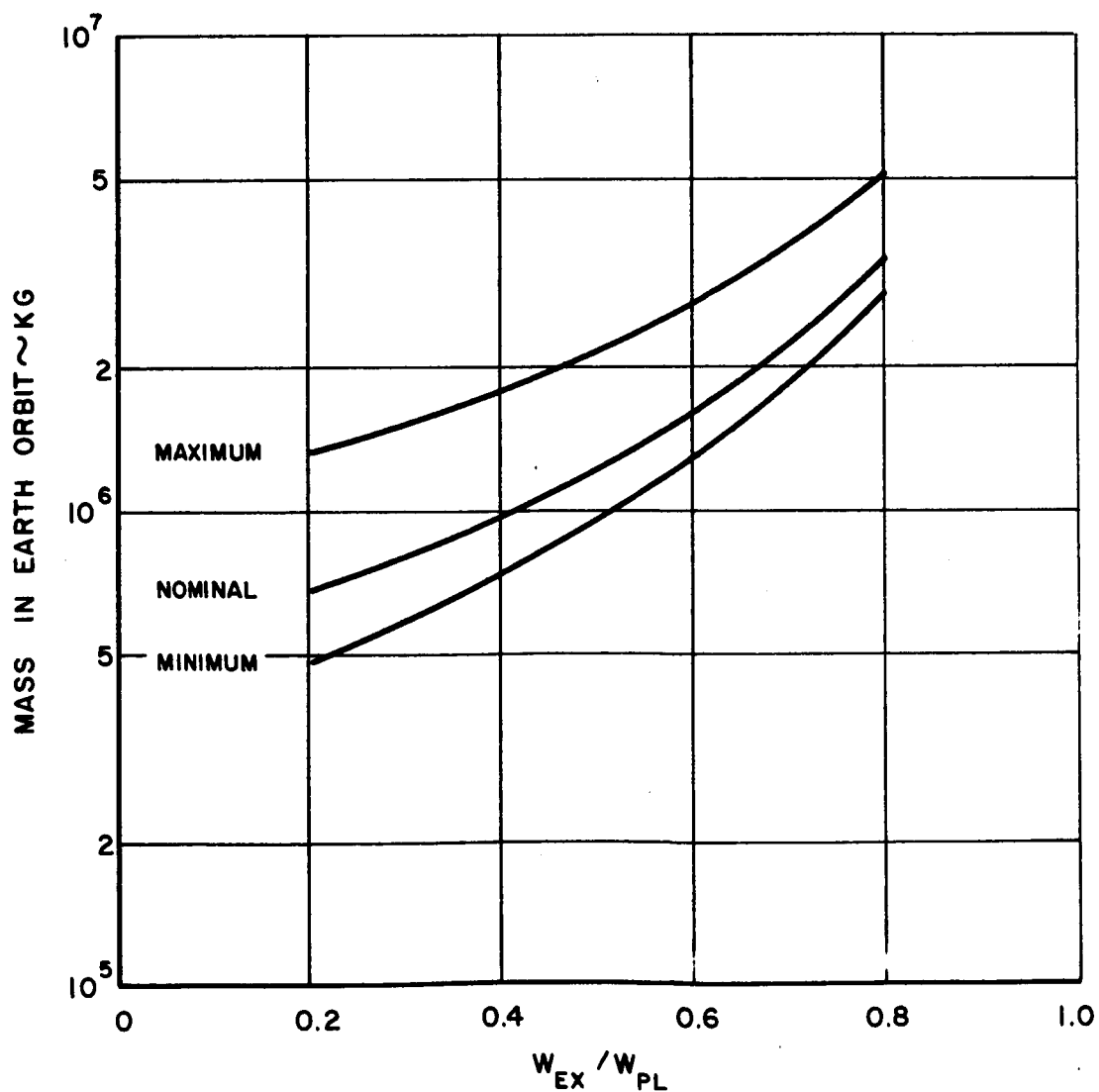
TITAN MANNED LANDING MISSIONS (SATURN VI)

LIGHTWEIGHT ADVANCED NUCLEAR

TRIP TIME = 4000 DAYS

STAY TIME = 0

$I_{SP} = 2000$ SEC



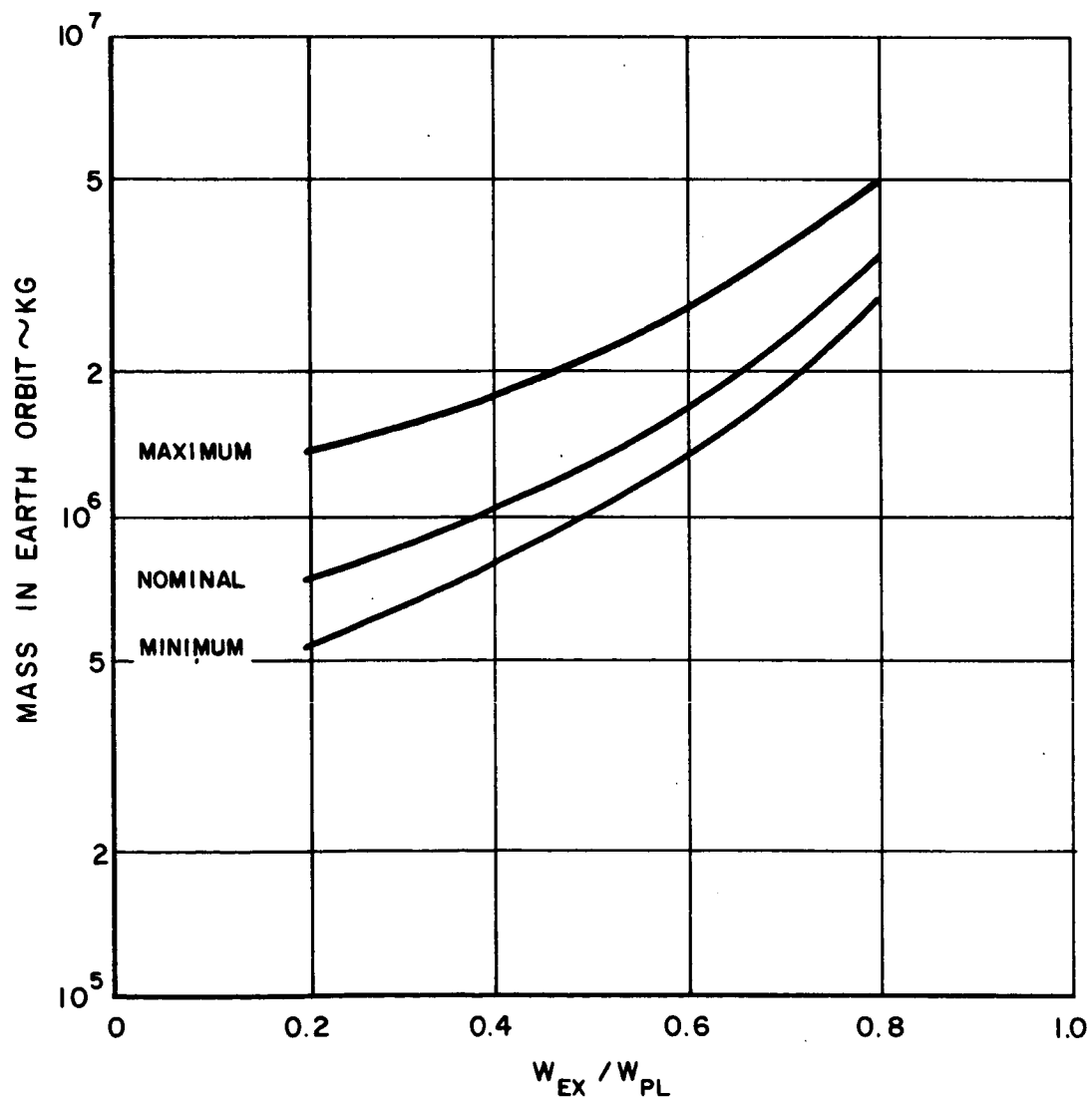
TITAN MANNED LANDING MISSIONS (SATURN VI)

GASEOUS-CORE NUCLEAR

TRIP TIME = 4000

STAY TIME = 0

$I_{sp} = 2000$ SEC



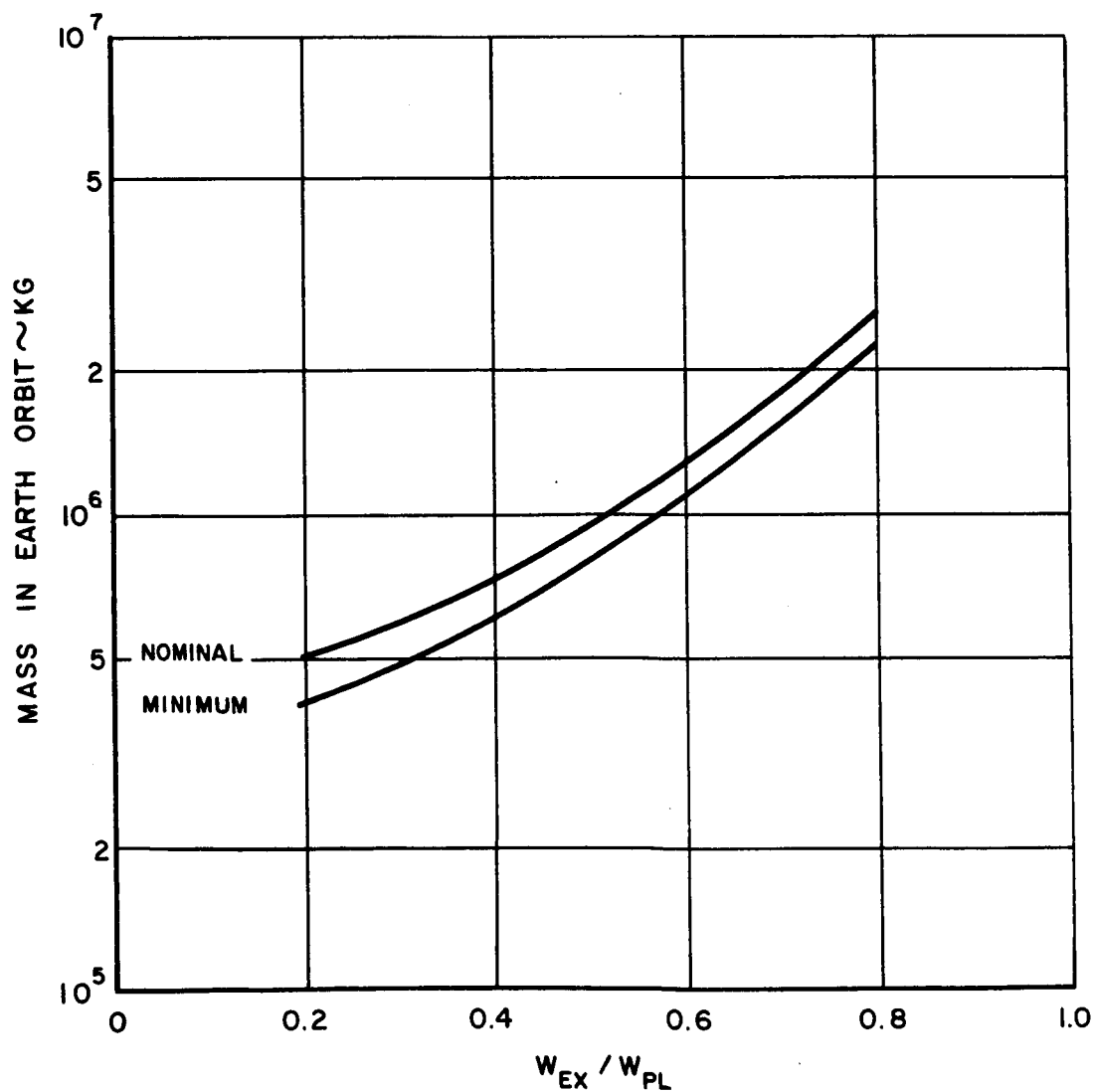
TITAN MANNED LANDING MISSIONS (SATURN VI)

GASEOUS-CORE NUCLEAR

TRIP TIME = 4000

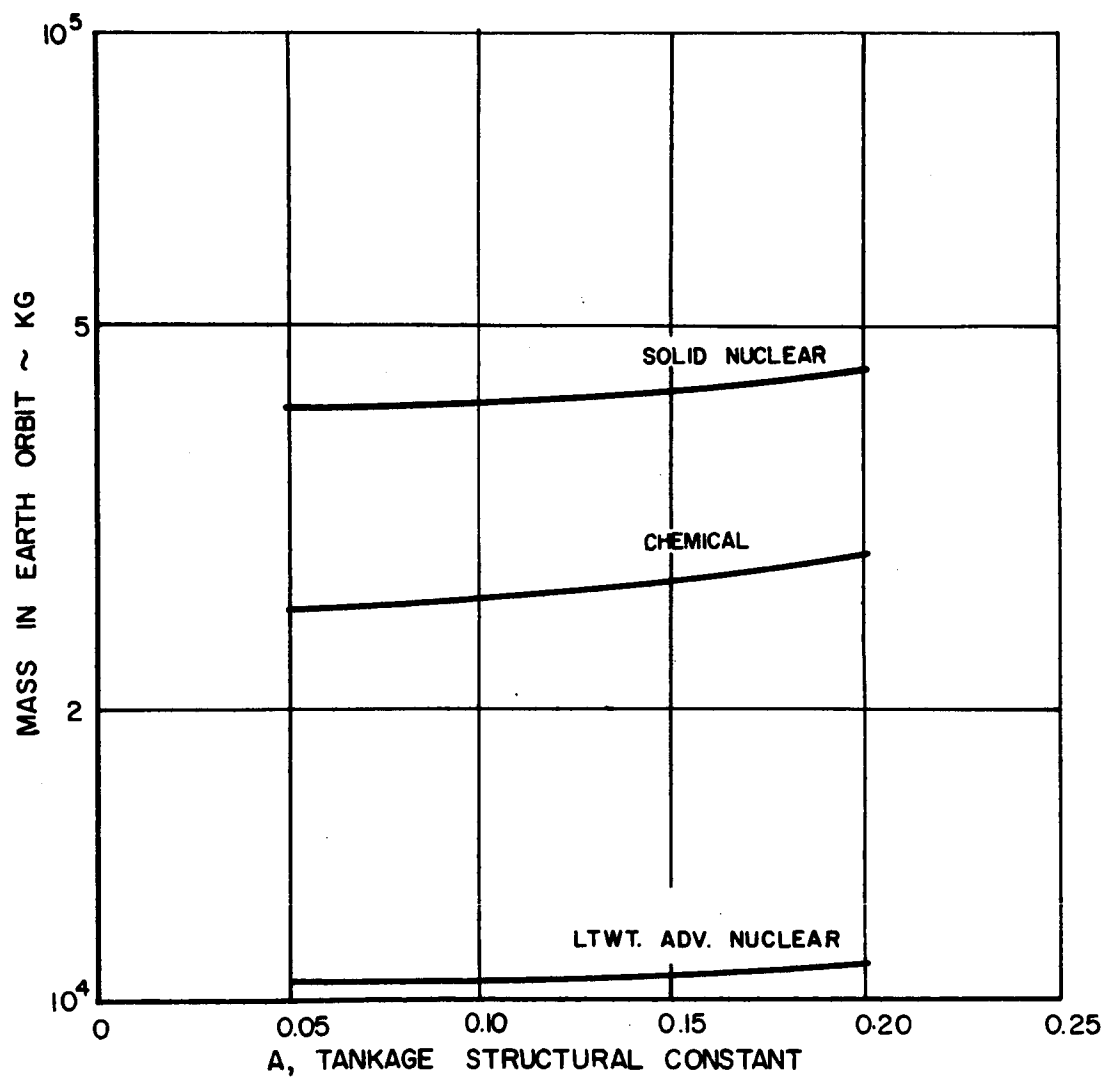
STAY TIME = 0

$I_{SP} = 3500$ SEC



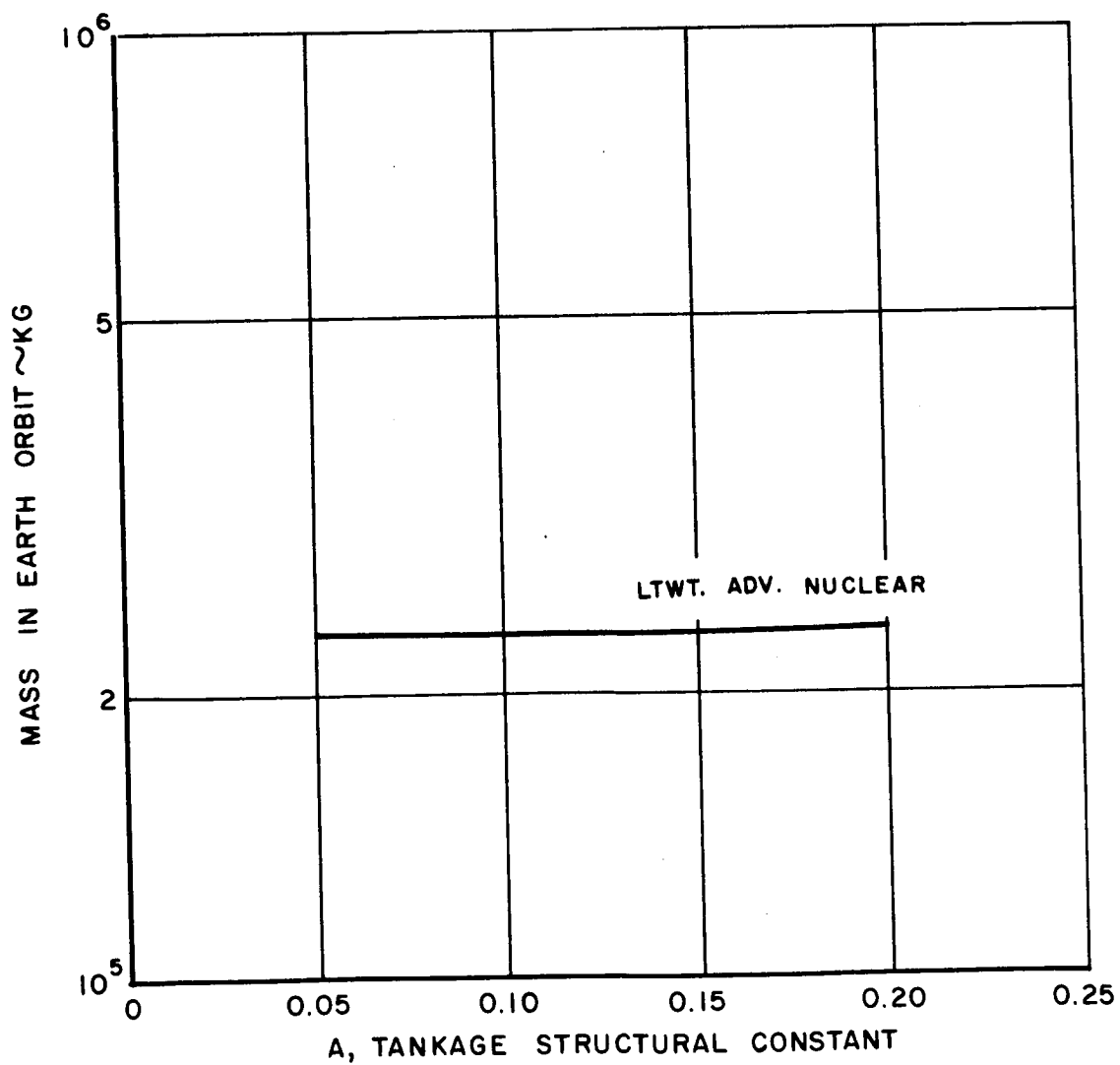
MARS ORBITAL STOPOVERS

$$W_{PL} = 4536 \text{ KG}$$



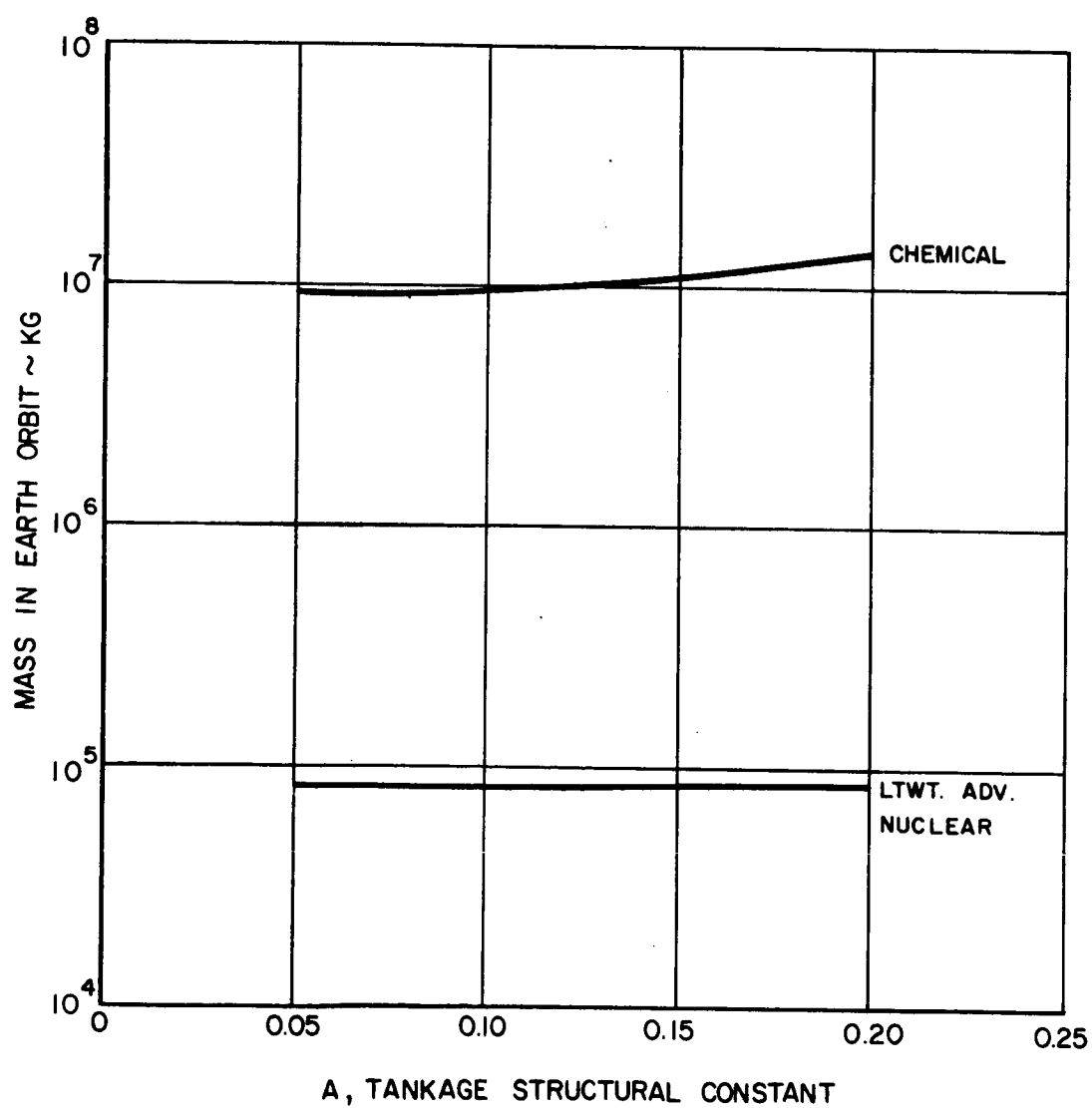
JUPITER ORBITAL STOPOVERS

$$W_{PL} = 4536 \text{ KG}$$



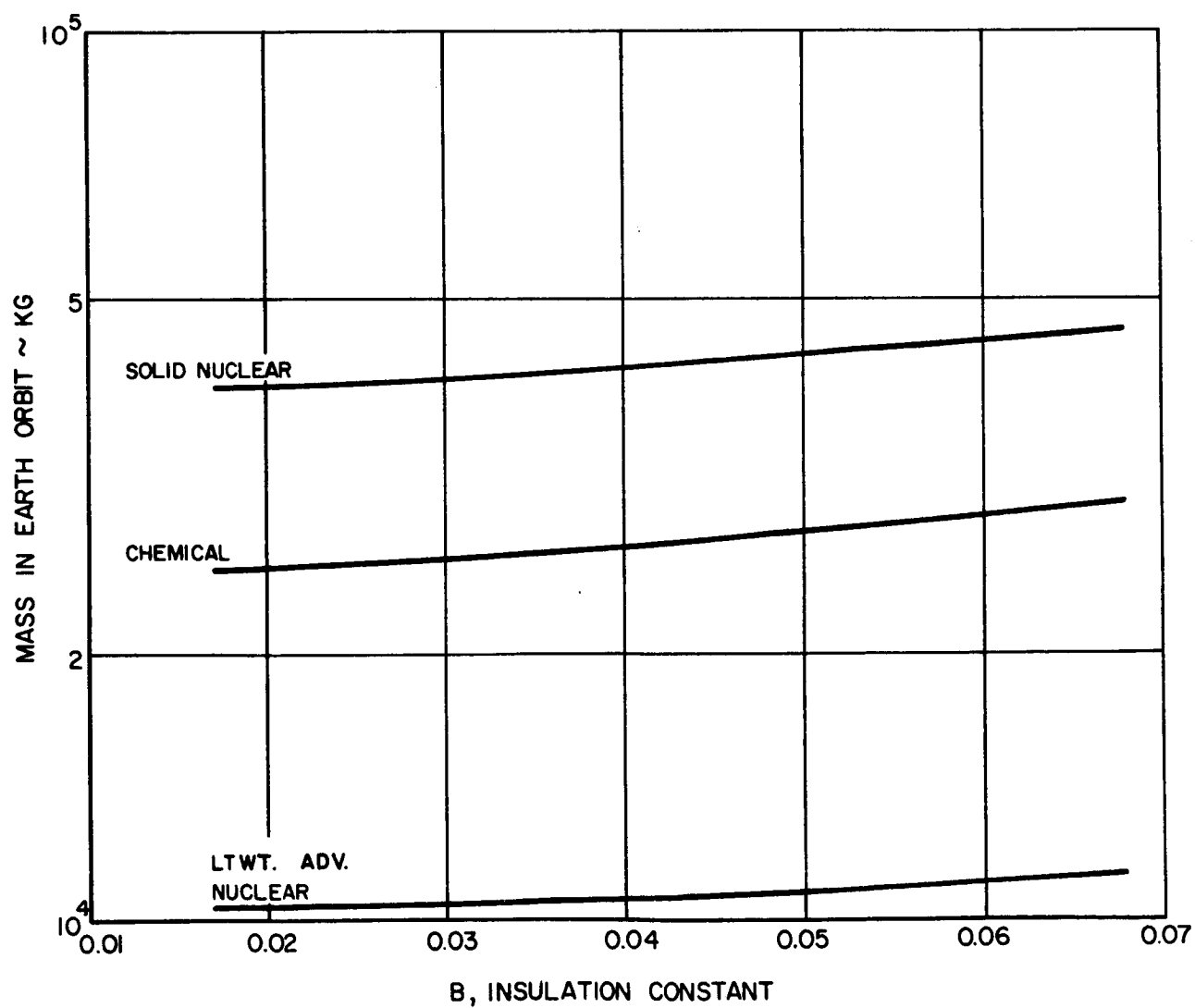
NEPTUNE ORBITAL STOPOVERS

$$W_{PL} = 4536 \text{ KG}$$



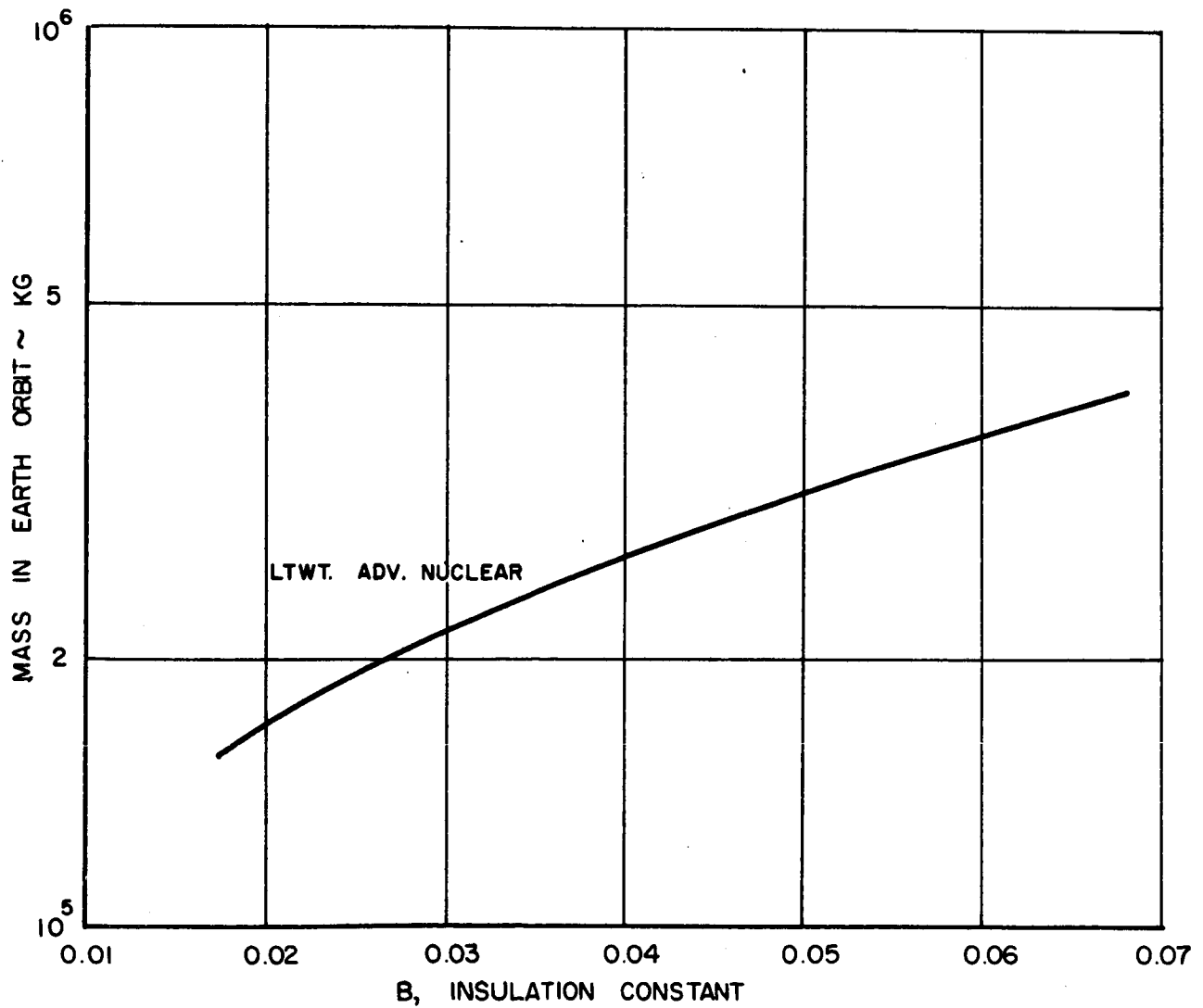
MARS ORBITAL STOPOVERS

$$W_{PL} = 4536 \text{ KG}$$



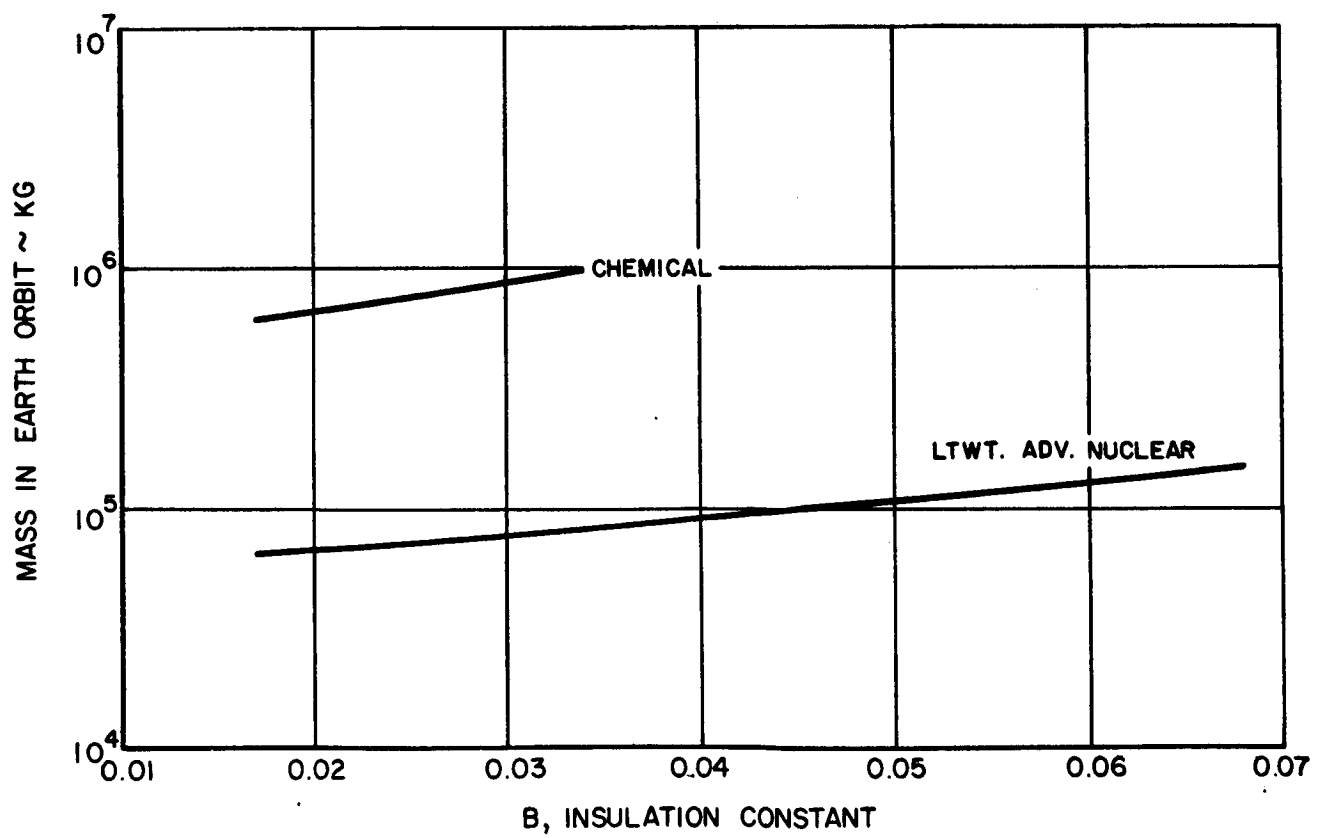
JUPITER ORBITAL STOPOVERS

$$W_{PL} = 4536 \text{ KG}$$



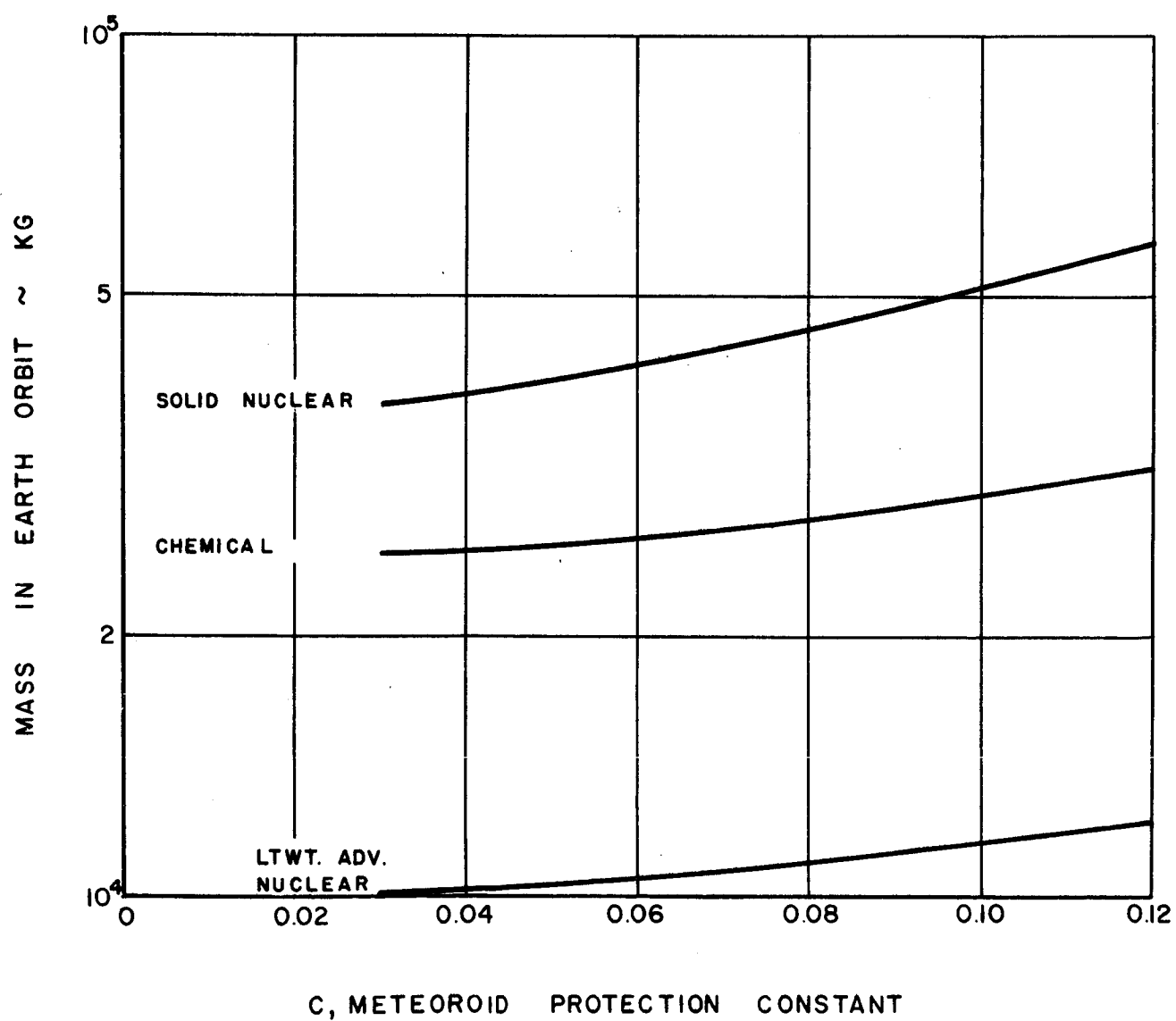
NEPTUNE ORBITAL STOPOVER

$$W_{PL} = 4536 \text{ KG}$$



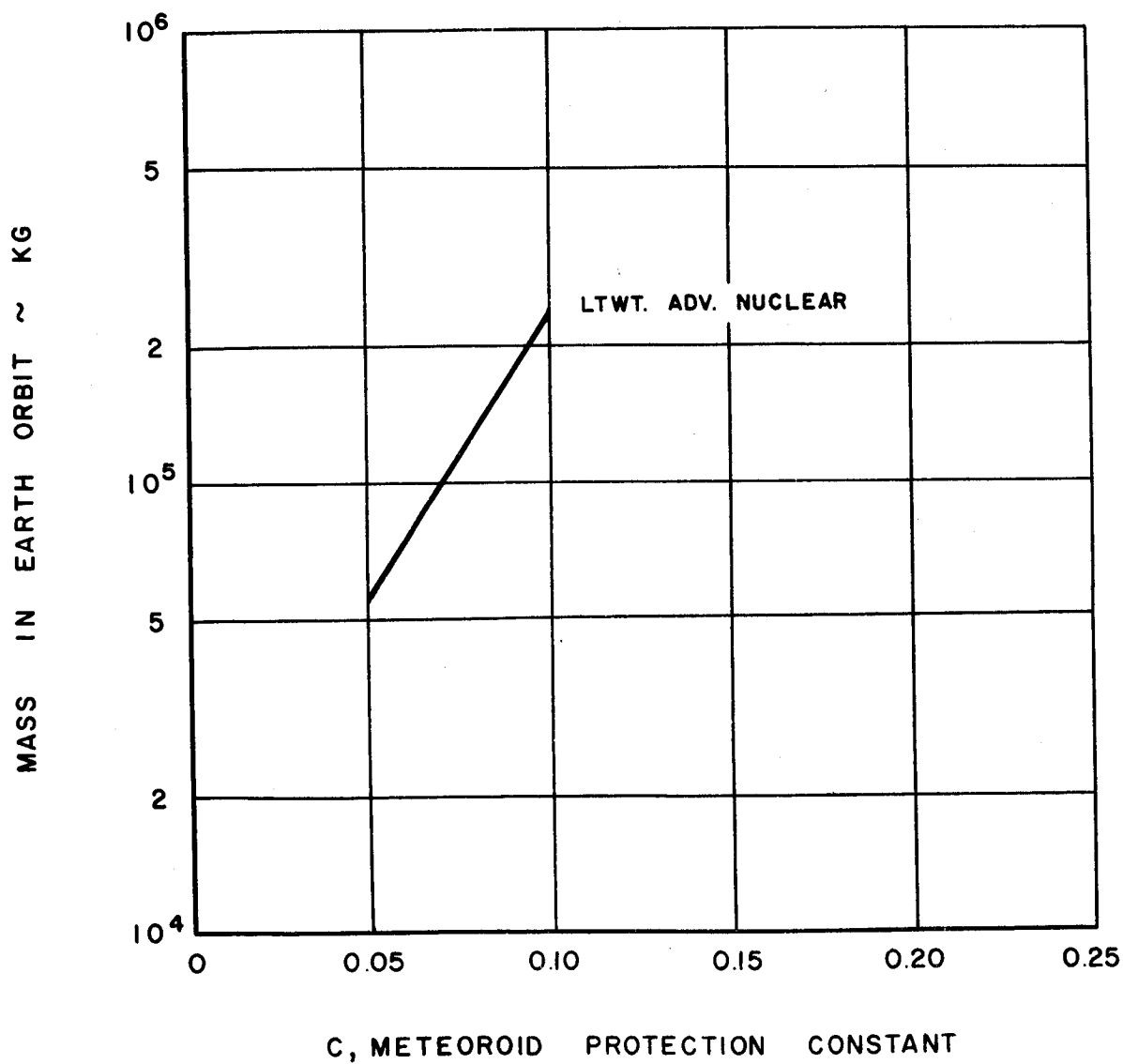
MARS ORBITAL STOPOVERS

$$W_{PL} = 4536 \text{ KG}$$



JUPITER ORBITAL STOPOVERS

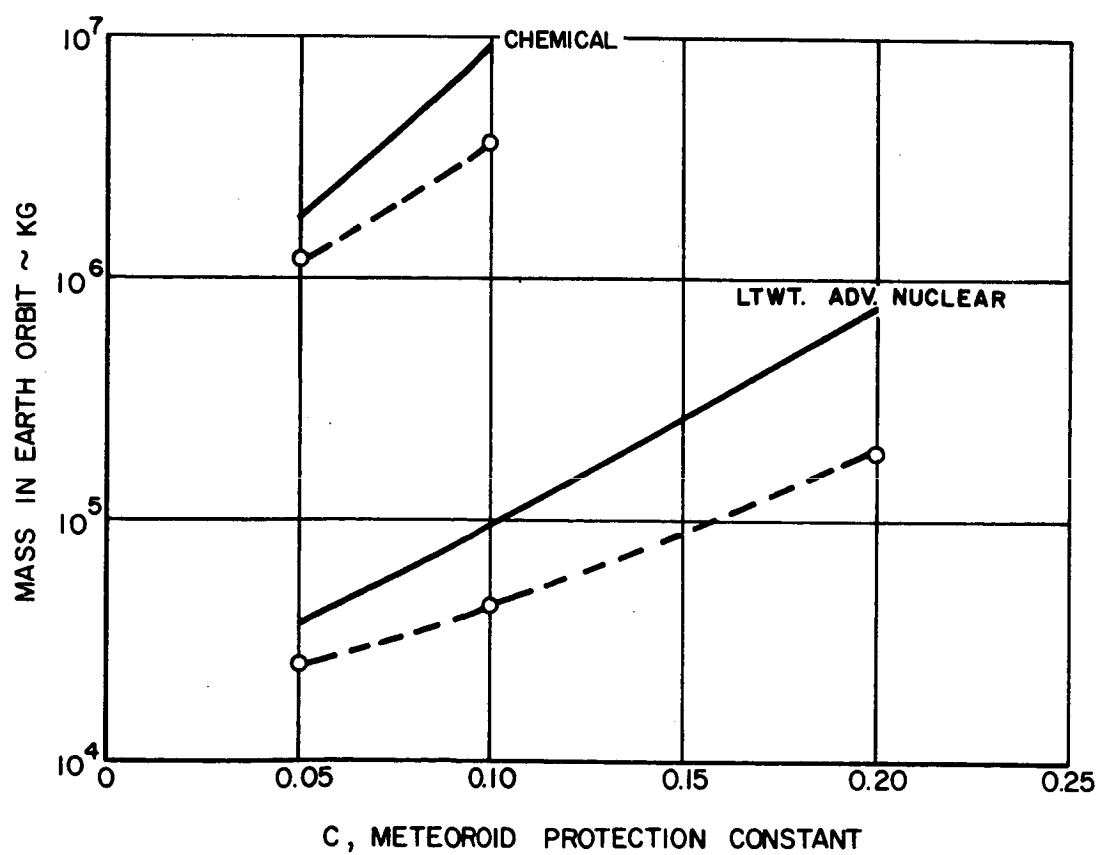
$$W_{PL} = 4536 \text{ KG}$$



NEPTUNE ORBITAL STOPOVERS

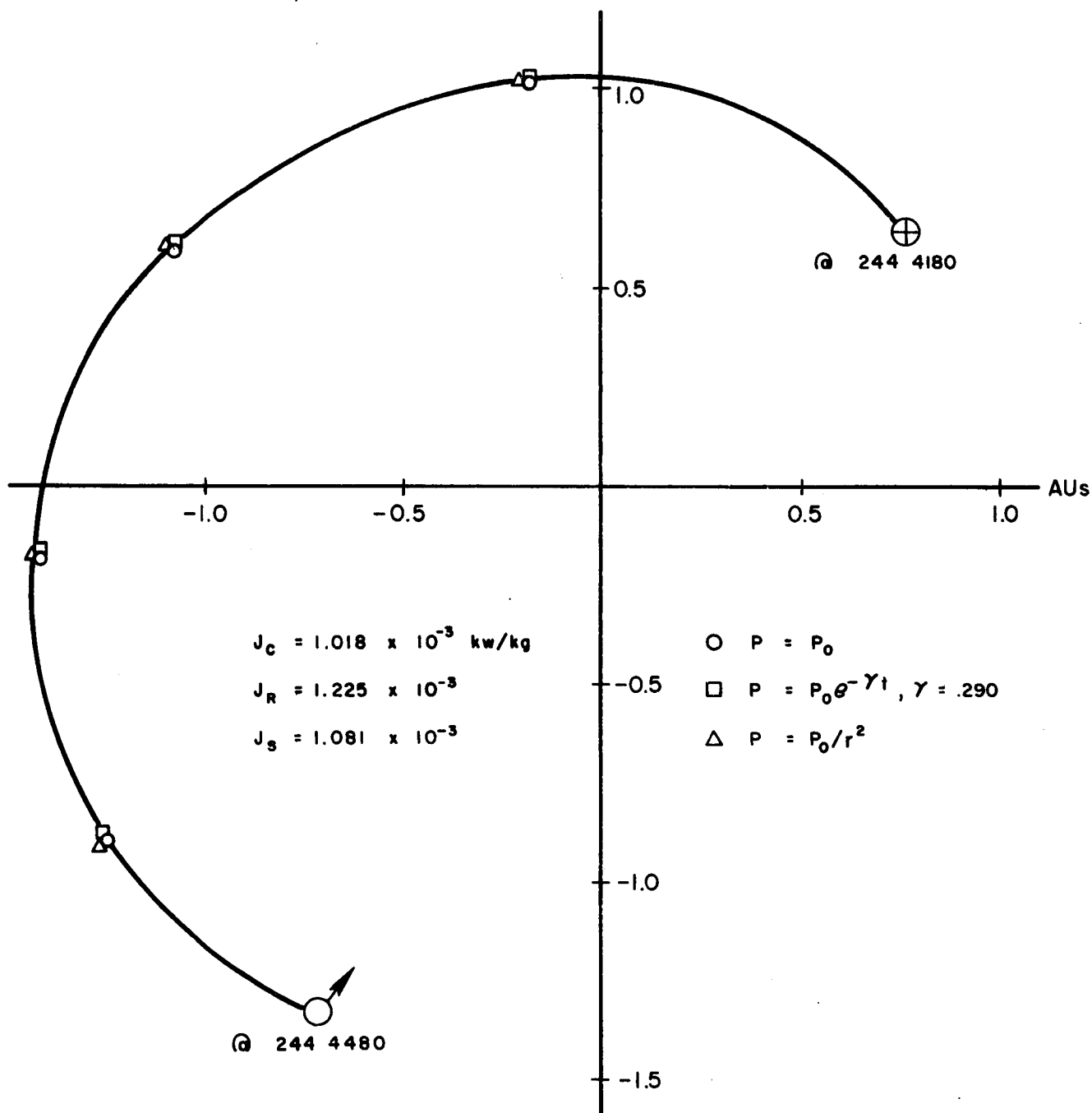
$$W_{PL} = 4536 \text{ KG}$$

— SINGLE SKIN
○ — — ○ WHIPPLE BUMPER



OPTIMAL LOW-THRUST TRAJECTORIES MARS FLYBY

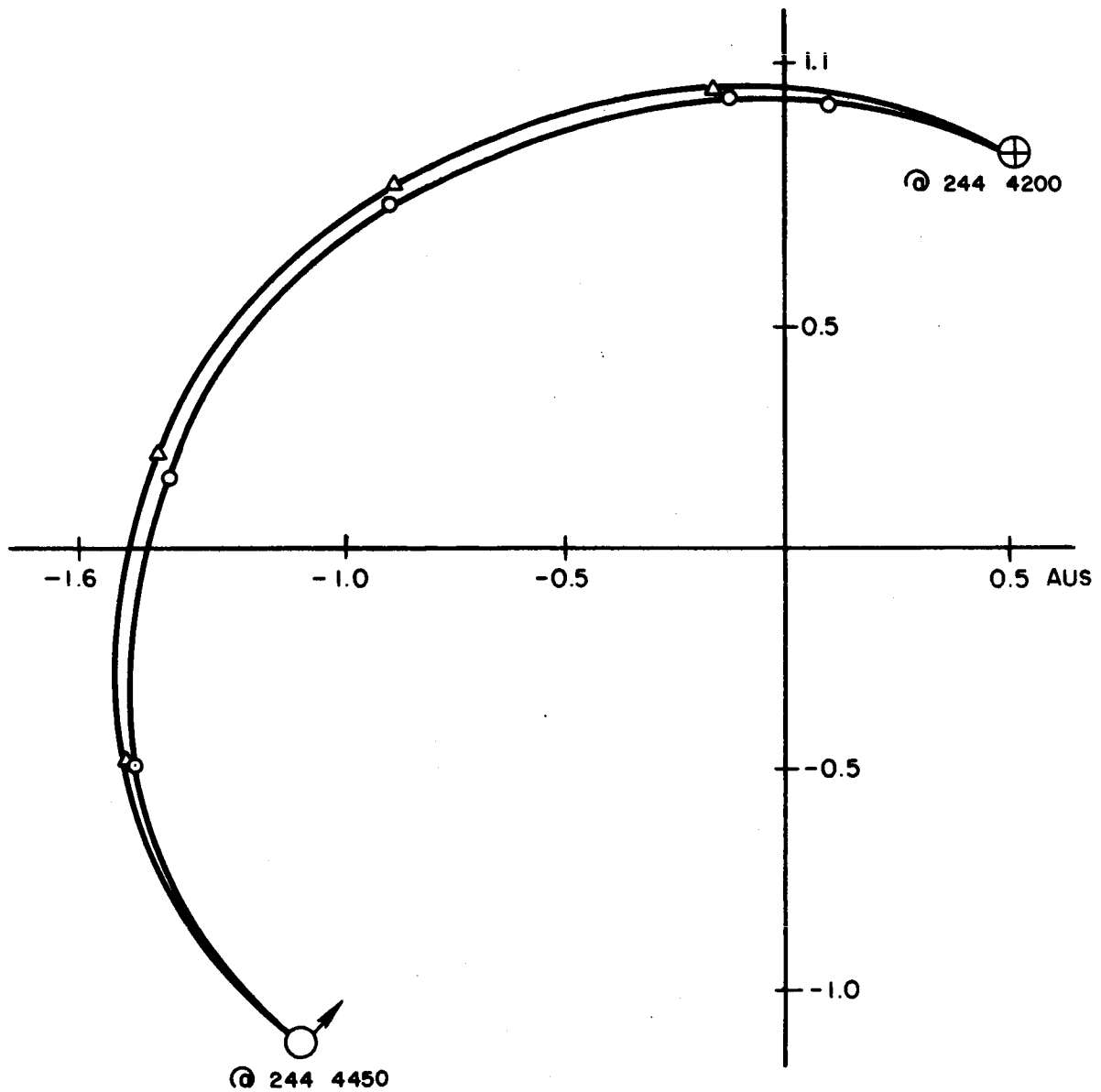
NOTE: THE TRAJECTORIES ARE REPRESENTED
HERE BY ONE CURVE AS THERE ARE
ONLY SLIGHT DIFFERENCES AMONG THEM



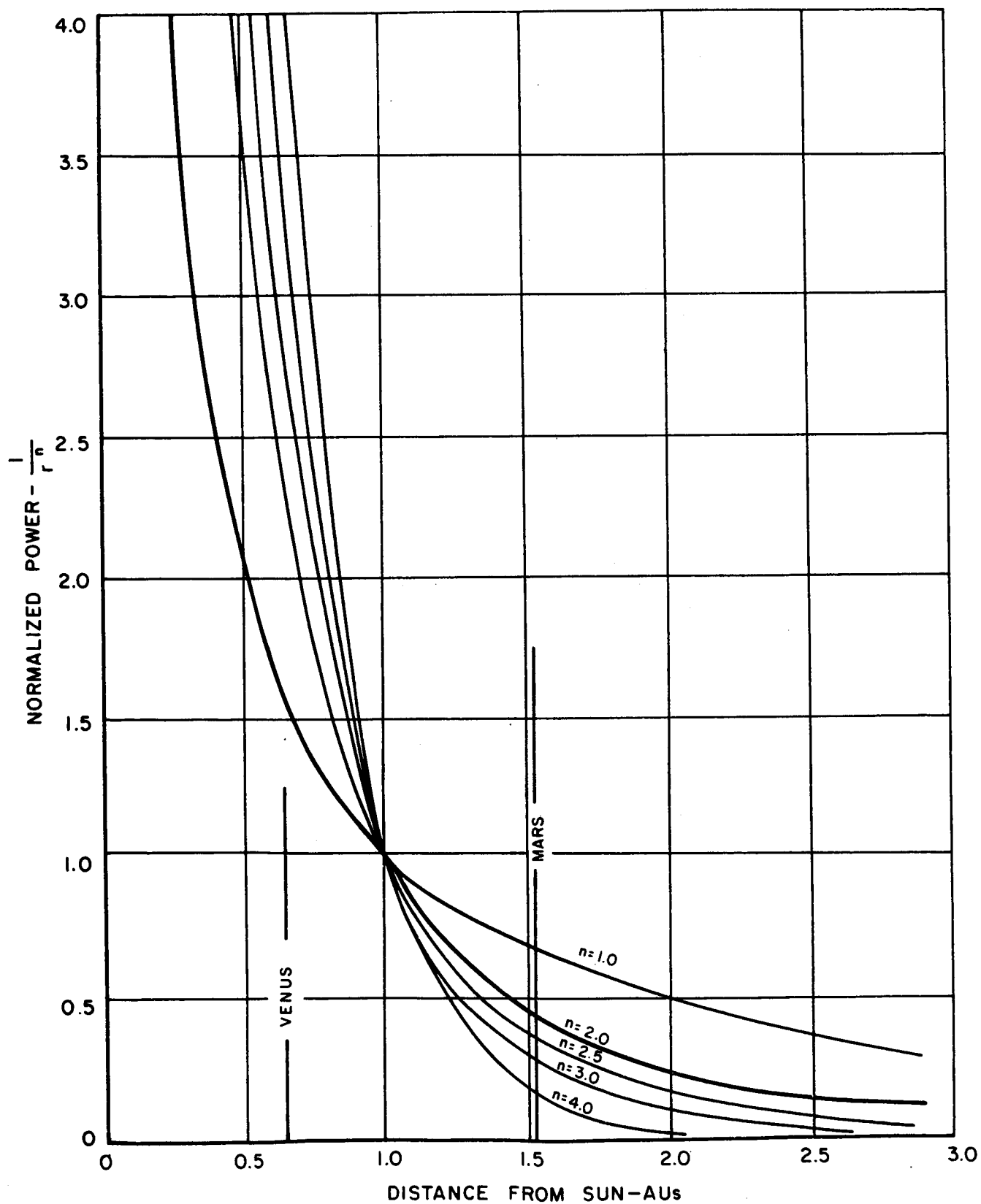
COMPARISON OF TRAJECTORIES FOR CONSTANT POWER AND RADIOISOTOPE POWER

O - CONSTANT POWER, P_0

Δ - RADIOISOTOPE POWER, P_0 / e^{at} (T_{m170} , .35 YR. HALF-LIFE)

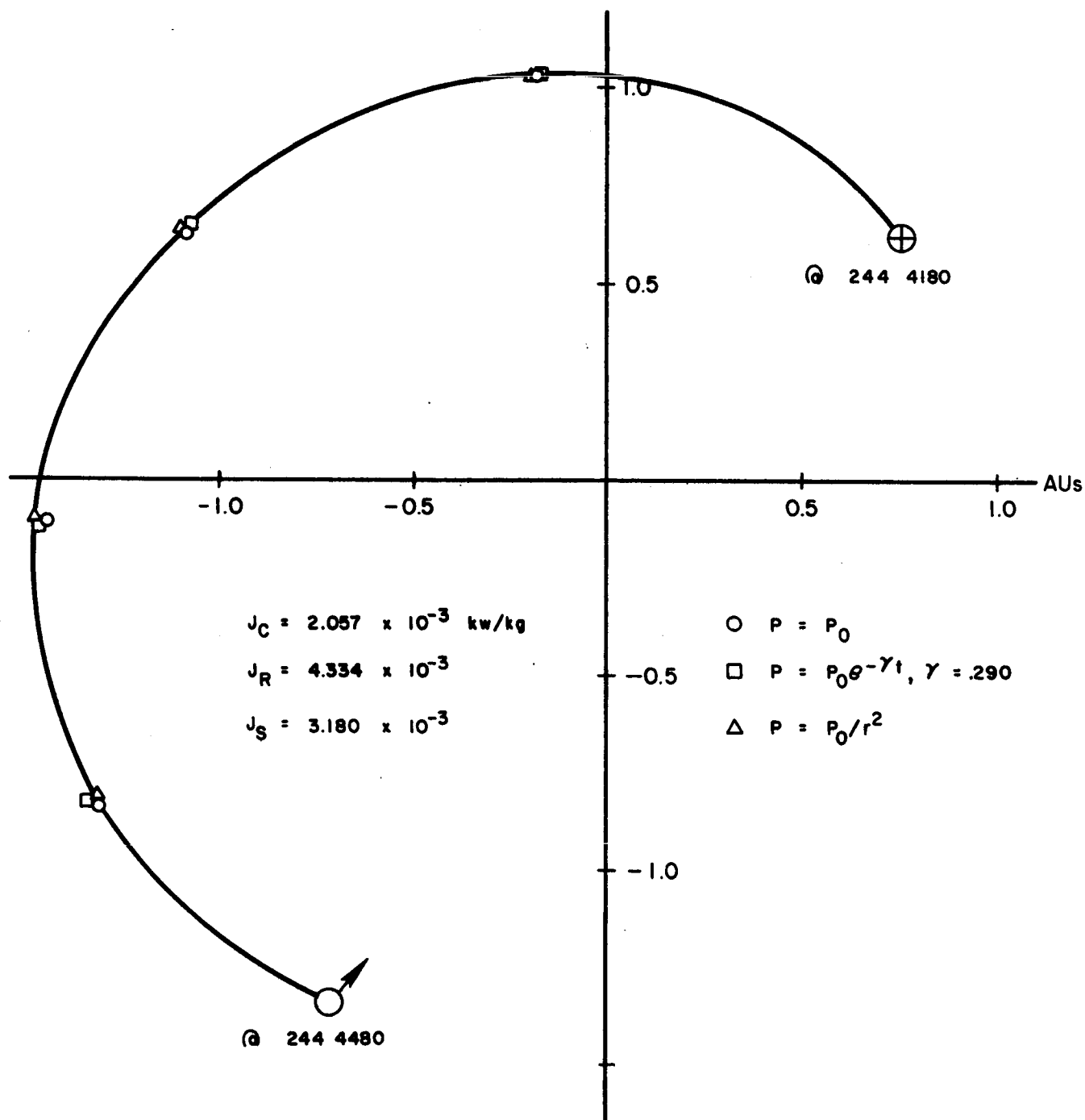


NORMALIZED POWER OF SOLAR CELL POWER SOURCE VS DISTANCE FROM SUN



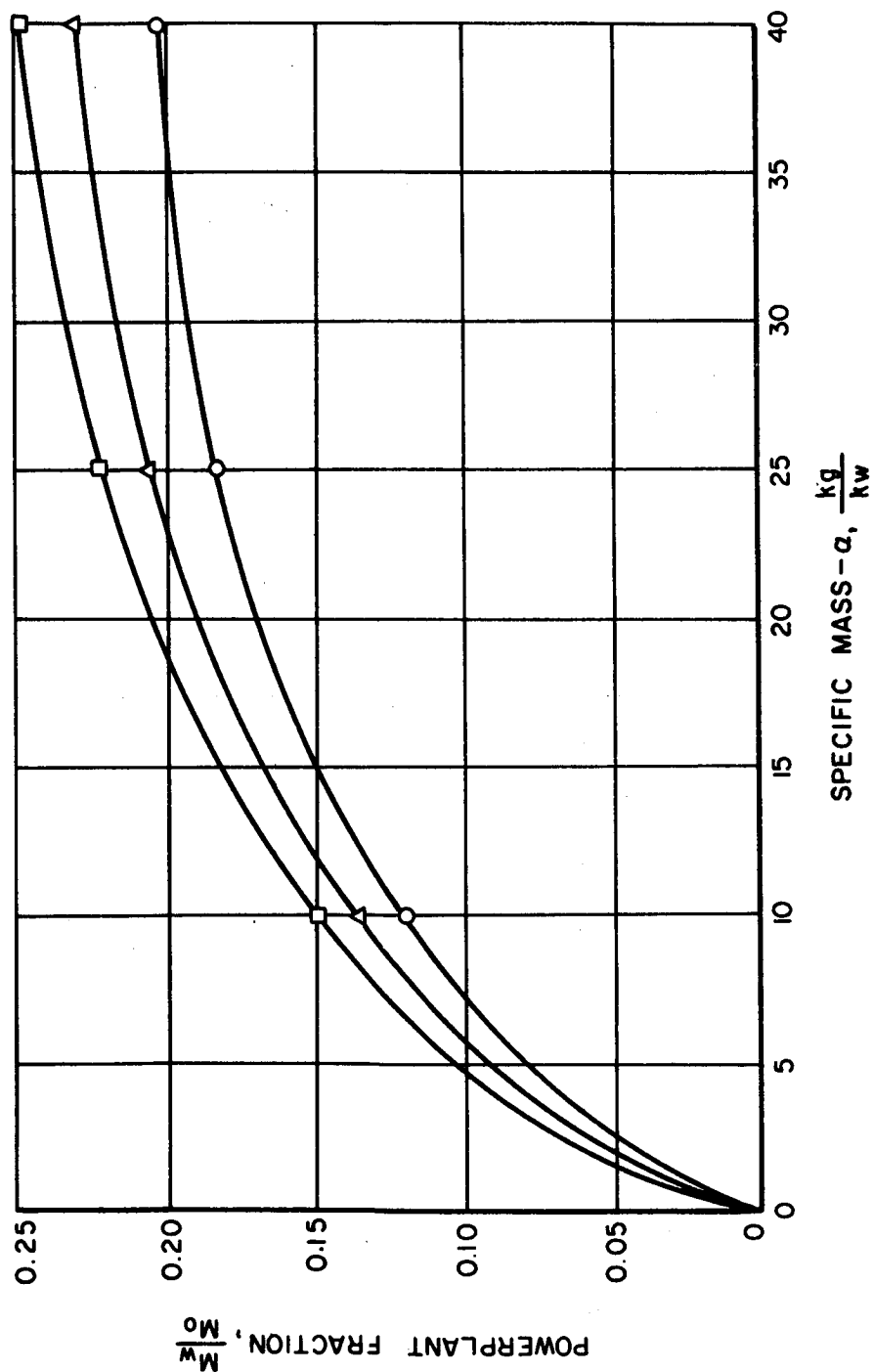
OPTIMAL LOW-THRUST TRAJECTORIES MARS RENDEZVOUS

NOTE: THE TRAJECTORIES ARE REPRESENTED
HERE BY ONE CURVE AS THERE ARE ONLY
SLIGHT DIFFERENCES AMONG THEM



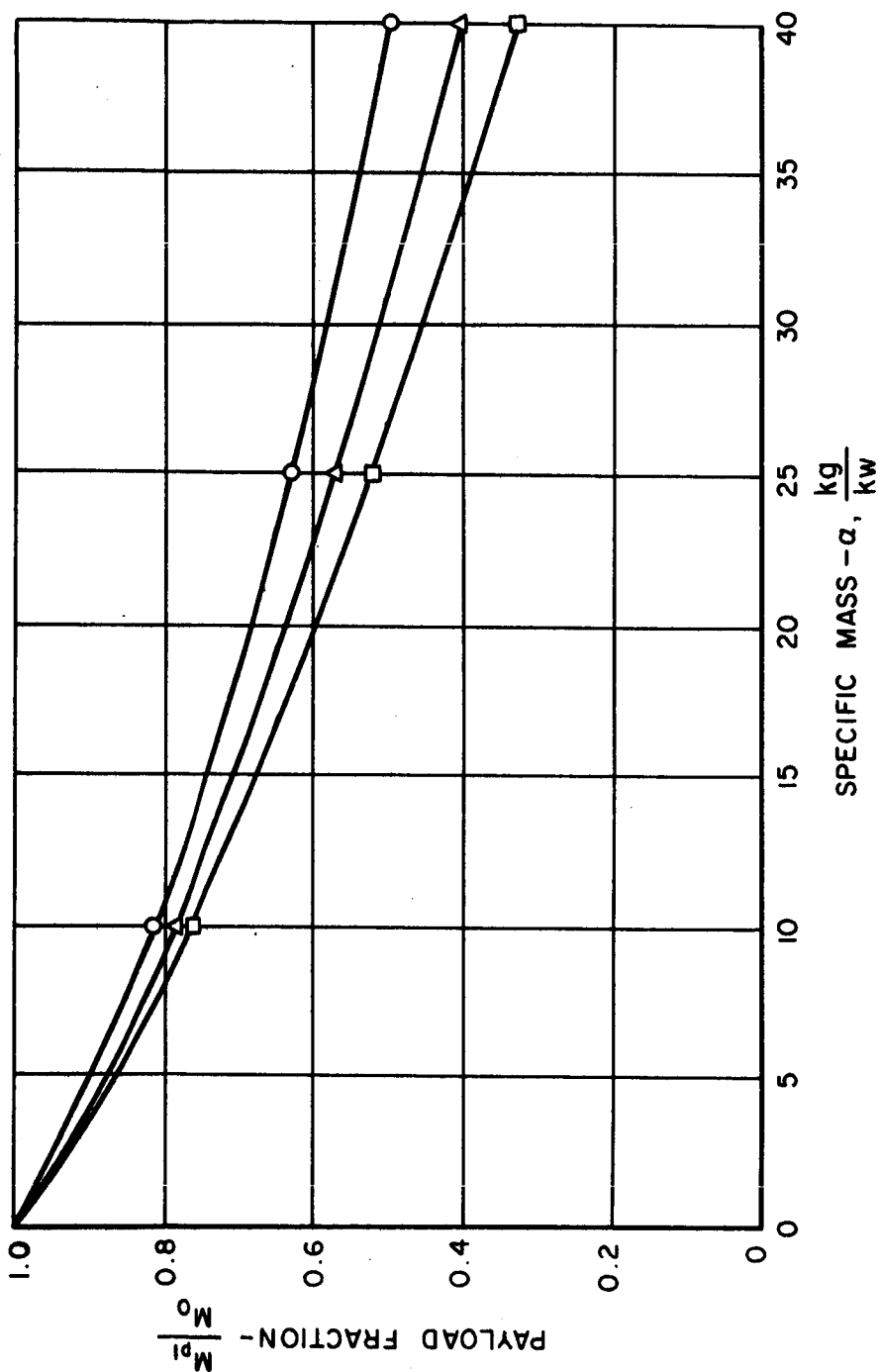
POWERPLANT FRACTION VS SPECIFIC MASS MARS RENDEZVOUS (4180 - 4480)

- O $P = P_0$
- $P = P_0 g^{-\gamma t}, \gamma = .290$
- △ $P = P_0 / r^2$



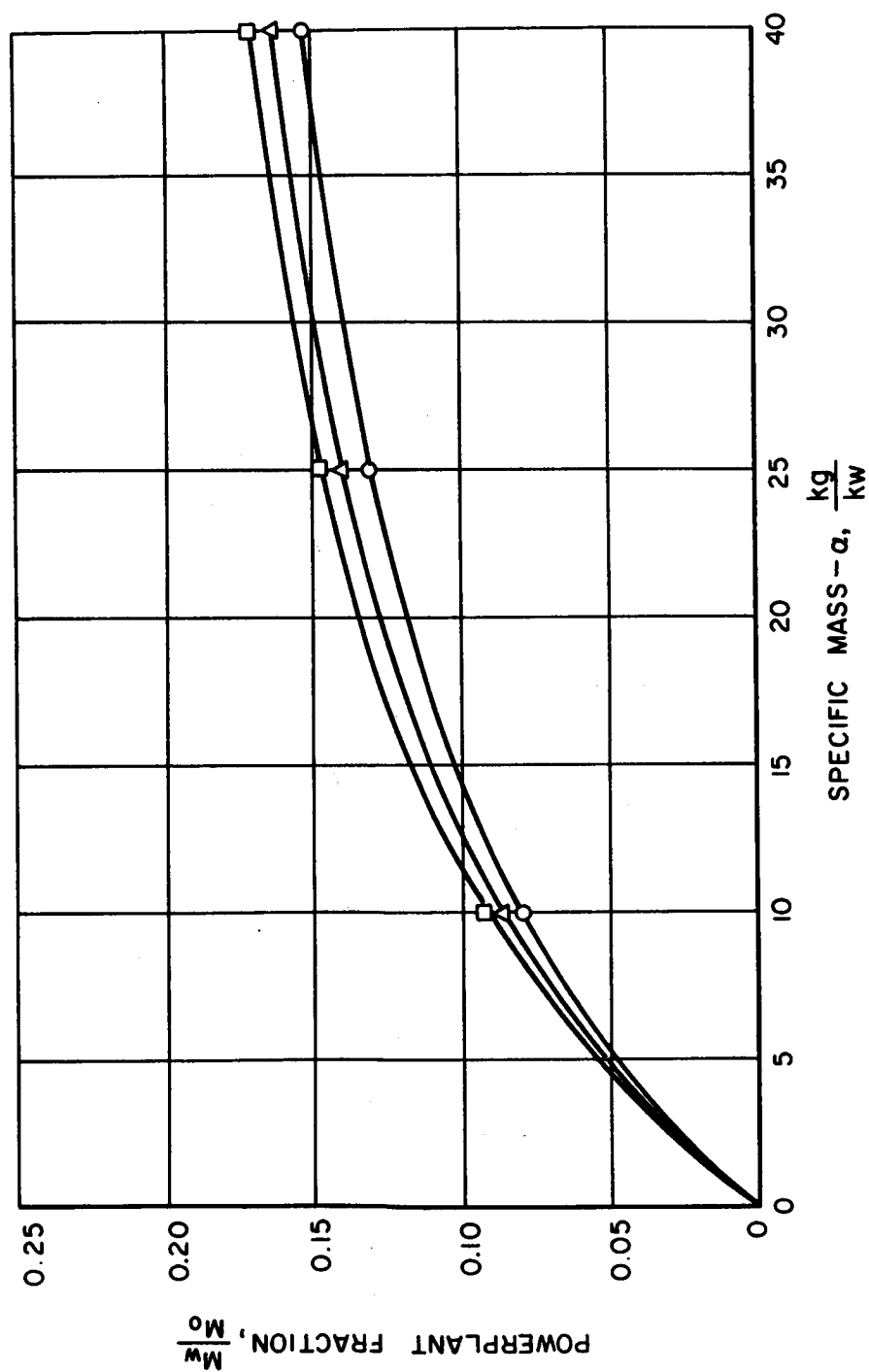
PAYLOAD FRACTION VS SPECIFIC MASS MARS RENDEZVOUS (4180-4480)

- $\circ J_c = 2.057 \times 10^{-3} \frac{Kw}{Kg}$
 $\square J_R = 4.334 \times 10^{-3}$
 $\triangle J_s = 3.180 \times 10^{-3}$



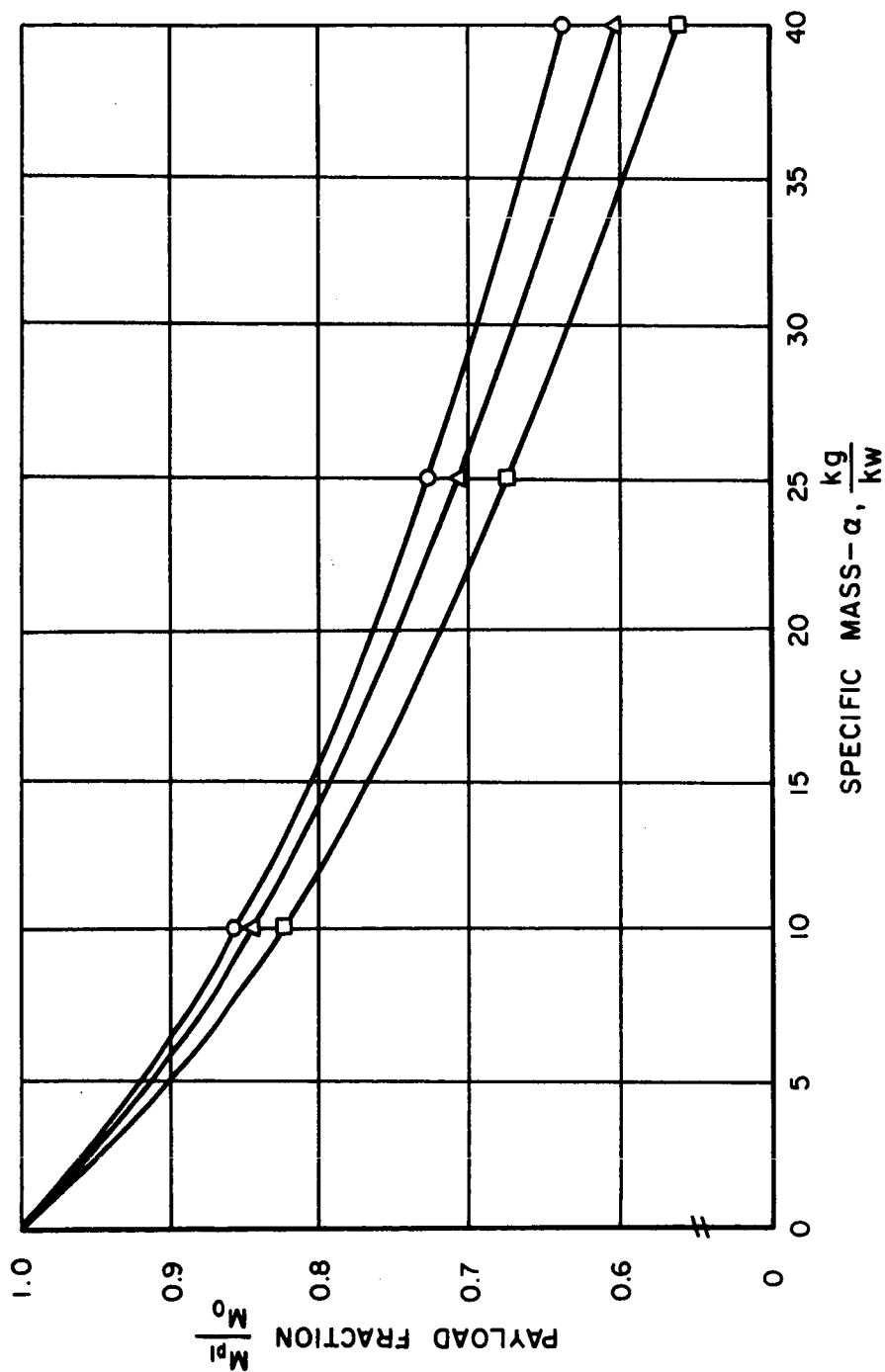
POWERPLANT FRACTION VS SPECIFIC MASS MARS FLYBY (4180-4480)

- O $P = P_0$
 □ $P = P_0 e^{-\gamma t}, \gamma = .290$
 Δ $P = P_0 / r^2$

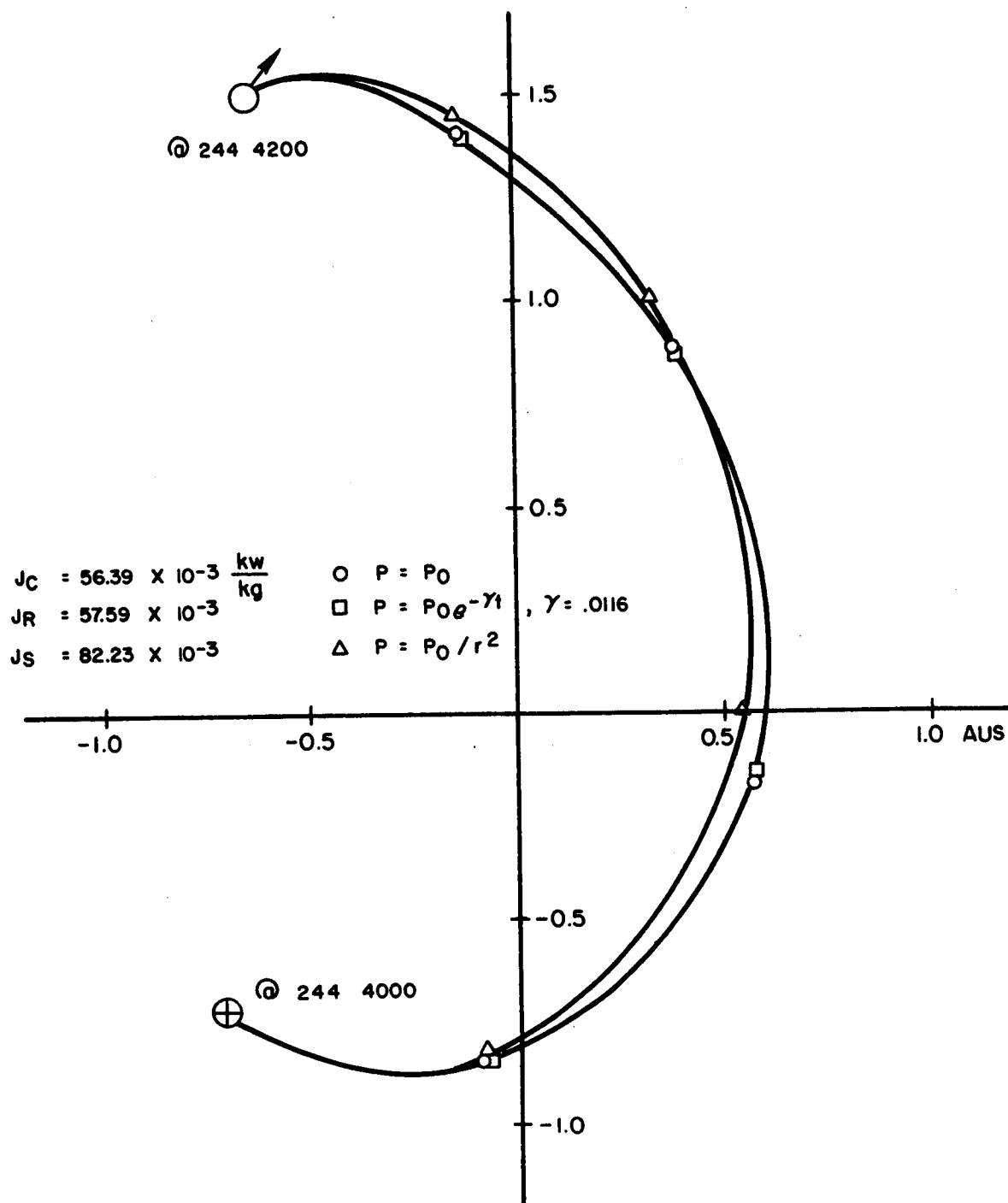


PAYLOAD FRACTION VS SPECIFIC MASS MARS FLYBY (4180-4480)

- $\circ J_c = 1.018 \times 10^{-3} \frac{\text{kg}}{\text{kg}}$
 $\square J_R = 1.225 \times 10^{-3}$
 $\triangle J_S = 1.081 \times 10^{-3}$

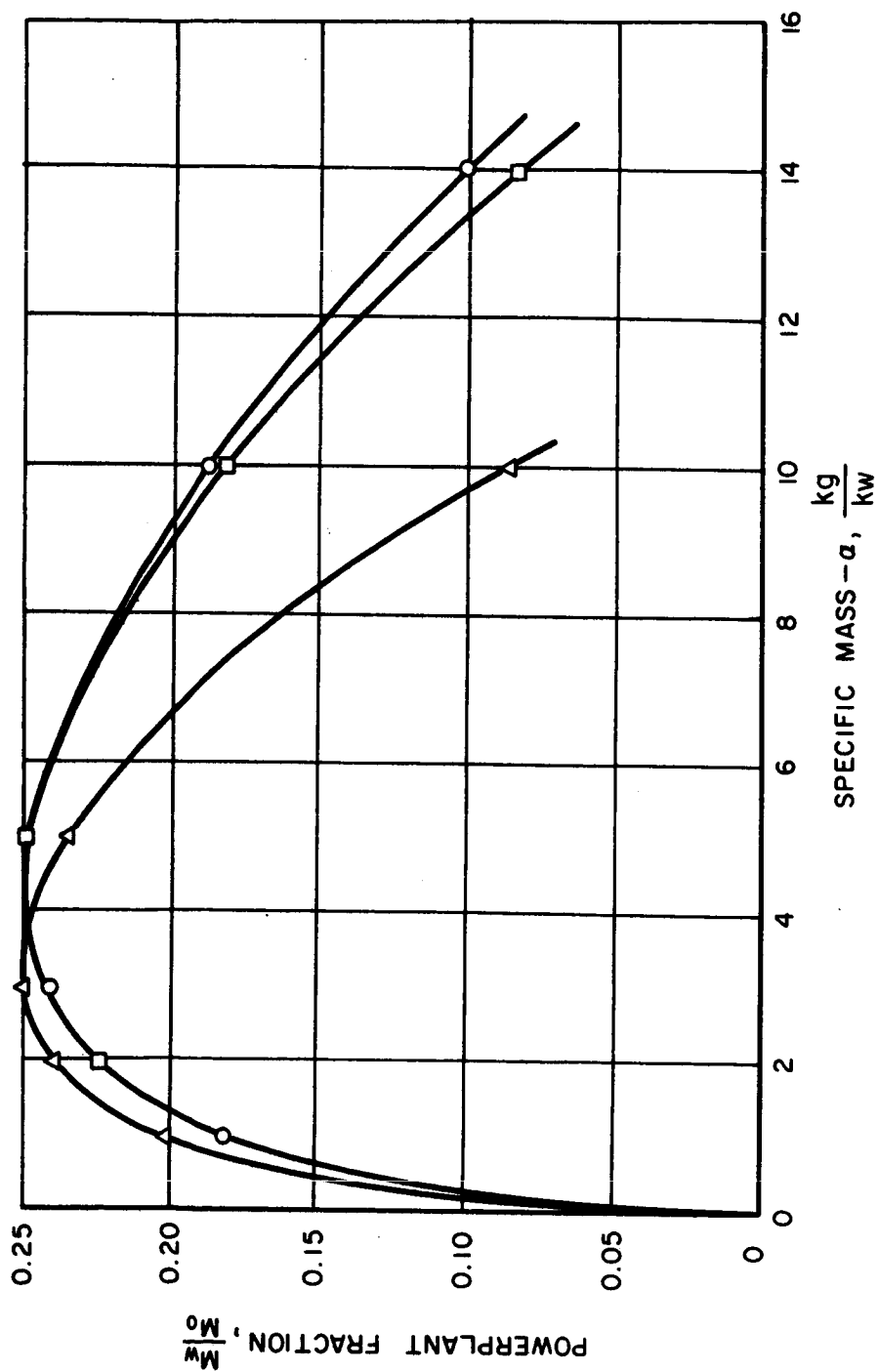


OPTIMAL LOW-THRUST TRAJECTORIES MARS RENDEZVOUS



POWERPLANT FRACTION VS SPECIFIC MASS MARS RENDEZVOUS (4000 - 4200)

- $\circ$ $P = P_0$
 $\square$ $P = P_0 e^{-\gamma_1}, \gamma = .0116$
 Δ $P = P_0 / r^2$

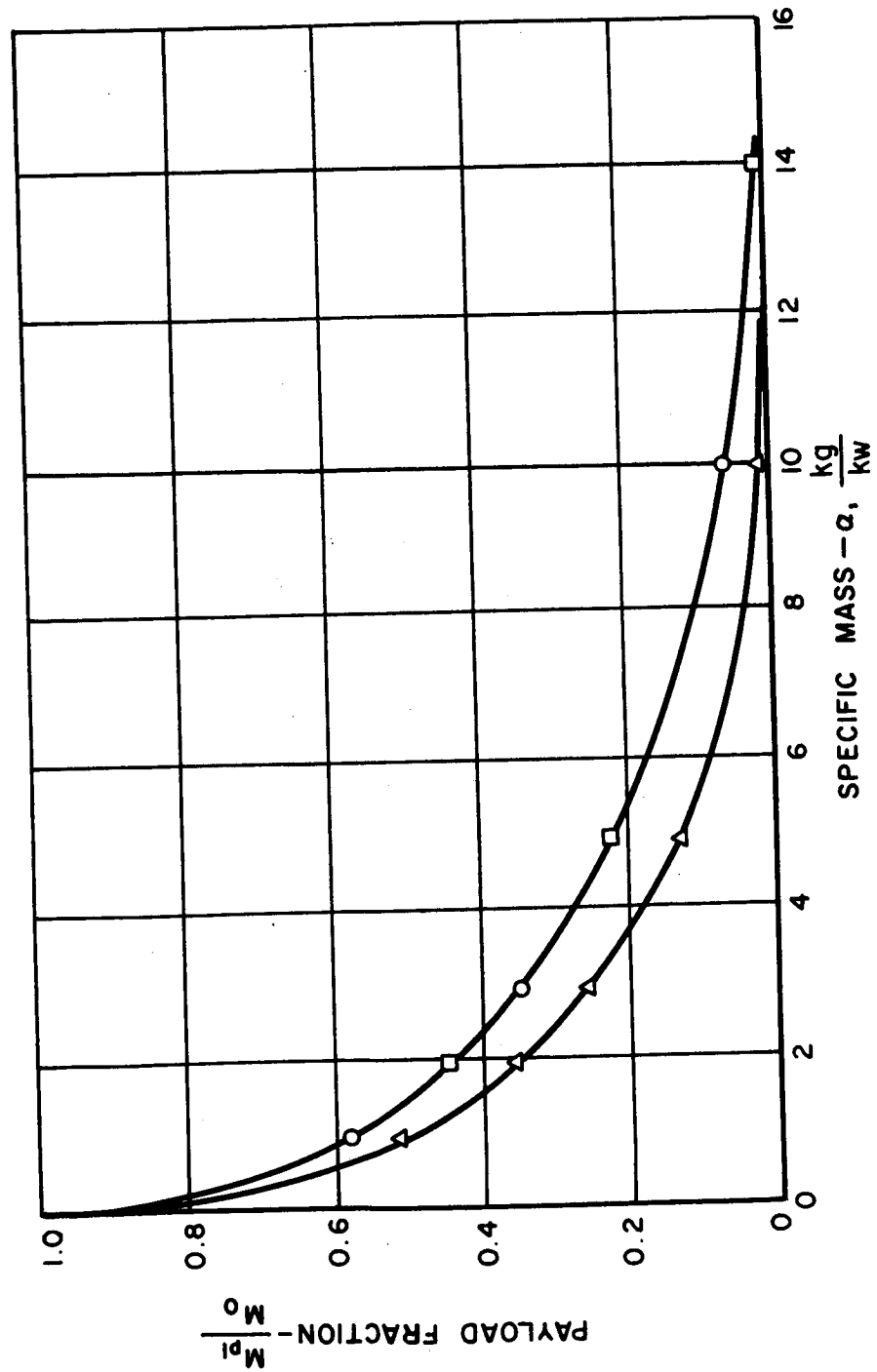


PAYLOAD FRACTION VS SPECIFIC MASS MARS RENDEZVOUS (4000-4200)

$$J_C = 56.39 \times 10^{-3} \frac{\text{kw}}{\text{kg}}$$

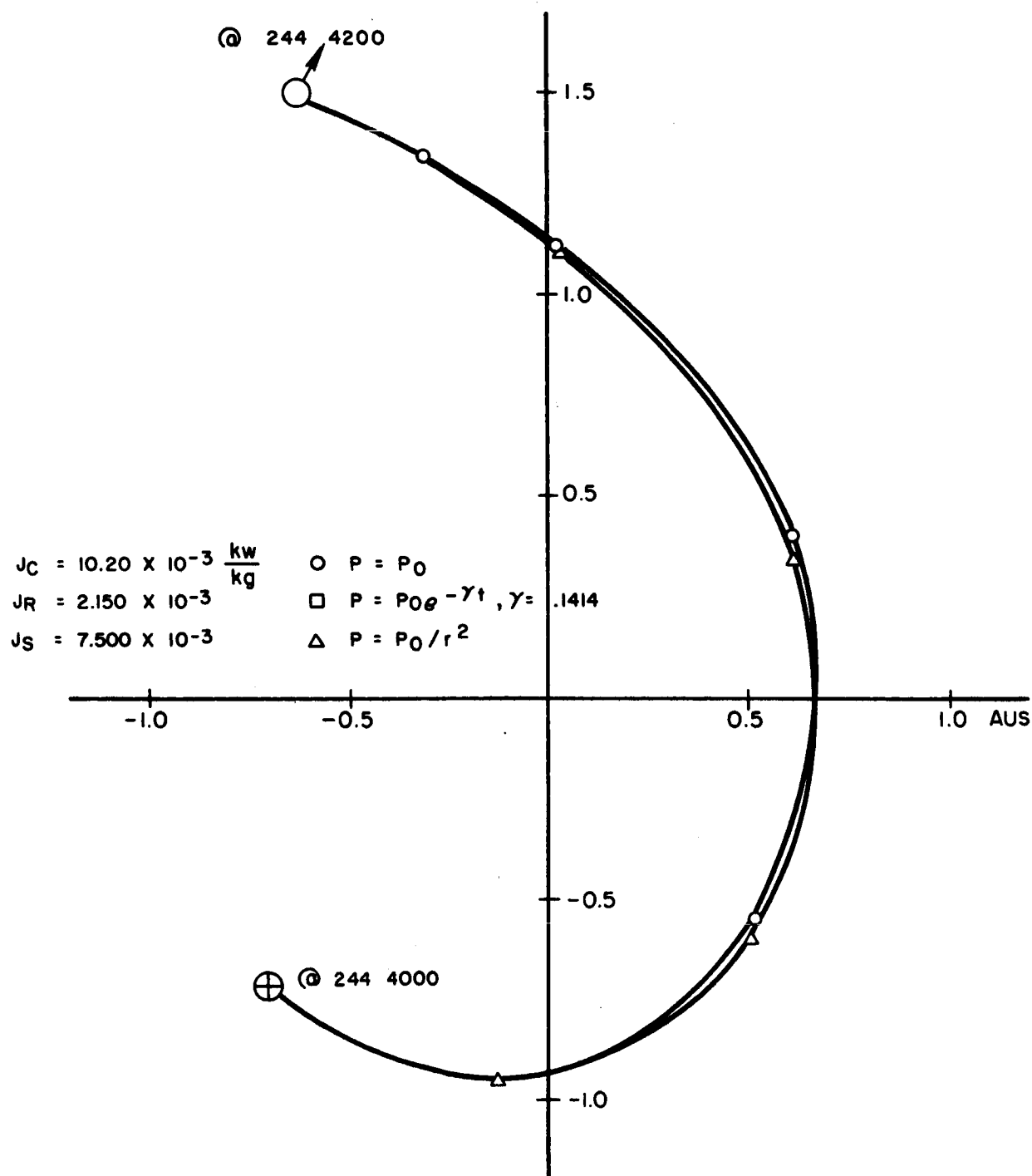
$$J_R = 57.59 \times 10^{-3}$$

$$J_S = 82.23 \times 10^{-3}$$



OPTIMAL LOW-THRUST TRAJECTORIES MARS FLYBY

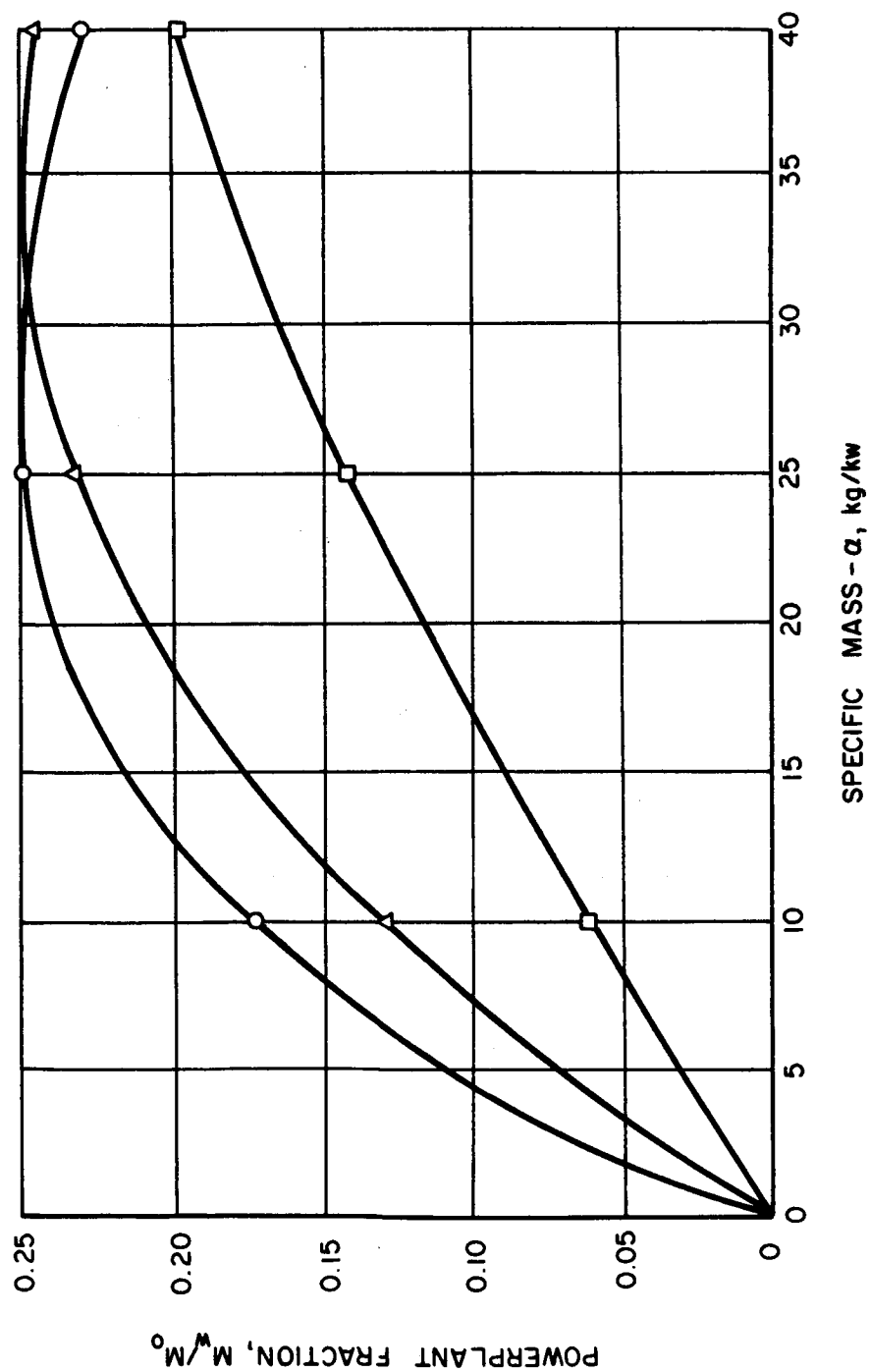
NOTE: THE CONSTANT POWER AND RADIOISOTOPE
POWER TRAJECTORIES COINCIDE
WITH SMALL ERROR



POWERPLANT FRACTION VS SPECIFIC MASS

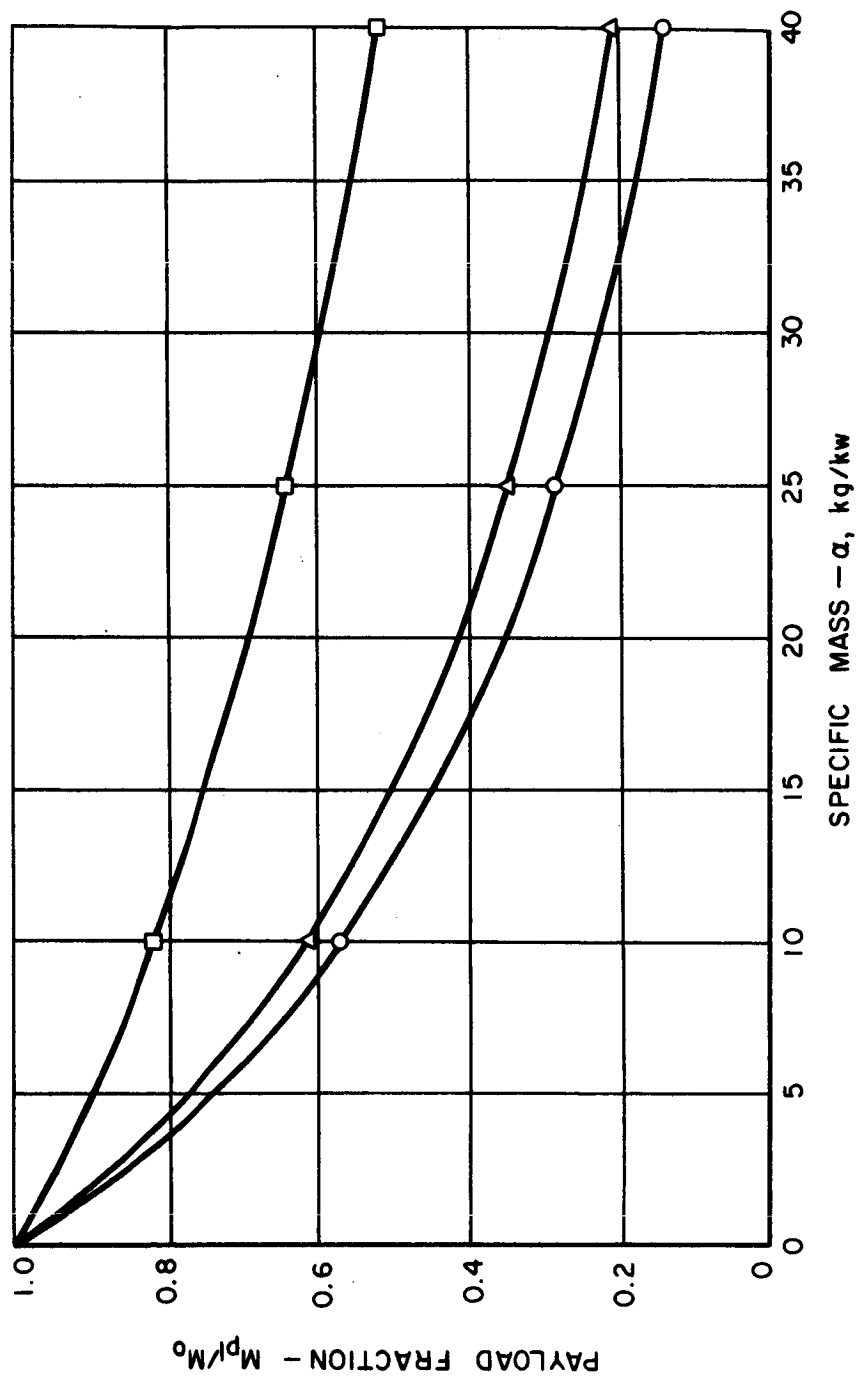
MARS FLYBY (4000-4200)

- O $P = P_0$
 □ $P = P_0 e^{-\gamma t}, \gamma = .1414$
 Δ $P = P_0 / r^2$



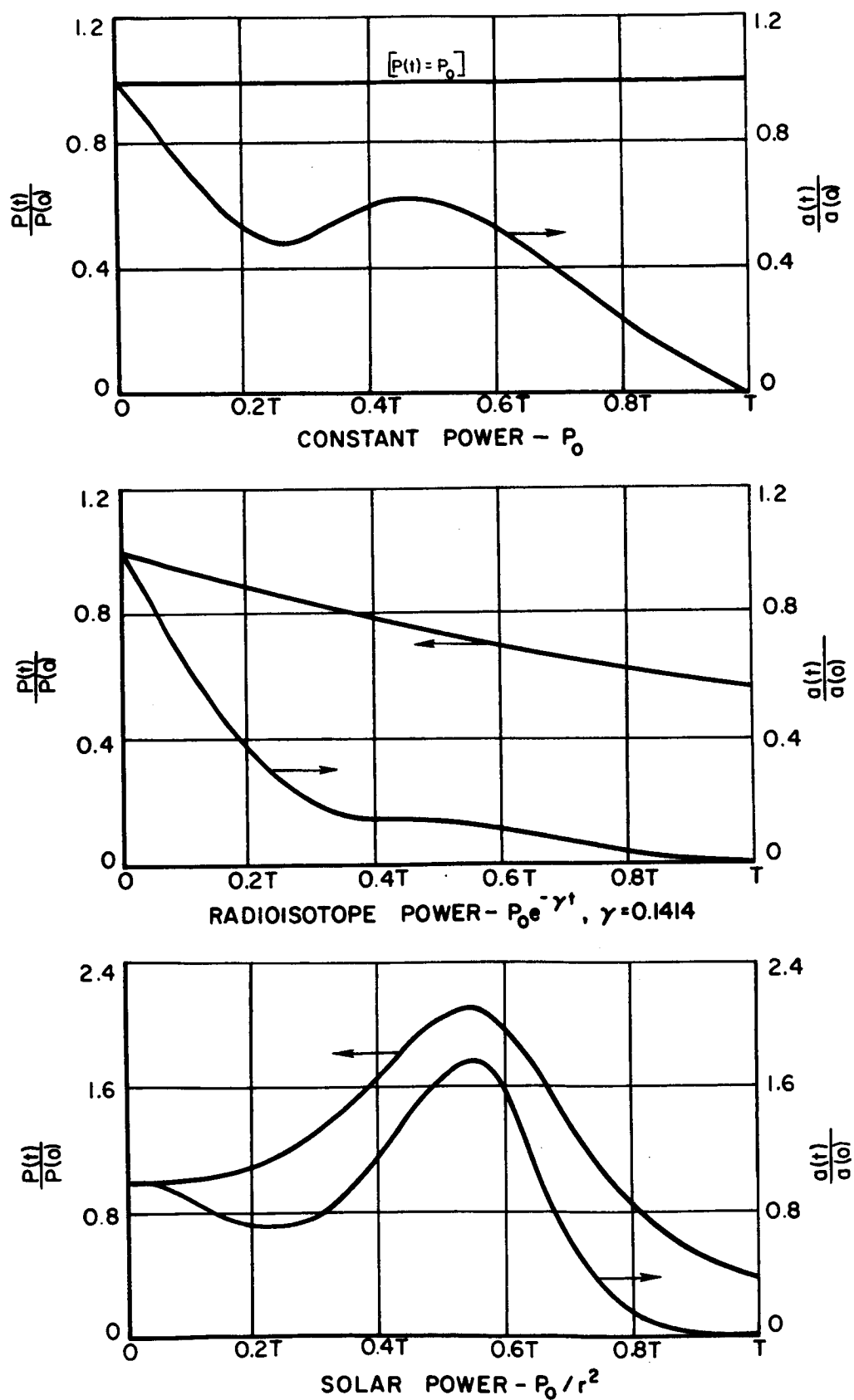
PAYLOAD FRACTION VS SPECIFIC MASS
MARS FLYBY (4000-4200)

- $J_c = 10.2 \times 10^{-3} \text{ kw/kg}$
□ $J_R = 2.15 \times 10^{-3}$
△ $J_g = 7.5 \times 10^{-3}$



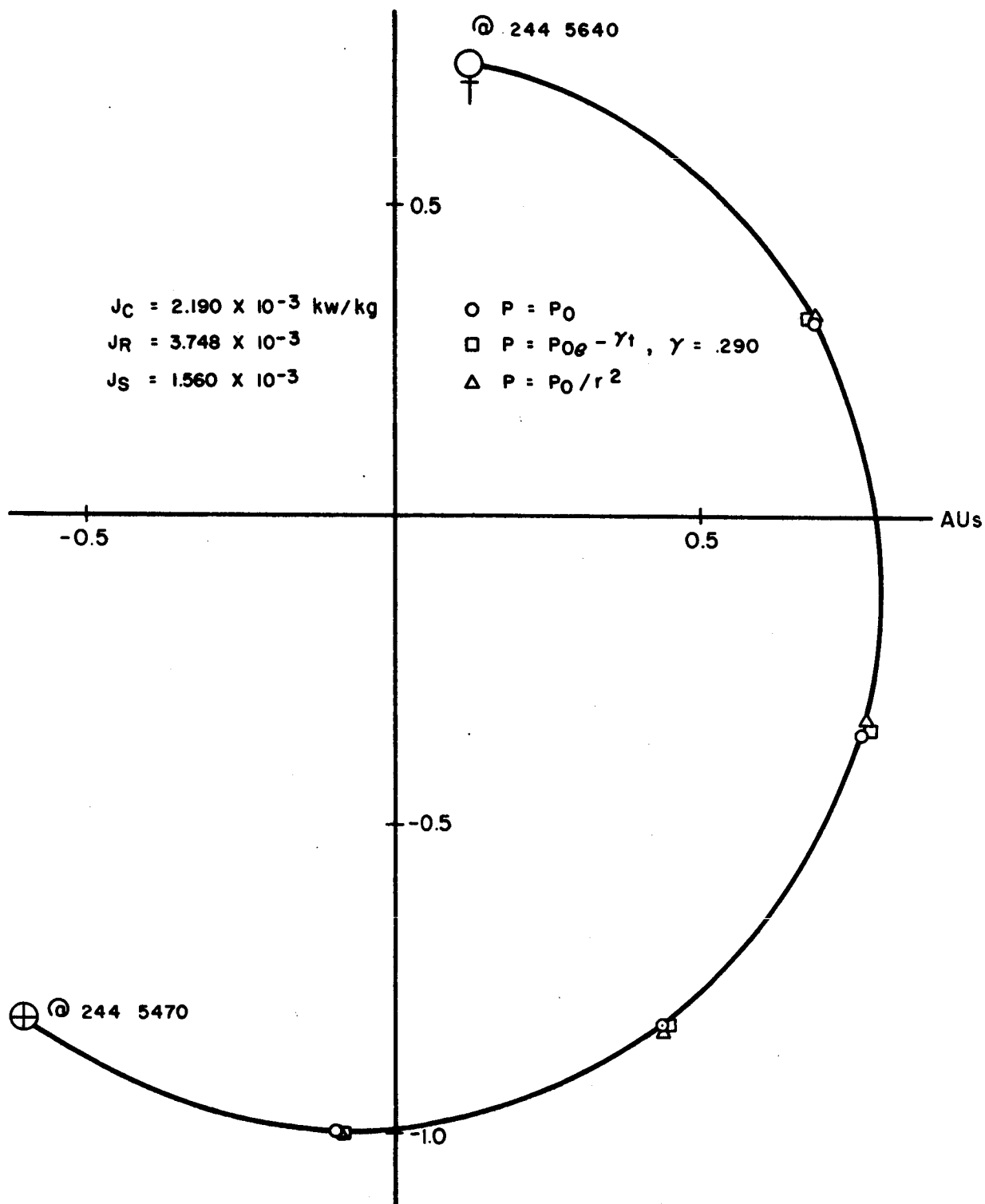
NORMALIZED POWER AND THRUST ACCELERATION
VS TIME FOR MARS FLYBY

(4400-4200)



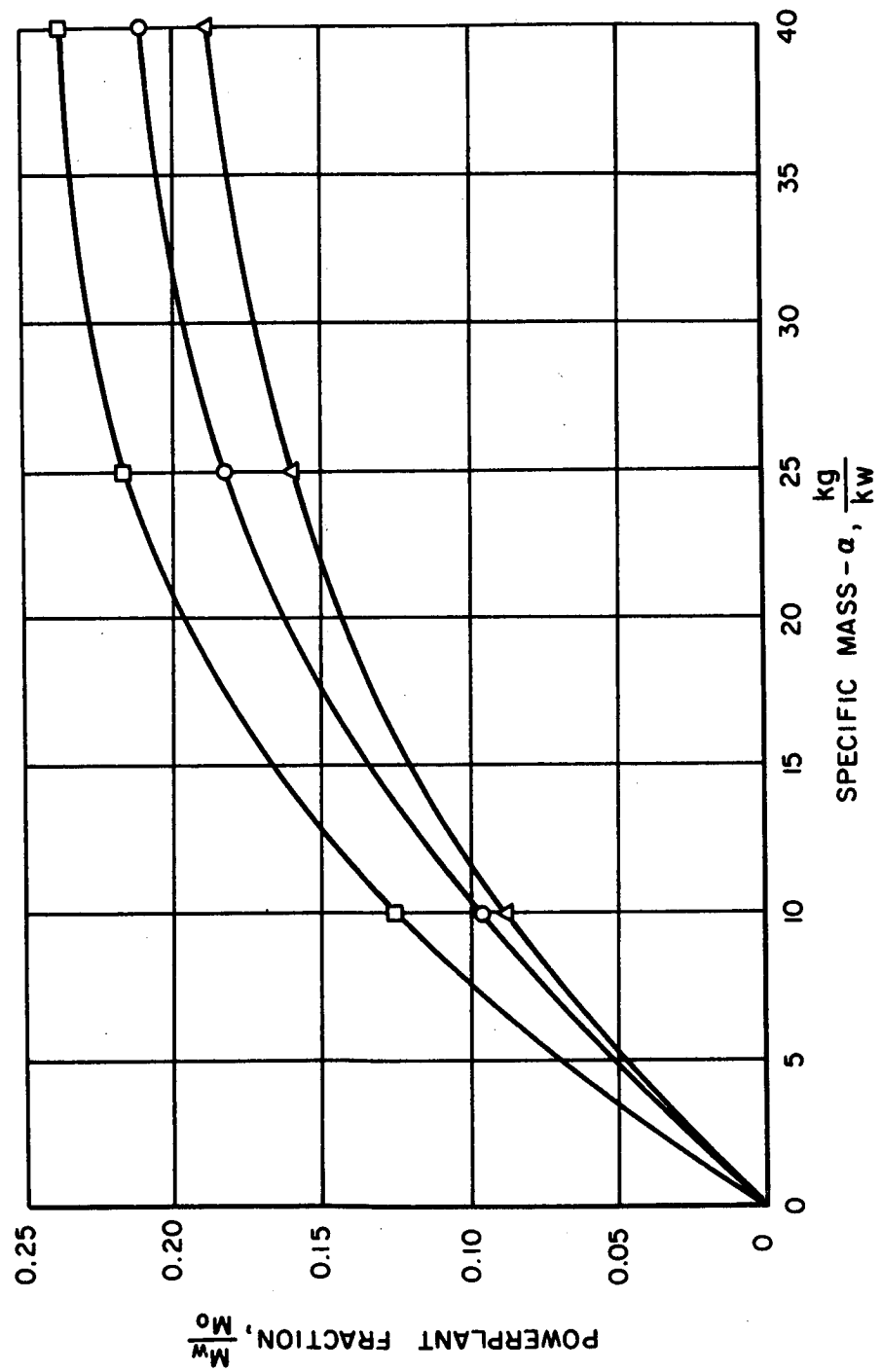
OPTIMAL LOW-THRUST TRAJECTORIES VENUS RENDEZVOUS

NOTE: THE TRAJECTORIES ARE REPRESENTED
HERE BY ONE CURVE AS THERE ARE
ONLY SLIGHT DIFFERENCES AMONG THEM



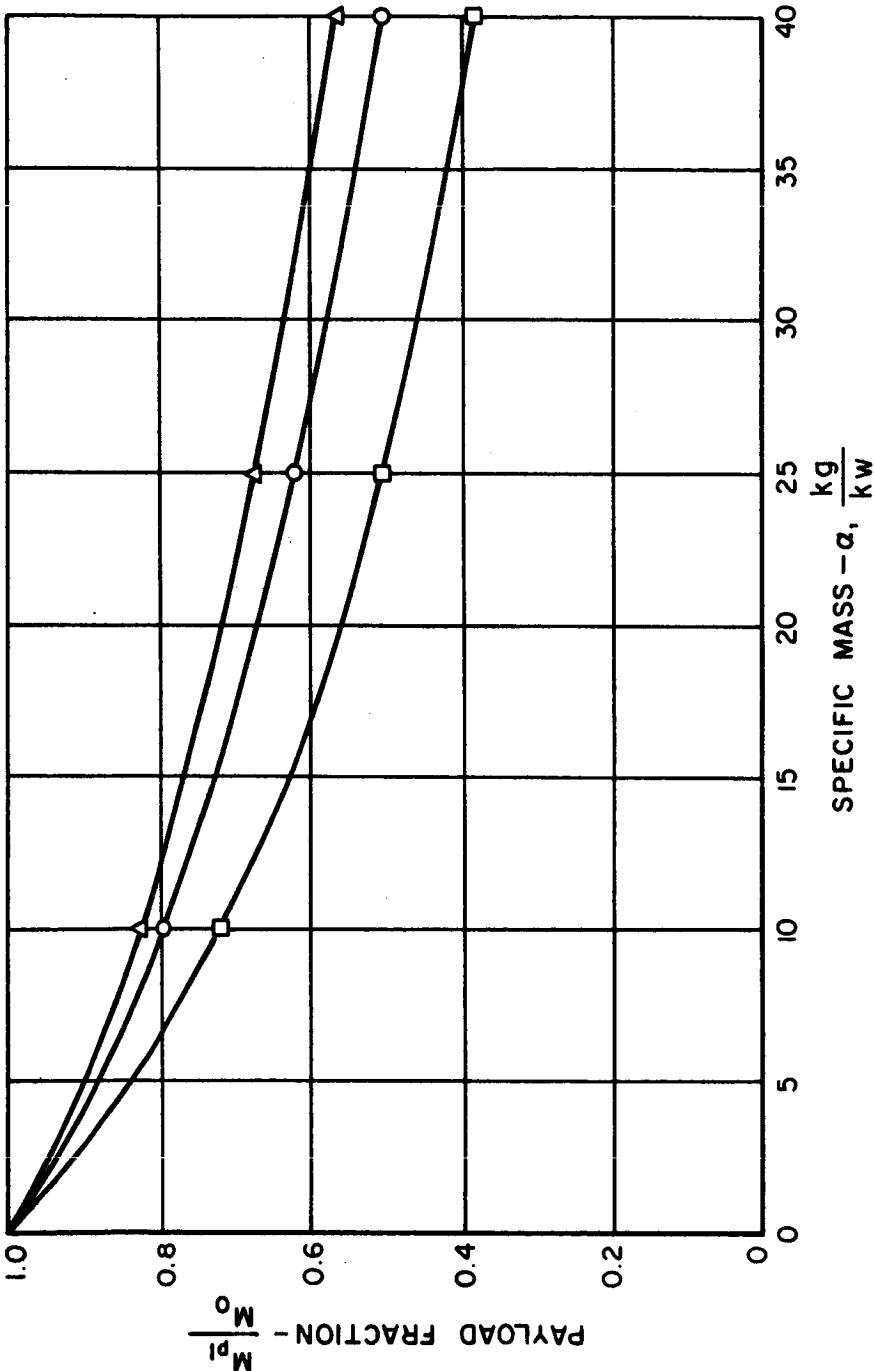
POWERPLANT FRACTION VS SPECIFIC MASS VENUS RENDEZVOUS (5470 - 5640)

- $\circ$ $P = P_0$
 $\square$ $P = P_0 \theta^{-\gamma_t}, \gamma = .290$
 $\triangle$ $P = P_0 / r^2$



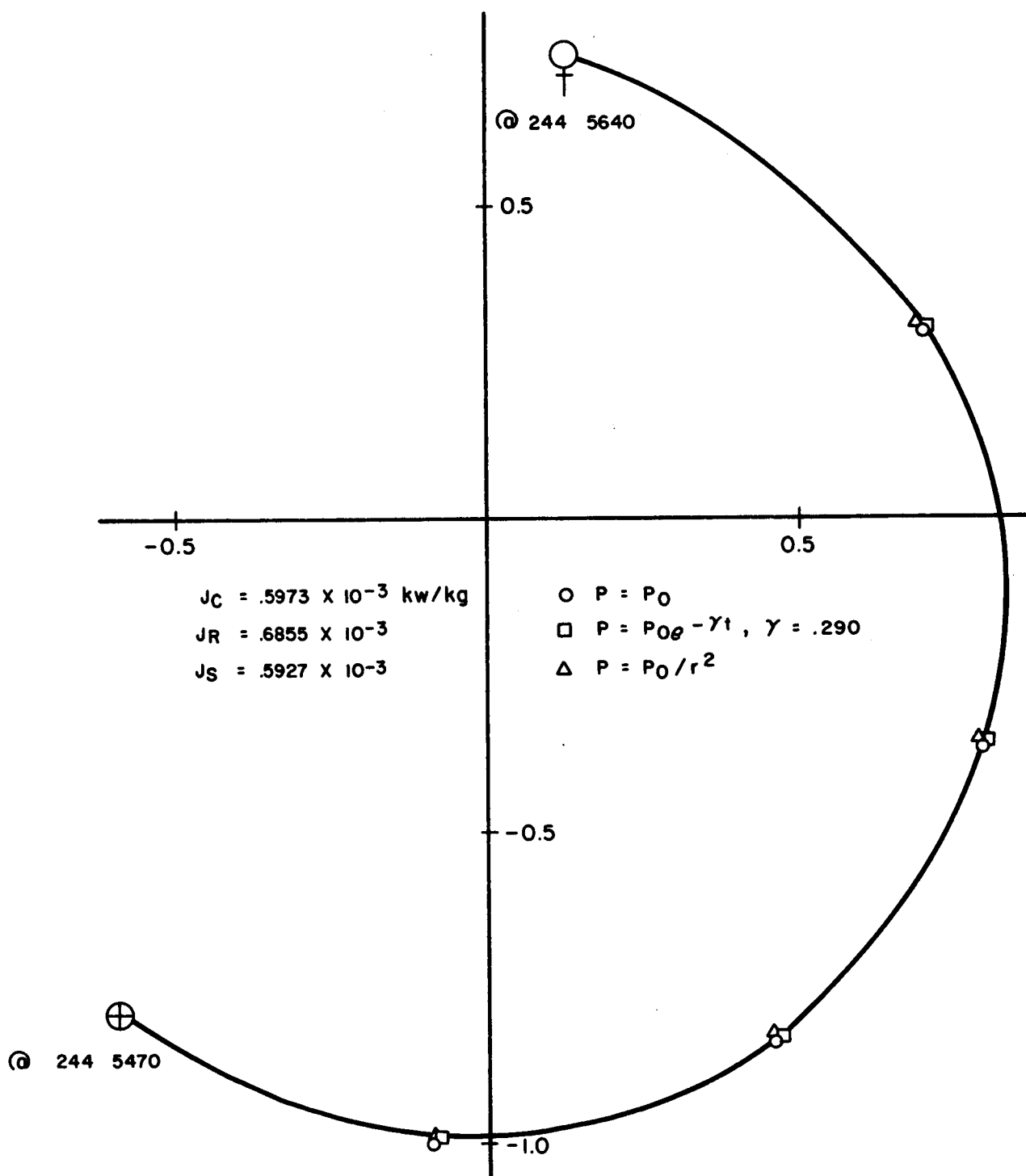
PAYLOAD FRACTION VS SPECIFIC MASS
VENUS RENDEZVOUS (5470-5640)

- $J_C = 2.190 \times 10^{-3} \frac{kg}{kW}$
- $J_R = 3.748 \times 10^{-3}$
- △ $J_S = 1.560 \times 10^{-3}$



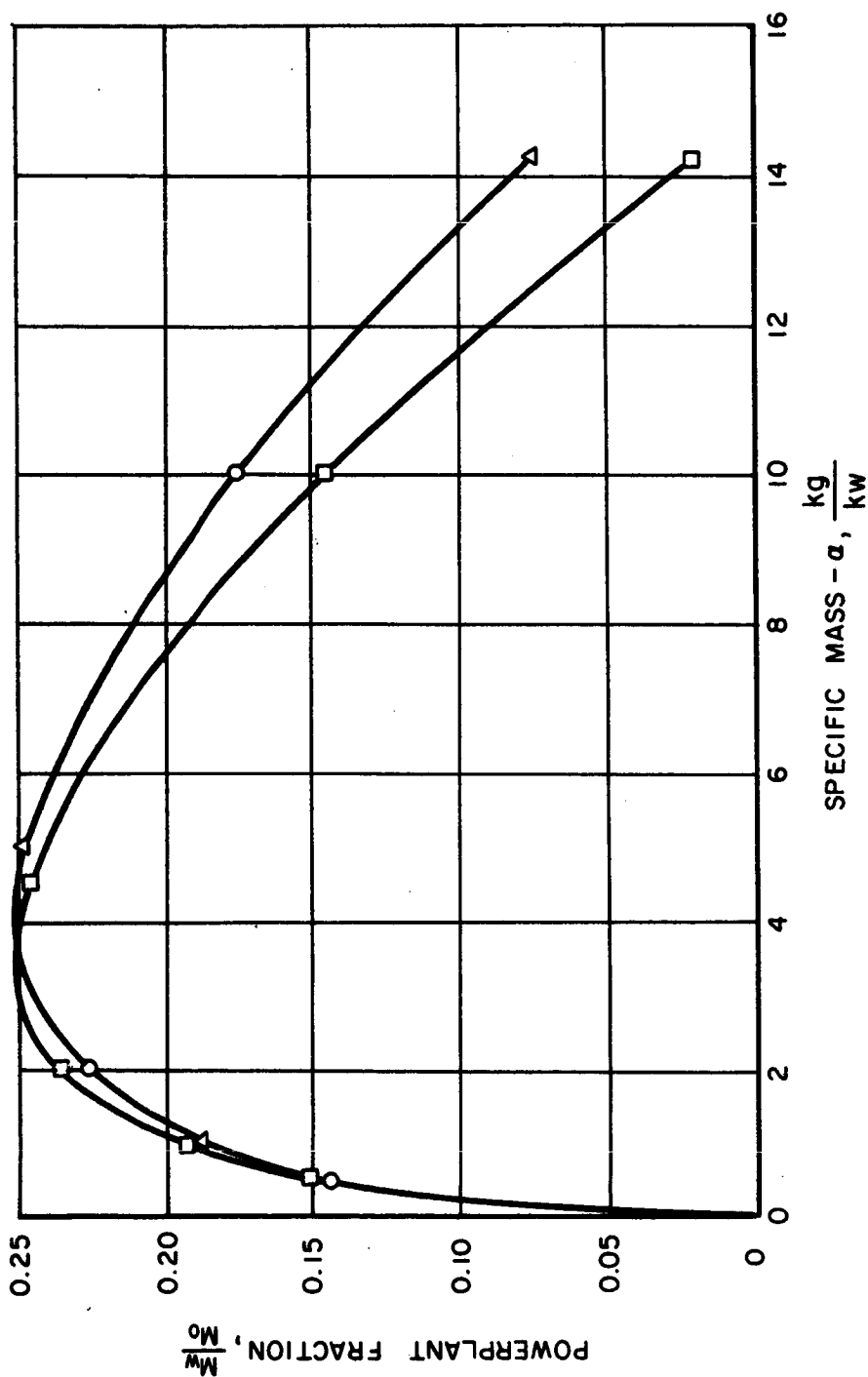
OPTIMAL LOW-THRUST TRAJECTORIES VENUS FLYBY

NOTE: THE TRAJECTORIES ARE REPRESENTED
HERE BY ONE CURVE AS THERE ARE
ONLY SLIGHT DIFFERENCES AMONG THEM



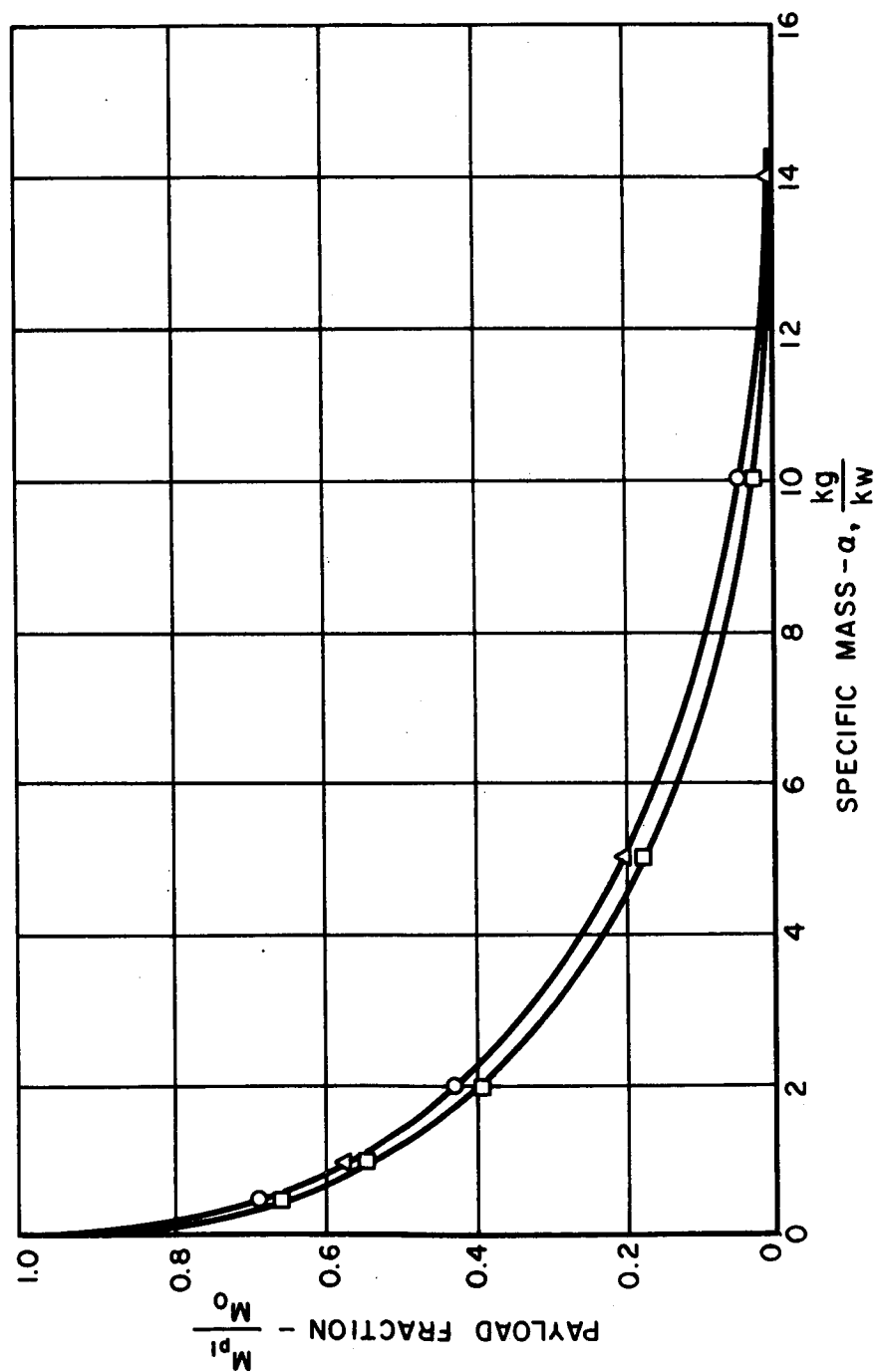
POWERPLANT FRACTION VS SPECIFIC MASS
VENUS FLYBY (5470-5640)

- $\circ$ $P = P_0$
 $\square$ $P = P_0 e^{-\gamma t}, \gamma = .290$
 Δ $P = P_0 / r^2$



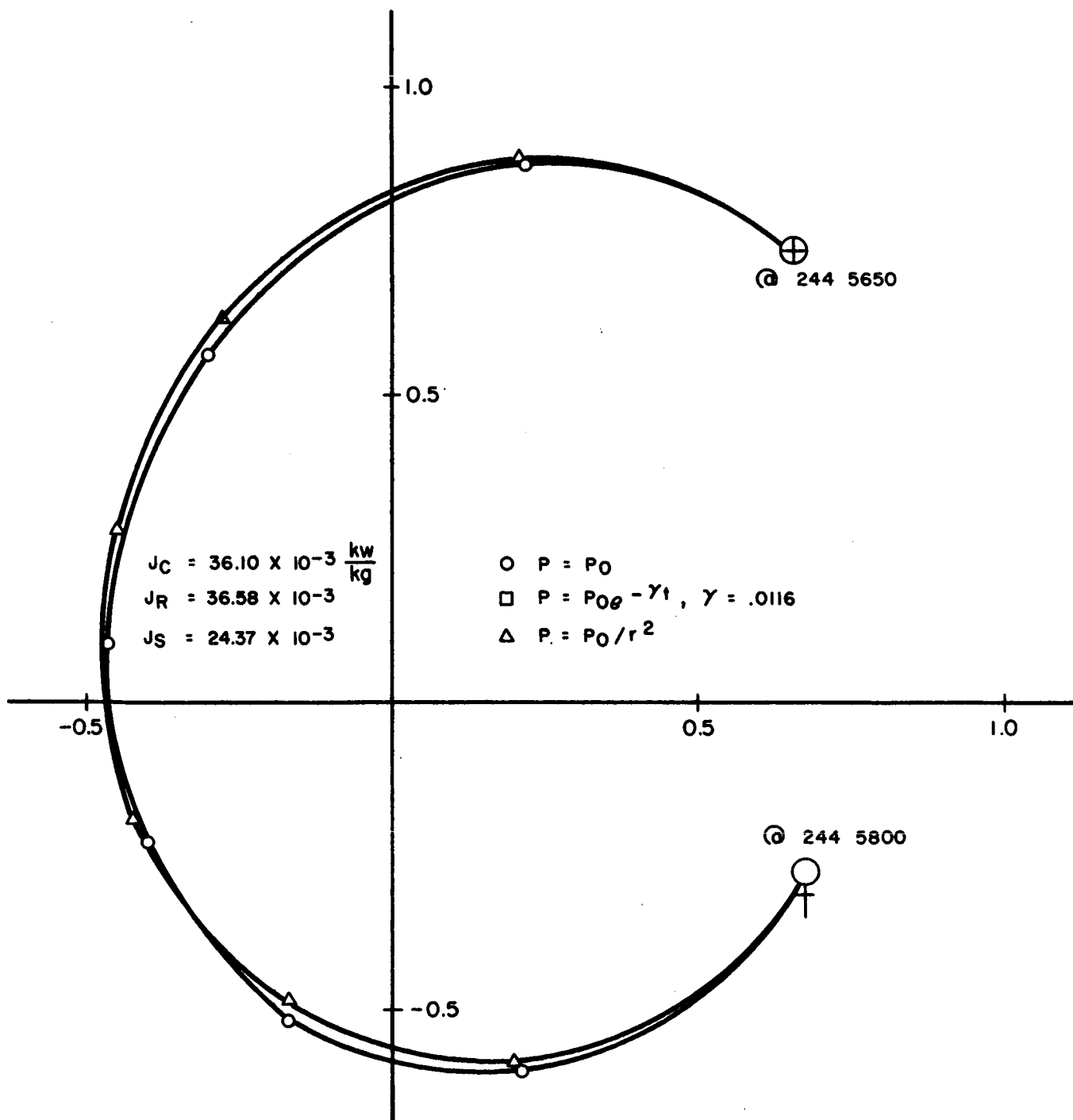
PAYLOAD FRACTION VS SPECIFIC MASS VENUS FLYBY (5470-5640)

- $\circ J_c = .5973 \times 10^{-3} \frac{kw}{kg}$
 $\square J_R = .6855 \times 10^{-3}$
 $\triangle J_S = .5927 \times 10^{-3}$



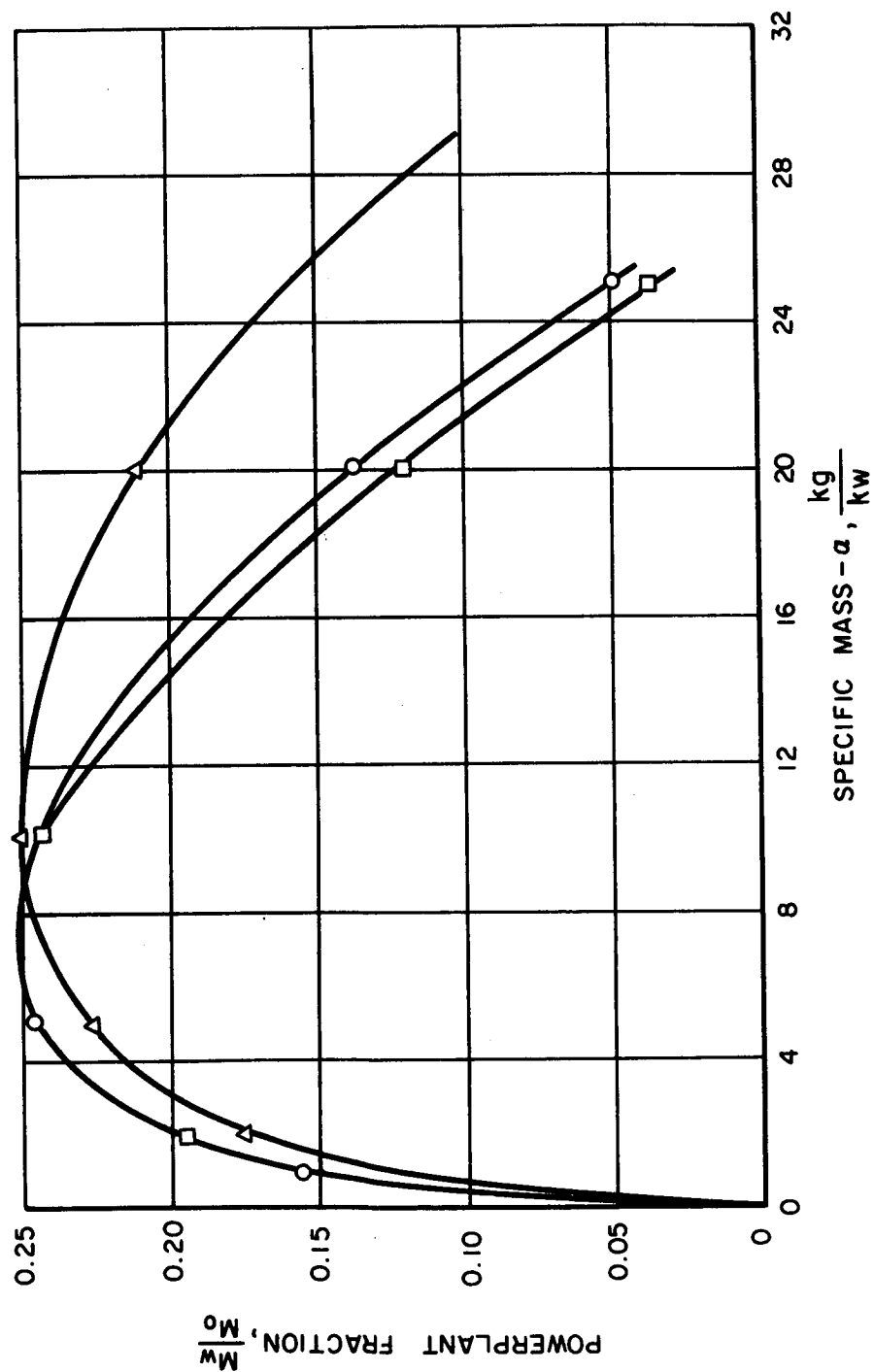
OPTIMAL LOW-THRUST TRAJECTORIES VENUS RENDEZVOUS

NOTE: THE CONSTANT POWER AND RADIOISOTOPE POWER
TRAJECTORIES COINCIDE WITH SMALL ERROR



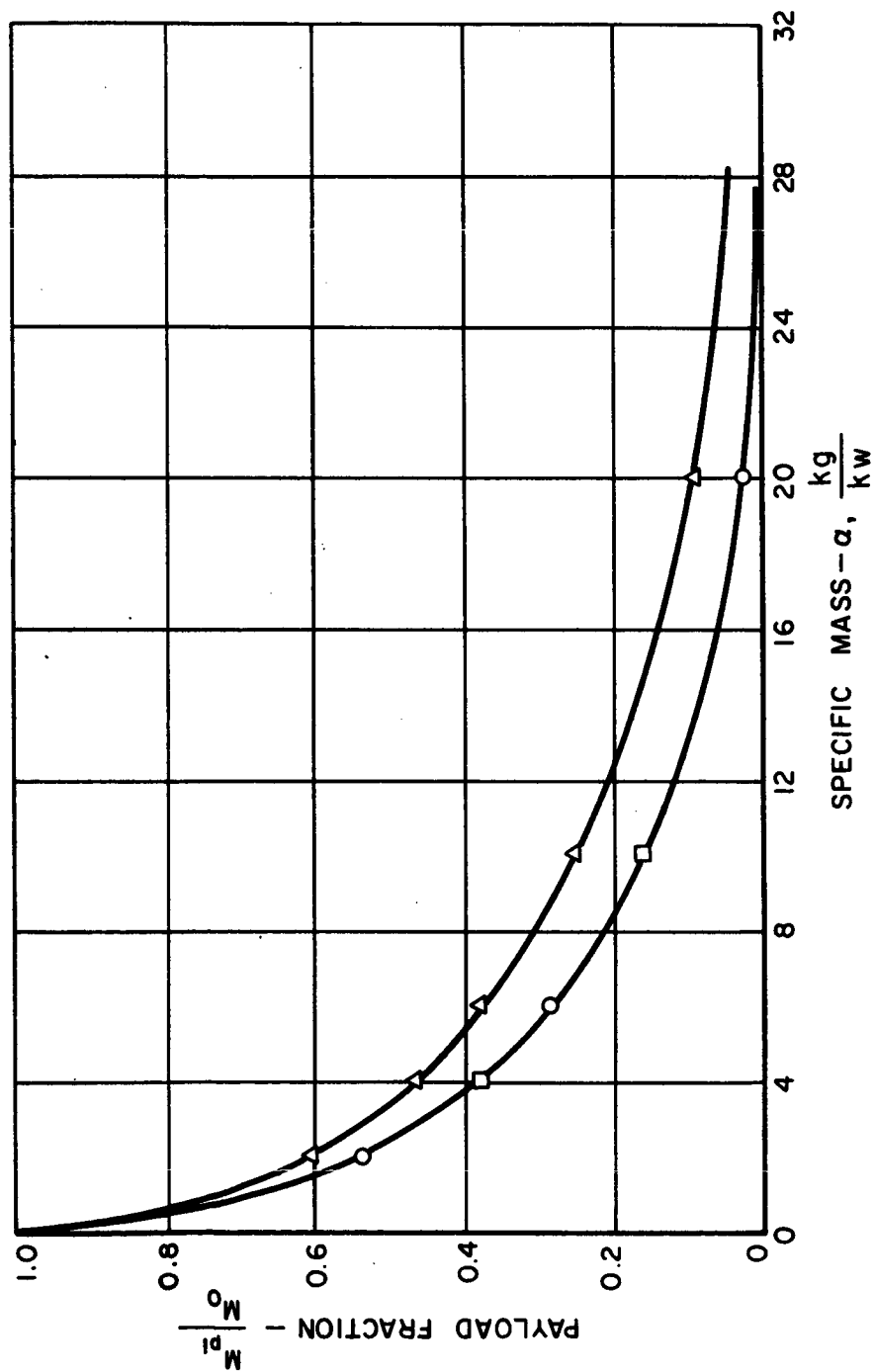
POWERPLANT FRACTION VS SPECIFIC MASS VENUS RENDEZVOUS (5650 - 5800)

- $\circ$ $P = P_0$
 $\square$ $P = P_0 \theta^{-\gamma_1}, \gamma = .0116$
 $\triangle$ $P = P_0 / r^2$



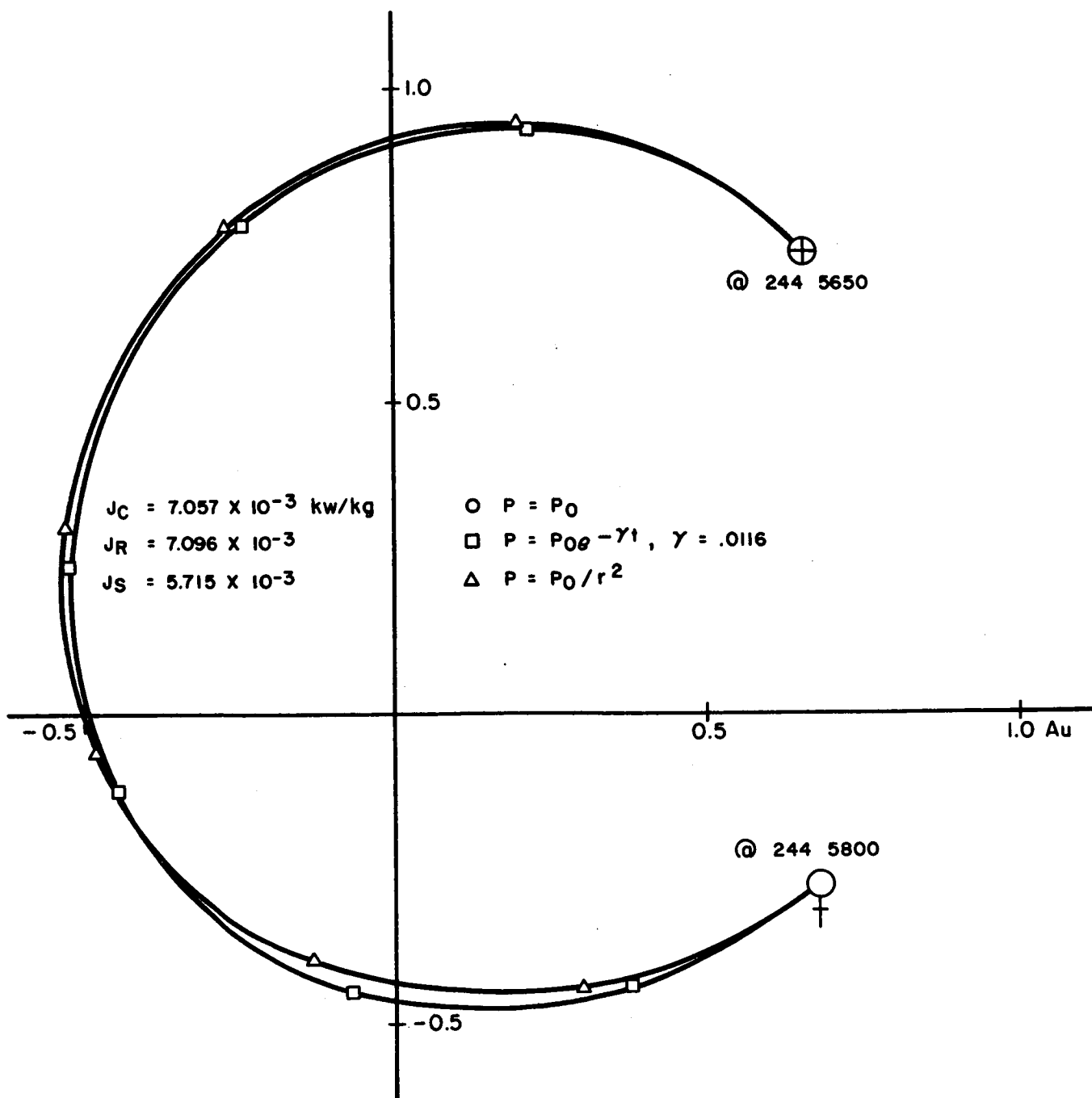
PAYLOAD FRACTION VS SPECIFIC MASS VENUS RENDEZVOUS (5650-5800)

- $\circ J_c = 36.10 \times 10^{-3} \frac{\text{kw}}{\text{kg}}$
 $\square J_R = 36.58 \times 10^{-3}$
 $\triangle J_S = 24.37 \times 10^{-3}$



OPTIMAL LOW-THRUST TRAJECTORIES VENUS FLYBY

NOTE: THE CONSTANT POWER AND RADIOISOTOPE POWER
TRAJECTORIES COINCIDE WITH SMALL ERROR



POWERPLANT FRACTION VS SPECIFIC MASS
VENUS FLYBY (5650-5800)

- $\circ$ $P = P_0$
 $\square$ $P = P_0 e^{-\gamma t}, \gamma = .0116$
 Δ $P = P_0 / r^2$

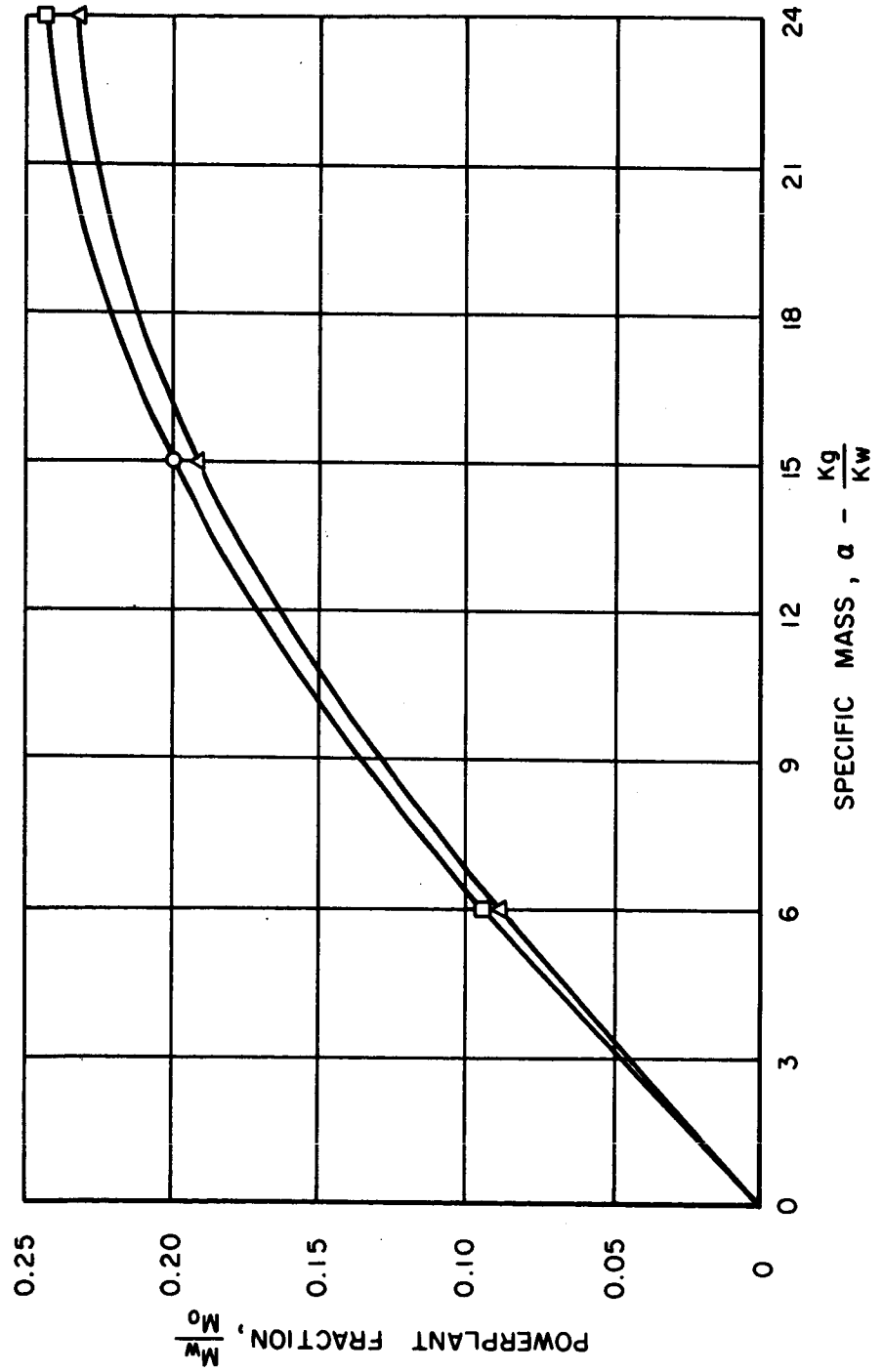


FIG 516

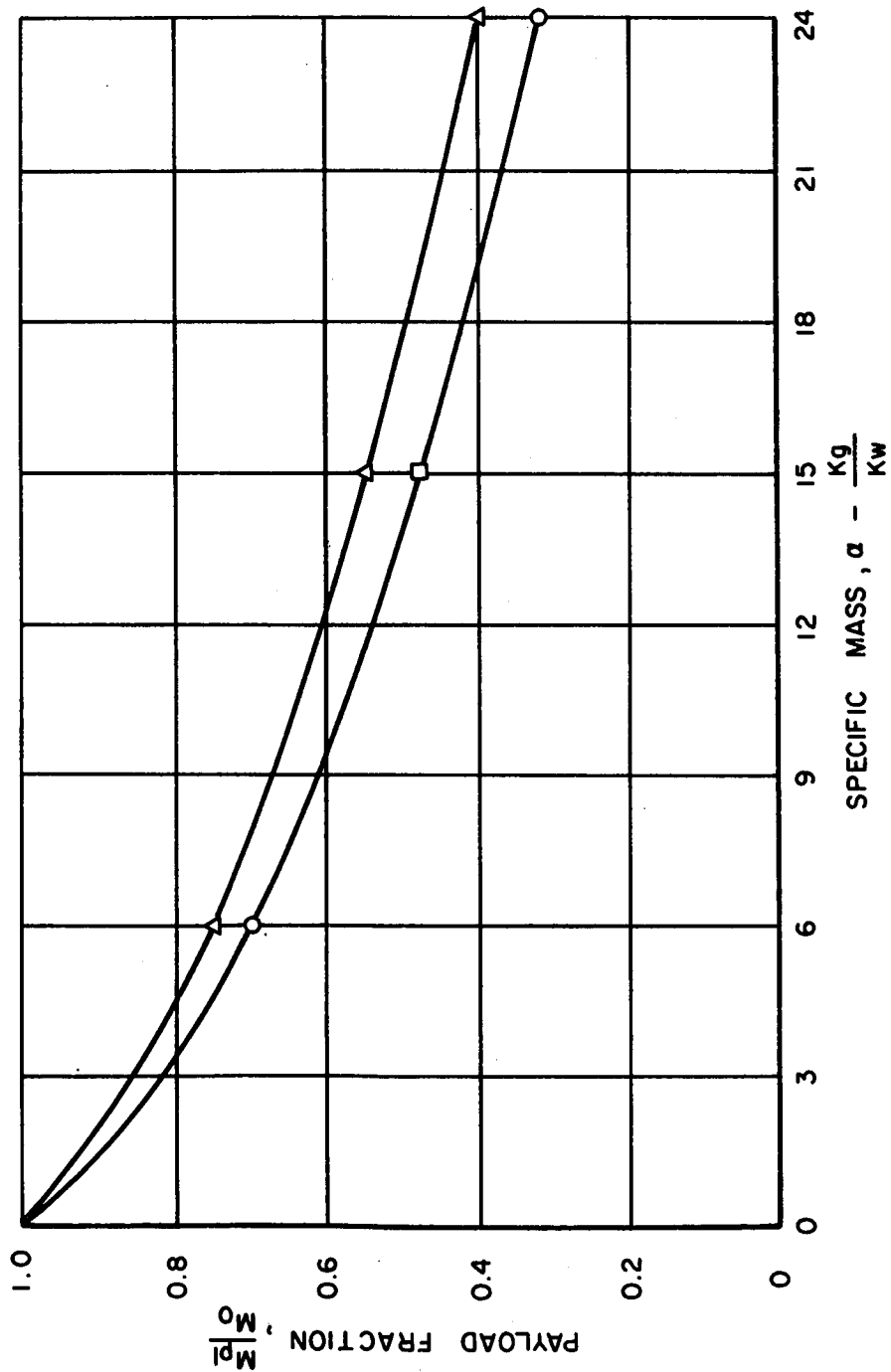
E-910352-9

PAYLOAD FRACTION VS SPECIFIC MASS
VENUS FLYBY (5650 - 5800)

$$J_C = 7.057 \times 10^{-3} \frac{Kw}{Kg}$$

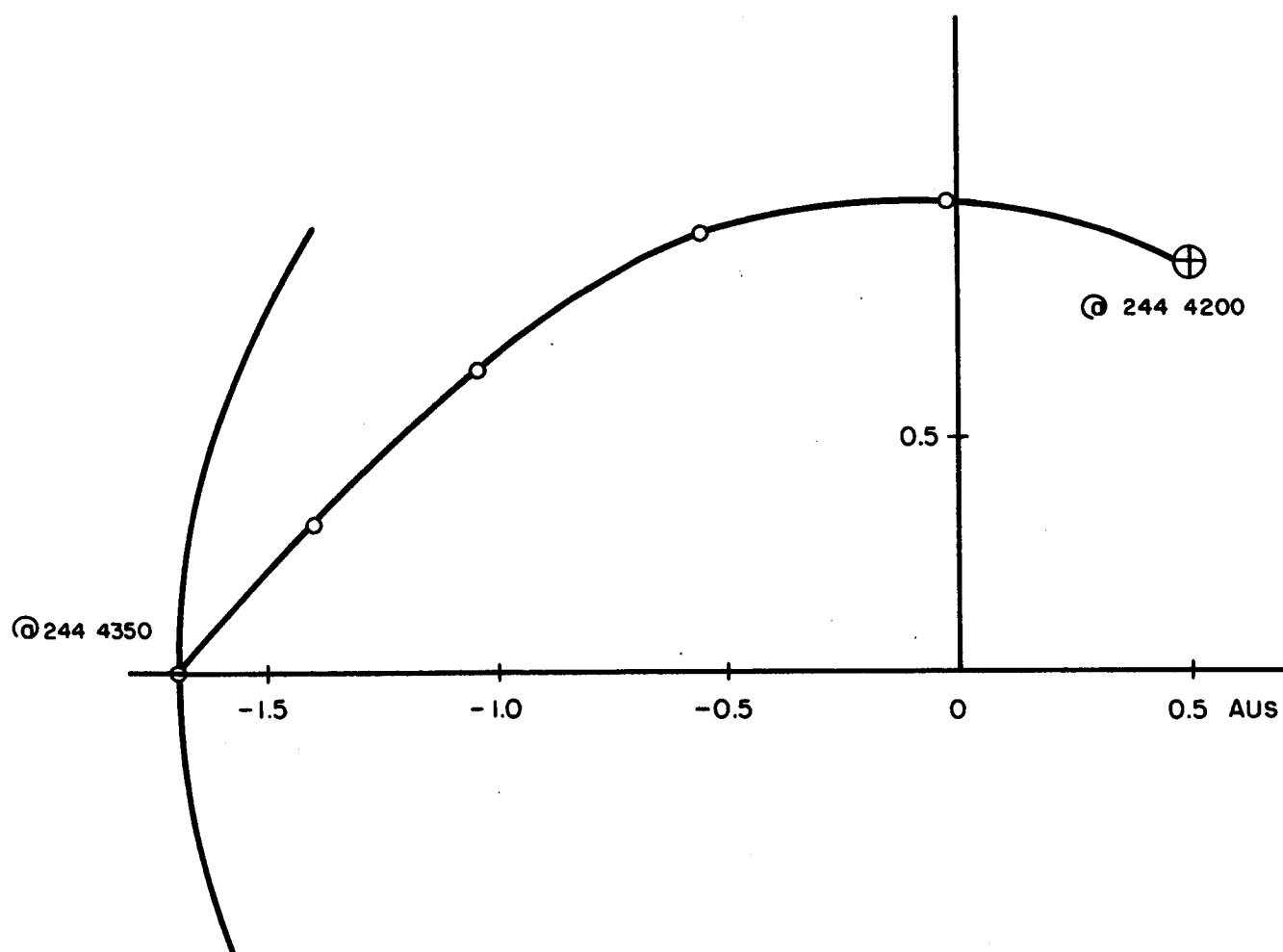
$$J_R = 7.096 \times 10^{-3}$$

$$J_S = 5.715 \times 10^{-3}$$



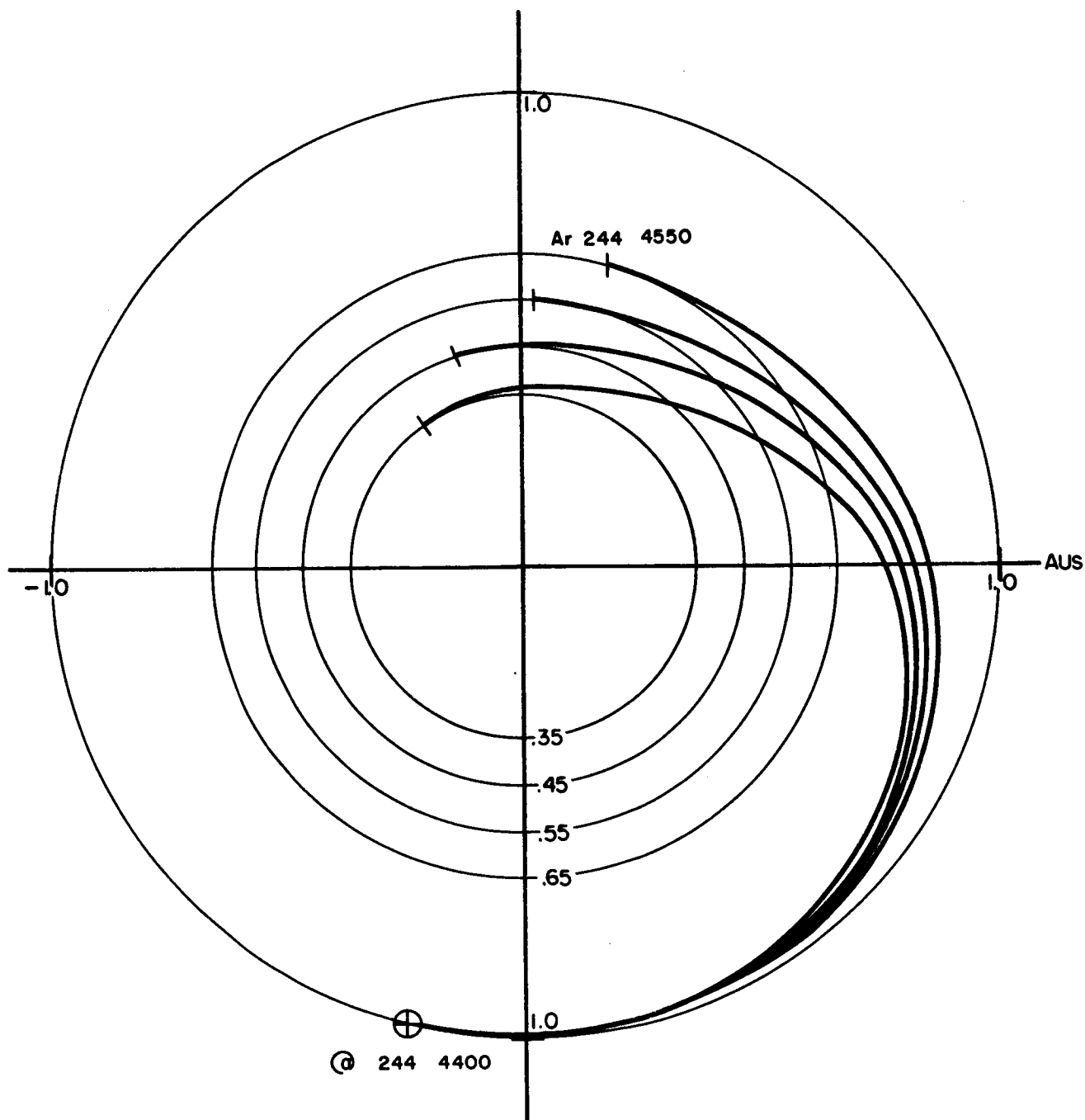
OPTIMUM VARIABLE THRUST PROBE
TO SPECIFIED RADIUS

$R = 1.70 \text{ AU}$



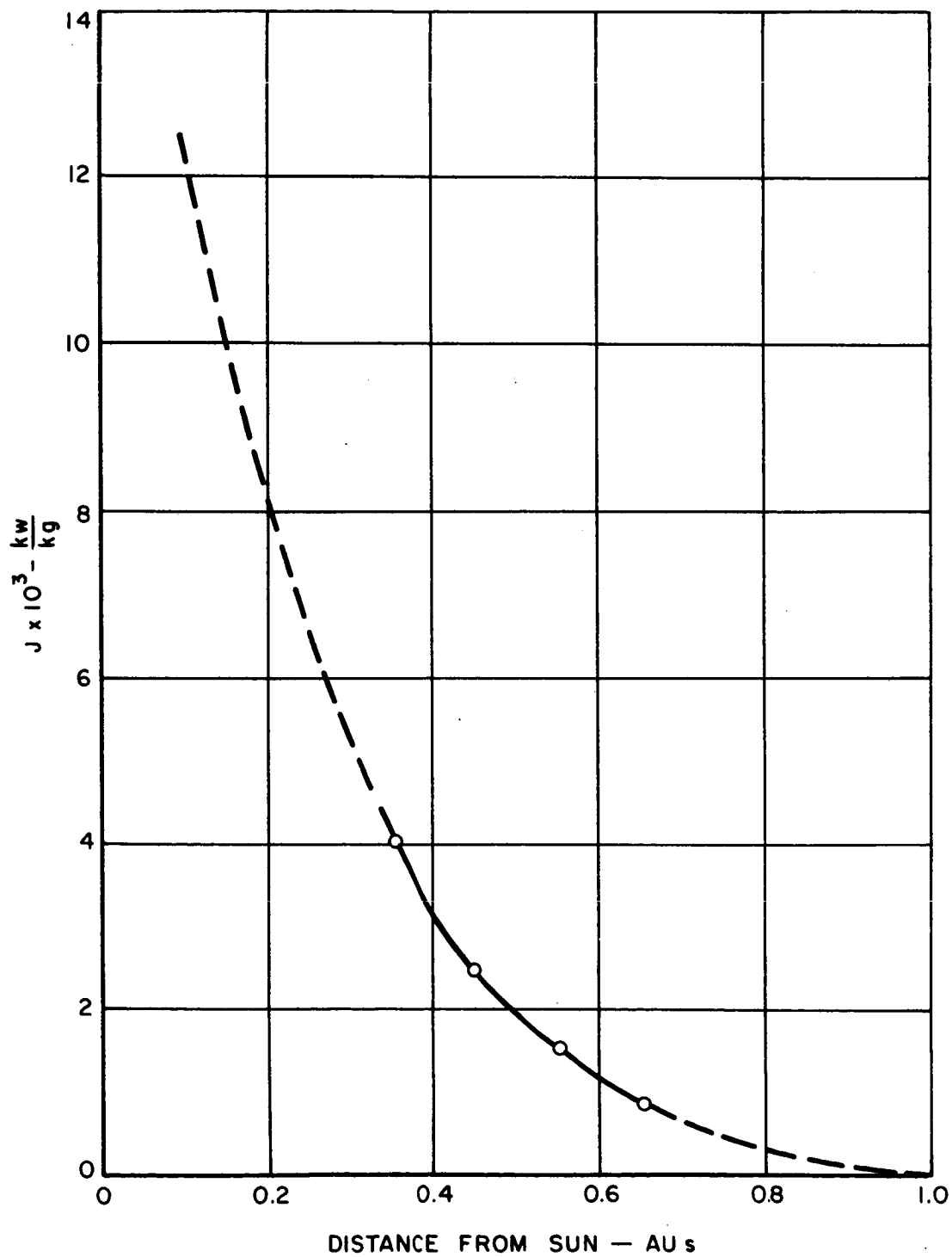
OPTIMUM VARIABLE THRUST PROBE TO SPECIFIED RADIUS

$R = .35 \text{ AU}$



J VS RADIUS OF PROBE

(4400 - 4550)

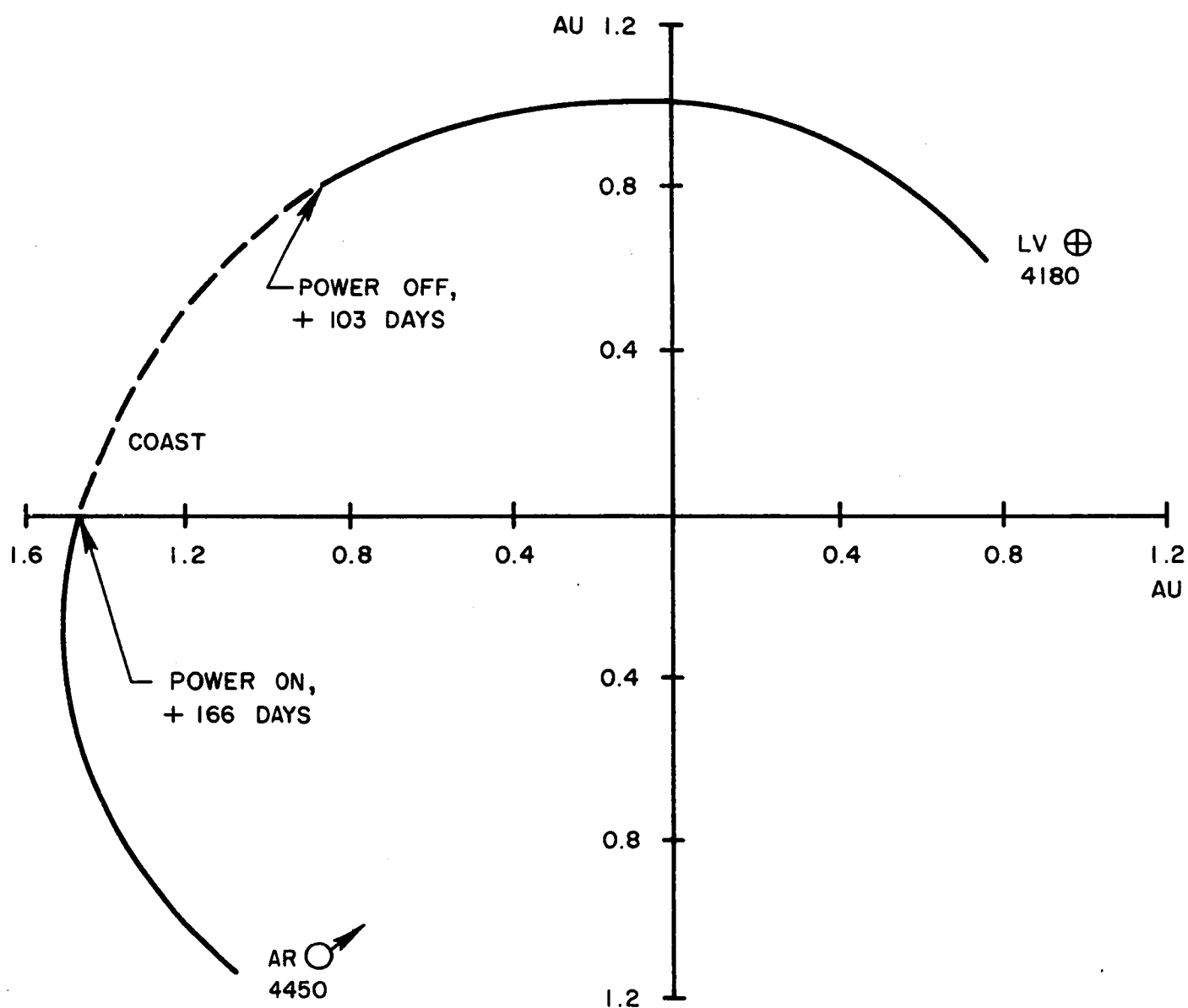


CONSTANT THRUST WITH COAST TRAJECTORY

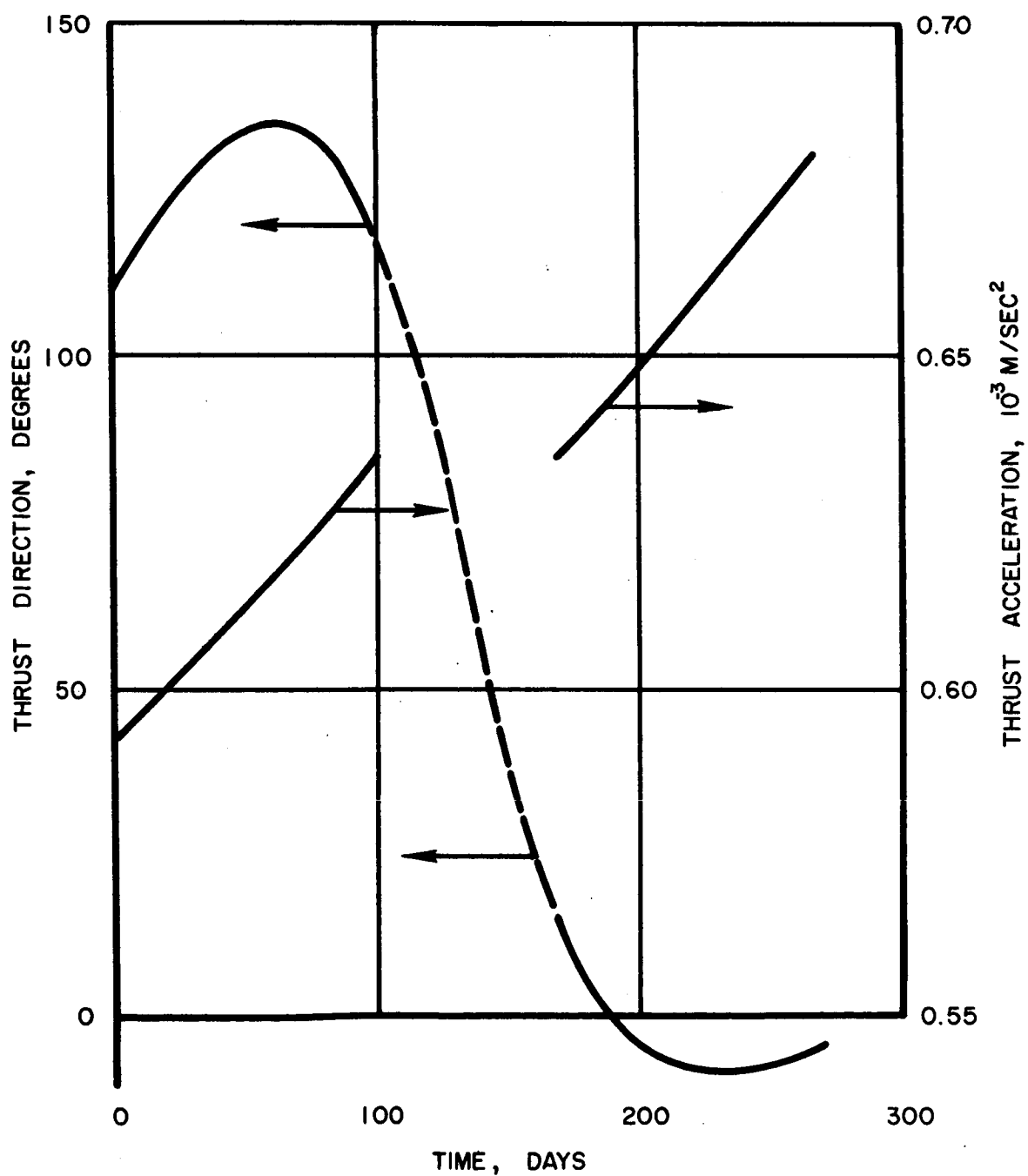
$$I_{sp} = 8000 \text{ SEC}$$

$$a_0 = 0.593 \times 10^3 \text{ M/SEC}^2$$

$$J = 6.976 \text{ M}^2/\text{SEC}^3$$



CONSTANT THRUST CONTROL TIME HISTORY

LV $\oplus$ 4180AR $\bigcirc$ 4450 $I_{sp} = 8000 \text{ SEC}$ $a_0 = 0.593 \times 10^{-3} \text{ M/SEC}^2$ $J = 6.976 \text{ M}^2/\text{SEC}^3$ 

COMPARISON OF VEHICLE PARAMETERS FOR MARS RENDEZVOUS MISSION:
RESULTS OF MELBOURNE AND SAUER VS RESULTS OF THE MODIFIED NEWTON-RAPHSON ALGORITHM

T (days)	α (kg/kw)	μ_{p1}		μ_w		μ_1		$C \times 10^{-2}$ (km/sec)		$\int_0^T a^2 dt$ (m ² /sec ³)	
		M-S	N-R	M-S	N-R	M-S	N-R	M-S	N-R	M-S	N-R
270	1	0.935	0.931	0.033	0.035	0.967	0.965	1.847	1.620	2.178	2.450
	5	0.854	0.844	0.075	0.081	0.928	0.924	0.834	0.730	2.180	2.450
	10	0.791	0.775	0.110	0.122	0.901	0.897	0.602	0.526	2.185	2.450
	20	0.695	0.668	0.171	0.194	0.866	0.863	0.443	0.387	2.199	2.450
240	1	0.922	0.920	0.039	0.040	0.961	0.960	1.653	1.480	3.085	3.240
	5	0.826	0.820	0.089	0.094	0.915	0.914	0.751	0.668	3.090	3.240
	10	0.750	0.740	0.133	0.143	0.834	0.883	0.546	0.483	3.103	3.250
	20	0.634	0.613	0.208	0.232	0.842	0.845	0.408	0.359	3.136	3.250
210	1	0.904	0.904	0.048	0.048	0.952	0.951	1.488	1.358	4.758	4.840
	5	0.784	0.781	0.111	0.114	0.895	0.895	0.682	0.613	4.772	4.830
	10	0.690	0.682	0.166	0.176	0.856	0.859	0.500	0.444	4.804	4.830
	20	0.545	0.525	0.262	0.290	0.807	0.815	0.376	0.331	4.889	4.830
180	1	0.876	0.875	0.063	0.062	0.939	0.937	1.398	1.242	8.082	8.080
	5	0.722	0.719	0.144	0.147	0.866	0.866	0.632	0.561	8.118	8.100
	10	0.600	0.591	0.214	0.228	0.814	0.819	0.459	0.407	8.205	8.120
	20	0.413	0.386	0.338	0.379	0.750	0.764	0.347	0.304	8.420	8.150

MIXED-THRUST EARTH-MARS TRANSFER

LV $\oplus$ 4215AR $\odot$ 4375 $V_{\infty A} = 0.163$ EMOS $V_{\infty B} = 0.260$ EMOS $J_m = 37.96 \text{ m}^2/\text{sec}^3$

$$\Gamma^2 = \frac{J}{J_m}$$

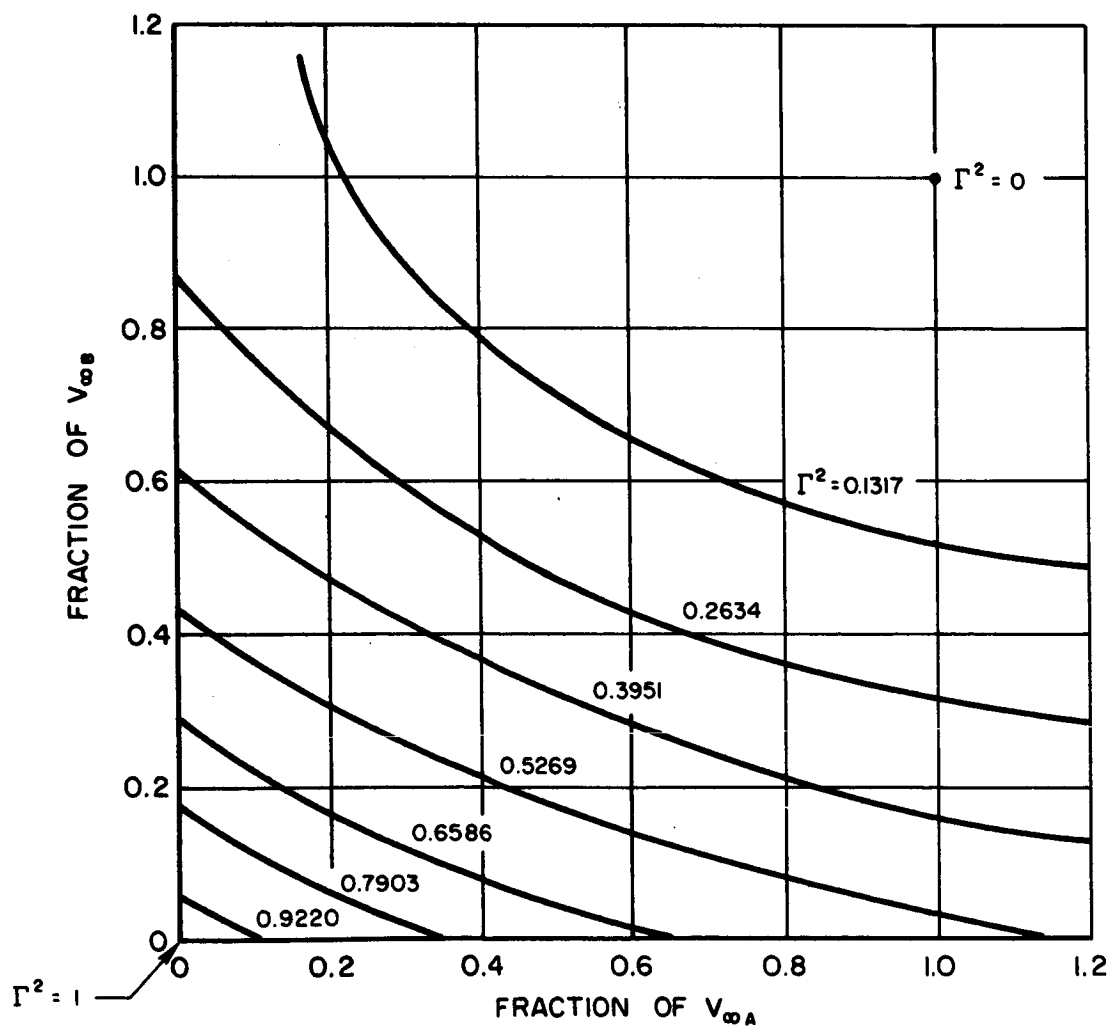


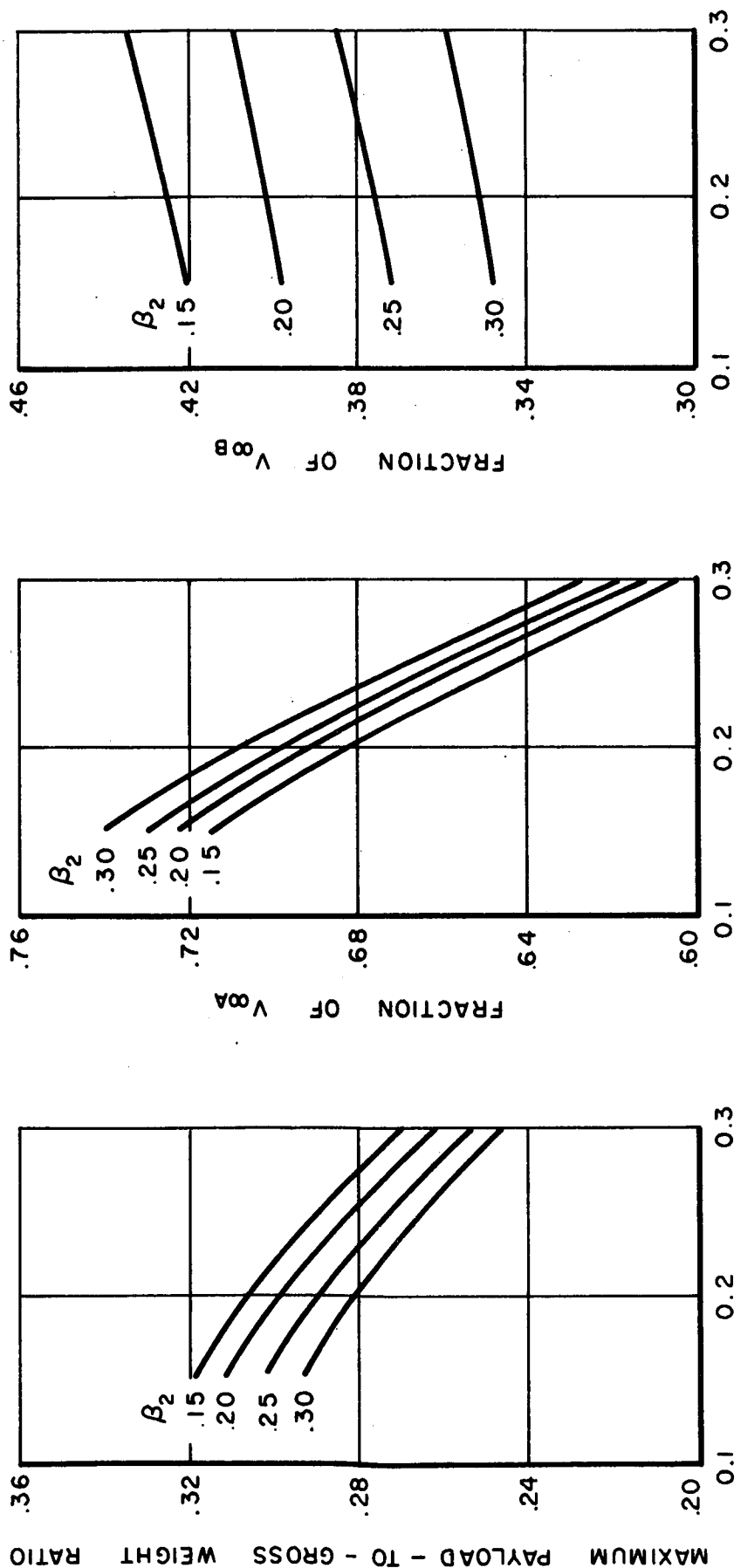
FIG. 524

E-910352-9

OPTIMIZATION OF HIGH - LOW - HIGH THRUST OPERATION

LV ⊕ 244 4215 AR ○ 244 4375
 $V_{\infty A} = 0.163$ EMOS $V_{\infty B} = 0.260$ EMOS

β_2 = ARRIVAL STEP STRUCTURAL FACTOR



HIGH THRUST STEP STRUCTURAL FACTOR

EFFECT OF SPECIFIC WEIGHT ON VEHICLE MASS

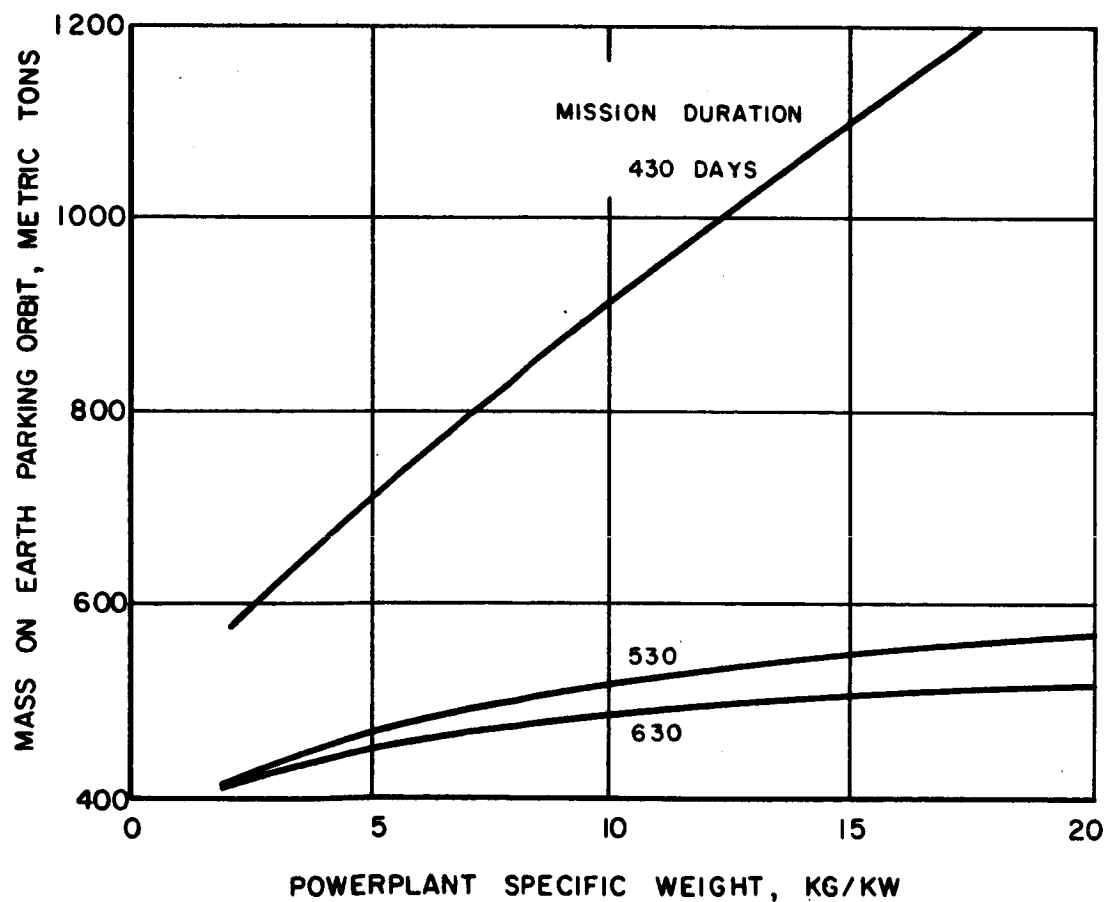
MARS ROUNDTRIP MISSION

OPTIMUM HYBRID-THRUST OPERATION

VARIABLE LOW-THRUST

NO POWER LOSS

30-DAY STAYTIME



EFFECT OF TRIP TIME ON VEHICLE MASS

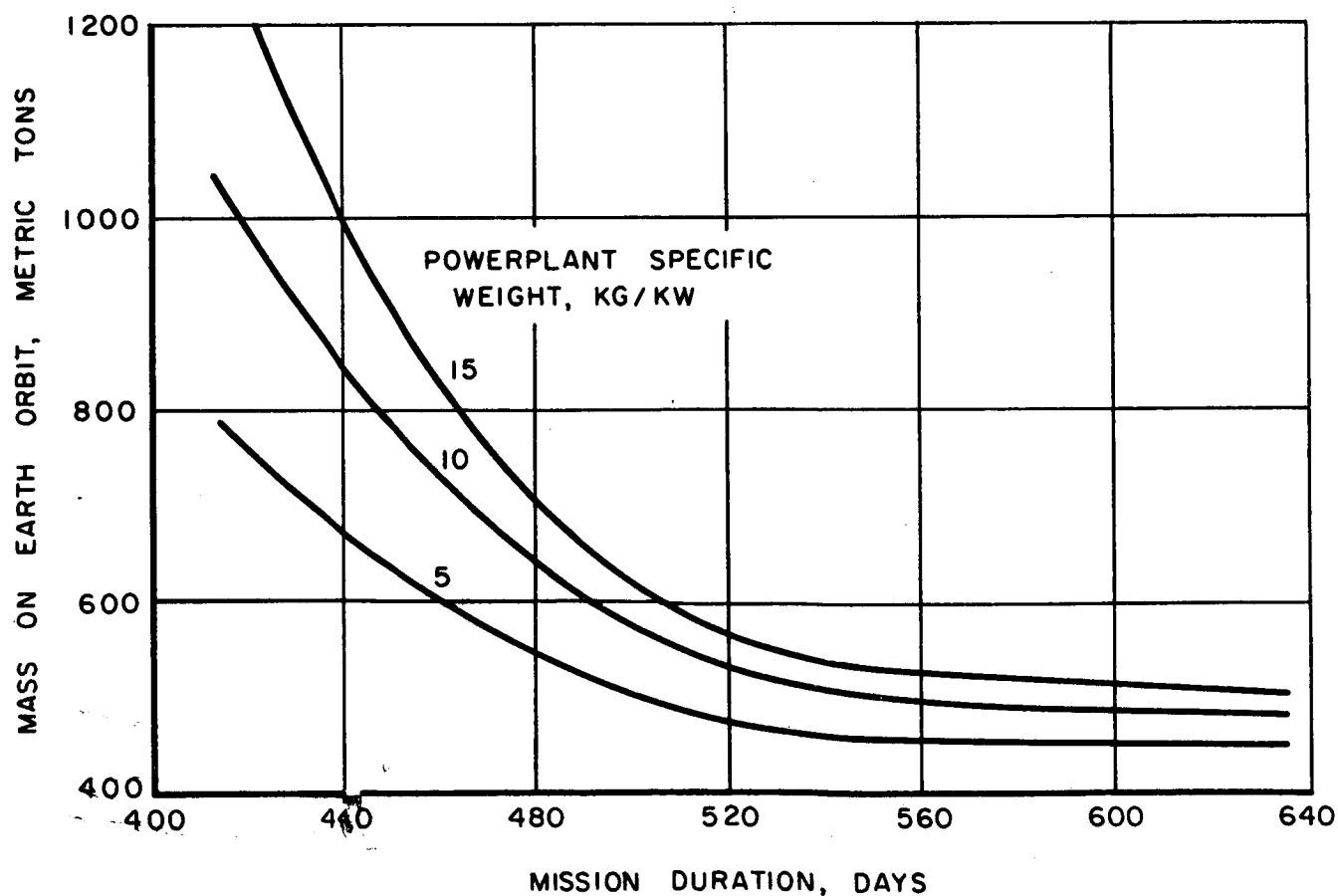
MARS ROUNDTRIP MISSION

OPTIMUM HYBRID-THRUST OPERATION

VARIABLE LOW THRUST

NO POWER LOSS

30 - DAY STAYTIME

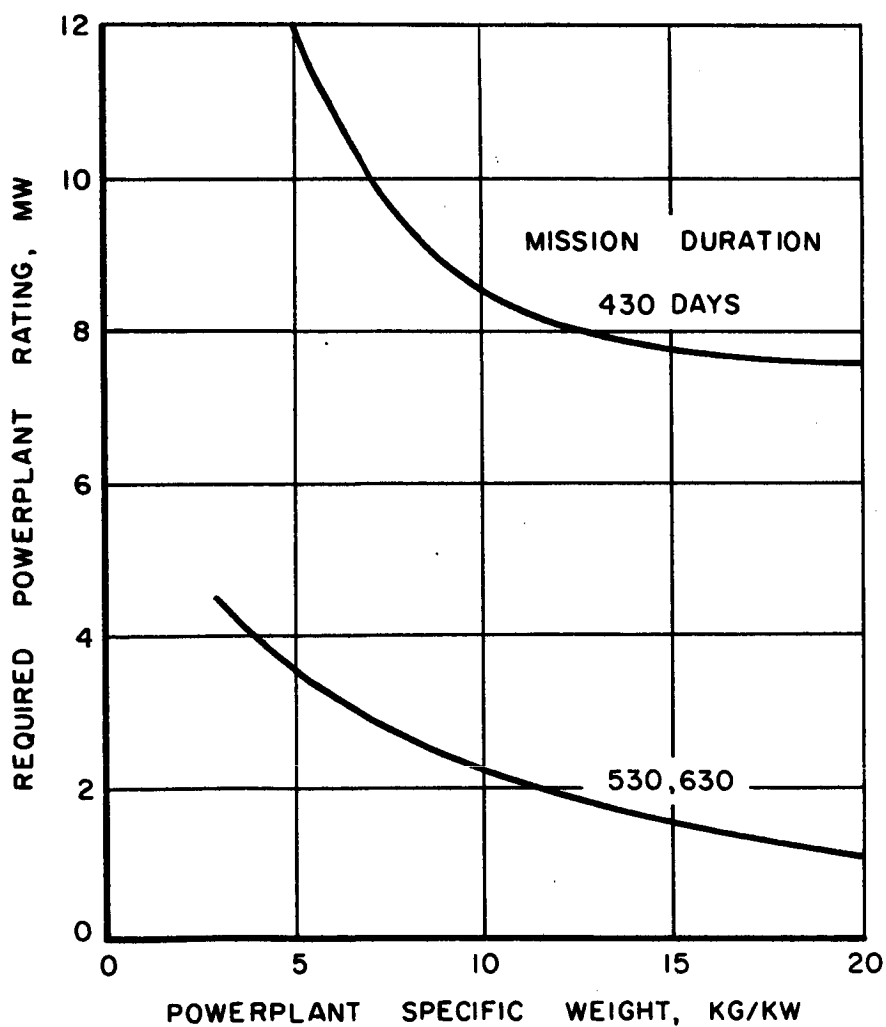


POWER REQUIREMENTS FOR MARS MISSION

OPTIMUM HYBRID-THRUST OPERATION

VARIABLE LOW THRUST

30 - DAY STAYTIME



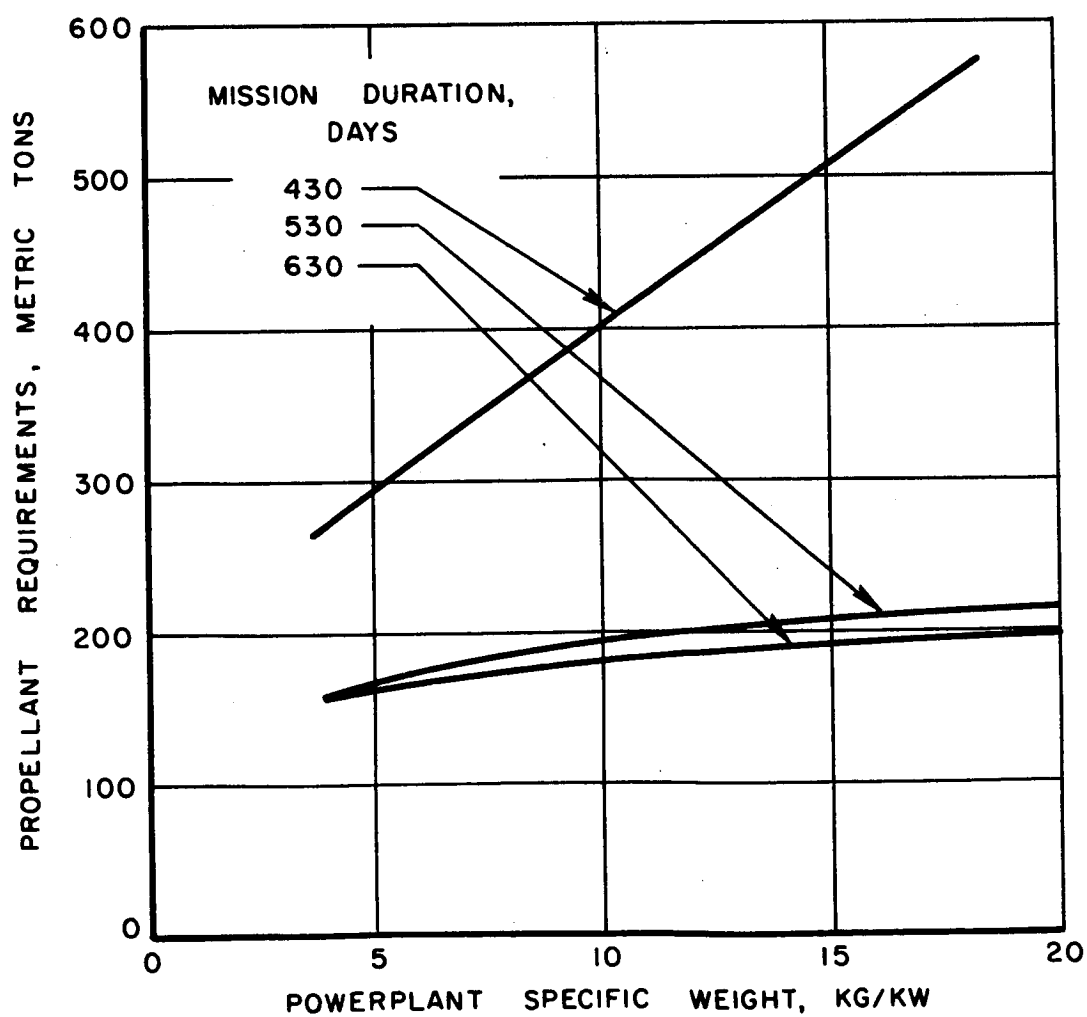
EFFECT OF POWERPLANT ON EARTH DEPARTURE NUCLEAR SYSTEM

OPTIMUM NUCLEAR + ELECTRIC OPERATION

MARS ROUNTRIP MISSION

VARIABLE LOW THRUST

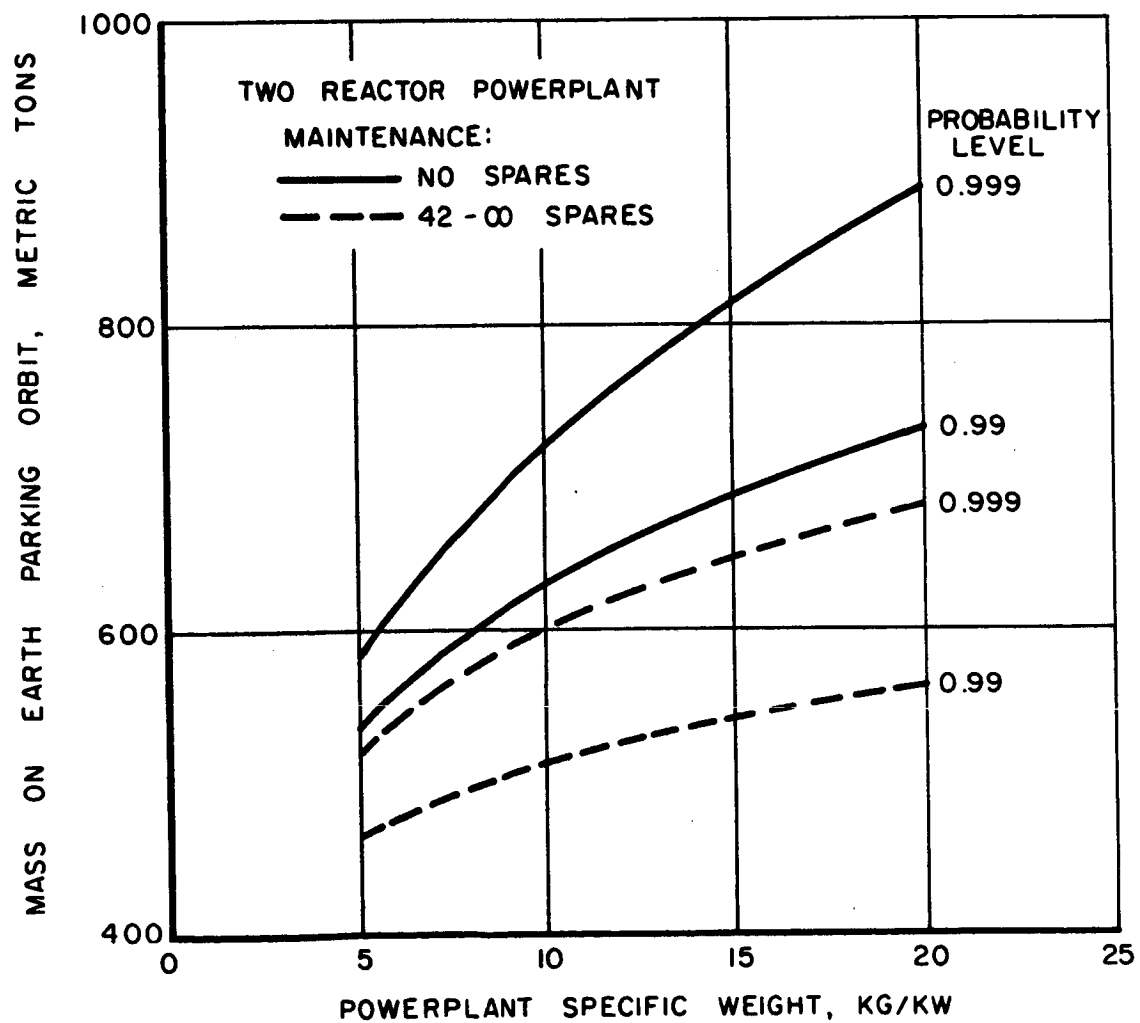
30 - DAY STAYTIME



EFFECT OF POWERPLANT ON VEHICLE MASS

530 DAY MISSION

OPTIMUM NUC + ELECT OPERATION

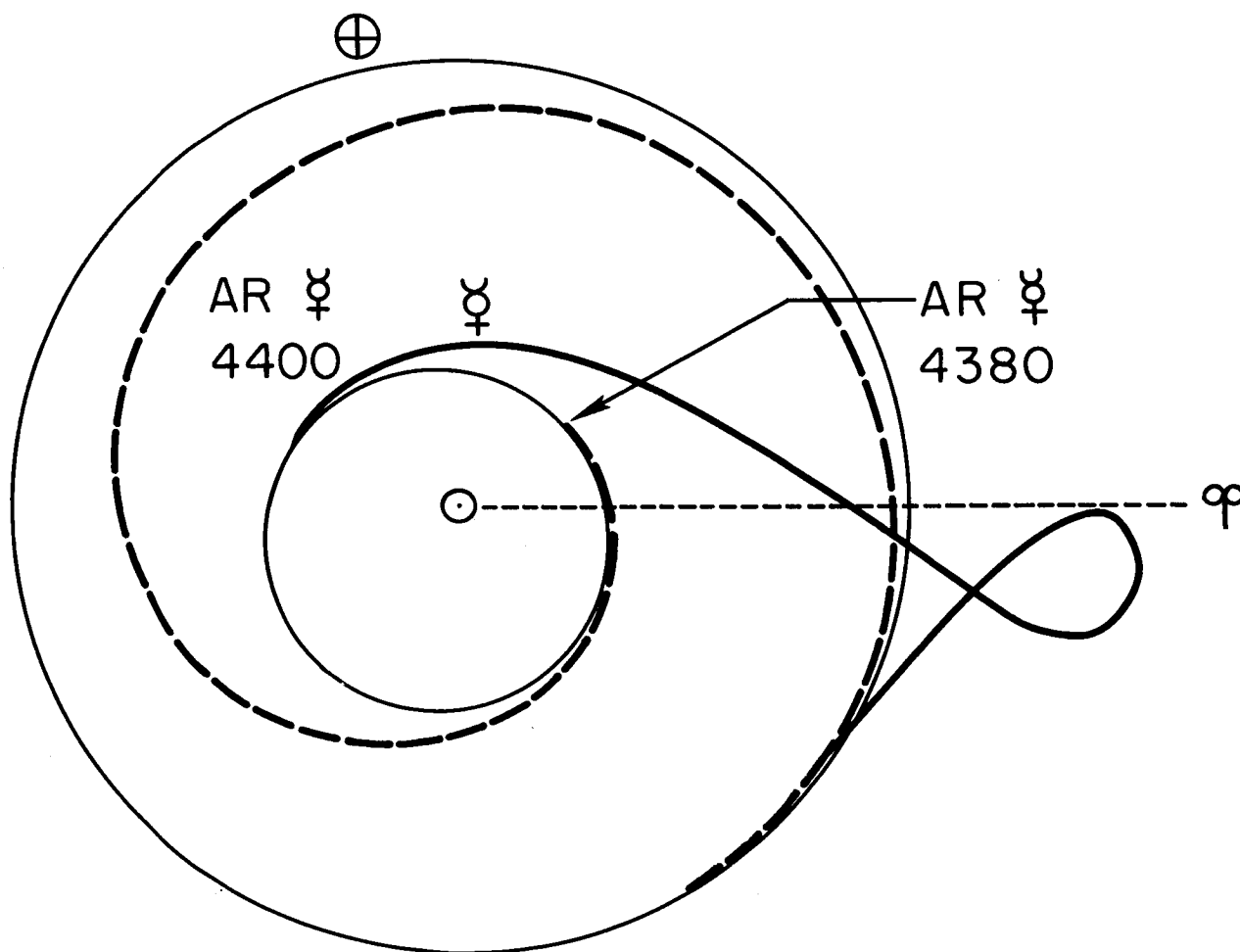


STATIONARY SOLUTIONS

300-DAY MERCURY TRANSFERS

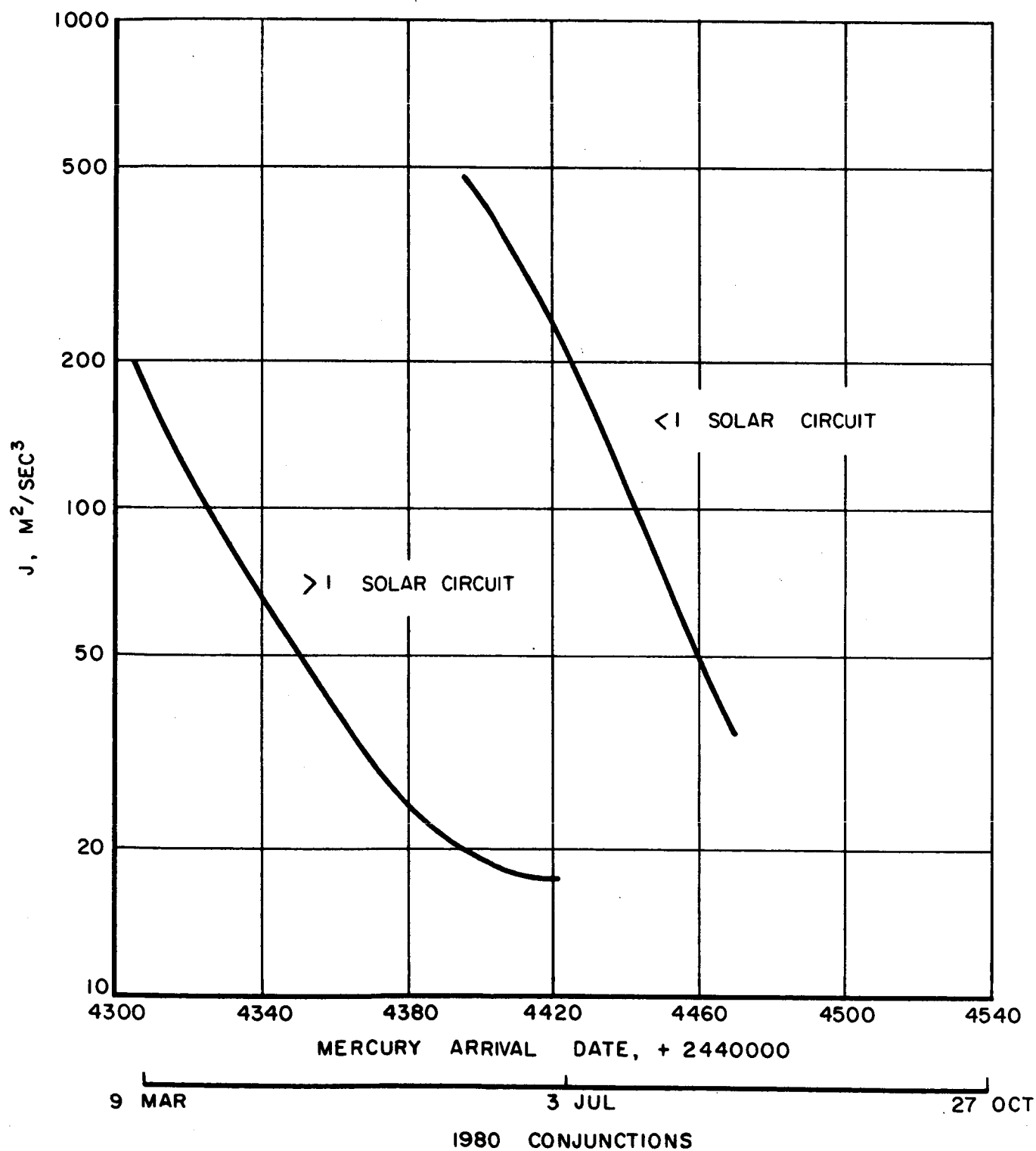
———— $J = 438 \text{ m}^2/\text{sec}^3$

----- $J = 24.5 \text{ m}^2/\text{sec}^3$



EARTH - MERCURY TRAJECTORY REQUIREMENTS

VARIABLE LOW THRUST
300 - DAY TRANSFER TIME



MERCURY ORBITAL PROBES

HIGH + LOW + HIGH - THRUST

CHEMICAL PROPULSION, $I_{sp} = 430$ SECS

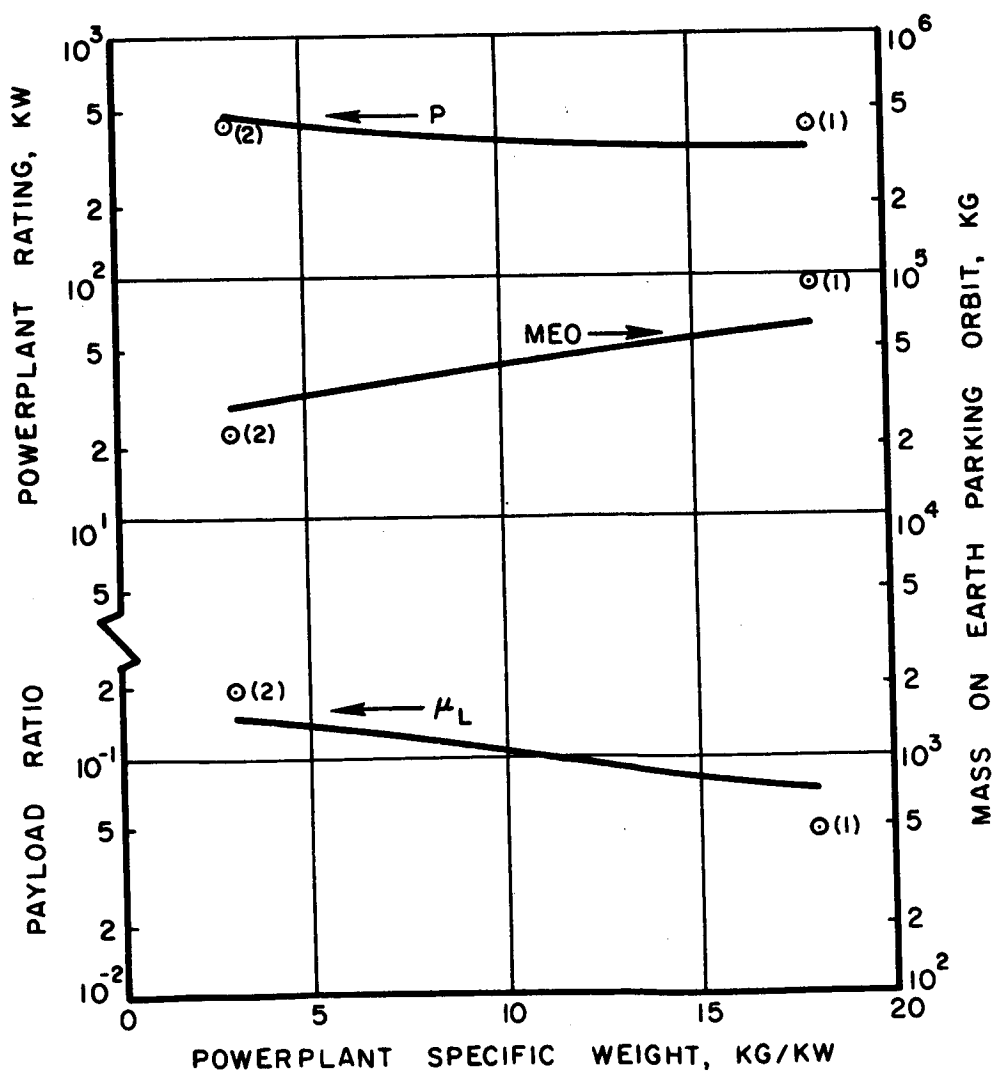
PAYLOAD = 4536 KG

NOMINAL A,B,C,Z

300 - DAY TRANSFER

(1) PESSIMISTIC SYSTEM

(2) OPTIMISTIC SYSTEM



MERCURY ORBITAL PROBES

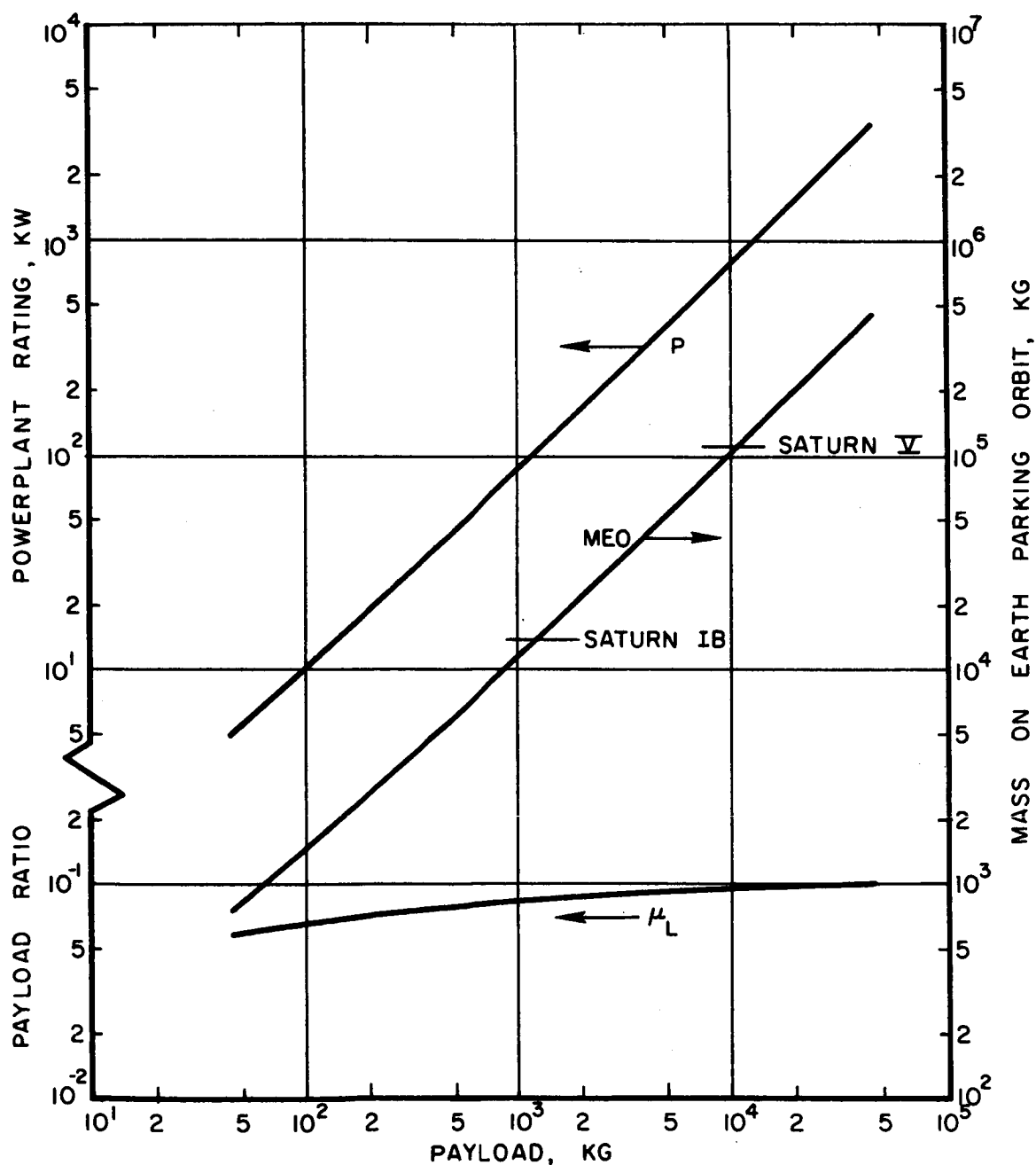
HIGH + LOW + HIGH - THRUST

CHEMICAL PROPULSION, $I_{sp} = 430$ SECS

POWERPLANT SPECIFIC WEIGHT = 12 KG/KW

NOMINAL A,B,C,Z

300 - DAY TRANSFER



MERCURY ORBITAL PROBES

HIGH + LOW + HIGH - THRUST

NUCLEAR PROPULSION, $I_{sp} = 800$ SECS

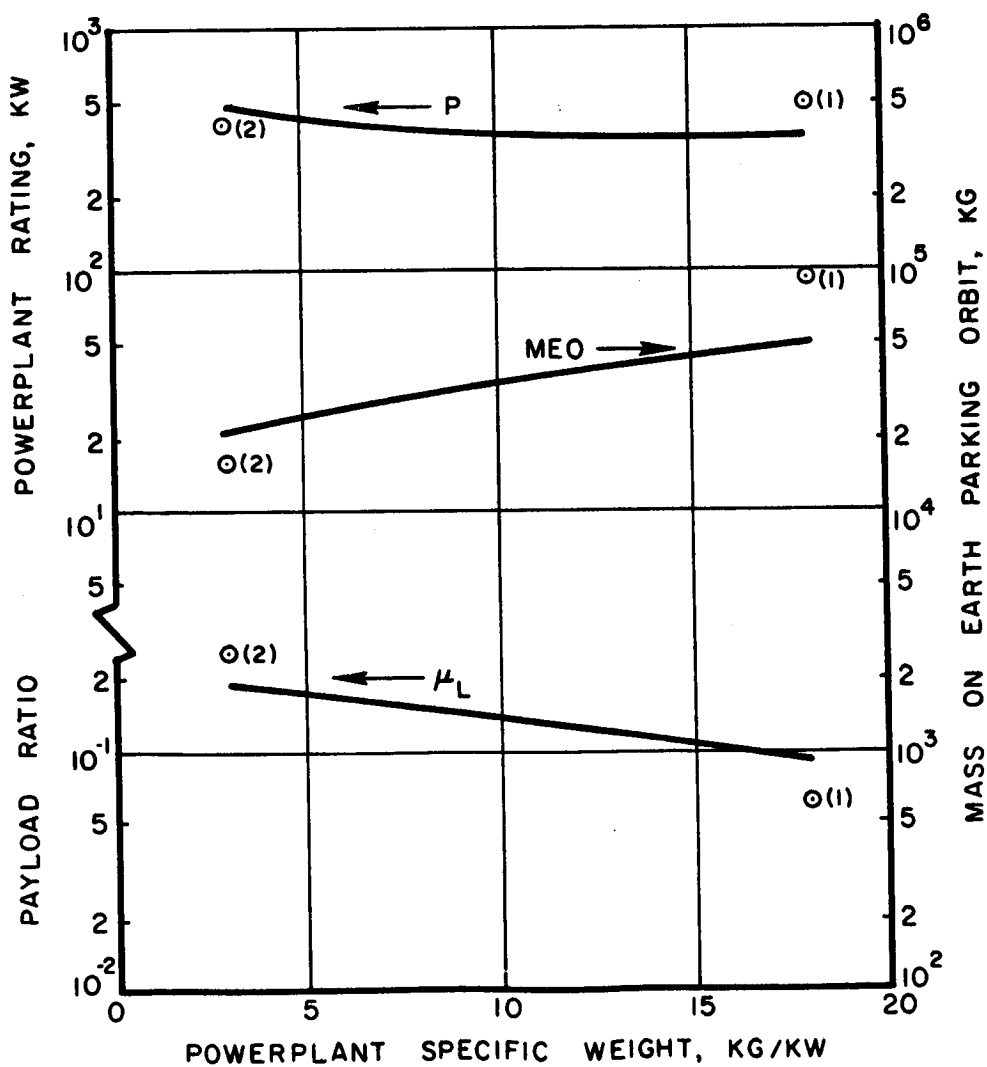
PAYLOAD = 4536 KG

NOMINAL A,B,C,Z

300 - DAY TRANSFER

(1) PESSIMISTIC SYSTEM

(2) OPTIMISTIC SYSTEM



MERCURY ORBITAL PROBES

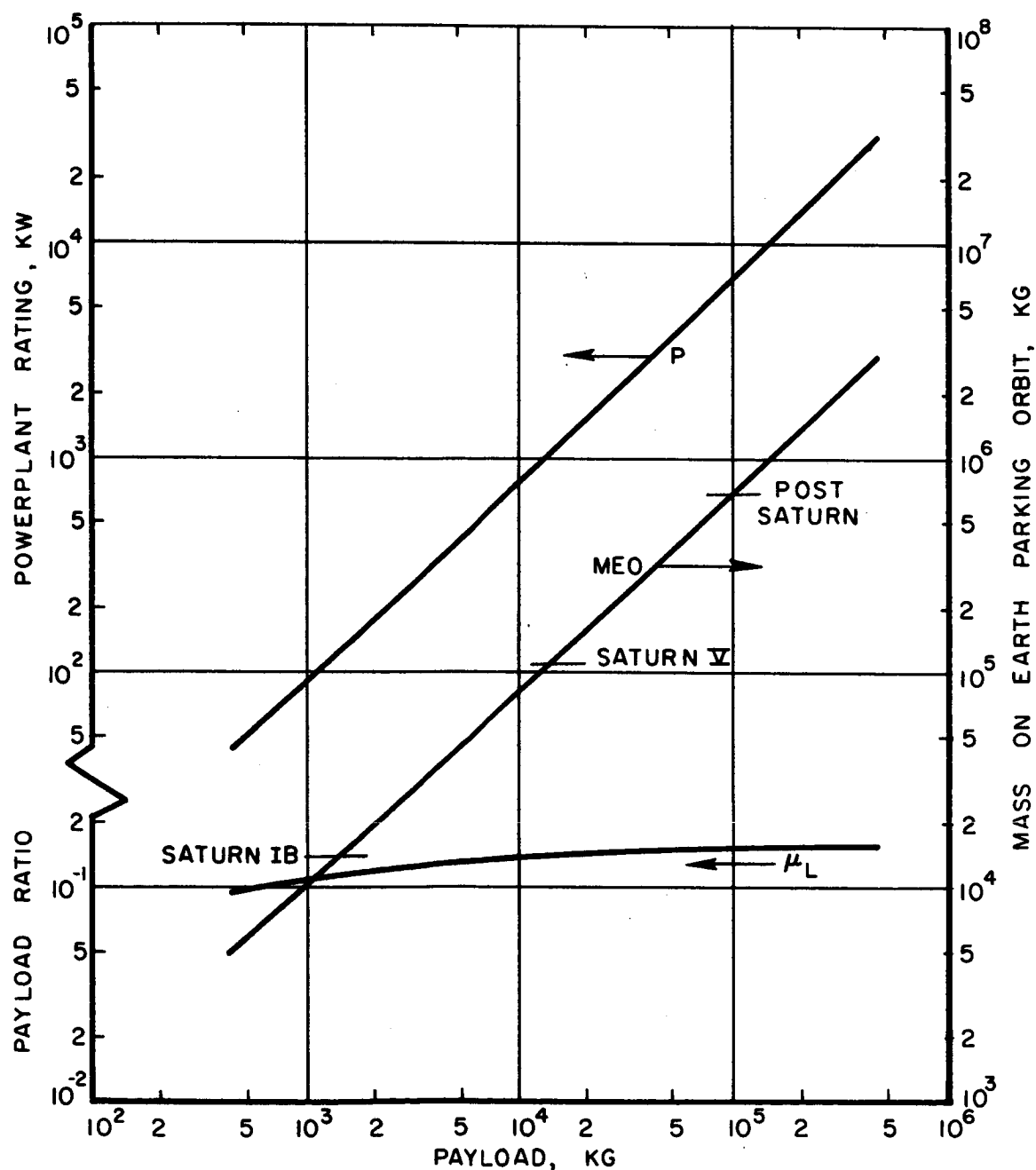
HIGH + LOW + HIGH - THRUST

NUCLEAR PROPULSION, $I_{sp} = 800$ SECS

POWERPLANT SPECIFIC WEIGHT = 12 KG/KW

NOMINAL A,B,C,Z

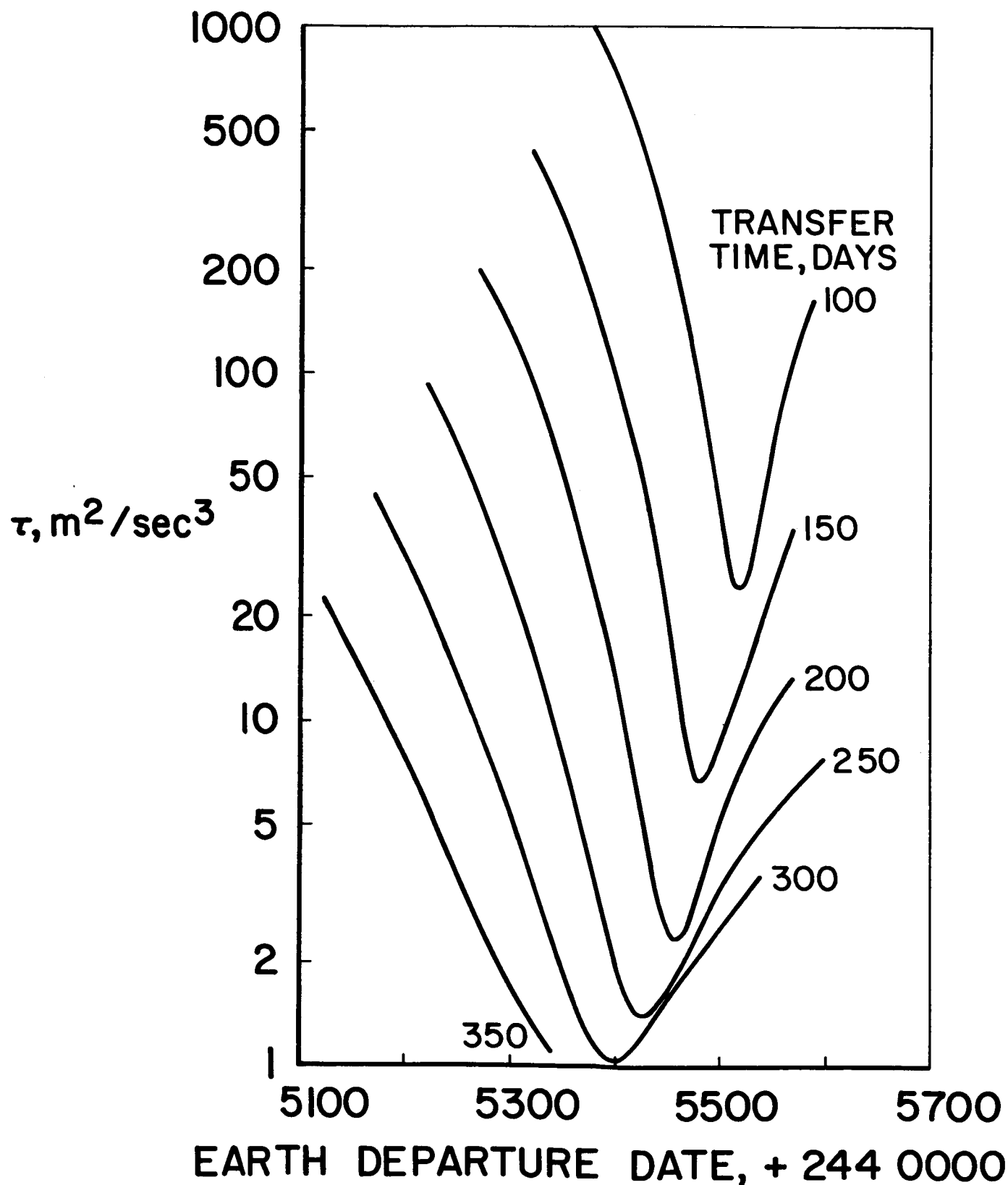
300 - DAY TRANSFER



EARTH-VENUS TRAJECTORY REQUIREMENTS

VARIABLE LOW THRUST

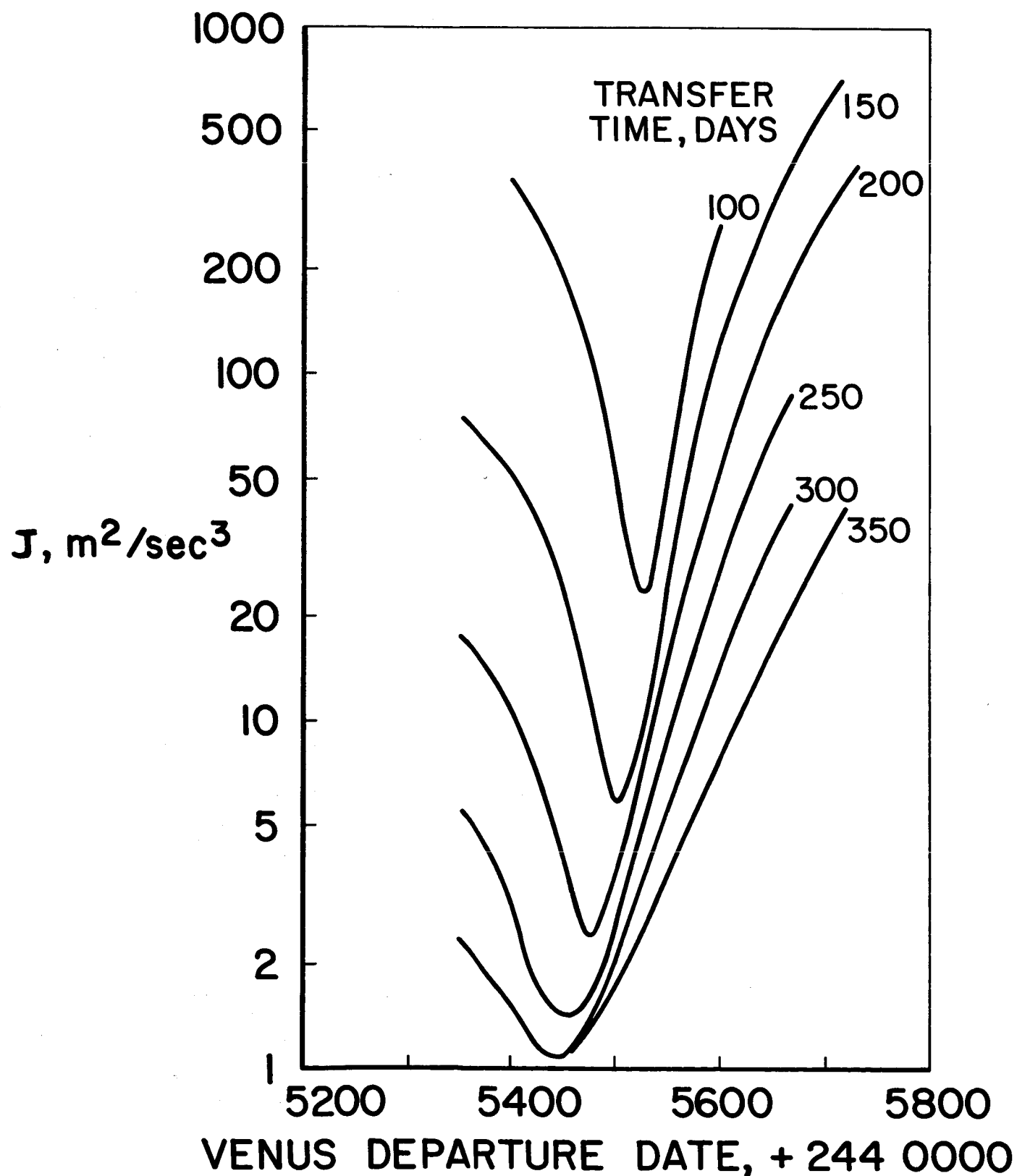
CONJUNCTION 25 AUG 1983



VENUS-EARTH TRAJECTORY REQUIREMENTS

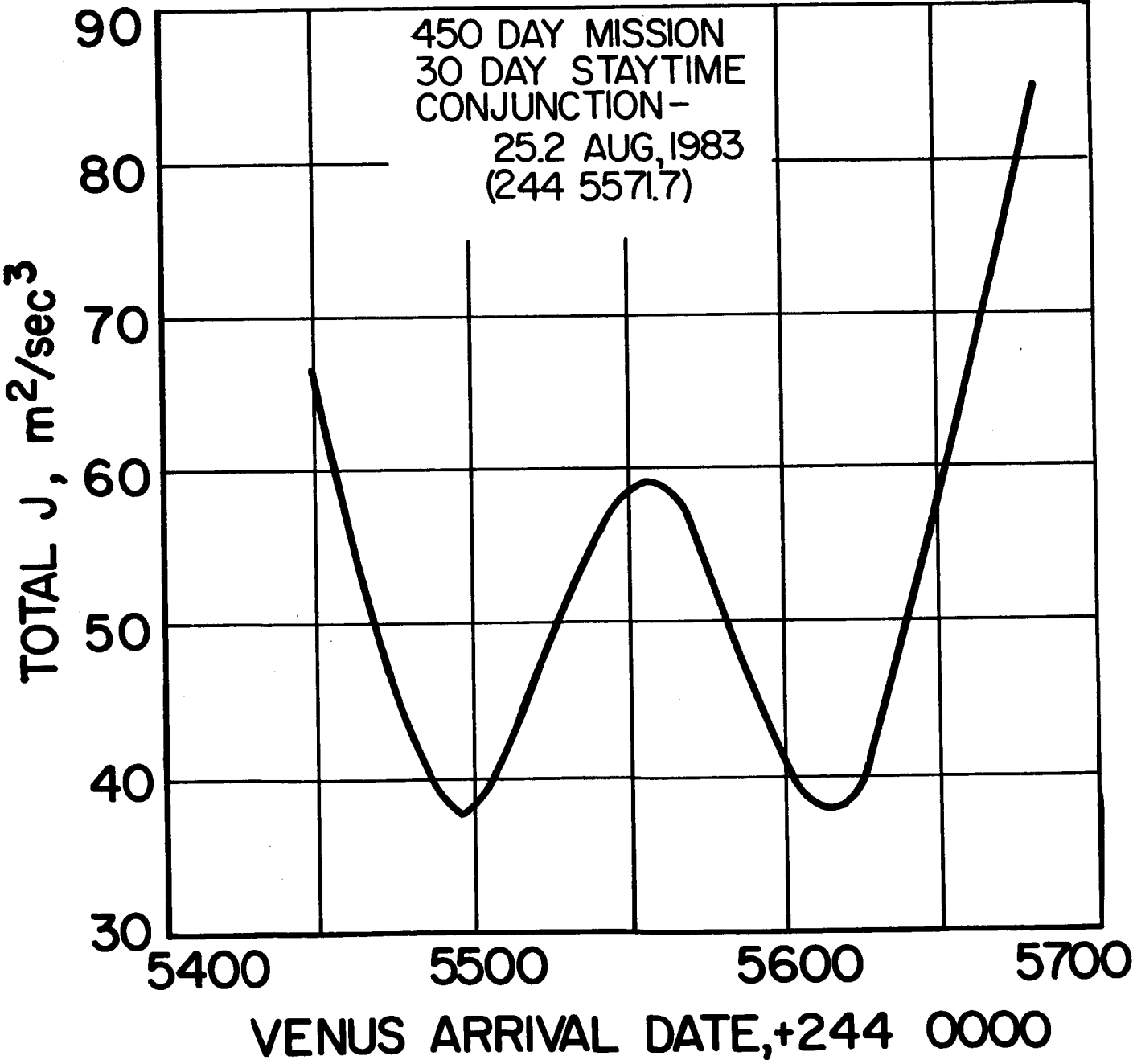
VARIABLE LOW THRUST

CONJUNCTION 25 AUG 1983



VENUS ROUND TRIP REQUIREMENTS

VARIABLE LOW THRUST



VENUS ORBITAL PROBES

HIGH + LOW + HIGH - THRUST

CHEMICAL PROPULSION, $I_{sp} = 430$ SECS

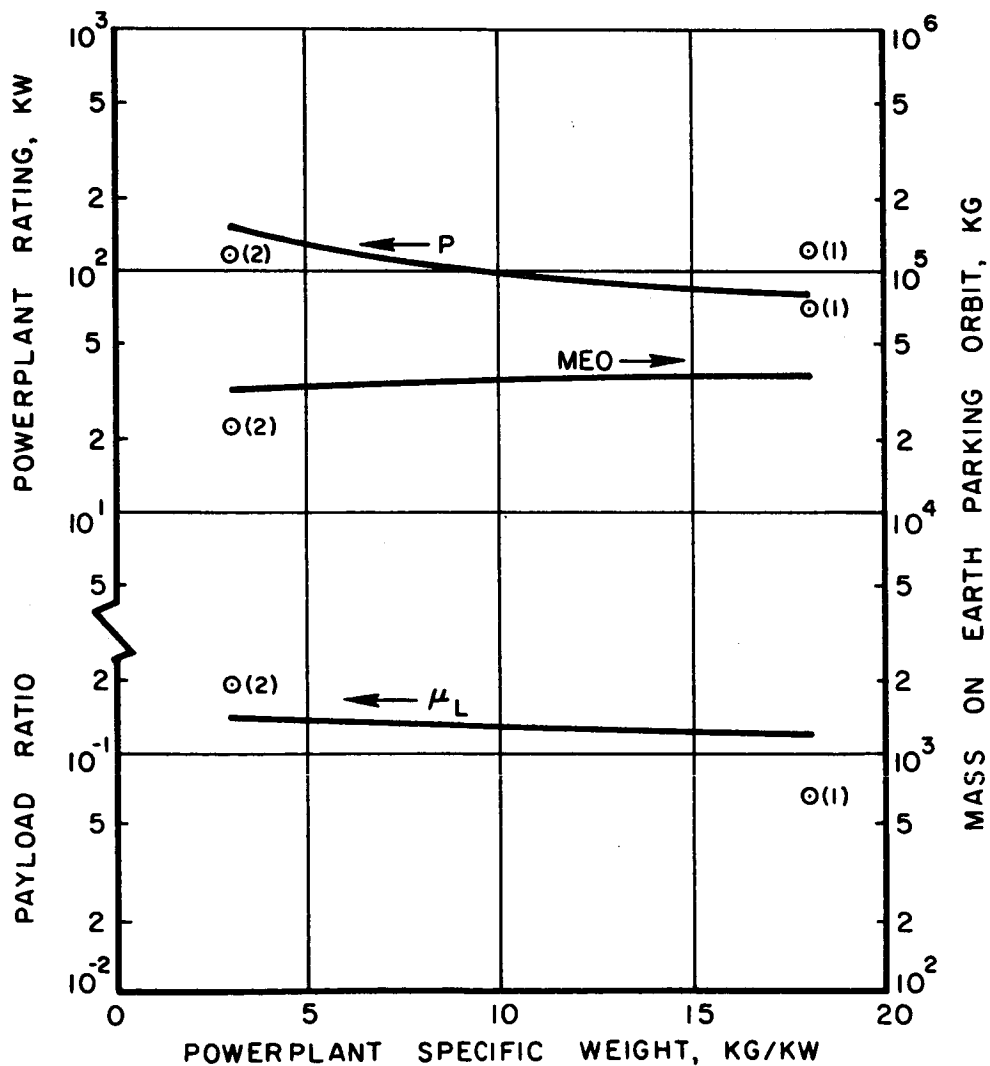
PAYLOAD = 4536 KG

NOMINAL A,B,C, Z

300 - DAY TRANSFER

(1) PESSIMISTIC SYSTEM

(2) OPTIMISTIC SYSTEM



VENUS ORBITAL PROBES

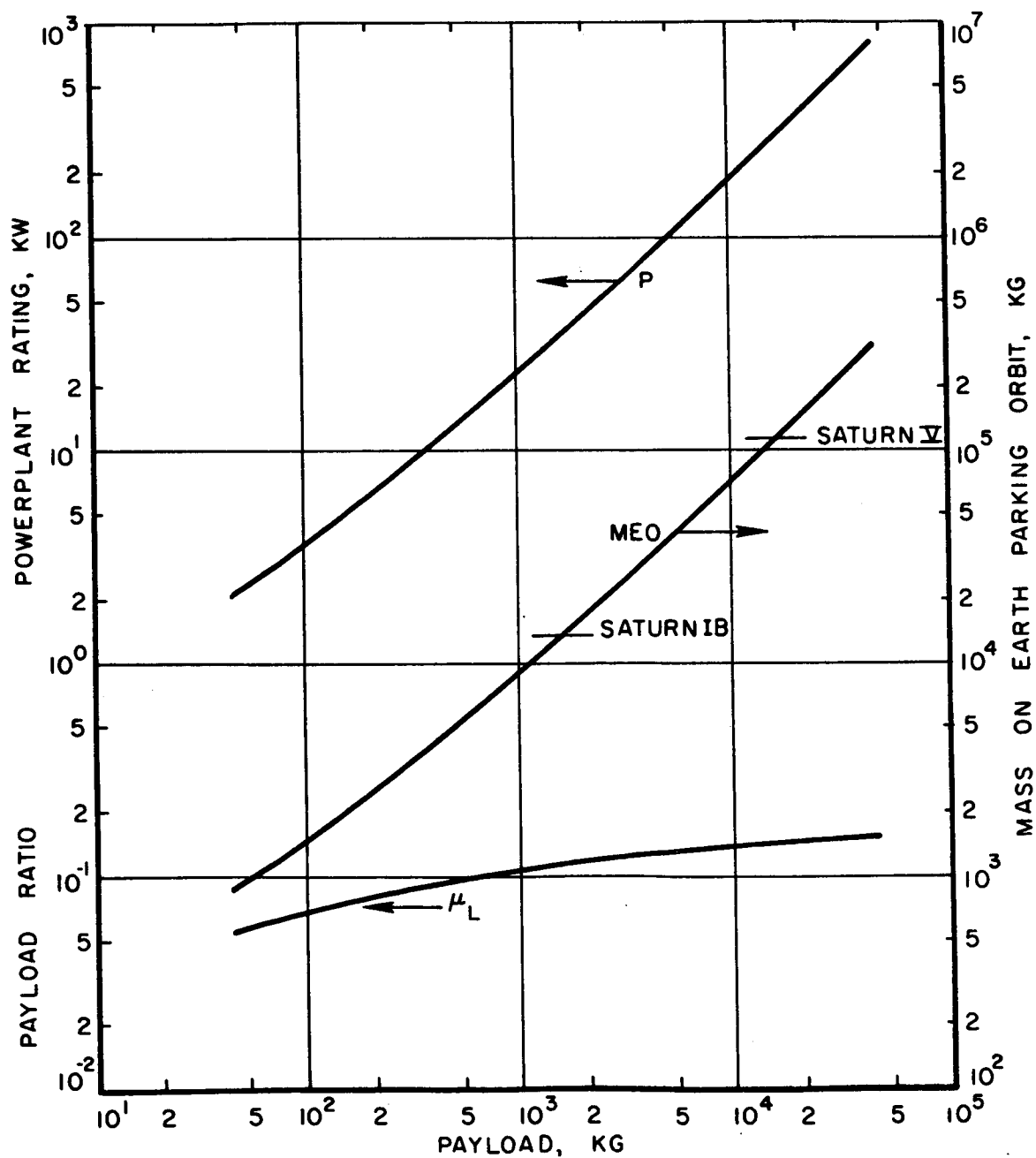
HIGH + LOW + HIGH - THRUST

CHEMICAL PROPULSION, $I_{sp} = 430$ SECS

POWERPLANT SPECIFIC WEIGHT = 12 KG/KW

NOMINAL A,B,C,Z

300 - DAY TRANSFER



VENUS ORBITAL PROBES

HIGH + LOW + HIGH - THRUST

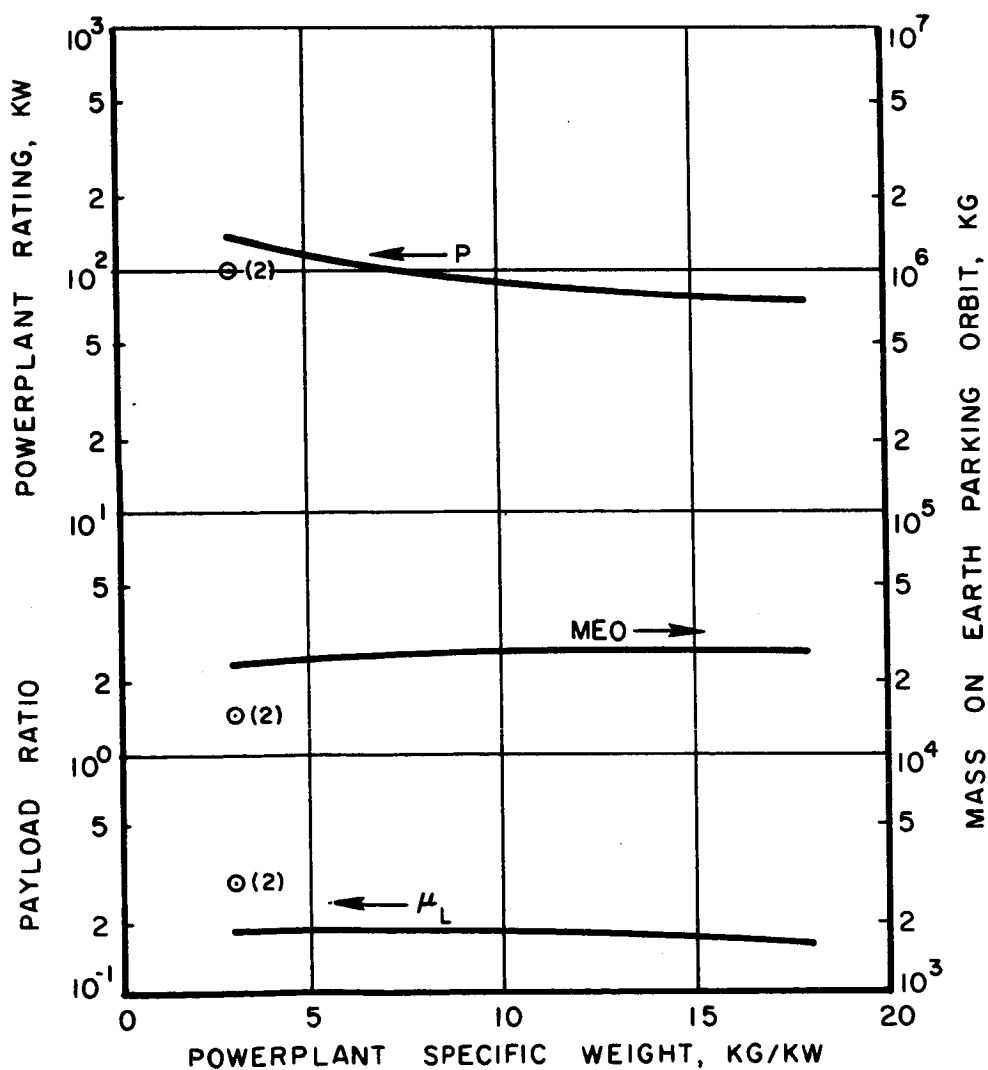
NUCLEAR PROPULSION, $I_{sp} = 800$ SECS

PAYLOAD = 4536 KG

NOMINAL A,B,C,Z

300 - DAY TRANSFER

(2) OPTIMISTIC SYSTEM



VENUS ORBITAL PROBES

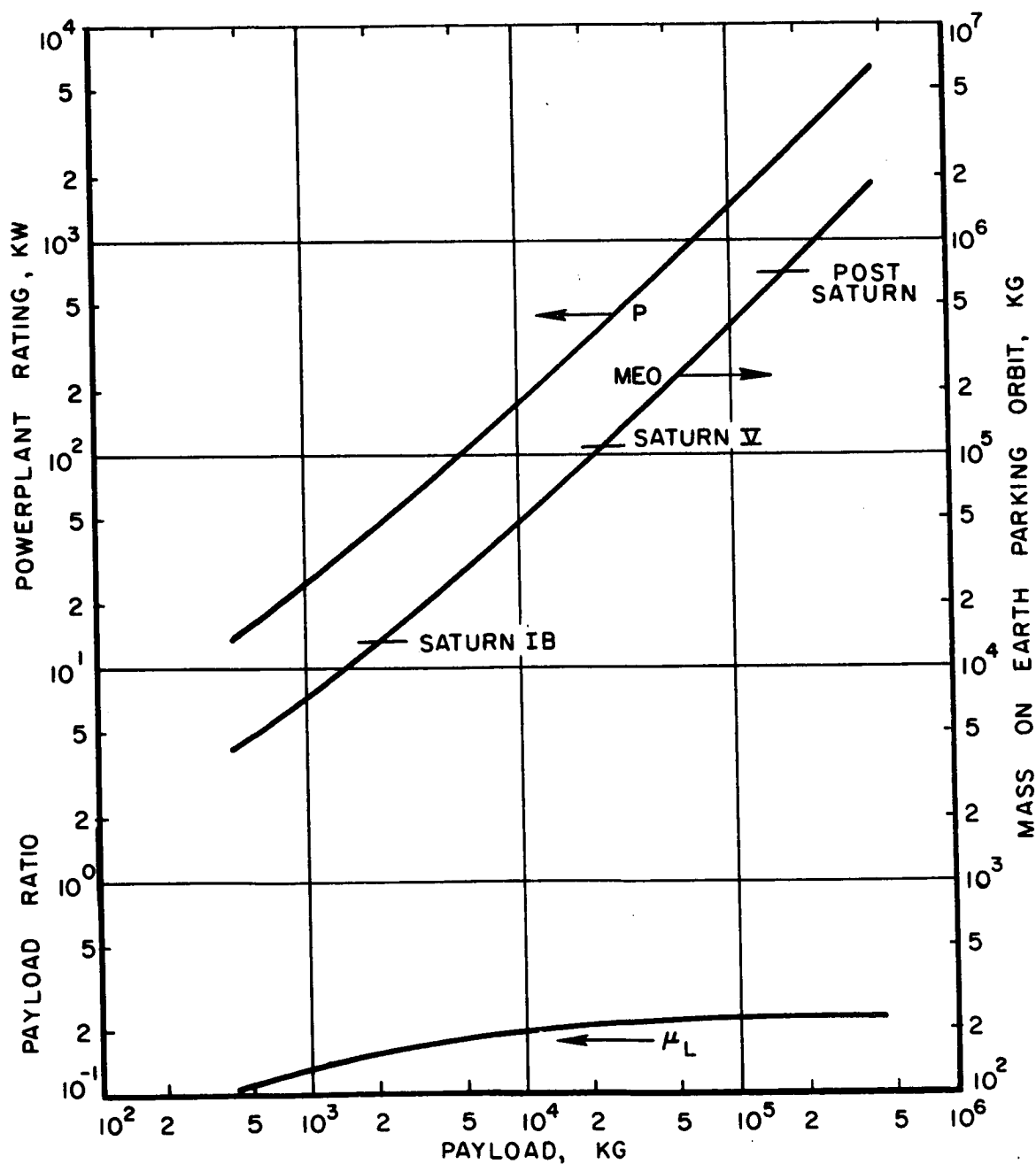
HIGH + LOW + HIGH - THRUST

NUCLEAR PROPULSION, $I_{sp} = 800$ SECS

POWERPLANT SPECIFIC WEIGHT = 12 KG/KW

NOMINAL A,B,C,Z

300 - DAY TRANSFER

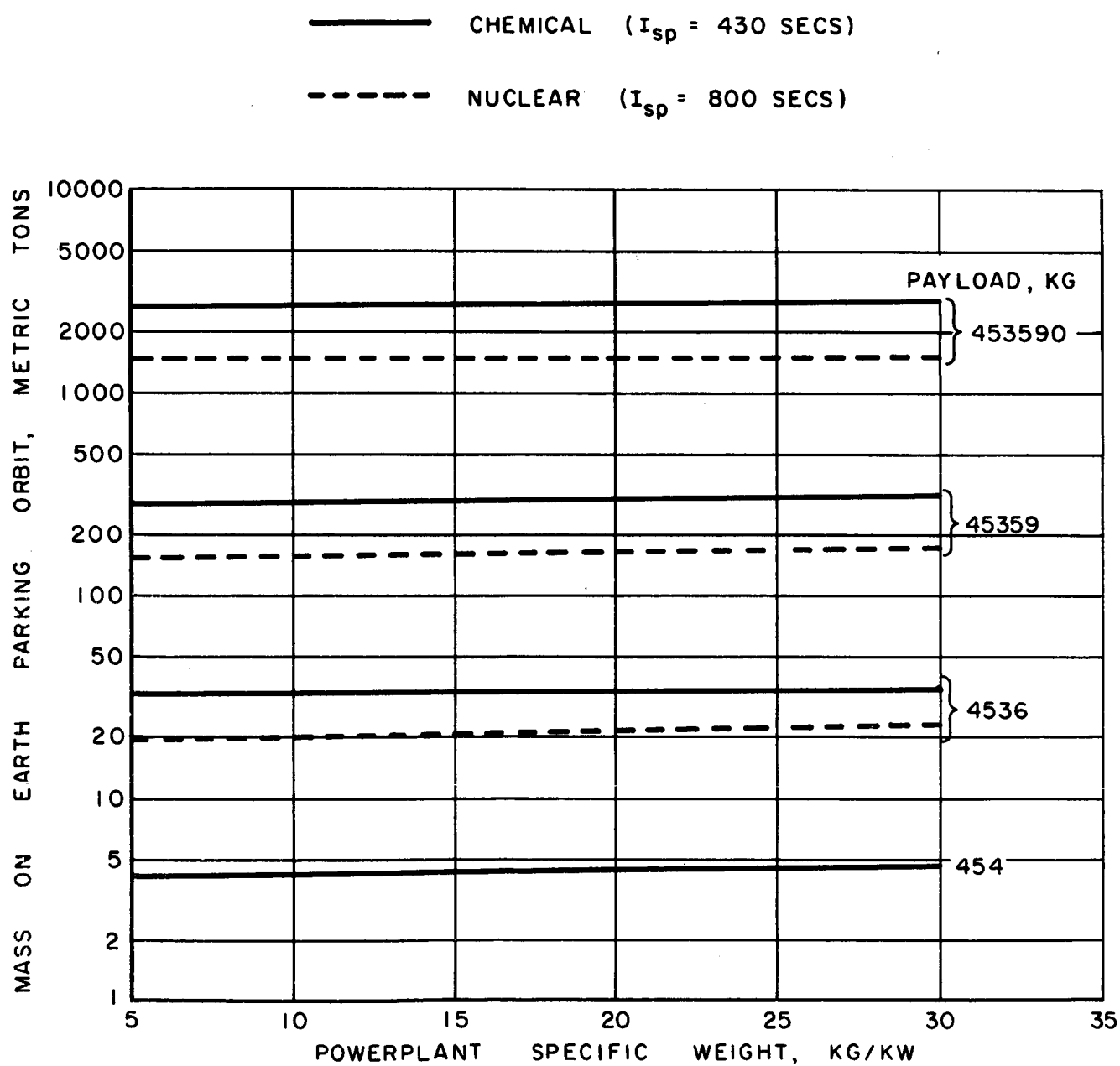


MISSION REQUIREMENTS FOR VENUS ORBITAL PROBES

OPTIMUM HYBRID - THRUST OPERATION

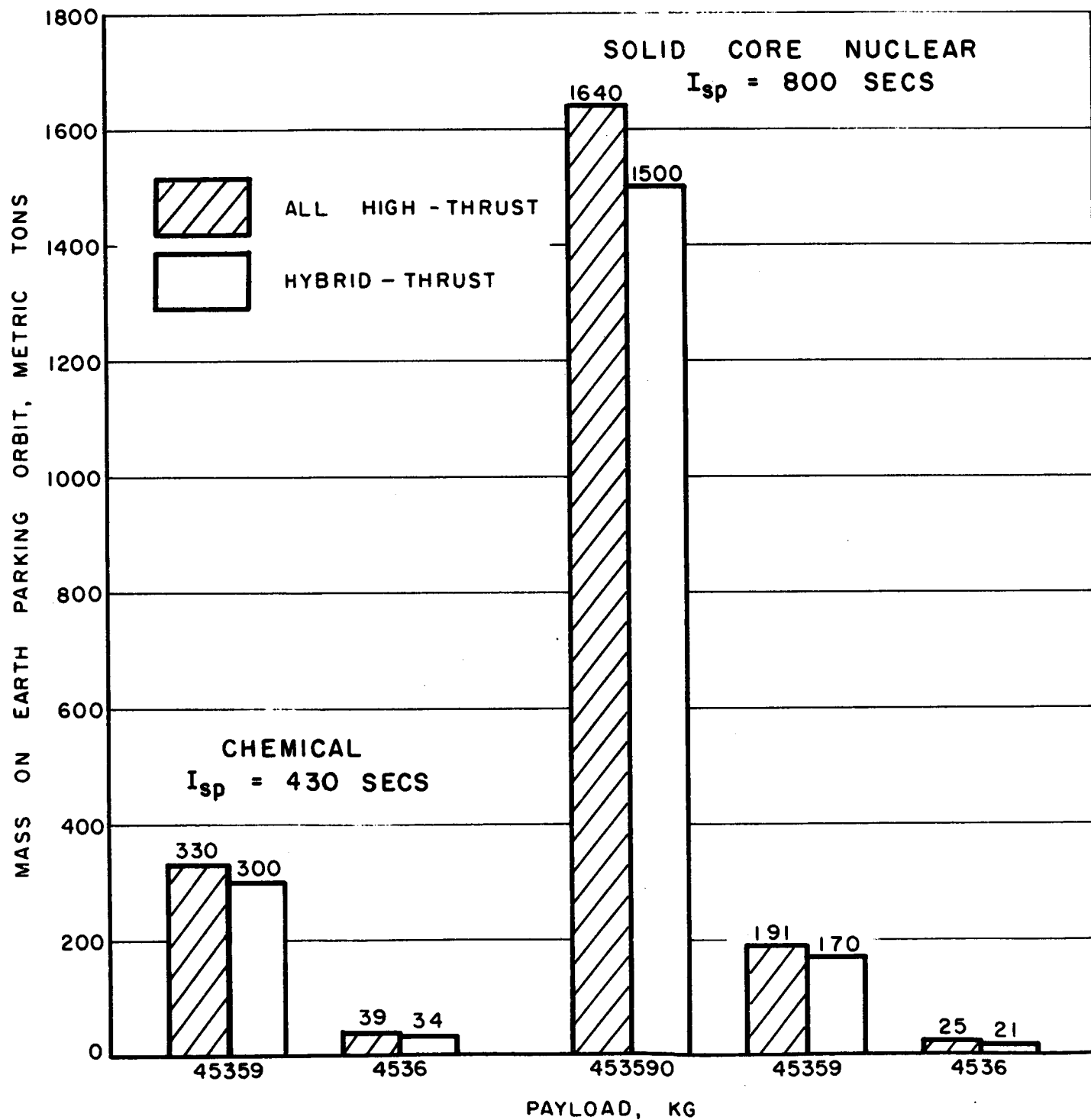
LV ⊕ 5490

AR ♀ 5640



COMPARISON OF HYBRID - AND HIGH - THRUST OPERATION

TRIP TIME = 150 DAYS

 $\alpha = 20 \text{ KG/KW}$ 

EFFECT OF POWERPLANT SPECIFIC WEIGHT ON EARTH DEPARTURE PROPULSION

VENUS ORBITAL PROBE

150 - DAY TRANSFER

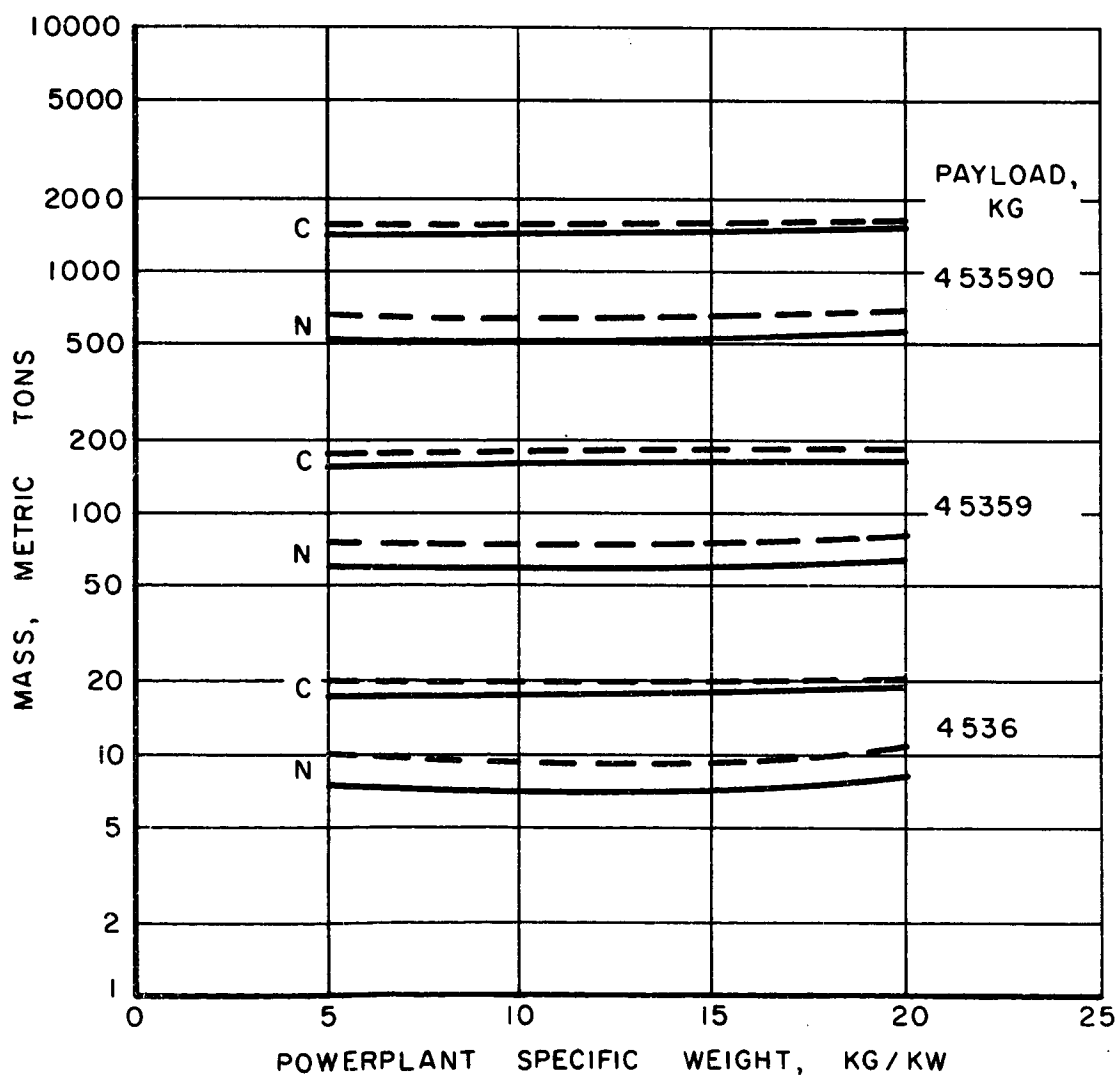
OPTIMUM HYBRID - THRUST OPERATION

--- EARTH DEPARTURE STEP

— EARTH DEPARTURE PROPELLANT

C CHEMICAL, $I_{sp} = 430$ SECS

N NUCLEAR, $I_{sp} = 800$ SECS

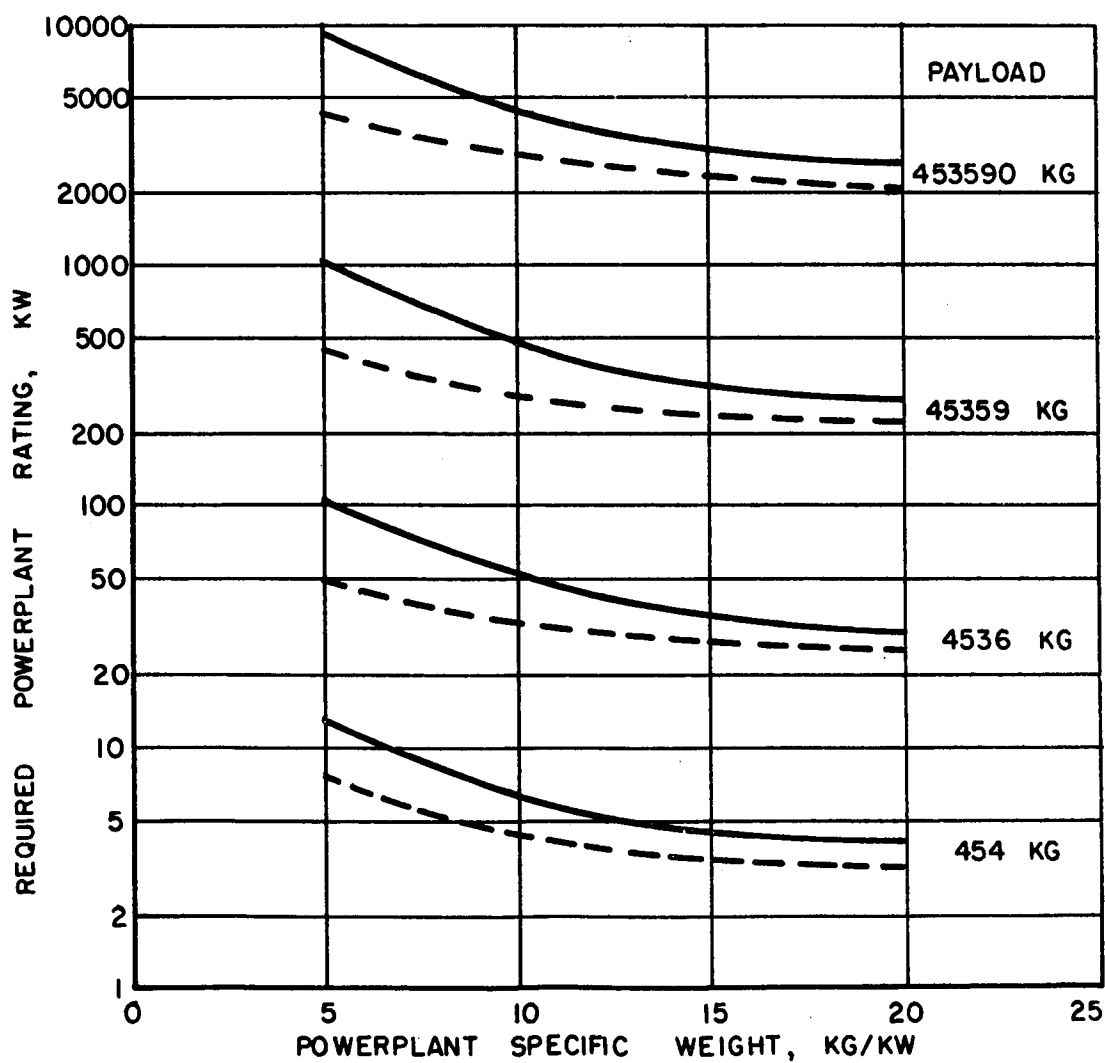


POWERPLANT REQUIREMENTS FOR VENUS ORBITAL PROBES

150 - DAY TRANSFER

OPTIMUM HYBRID - THRUST OPERATION

— CHEMICAL
- - - NUCLEAR

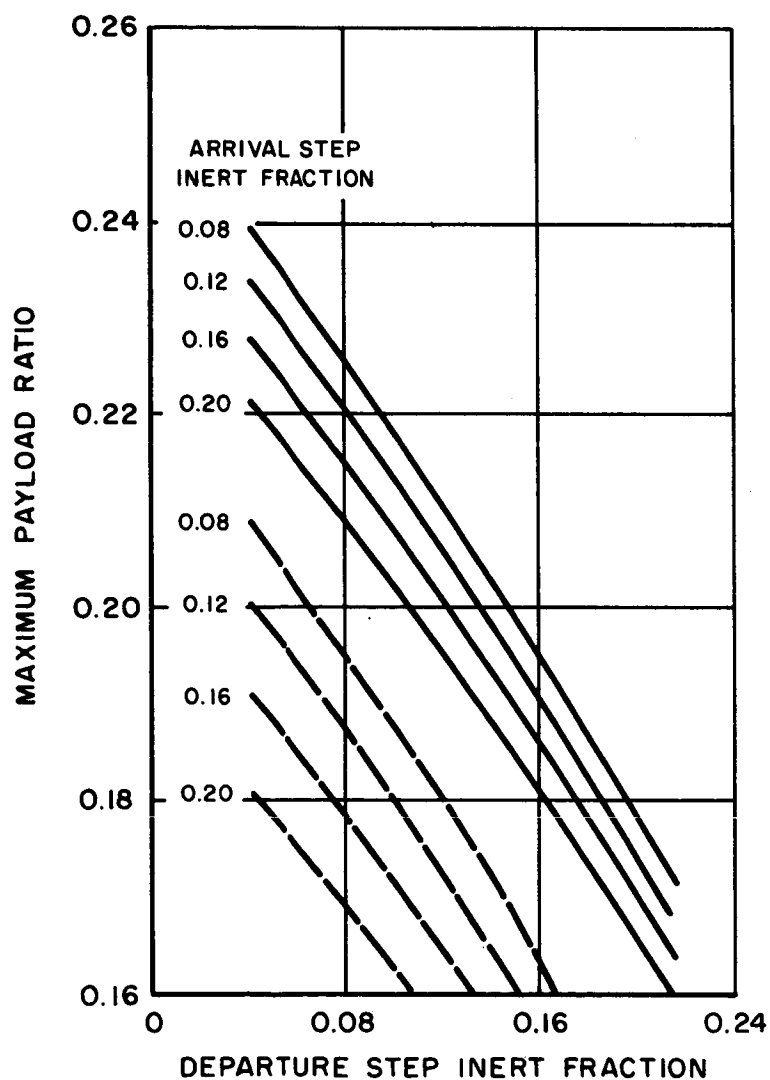


MAXIMUM PAYLOAD RATIOS

$$Q_w = 10 \text{ KG/KW}$$

LV $\oplus$ 4170AR $\odot$ 4450

———— CHEMICAL + ELECTRIC + CHEMICAL
- - - - CHEMICAL + CHEMICAL



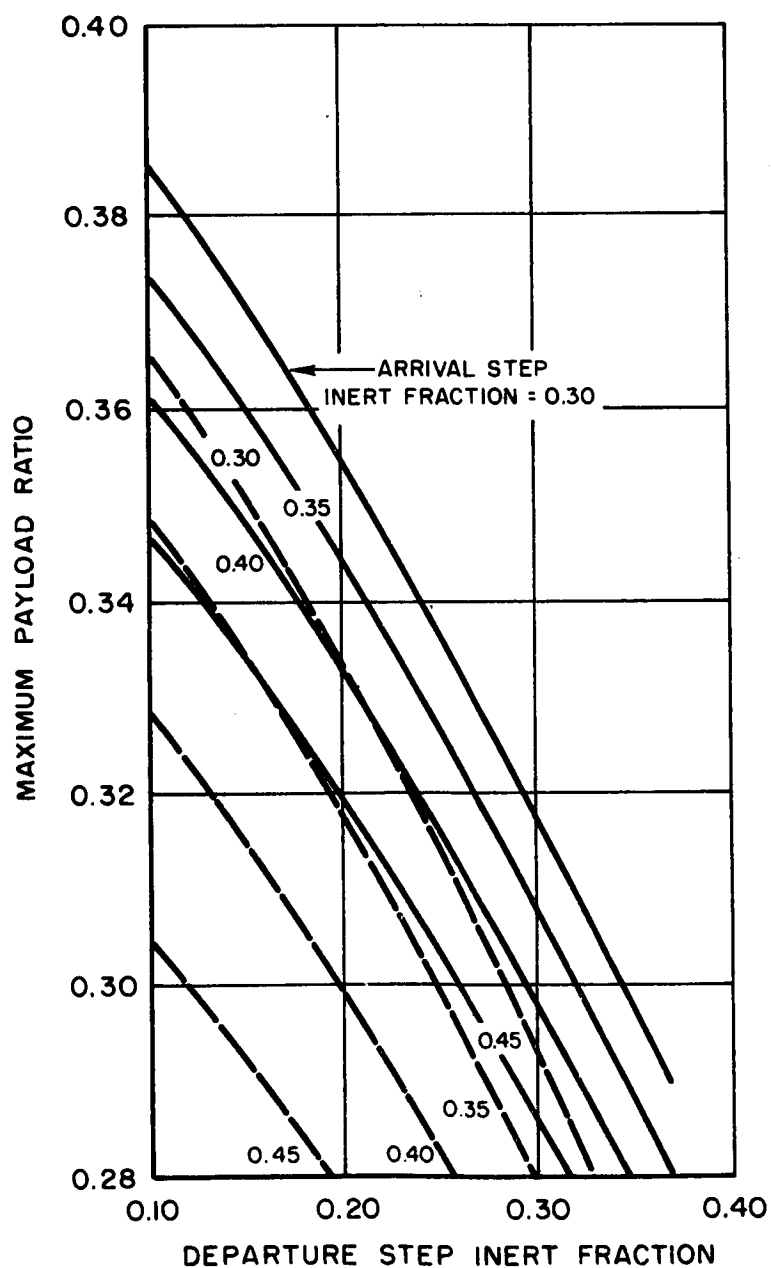
MAXIMUM PAYLOAD RATIOS

$$Q_w = 10 \text{ KG/KW}$$

LV $\oplus$ 4170AR $\odot$ 4450

—— NUCLEAR + ELECTRIC + NUCLEAR

- - - NUCLEAR + NUCLEAR

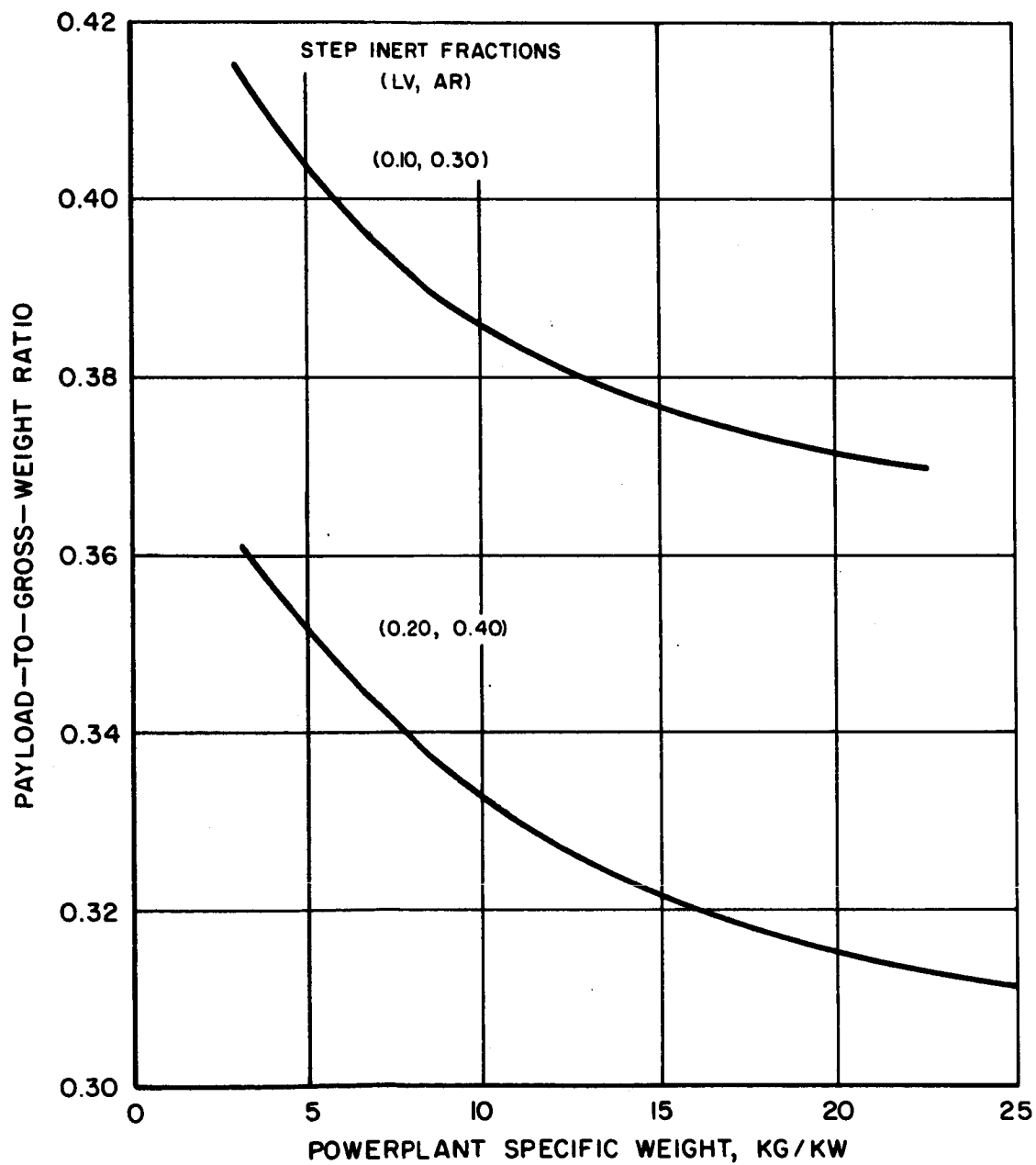


INFLUENCE OF POWERPLANT SPECIFIC WEIGHT

NUCLEAR + ELECTRIC + NUCLEAR

LV ⊕ 4170

AR ○ 4450



MASS REQUIREMENTS FOR MARS ORBITAL PROBES

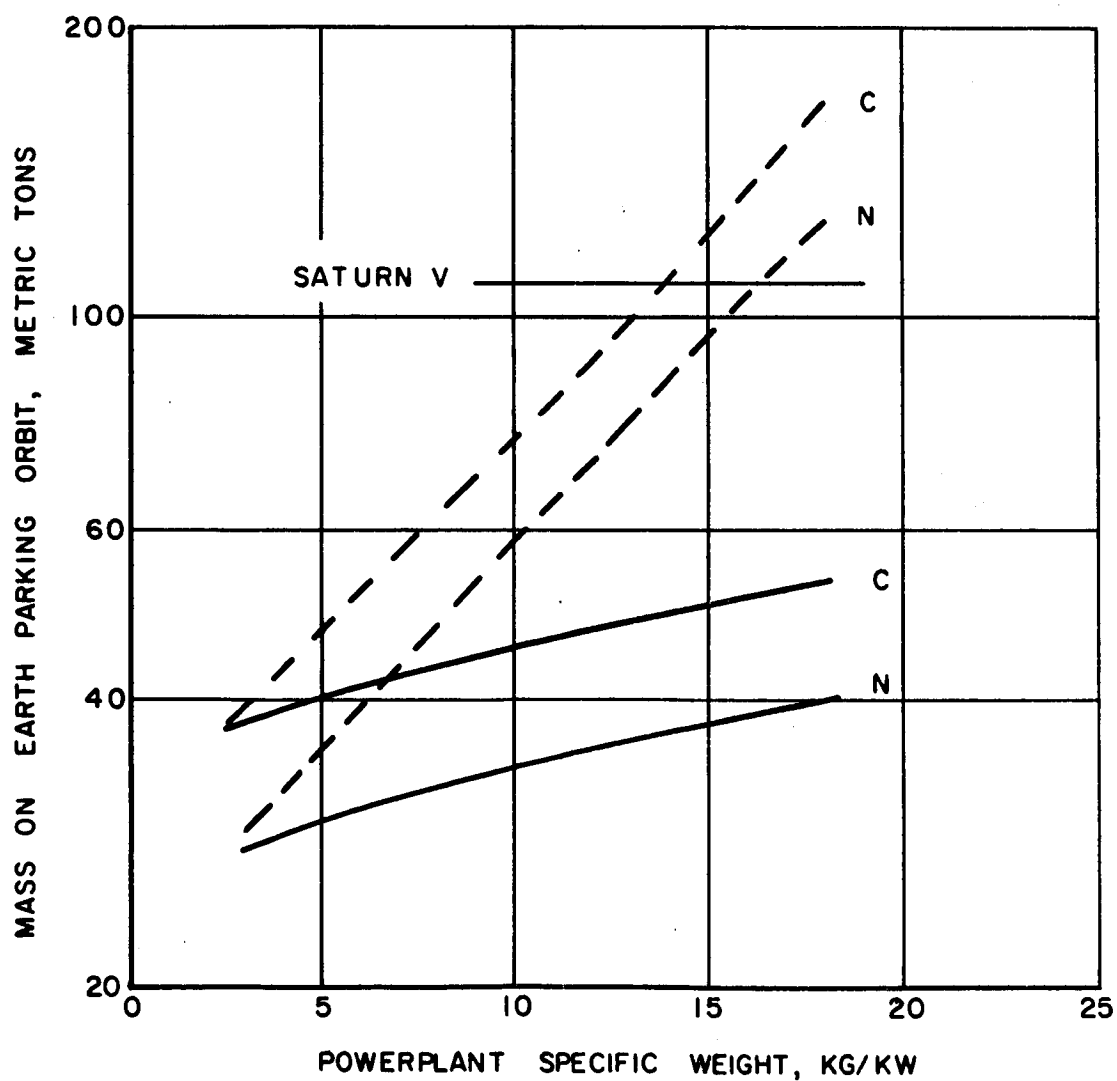
160 DAY TRANSFER

PAYLOAD = 4536 KG

C = CHEMICAL, $I_{sp} = 430$ SECSN = NUCLEAR, $I_{sp} = 800$ SECS

— OPTIMUM

- - - PARABOLIC



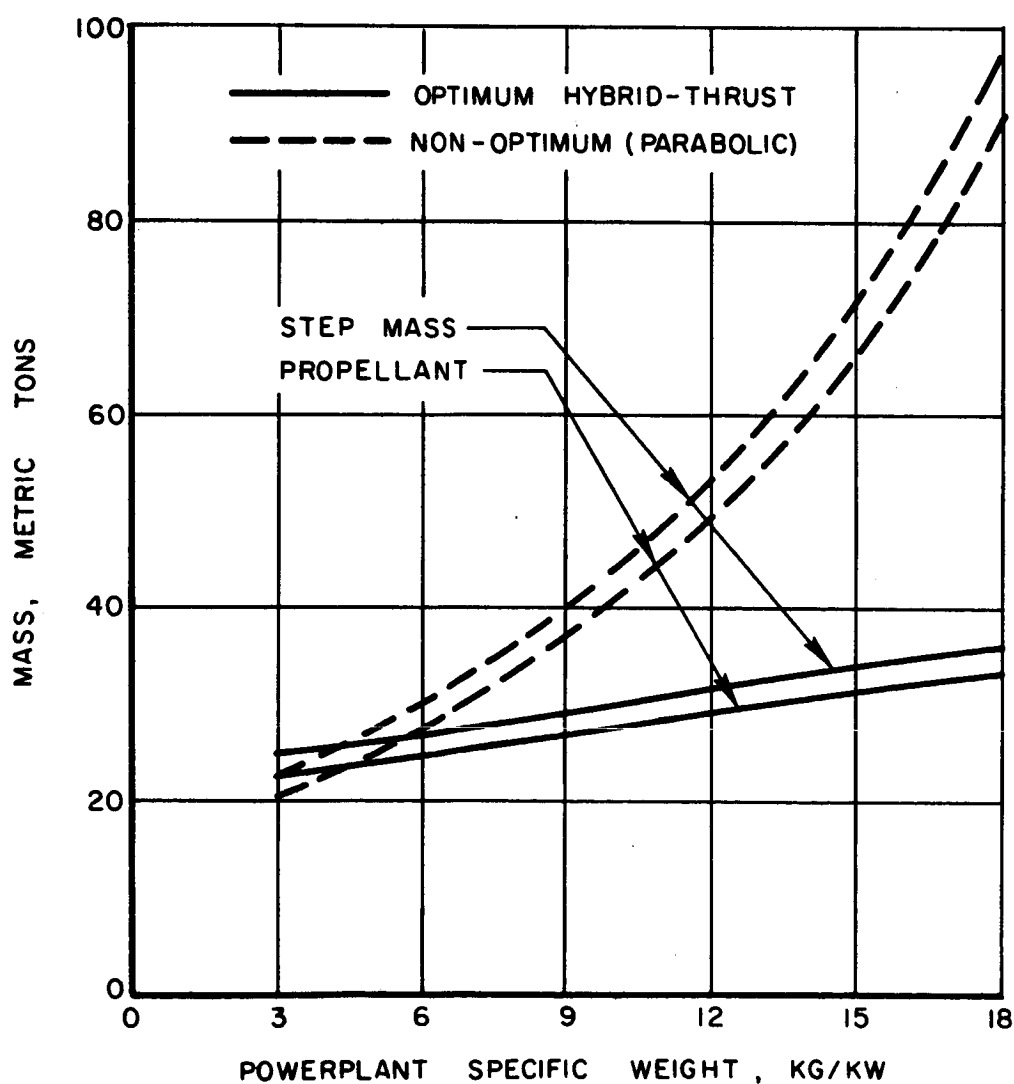
EFFECT OF POWERPLANT SPECIFIC WEIGHT ON EARTH DEPARTURE PROPULSION

MARS ORBITAL PROBE

160 - DAY TRANSFER

PAYLOAD = 4536 KG

CHEMICAL PROPULSION, $I_{sp} = 430$ SECS



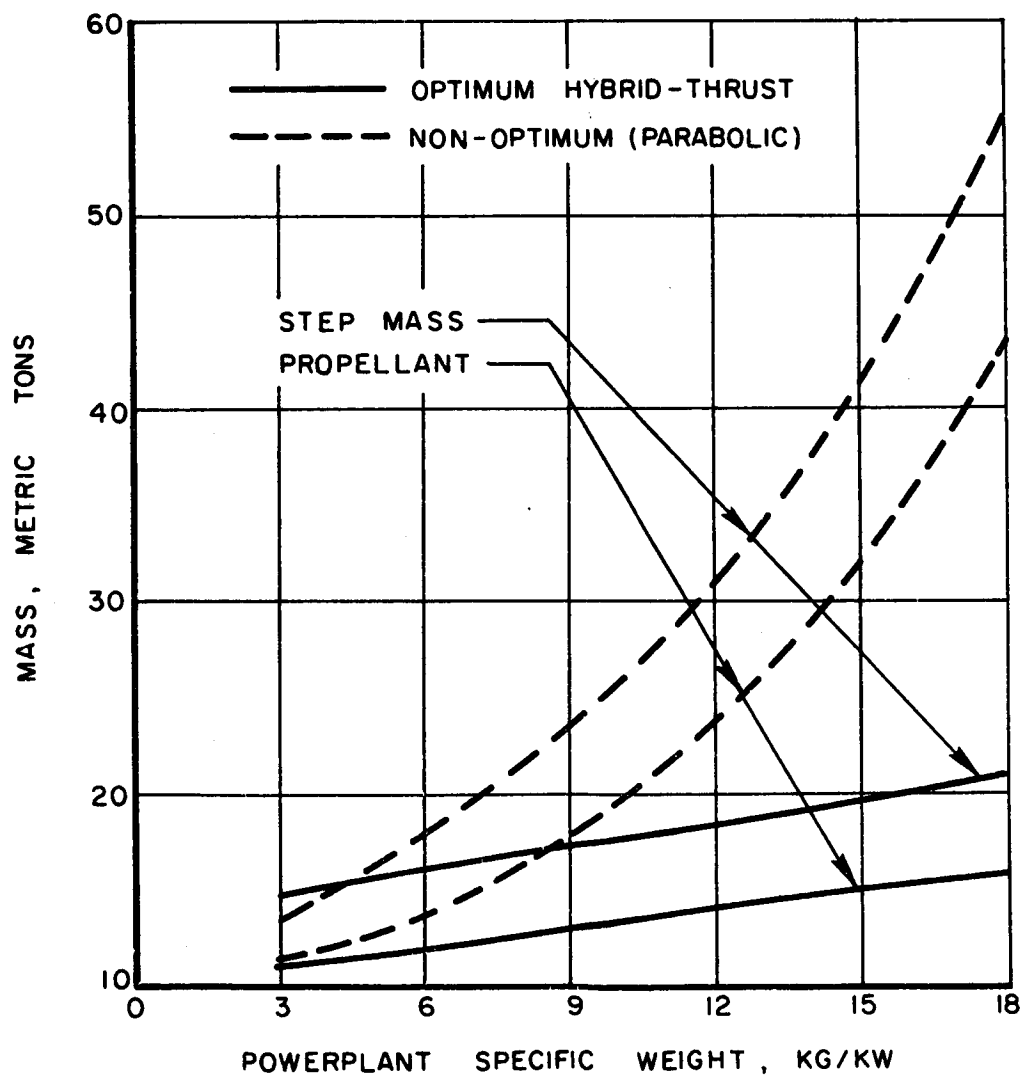
EFFECT OF POWERPLANT SPECIFIC WEIGHT ON EARTH DEPARTURE PROPULSION

MARS ORBITAL PROBE

160 - DAY TRANSFER

PAYLOAD = 4536 KG

NUCLEAR PROPULSION, $I_{sp} = 800$ SECS



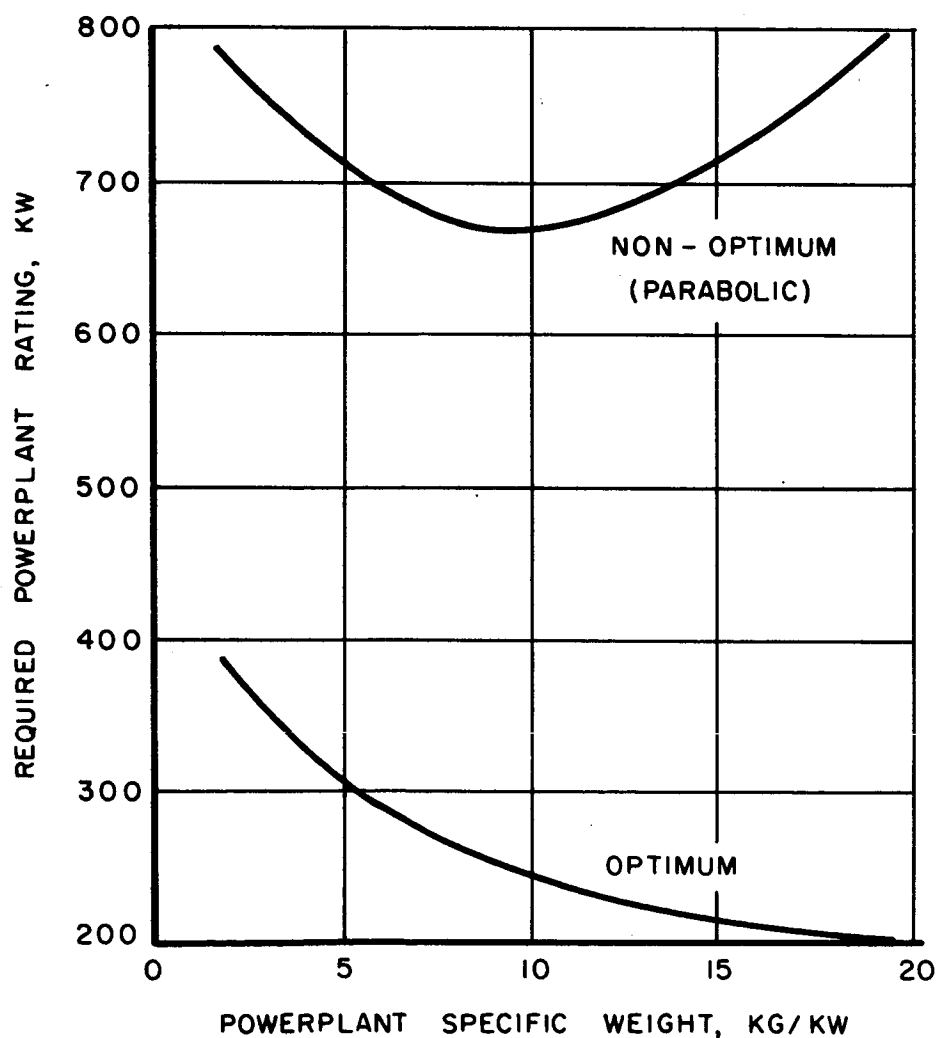
COMPARISON OF POWERPLANT RATINGS
FOR
OPTIMUM AND NON - OPTIMUM HYBRID - THRUST

MARS ORBITAL PROBE

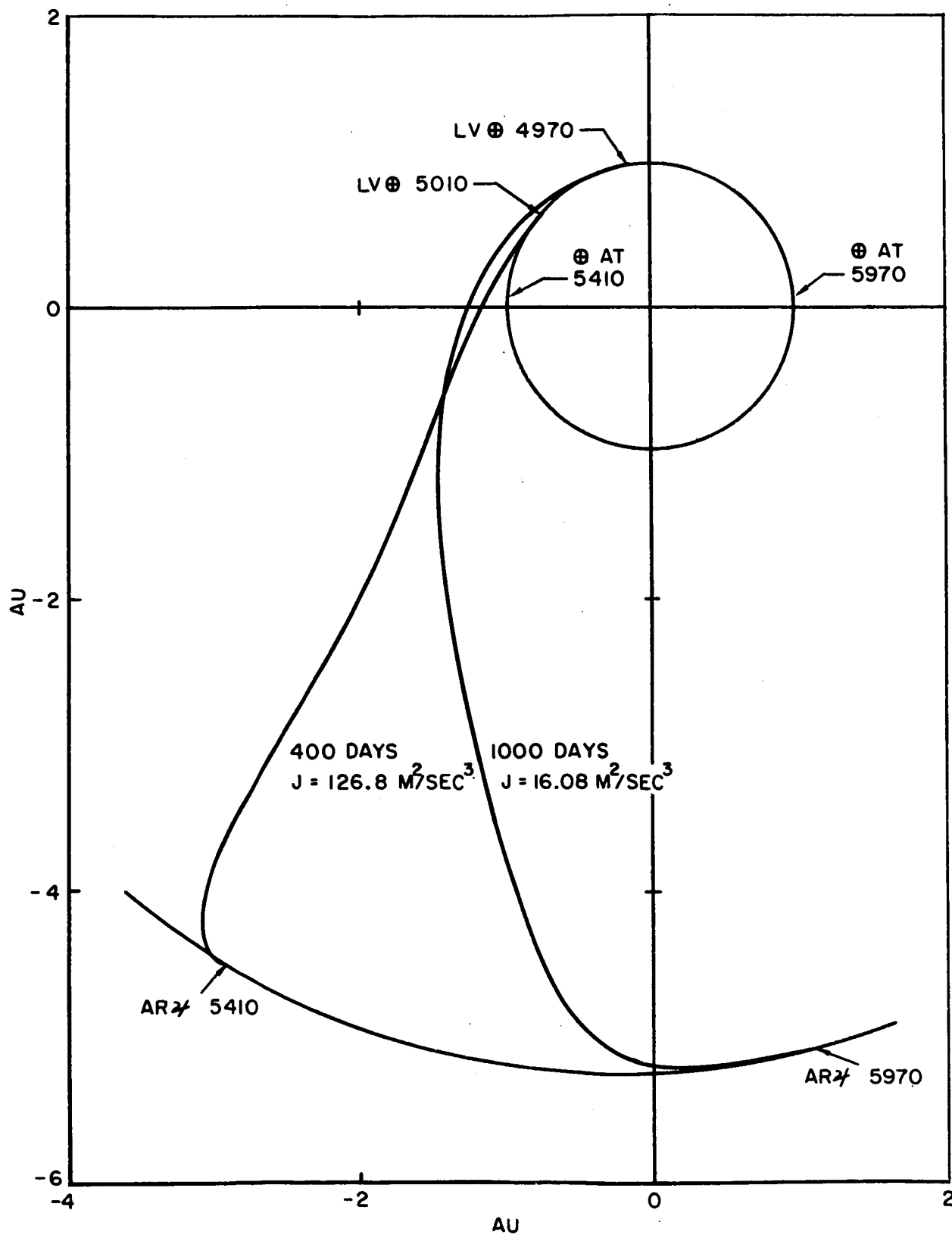
NUC + ELECT HYBRID

160 - DAY MISSION

PL = 4536 KG



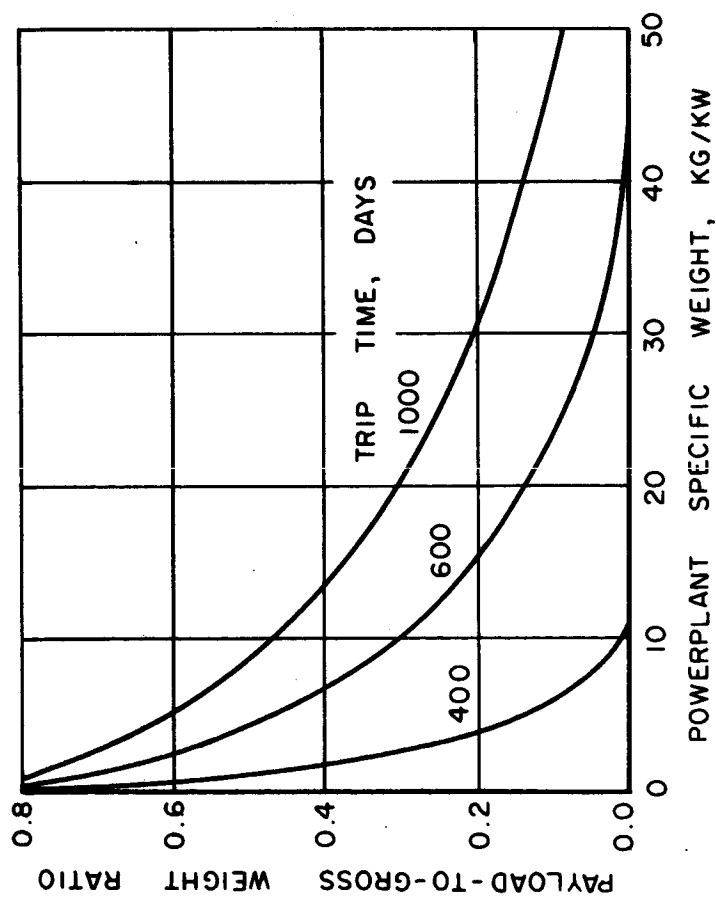
LOW THRUST RENDEZVOUS TRAJECTORIES



EFFECT OF TRIP TIME ON ELECTRIC PROPULSION SYSTEM

1982 JUPITER RENDEZVOUS PROBE

AVERAGE THRUSTOR EFFICIENCY = 0.8



AVERAGE THRUSTOR EFFICIENCY = 0.5

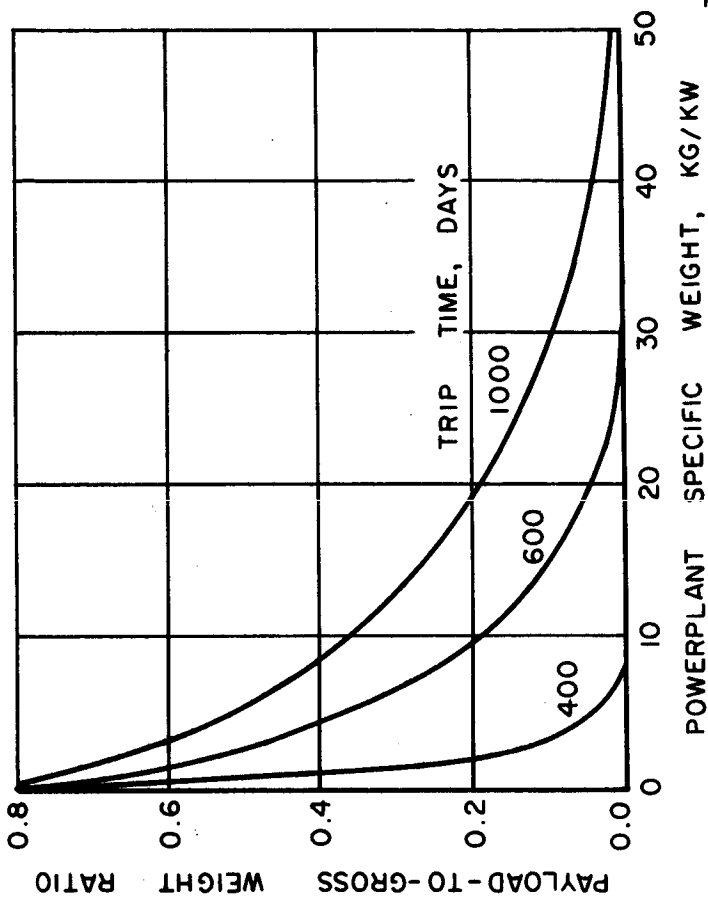


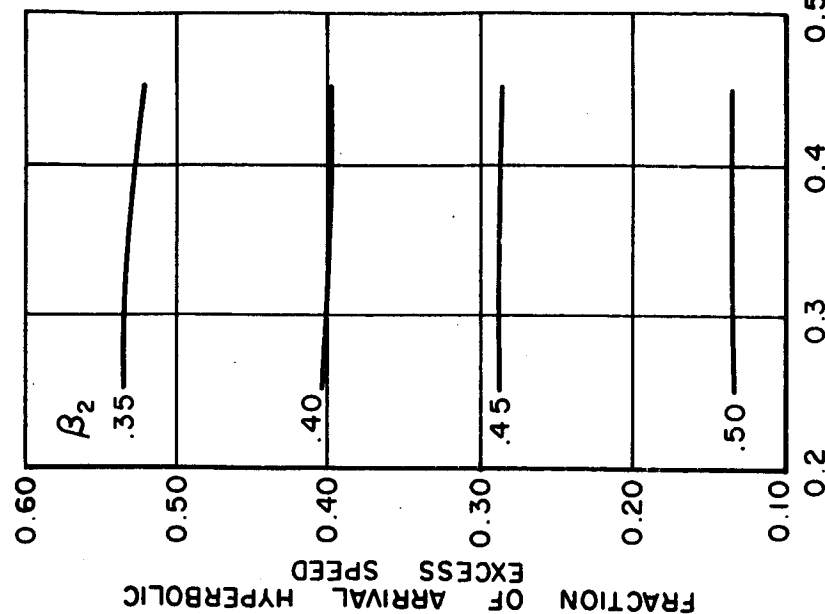
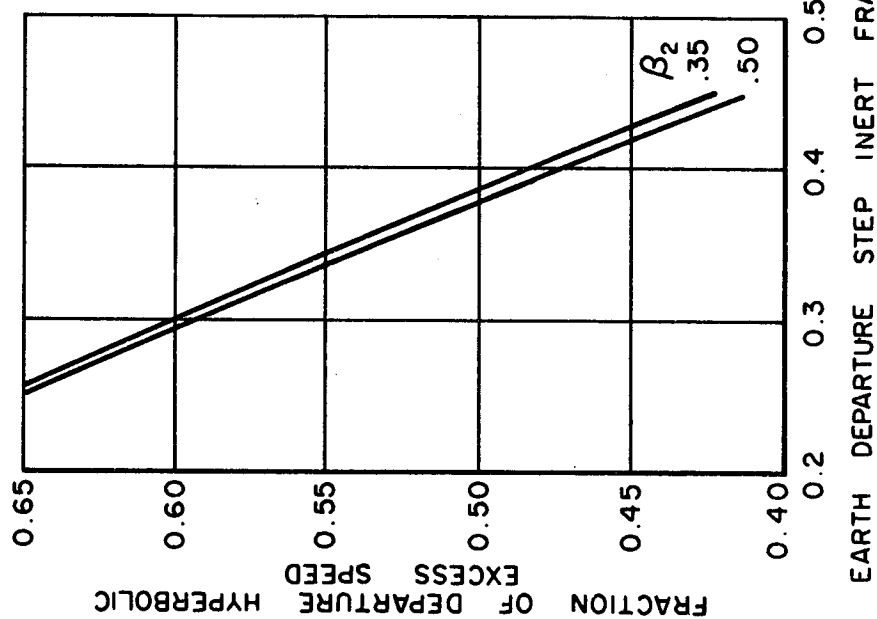
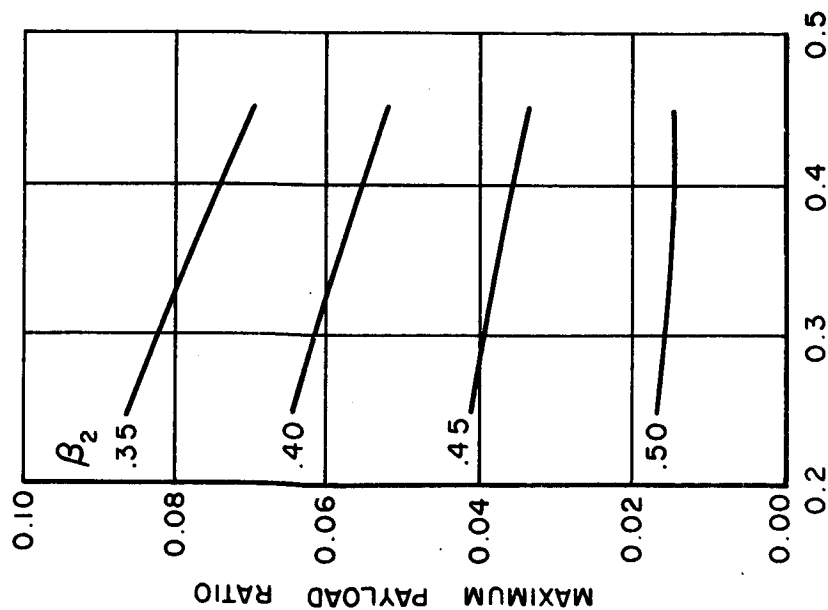
FIG. 555

JUPITER ORBITAL STOPOVER MISSIONS EFFECT OF HIGH-LOW THRUST ON PAYLOAD RATIO

LIQUID CORE NUCLEAR ROCKET

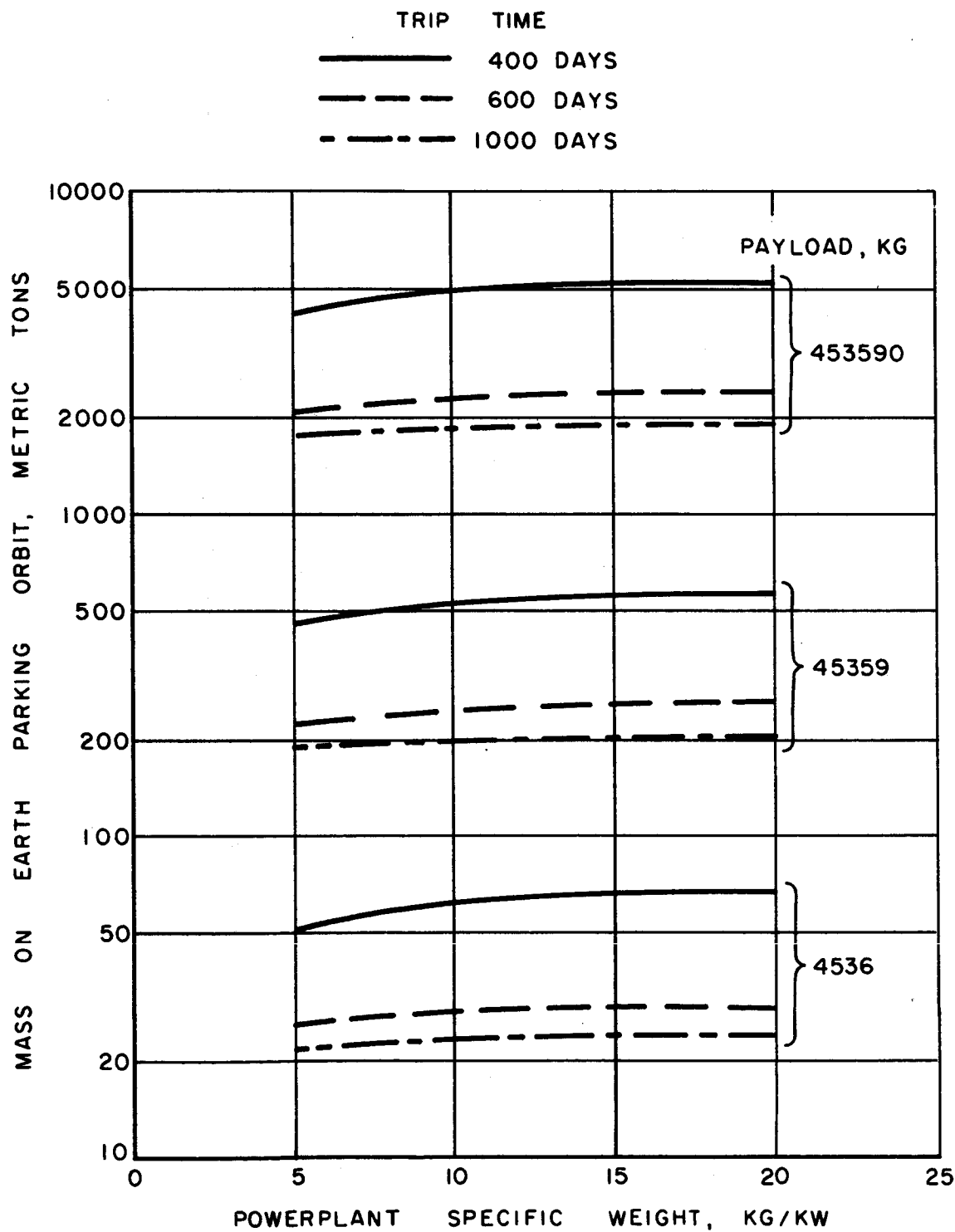
POWERPLANT SPECIFIC WEIGHT = 5 KG/KW

TRIP TIME = 400 DAYS

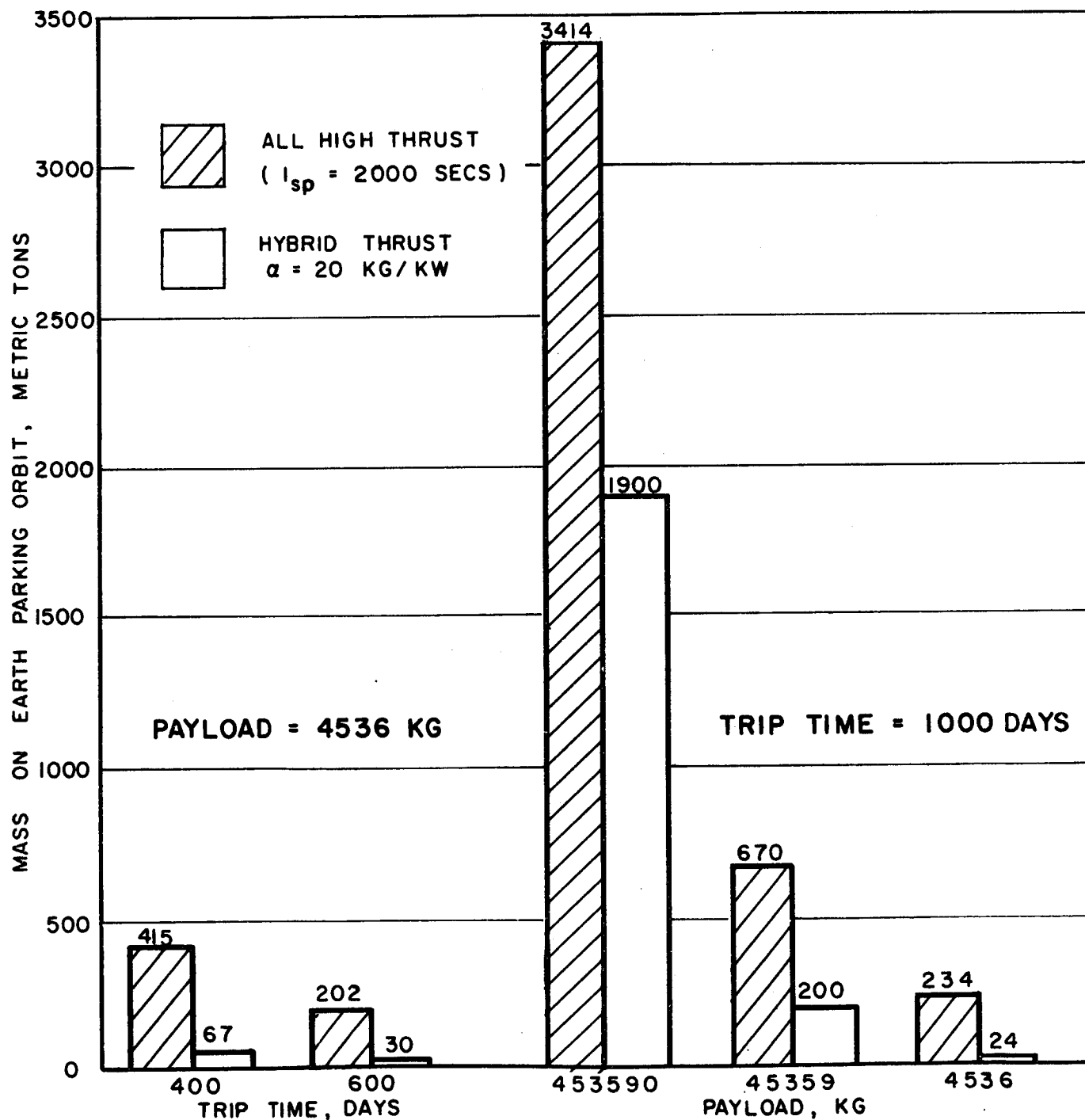


MISSION REQUIREMENTS FOR JUPITER ORBITAL PROBE MISSIONS

OPTIMUM LIQUID CORE NUCLEAR + ELECTRIC OPERATION

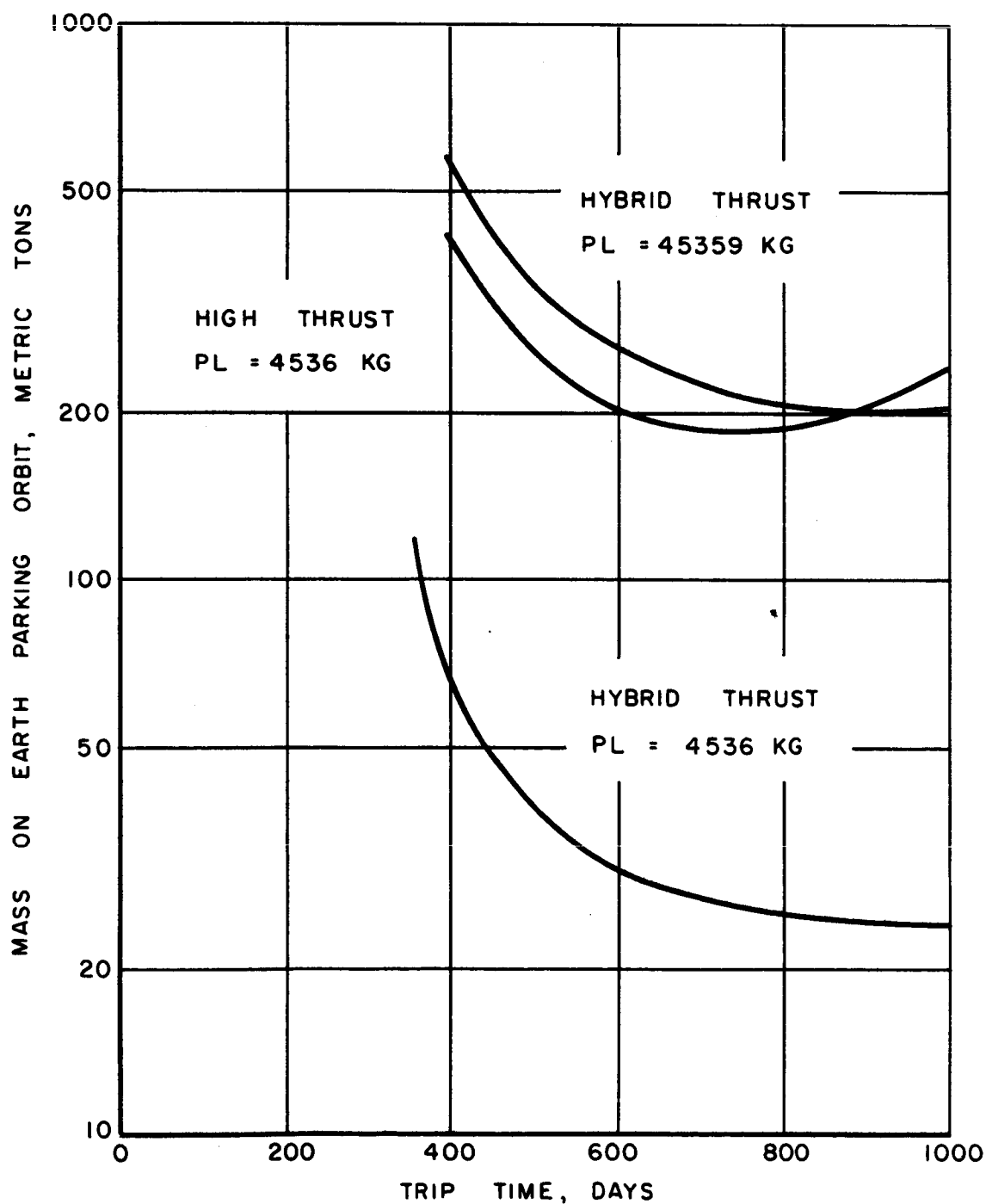


JUPITER ORBITAL PROBES

MASS SAVINGS DUE TO OPTIMUM HYBRID THRUST
OPERATIONLIQUID CORE NUCLEAR HIGH THRUST
LIQUID CORE NUCLEAR + ELECTRIC HYBRID THRUST

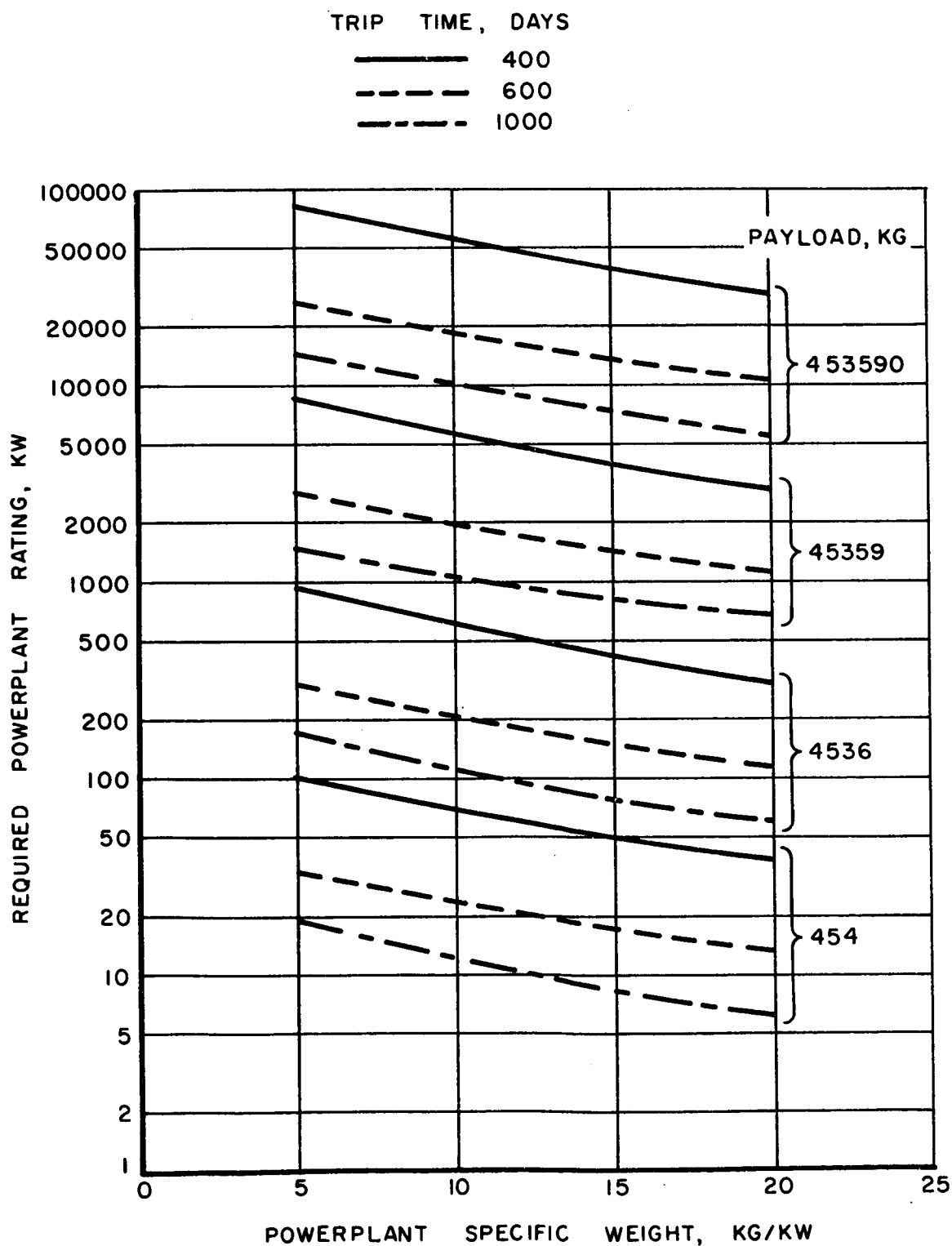
JUPITER ORBITAL PROBES

COMPARISON OF HYBRID AND NON-HYBRID OPERATION

LIQUID CORE NUCLEAR HIGH THRUST, $I_{sp} = 2000$ SECSLIQUID CORE NUC + ELECT HYBRID - THRUST, $\alpha = 20$ KG/KW

JUPITER ORBITAL PROBES

OPTIMUM HYBRID - THRUST OPERATION

LIQUID CORE NUCLEAR, $I_{sp} = 2000$ SECS

EFFECT OF POWERPLANT SPECIFIC WEIGHT ON EARTH DEPARTURE PROPULSION

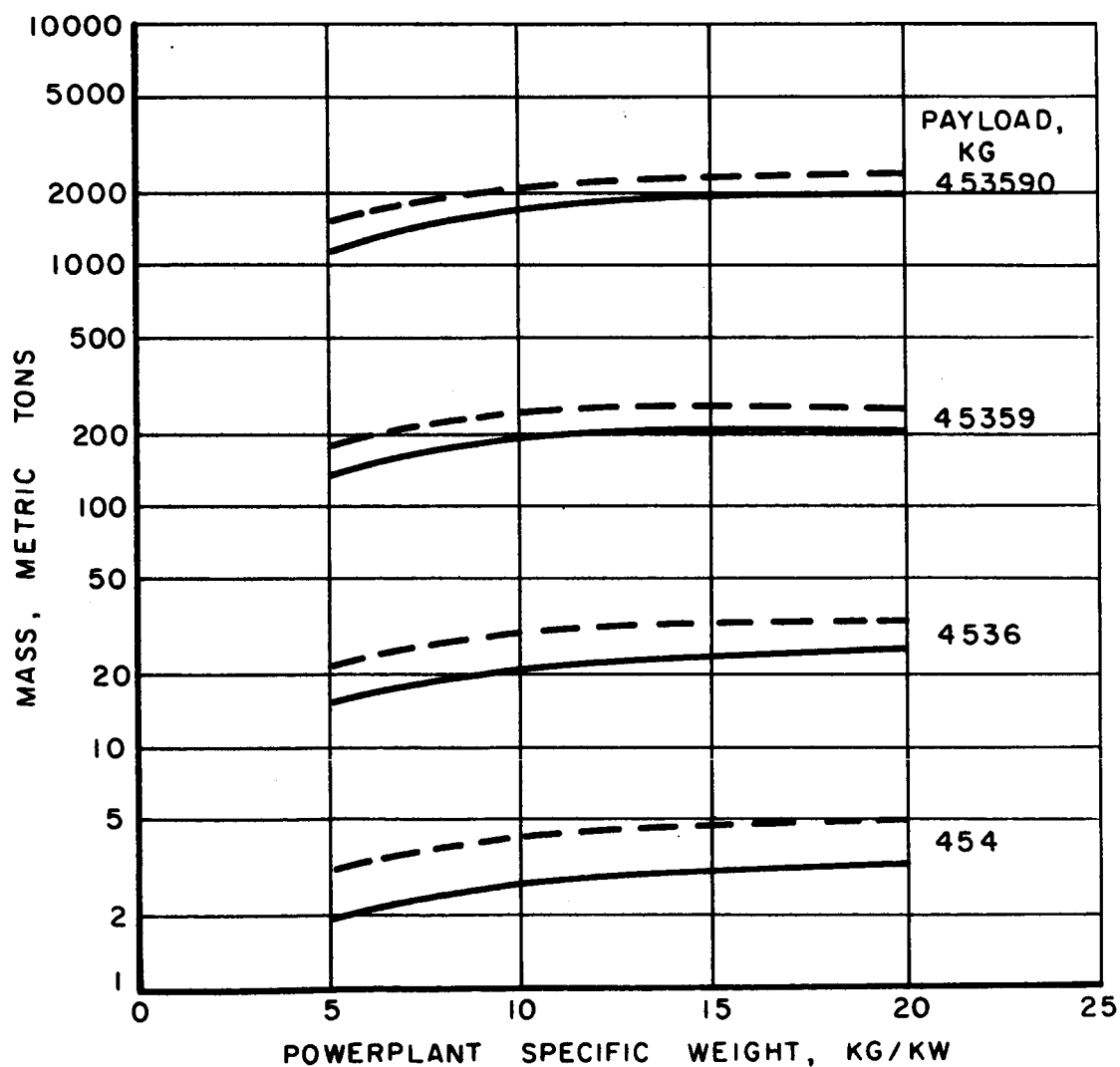
JUPITER ORBITAL PROBE

400-DAY TRANSFER

OPTIMUM LIQUID CORE + ELECTRIC OPERATION

----- EARTH DEPARTURE STEP

———— EARTH DEPARTURE PROPELLANT



ON EARTH DEPARTURE PROPULSION

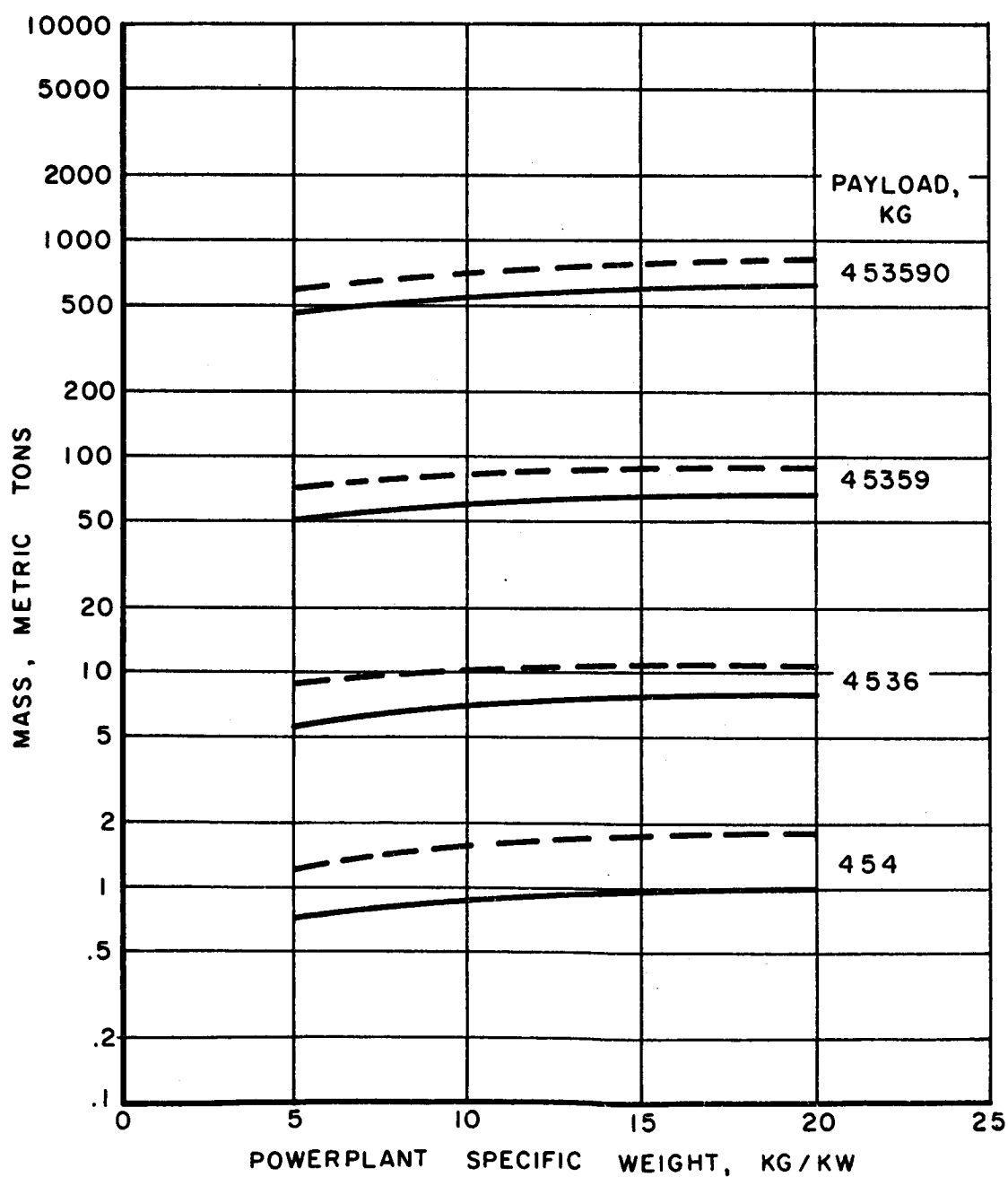
JUPITER ORBITAL PROBE

600-DAY TRANSFER

OPTIMUM LIQUID CORE + ELECTRIC OPERATION

- - - - EARTH DEPARTURE STEP

———— EARTH DEPARTURE PROPELLANT



EFFECT OF POWERPLANT SPECIFIC WEIGHT ON EARTH DEPARTURE PROPULSION

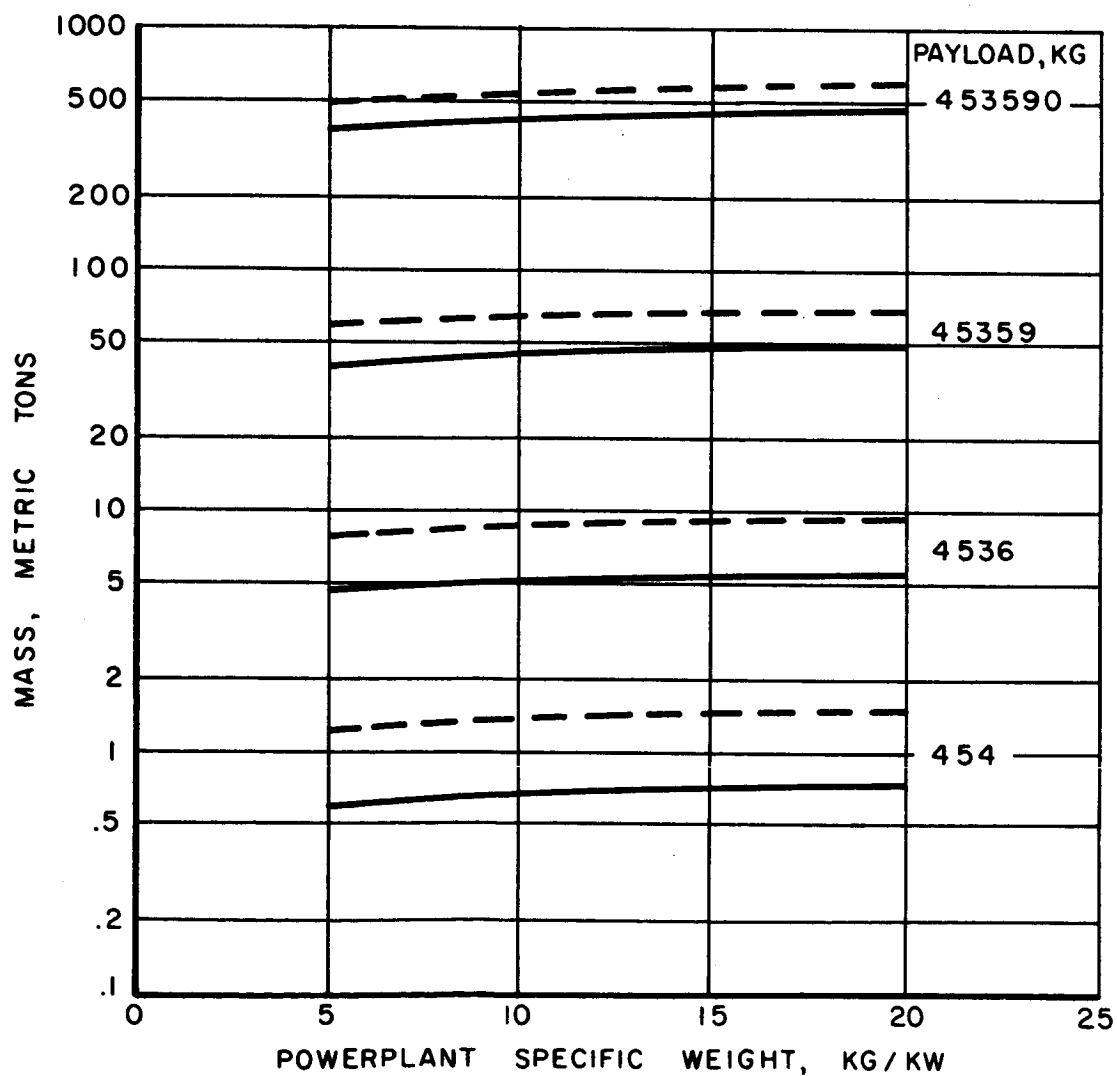
JUPITER ORBITAL PROBE

1000-DAY TRANSFER

OPTIMUM LIQUID CORE + ELECTRIC OPERATION

----- EARTH DEPARTURE STEP

———— EARTH DEPARTURE PROPELLANT



APPENDIX A

The scaling laws which define the weight requirements for propellant tank insulation and boil-off are expressed as

$$W_{Bo} + W_i = B \left(\frac{W_p}{\sigma} \right)^{\frac{2}{3}} \left(\frac{t \Delta T}{L} \right)^{\frac{1}{2}}$$

where

W_i = insulation weight, kg

W_{Bo} = boil-off weight, kg

B = constant

W_p = propellant weight, kg

σ = propellant specific gravity

t = exposure time, days

ΔT = average temperature differential, K

L = heat of vaporization, kcal/kg

The exposure time and temperature differential are functions of the mission. The temperature environment consists of three sources: the direct solar radiation, the radiation reflected from the planet's surface, and the radiated heat emitted from the planet. When the vehicle is orbiting a planet, all three sources must be considered. Conversely, the vehicle in transit is concerned only with the direct solar radiation. A black sphere in space will have an equilibrium temperature as shown in Fig. A-1. It is necessary to modify this temperature profile for tank configuration and effects of surface material. First, however, it should be determined whether the assumption of equilibrium temperature over the entire tank surface is valid.

The insulation blanket forms the outer layer of the tank. Since superinsulation is being utilized, its conductivity characteristics must be considered. It is undetermined at this time whether the superinsulation will require an outer protective shell to guard against damage of the thin aluminum foil sandwich construction. If such protection is assumed, the outer skin will provide substantial lateral heat transfer such that equilibrium temperature can be assumed.

If no protection is required, it is still questionable as to whether the high lateral conductivity will exist in the outer layers of the insulation. In Ref. 17 it is stated that "the outer skin of the storage tank has an infinite thermal conductivity in a direction orthogonal to the flow of heat

tending to enter or leave the propellant; hence, the surface temperature of the storage tank is uniform at any given time..." In discussing this assumption, the authors stated that "considerable variations of temperature will exist on the storage tank surface; however, it is felt that an average temperature would provide a representative temperature of the tank surface..."

In another NASA publication (Ref. 22), the authors assumed that "the outer (superinsulating) foil has infinite conductivity laterally". However, another reference (Ref. 23) cited the following. "As noted previously, multiple-foil radiation shielding is anisotropic in that heat conduction perpendicular to the surface is markedly less than parallel to the surface. There are two extreme conditions which may be considered. In one, the conduction parallel to the surface is infinite, and as a result, the skin assumes a uniform temperature. The other extreme is to consider the conduction parallel to the surface to be appreciable compared to the conduction perpendicular to the surface, but nevertheless a small absolute value. This is equivalent to stating that each section of the surface comes to an equilibrium temperature which is determined by the heat flux to that section of the surface and is not influenced by the adjacent sections. This case is a conservative one in that the calculated heat leak to the liquid is greater than would be experienced in practice. The assumption of a uniform skin temperature (infinite surface conduction) predicts smaller heat leaks than would actually occur".

A factor that would be considered in a highly detailed analysis is the heat leakage through any radiation shields, for this factor alone may completely invalidate whatever assumption is made relative to the skin temperature. In Ref. 24 it is stated that "conductive paths through multiple-foil shields, such as rivets, studs, struts, etc., may short-circuit the entire shield. Surprisingly small items may provide these shorts, e.g., a quarter-inch aluminum bolt or rivet may provide enough conductivity to ruin the performance of a good shield". In addition, the authors of Ref. 24 stated that work done at Arthur D. Little, Inc., "... indicates that more realistic models and systems will have to be considered in order to provide meaningful information. For example, conductive heat leaks through the insulation itself and through structural members may defeat the insulation and thus must be considered in even strongly idealized models".

It appears that the validity of the basic assumption of infinite lateral conductivity in the outer skin will depend upon actual tank design. This assumption yields the lowest skin temperature, but the entire tank area is tending to heat the propellant. If small lateral conductivity is assumed, the temperature on the lighted surface is considerably higher but on the exposed area is tending to heat the propellant. The latter assumption results in lower insulation weights since only the exposed surface requires insulating. However, it has been stated that the high heat leaks associated with the latter assumption will greatly increase the propellant boil-off. Conversely the heat leaks associated with the initial assumption will be minimal. The net effect is that the two assumptions will yield comparable results. It therefore appears

more feasible to use the temperature model where the large unknown (heat leaks) is minimal.

Now that the equilibrium temperature model is used, the temperature is a function of the surface absorptivity and emissivity and the tank configuration such that:

$$T \approx \left(\frac{\alpha}{\epsilon} \right)^{1/4} \left(\frac{A}{S} \right)^{1/4}$$

where

T = surface equilibrium temperature

α = absorptivity of surface

ϵ = emissivity of surface

A = projected exposed area

S = total surface area

For a sphere, $A/S = 0.25$. The cylindrical tanks used for this study will have various exposures to the sun and may also be clustered, thus providing some shadow-shielding effects. The following table summarizes the values of A/S for broadside exposures

Number of Tanks in Cluster	A/S	$\frac{\frac{A}{S_{\text{tank}}}}{\frac{A}{S_{\text{sphere}}}}$	$\frac{T_{\text{tank}}}{T_{\text{sphere}}}$
1	0.296	1.183	1.041
2	0.148*	0.592	0.876
3	0.197	0.788	0.941
4	0.148	0.592	0.876
5	0.118**	0.472	0.828
6	0.148	0.592	0.876

* one tank completely shielded

** four tanks around central tank

An axial alignment to the sun would result in an A/S value of 0.0833 for all cases, with a corresponding temperature ratio of 0.76.

The next consideration is the surface material of the tanks. The most efficient surface would be white, such as a magnesium oxide coating. Although a highly polished metal surface will reflect 90% to 95% of the radiation, the emissivity at the very low temperatures is so low that the resultant equilibrium temperature is high. The following table illustrates the equilibrium

temperature for three flat-plate surfaces where the back of the plate is perfectly insulated.

<u>Surface</u>	<u>Temperature</u>
White	-51 C
Black	122 C
Polished	428 C

Since the black surface has an α/ϵ ratio of unity, the corresponding value for the white surface is derived as:

$$\left(\frac{\alpha}{\epsilon}\right)_{\text{white}} = \left(\frac{T_{\text{white}}}{T_{\text{black}}}\right)^4 \left(\frac{\alpha}{\epsilon}\right)_{\text{black}} = \left(\frac{222}{395}\right)^4 (1) = 0.10$$

Other data available are from Ref. 25, in which an α/ϵ of 0.17 was measured for a white coating. The substrate was type 231 stainless steel. The coating material was a melamine resinrutile titanium dioxide paint of 0.010 in. thickness, applied in two coats. It is felt that the 1980 state of the art will permit materials with α/ϵ values of less than 0.17 and possibly approaching 0.10. However, a value of 0.17 is assumed for this study. Integrating this effect with the tank configuration parameters results in a net temperature ratio as shown in the following table for broadside exposure.

<u>Number of Tanks in Cluster</u>	<u>$\frac{T_{\text{white tank}}}{T_{\text{black sphere}}}$</u>
1	0.668
2	0.563
3	0.604
4	0.563
5	0.532
6	0.563

The corresponding temperature ratio for axial alignment is 0.344. The tank orientation to the sun in heliocentric space will vary with the mission. If it is assumed that the tank is always axially aligned to the flight path, the orientation to the sun will be nearly broadside for low-energy trips to the inner planets, and more axial for trips to the outer planets and high-energy trips to the inner planets. It is convenient and reasonable to combine all orientation effects and clustering effects, and assume an average temperature ratio of 0.57. This temperature profile is shown in Fig. A-1.

The heliocentric portion of a mission is only one contributor to the total temperature environment. A temperature analysis for a propellant tank in planetary orbit must also be performed. In a highly detailed analysis, one would consider the orientation time history of the tank with respect to both the sun and the planet. For this study, it is assumed that the tank is always parallel to the orbital path. Therefore the direct solar radiation varies from broadside to axial as the tank circles the planet. Likewise the reflected radiation is nearly broadside and varies from some maximum intensity to near zero at a point past the terminator, the point depending upon the orbital altitude.

Since the tanks may be in various-size orbits, the effects of orbital altitude can be considered and expressed as in Ref. 25, such that

$$F = 1 - \frac{\sqrt{h(h+2R)}}{h+R}$$

where F = intensity at altitude ÷ intensity at surface

h = orbital altitude

R = planet radius

The following table summarizes the effects of orbital altitude for the conditions used in this study.

<u>Planet</u>	<u>R*</u>	<u>h*</u>	<u>F</u>
Mercury	1350	50	0.735
Venus	3350	500	0.510
Earth	3440	150	0.715
Mars	1790	500	0.375
Jupiter	37,700	580,000	0.0018
Saturn	31,100	661,000	0.0010
Uranus	13,800	223,000	0.0018
Neptune	13,500	177,650	0.0021
Pluto	1580	1580	0.134

* R and h in nautical miles

Since an orbiting vehicle will be entering and leaving the planet's shadow, it is necessary to formulate approximate view factors. Figure A-2 is a schematic of an orbiting vehicle, where a view factor of unity corresponds to a broadside exposure and an A/S value of 0.296 (single tank). When the tank enters the shadow, it is assumed that the view factor has an equivalent

value of zero, instead of letting the intensity go to zero. The point at which the shadow occurs will vary with orbital altitude, but for low orbits it will occur just past the terminator. If the terminator is used as the shadow point, the average view factor for one revolution will be

$$x = \left(\frac{1+0.28}{2} \right) \left(\frac{180}{360} \right) = 0.32$$

Since the shadow points are beyond the terminator, the view factor will be slightly higher, as shown in Fig. A-2 where $x = 0.35$.

For clustered tanks, where A/S is lower, the average view factor is about 0.20. Also, for large orbits, it is assumed that no shadow exists. Hence, single tanks have a view factor of 0.64 and clustered tanks a value of 0.39 (see Fig. A-2).

For the outer planets of Jupiter and Saturn, the tanks are not only in large orbits about the planet, but are simultaneously in orbits about one of the planet's moons. It is assumed that for half the stay time, the moon is in the planet's shadow. Thus the view factors become 0.17 and 0.10 for single and clustered tanks, respectively. Since these view factors are small, and since the total heat pulse while in planetary orbit is small compared to the total heat pulse of the heliocentric transfer, the effects of the heat flux from the moons will be neglected.

The direct solar radiation to a tank in planetary orbit is then

$$I_s = xT_s^4$$

where

I_s = average flux intensity

x = direct radiation view factor

T_s = equilibrium temperature of tank in space

The reflected solar radiation will impinge nearly broadside at all times, except in the planet's shadow. Therefore a view factor of 0.80 is assumed for the tanks while receiving reflected radiation. It is also assumed that the tanks are in shadow 40% of the time. Thus an average view factor of 0.50 is assumed for a single tank and 0.25 for clustered tanks. For large orbits, the view factors become 0.80 and 0.40 for single and clustered tanks, respectively.

The reflected solar radiation has an intensity expressed as:

$$I_r = ayFT_s^4$$

where

I_r = average reflected solar flux intensity

a = albedo

y = reflected radiation view factor

F = altitude parameter

T_s = equilibrium temperature of tank in space

The heat emitted from each planet (except Mercury) is assumed to be constant over its surface, where the average surface temperature is defined as:

$$T_p = \frac{394 (1-a)^{1/4}}{\sqrt{2R}}$$

Mercury's period of revolution is assumed equal to its orbital period. Hence, one side is always in the shadow and has a temperature of 20 deg. Kelvin. The maximum temperature on the sunlit side is 620 K. An average surface temperature of 310 K is assumed.

The planet-emitted radiation can then be expressed as

$$I_p = zFT_p^4$$

where

I_p = planet-emitted flux intensity

z = view factor

F = altitude factor

T_p = average surface temperature of planet

The view factor, Z , is 1.00 and 0.50 for single and clustered tanks respectively.

The total radiation to an orbiting tank can be expressed as

$$I_T = xT_s^4 + ayFT_s^4 + zFT_p^4$$

The resulting equilibrium temperature of the tank surface is then:

$$T_e = T_s \left[x + ayF + zF \left(\frac{T_p}{T_s} \right)^4 \right]^{1/4}$$

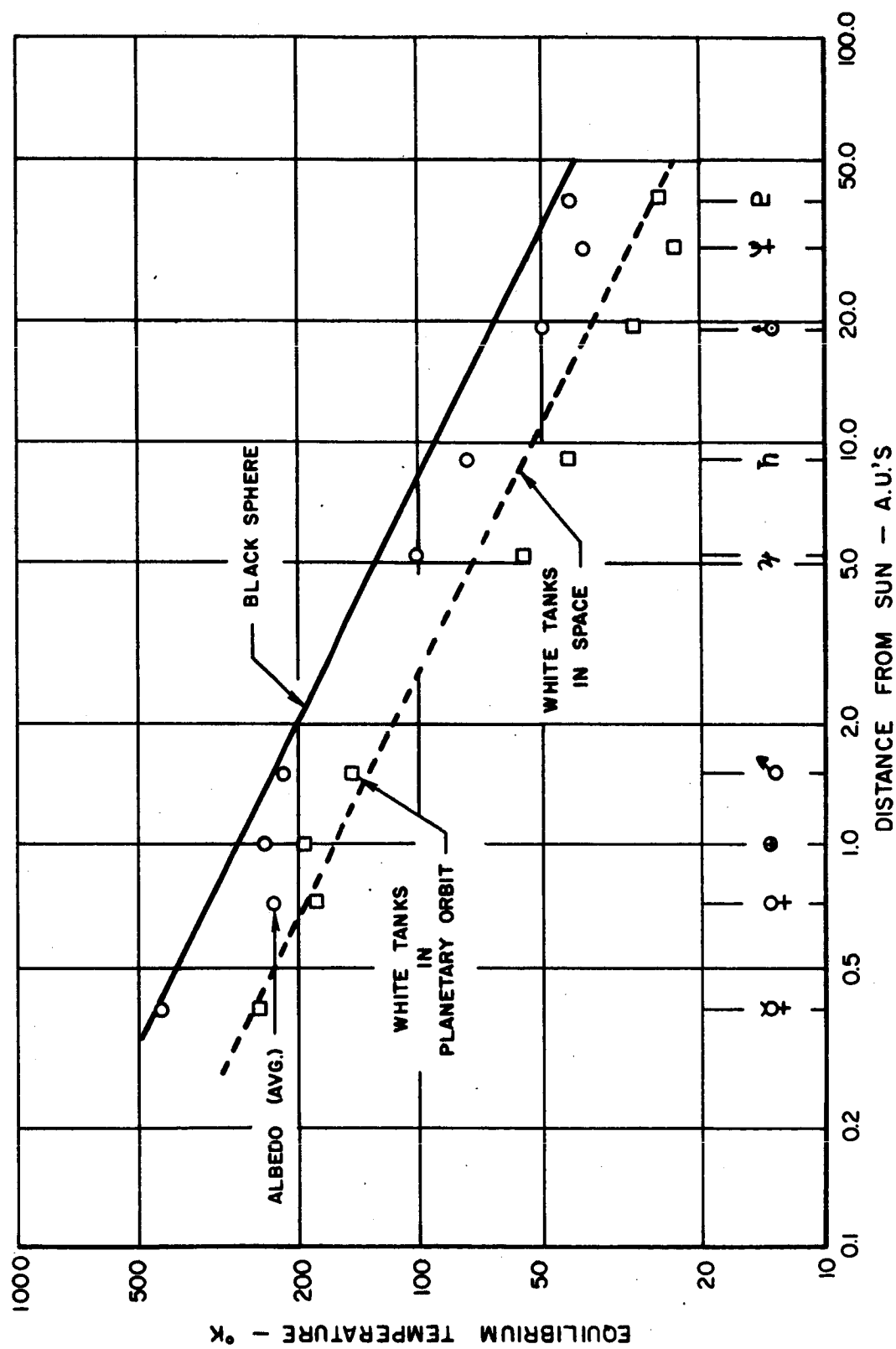
With an α/ϵ of 0.17 and clustered tanks, the temperature environment and prime parameters can be summarized as follows.

<u>Planet</u>	<u>F</u>	<u>a</u>	<u>T_p *</u>	<u>T_s *</u>	<u>T_e *</u>
Mercury	0.735	0.058	310	254	256
Venus	0.510	0.76	230	187	181
Earth	0.715	0.39	246	160	196
Mars	0.375	0.148	217	130	147
Jupiter	0.0018	0.51	102	70	55
Saturn	0.0010	0.50	78	53	42
Uranus	0.0018	0.66	49	37	29
Neptune	0.0021	0.62	40	29	23
Pluto	0.134	0.16	42	26	25

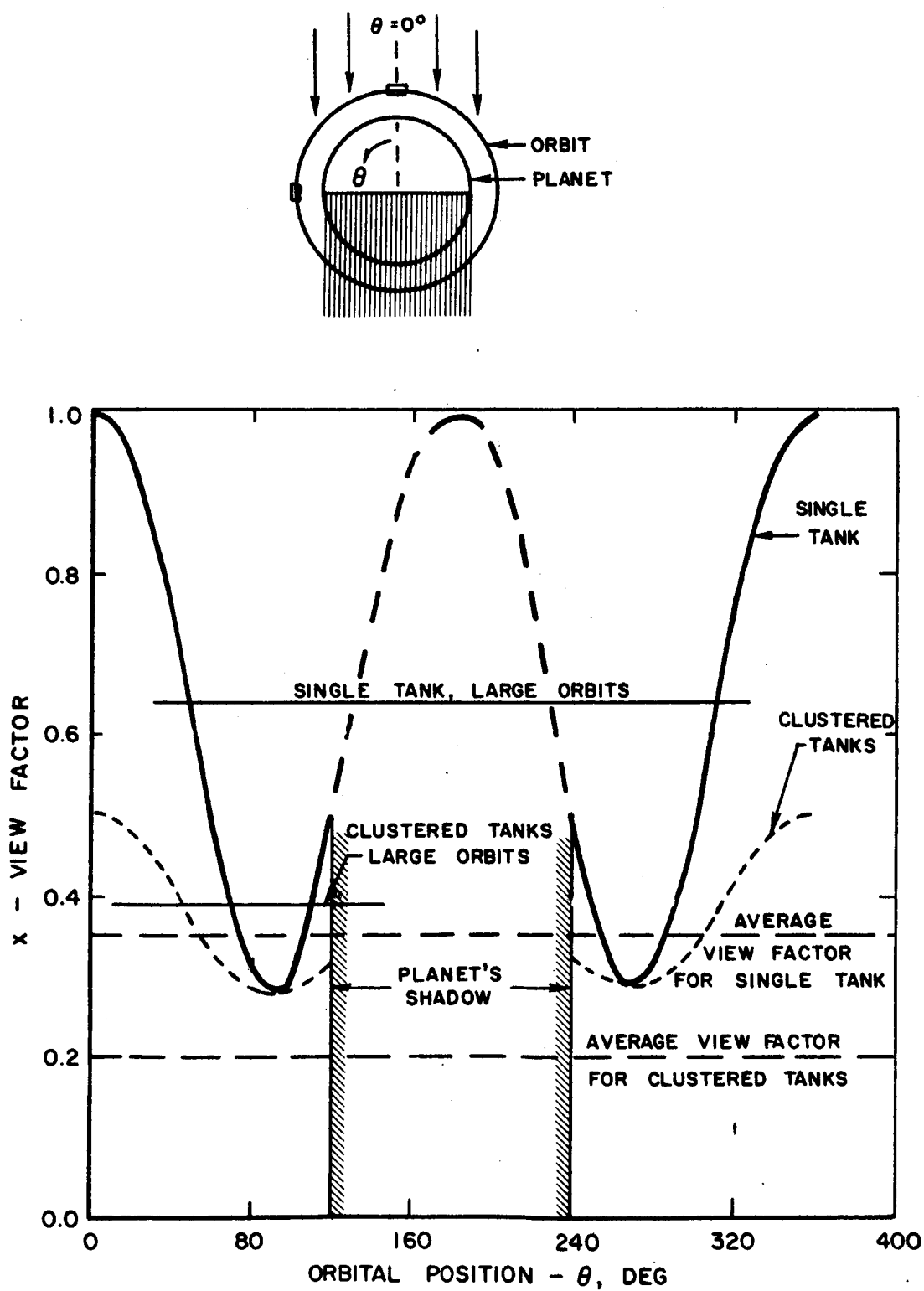
* degrees Kelvin

The data in the above table are also shown graphically in Fig. A-1. With these temperatures, it is necessary to establish the temperature time-history during a given mission. Examples are shown in Fig. A-3 for Mercury and Jupiter. From profiles such as these, average temperatures are established and used for the average temperature environment for the planetary escape stages. The Earth escape stages have a 10-day exposure and an average temperature of 196 K. The temperatures for the planetary capture stages are derived similarly to the planetary escape stages except that the effects of orbital stay time are omitted.

EQUILIBRIUM TEMPERATURES

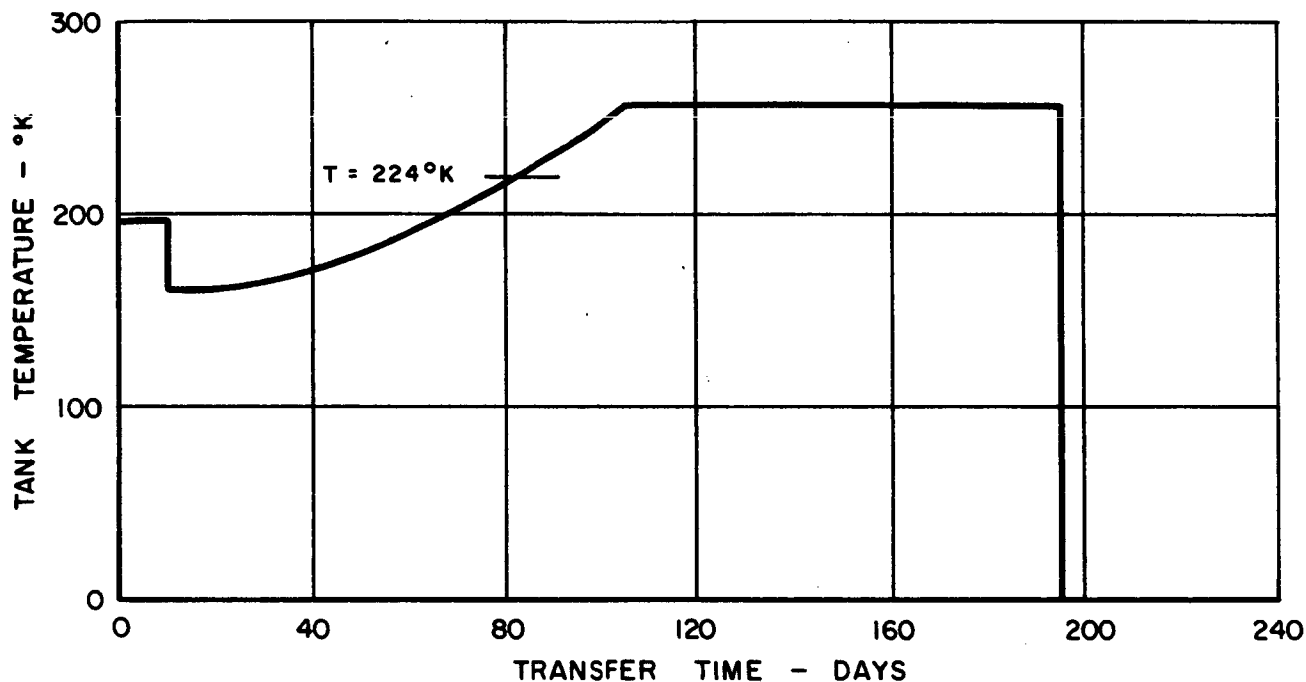


PROPELLANT TANK VIEW FACTOR FOR DIRECT SOLAR RADIATION

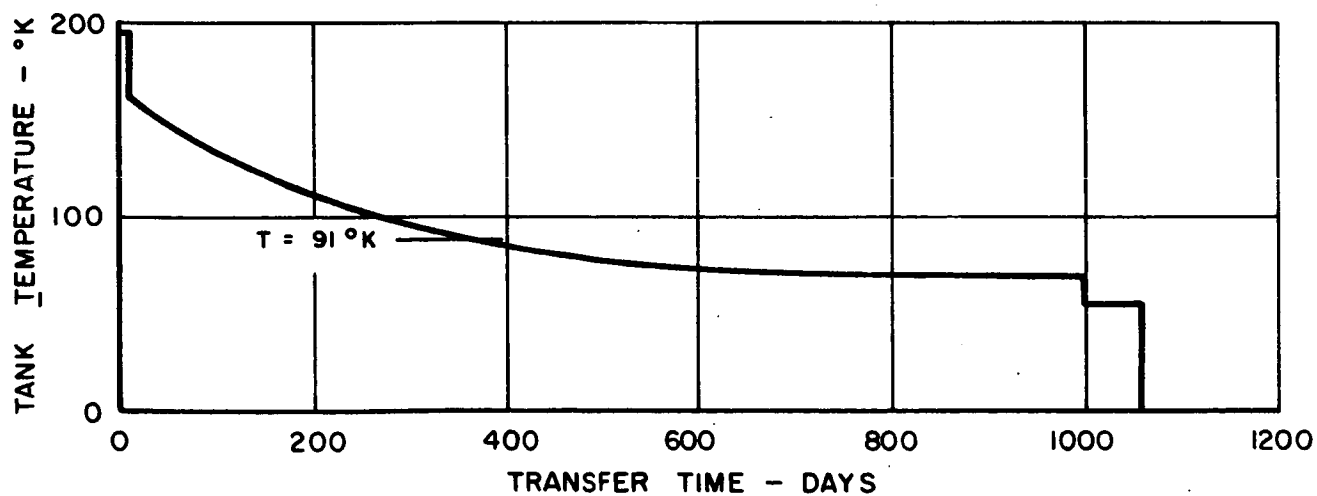


TEMPERATURE TIME HISTORIES

EARTH TO MERCURY



EARTH TO JUPITER



APPENDIX B

MASS COMPUTATIONAL PROGRAM

This mass computational program is based on a system of events which define a mission. The events are analyzed in reverse chronological order, with the last computation being the Earth departure maneuver. The following is a description of a manned landing mission. The computations start with the Earth return payload.

Midcourse Correction

A contingency is included for a return-leg, midcourse correction consisting of a 500 fps incremental velocity. This maneuver uses chemical propulsion with a step structural factor of 0.20. The midcourse correction step weight is then:

$$W_{MCI} = \left(\frac{\mu_{MCI}^{-1}}{1 - 0.2 \mu_{MCI}} \right) W_{ERPL} \quad (B-1)$$

where

$$\mu_{MCI} = e^{\frac{15.52}{I_{spMCI}}} \quad (B-2)$$

The payload leaving the planet can be written as:

$$W_{LPPL} = W_{ERPL} + W_{MCI} \quad (B-3)$$

Planetary Escape

The hyperbolic excess velocity from the trajectory analysis is fed into a subroutine to determine the gravity losses associated with escaping from a gravitational field. Since the gravity losses are a function of the initial acceleration, a value of 0.90 is used to initiate the iteration. The gravity loss subroutine outputs a corrected V_{∞} which includes the losses. This V_{∞} is then converted to ΔV , where

$$\Delta V_{LP} = (2V_{CP}^2 + V_{\infty P}^2)^{\frac{1}{2}} - V_{CP} \quad (B-4)$$

where V_{CP} is the circular velocity around the planet at the orbital altitude of departure. The ratio of initial mass to burn-out mass, μ , is computed

$$\mu_{LP} = e \frac{\Delta V_{LP}}{I_{sp} g} \quad (B-5)$$

where "g" is the acceleration of gravity at the Earth's surface. The inert weights of the propellant tanks are defined as

$$W_{ILP} = \frac{\frac{AW_p^{0.9}}{\sigma^{0.533}} + B\left(\frac{W_p}{\sigma}\right)^{\frac{2}{3}}\left(\frac{t \Delta T}{L}\right)^{\frac{1}{2}} + C\left(\frac{W_p}{\sigma}\right)^{\frac{5}{6}} T^{\frac{1}{4}} + Zn^{1.12} + \tau g_1 (W_{PL} + W_p) n^{0.12} + 500}{1 - \tau g_1 n^{0.12}} \quad (B-6)$$

In Eq. (6), only the larger of the first or third terms are used. If the third term is the largest, it means that the meteoroid protection weights, and thus the skin thickness, is greater than the tank walls. Therefore, the meteoroid protection material will also suffice for the tank walls. Conversely, if the first term is largest, it indicated that no further meteoroid protection need be added to the tank skin.

An initial approximation is made to the propellant weight, W , in Eq. (6), such that

$$W_{PO} = (\mu_{LP}^{-1}) W_{LPL}$$

With the resulting inert weight, W_{ILP} , the inert weight structural factor can be expressed as

$$\beta = \frac{W_{ILP}}{W_{ILP} + W_{PO}} \quad (B-7)$$

A limit is put on β such that

$$\beta_{LIM} = \frac{1}{\mu_{LP}}$$

If $\beta \geq \beta_{LIM}$, the initial guess at propellant weight is too small. If $\beta < \beta_{LIM}$, then a corresponding propellant weight can be defined as,

$$W_{PPL} = W_{PO} \left(\frac{1 - \beta}{1 - \mu_{LP} \beta} \right)$$

Since $W_{PPL} \neq W_{PO}$, another iteration is taken by increasing the propellant weight greater than W_{PO} . This procedure is continued until the propellant

weight input into Eq. (6) is within one-tenth of 1% of the propellant weight computed by

$$W_p = \left[\frac{(1-\beta)(\mu-1)}{1-\mu\beta} \right] W_{LPPL} \quad (B-8)$$

If $\beta \geq \beta_{LIM}$, instead of guessing more values of propellant weight, values of β are assumed which are slightly less than β_{LIM} . A propellant weight is computed from Eq. (8) and substituted into Eq. (6). With the resultant inert weight, a structural factor is computed from Eq. (7).

The iteration is completed by computing the propellant weight from the last β . Again, the initial and final values of propellant weight are compared to the one-tenth of 1% tolerance.

Values of β below β_{LIM} are continually decreased until the convergence on propellant weight is reached.

The next iteration consists of reducing the initial acceleration, g_i , and repeating the above procedure. If the magnitude of $W_p + W_I$ decreased, g_i is decreased until a minimum value of $W_p + W_I$ is found. If, initially, the value of $W_p + W_I$ increases, then g_i is increased until the minimum value of $W_p + W_I$ is found.

The total weight in planetary orbit prior to departure is then

$$W_{POF} = W_{PLP} + W_{ILP} + W_{LPPL}$$

Planetary Surface Operations

The surface operations are performed with an excursion module whose weight, minus all propulsion, is defined as a function of the spacecraft weight, W_{ERPL} . The total velocity required to ascend from the planet's surface and establish orbit is defined as:

$$\Delta V_{PTO} = V_{cp} \{1 + \sqrt{2/r_c} + 1\} (r_c - 1)$$

The mass ratio is defined as:

$$\mu_{PTO} = e \frac{\Delta V_{PTO}}{I_{sp} g}$$

The initial acceleration is set at 1.2 g's. With this value, the inert weight is computed, using an initial approximation for propellant weight

$$W_{PPTO} = (\mu_{PTO} - 1) W_{EXM}$$

The convergence on propellant weight is performed as for the planetary escape maneuver. The total weight on the planet's surface is:

$$W_{PS} = W_{PPTO} + W_{IPTO} + W_{EXM}$$

The total velocity requirements to descend from orbit are assumed equal to those for ascending to orbit. Therefore

$$\Delta V_{OTP} = \Delta V_{PTO}$$

The propulsion requirements are derived in a manner similar to that for ascent. The total mass in planetary orbit prior to the surface operation is:

$$W_{POI} = W_{POP} + W_{PS} + W_{POTP} + W_{IOTP}$$

which is the sum of the spacecraft weight, planetary escape propulsion, excursion module weight, and all propulsion for the ascent and descent maneuvers to the planet's surface. This total weight is also the planetary escape payload.

Planetary Capture

The computations for the planetary capture maneuver are identical to that for planetary departure. The hyperbolic excess speed is increased by the gravity-loss subroutine to account for the gravitational field. The optimum initial T/W is determined along with the corresponding propellant and inert weights. The total mass approaching the planet prior to capture is

$$W_{AP} = W_{POI} + W_{PAP} + W_{IAP}$$

Midcourse Correction

As with the return leg, a midcourse correction contingency is included for the outbound transfer. Again a ΔV of 500 fps is assumed along with a step structural factor of 0.20. The Earth escape payload then becomes

$$W_{EEPL} + W_{MCO} + W_{AP}$$

Earth Escape

All Earth escape maneuvers start from a 278-km circular orbit after orbiting for 10 days. Again, the optimum initial T/W ratio, propellant weight, and inert weights are determined as in the planetary escape maneuver. The initial mass required in Earth orbit is defined as

$$MEO = W_{EEPL} + W_{PLE} + W_{ILE}$$

The corresponding payload fraction is

$$\omega = \frac{W_{ERPL} + W_{EXM}}{MEO}$$

APPENDIX C

THE FINITE-DIFFERENCE NEWTON-RAPHSON ALGORITHM

The finite-difference Newton-Raphson method for solving two-point boundary value problems may be explained by considering the following system of nonlinear second order equations:

$$\ddot{x}_i = f_i(x_1, \dots, x_{2m}, t) \quad i = 1, \dots, 2m \quad (C-1)$$

with associated boundary conditions

$$\begin{aligned} x_i(a) &= \alpha_{i0} & \dot{x}_i(a) &= \dot{\alpha}_{i0} \\ x_i(b) &= \alpha_{i(N+1)} & \dot{x}_i(b) &= \dot{\alpha}_{i(N+1)} \end{aligned} \quad i = 1, \dots, m \quad (C-2)$$

on half of the variables and their derivatives at fixed initial and terminal times.

A particular problem of this form arises from a variational trajectory problem where the variables having boundary conditions are the positions in space, and the variables whose boundary values are not specified are the Lagrange multipliers. The method may also be used to solve classical problems in celestial mechanics where boundary conditions would be imposed on all the state variables (orbital elements) but not their derivatives (Ref. C-1).

The second-order form of the equations has been purposely chosen so that a simple, stable finite-difference approximation can be used. Variational problems and celestial mechanics problems are often stated in a first-order Hamiltonian formulation which can generally be transformed into a second-order Lagrangian formulation by the methods of classical mechanics. The resulting equations may contain first derivatives (unlike Eq. (C-1)), but the method can easily handle this case (Ref. C-1).

It is assumed that the Euler-Lagrange equations for the control variables have been explicitly solved in terms of the Lagrange multipliers, so that no control variables appear in Eq. (C-1). For problems where this cannot be done, the generalized Hamiltonian formulation of Ref. C-2 may be used.

The solution of the boundary value system represented in Eqs. (C-1) and (C-2) is approached as follows. If an appropriate space of $2m$ -tuples of functions

x_1 is defined on the interval $a \leq t \leq b$, (C-1) may be considered as defining the operator equation

$$L(X) - F(X) = P(X) = 0. \quad (C-3)$$

L represents the linear operator defined by the second derivatives, and F is the nonlinear operator defined by the right-hand sides. Proceeding formally, the Newton-Raphson iteration,

$$X_{n+1} = X_n - [P'(X_n)]^{-1}P(X_n), \quad (C-4)$$

can be applied to (C-3) when an approximate solution, X_0 , is known. Letting $\Delta_n = X_{n+1} - X_n$, multiplying through by the derivative operator $P'(X_n)$, and relating the notation to Eq. (C-3), this iteration becomes

$$[-L + F'(X_n)](\Delta_n) = L(X_n) - F(X_n). \quad (C-5)$$

The quantity $[-L + F'(X_n)]$ is a linear operator which when applied to the iterative change Δ_n yields the value of the operator at the previous iteration. Reference C-3 gives the general conditions under which this approach is applicable and convergent.

The algorithm amounts to putting (C-5) in the form of a large, but easily solved, matrix equation. This end is achieved by imposing a mesh of N points,

$$t_j = a + jh, \quad h = \frac{b-a}{N+1}, \quad \text{and } j = 1, \dots, N, \quad (C-6)$$

on the interval $[a, b]$. The value of the n^{th} iterate $x_1^n(t_j)$ is written $x_1^n_j$, x_n is thus a $2mN^{\text{th}}$ dimensional vector, and its elements $x_1^n_j$ are ordered such that i runs through its $2m$ values for each value of j . Some differences in this arrangement will be encountered on the ends of the vector to accommodate the boundary conditions on $\dot{x}_1$.

By approximating the second derivatives with the central difference quotient,

$$\ddot{x}_{1j} \approx \frac{x_{1j-1} - 2x_{1j} + x_{1j+1}}{h^2}, \quad (C-7)$$

2mN linear equations for the unknowns $\delta_{ij}^n = x_{ij}^{n+1} - x_{ij}^n$ are obtained as follows:

$$-\frac{\delta_{ij-1}^n - 2\delta_{ij}^n + \delta_{ij+1}^n}{h^2} + \sum_{v=1}^{2m} \left[\frac{\partial f_i}{\partial x_v}(x_{ij}^n, \dots, x_{2mj}^n, t) \right] \delta_{vj}^n = p_{ij}^n, \quad (C-8)$$

where

$$p_{ij} = \frac{x_{ij-1}^n - 2x_{ij}^n + x_{ij+1}^n}{h^2} - f_i(x_{ij}^n, \dots, x_{2mj}^n, t). \quad (C-9)$$

These equations form the major part of the matrix equation, Eq. (C-5). To complete the system, two additional mesh points are added, each being one mesh spacing outside the interval $[a, b]$. Defining only the state variables at these points, Eq. (C-8) can be written for $i = 1, \dots, m$ and $j = 0, N+1$ (i.e. for $t = a, b$). The boundary conditions on $\dot{x}_i$ can now be included in the system with the equations

$$\frac{\delta_{ij-1}^n - \delta_{ij+1}^n}{2h} = \frac{x_{ij-1}^n - x_{ij+1}^n}{2h} - \dot{x}_{ij} \quad (C-10)$$

$$i = 1, \dots, m \text{ and } j = 0, N+1.$$

The conditions on x_i are naturally met by using the given x_{ij} and noting that the corresponding $\delta_{ij} = 0$.

These equations form a block tri-diagonal matrix equation of the form:

$$\begin{pmatrix} B_0 & C_0 & & & & \\ A_1 & B_1 & -I & & & \\ & -I & B_2 & -I & & \\ & & & \ddots & & \\ & & & & -I & B_{N-1} & -I \\ & & & & & -I & B_N & C_N \\ & & & & & & A_{N+1} & B_{N+1} \end{pmatrix} \begin{pmatrix} D_0 \\ D_1 \\ D_2 \\ \vdots \\ \vdots \\ D_{N-1} \\ D_N \\ D_{N+1} \end{pmatrix} = \begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ \vdots \\ \vdots \\ P_{N-1} \\ P_N \\ P_{N+1} \end{pmatrix} \quad (C-11)$$

The matrix elements are each $2m \times 2m$ submatrices, and the vector elements represent a corresponding partitioning as follows:

$$P_j = h^2 \begin{pmatrix} p_{1,j} \\ \vdots \\ p_{2m,j} \end{pmatrix}, \text{ and } D_j = \begin{pmatrix} \delta_{1,j} \\ \vdots \\ \delta_{2m,j} \end{pmatrix}, \quad j = 1, \dots, N \quad (C-12)$$

This system must be formed and solved for each iteration, and

$$B_j = 2I + h^2 J_j^n, \quad j=1, \dots, N, \quad (C-13)$$

J_j^n being the $2m \times 2m$ Jacobian matrix with elements $\partial f_i / \partial x_j$, evaluated at the n^{th} iteration. The different blocks at the extremes of the matrix arise from the special boundary condition equations.

For the most part the subdiagonal and superdiagonal blocks are merely negative identity matrices. Denoting these submatrices generally by A_j and C_j ($j = 1, \dots, N+1$) the solution of Eq. (C-1) is obtained by the following definitions (Ref. C-4, pg. 196):

$$W_0 = B_0^{-1} C_0, \quad G_0 = B^{-1} P_0, \quad (C-14)$$

$$W_j = (B_j - A_j W_{j-1})^{-1} C_j, \quad G_j = (B_j - A_j W_{j-1})^{-1} (P_j - A_j G_{j-1}), \quad j=1, \dots, N+1.$$

These definitions make possible the recursive computation of the components of the solution

$$D_{N+1} = G_{N+1}, \quad D_j = G_j - W_{j+1}, \quad j = N, \dots, 1. \quad (C-15)$$

Of course, considerable simplification takes place when $-I$ is substituted for the appropriate A 's and C 's.

APPENDIX C REFERENCES

- C-1. Van Dine, C. P.: An Application of Newton's Method to the Finite Difference Solution of Nonlinear Boundary Value Systems. United Aircraft Research Laboratories Report UAR-D37, March 1965.
- C-2. Payne, M.: "Dirais Generalized Hamiltonian Dynamics and the Pontryagin Principle", in NASA TMX-53150, Huntsville, Ala., October 1964.
- C-3. Kantorovich, L. V. and G. P. Akilov: Functional Analysis in Normed Spaced Chap. XVIII, Pergamon Press, 1964.
- C-4. Varga, R. S.: Matrix Iterative Analysis. Chap. 6, Prentice-Hall, 1962.

APPENDIX D

LOW-THRUST TRAJECTORY ANALYSIS

Variable Thrust

The variable-thrust trajectory analysis and all computations were performed in three dimensions. The analysis was carried out under the following assumptions:

1. The vehicle thruster is capable of completely variable I_{sp} . The thruster efficiency, $\eta(I_{sp})$, is taken as unity throughout the powered flight.
2. The departure and destination planets associated with the trajectories are nongravitating points in space; the only gravitation force acting upon the vehicle is that induced by the Sun's mass.
3. The planetocentric and heliocentric trajectories are computed separately and matched such that the asymptotic velocity of the vehicle in the planetocentric frame is added vectorially to the heliocentric velocity of the planet to give the boundary value of velocity for the interplanetary trajectory.

The methods of the calculus of variations are applied to minimizing the integral

$$J = \int_0^T \frac{a^2}{2P(X,t)} dt$$

subject to the dynamical constraints

$$g_1(\dot{X}, X) = \dot{x} - q = 0$$

$$g_2(\dot{X}, X) = \dot{y} - r = 0$$

$$g_3(\dot{X}, X) = \dot{z} - s = 0$$

$$g_4(\dot{X}, X) = \dot{q} - a_x + \frac{x}{R^3} = 0$$

$$g_5(\dot{X}, X) = \dot{r} - a_y + \frac{y}{R^3} = 0$$

$$g_6(\dot{X}, X) = \dot{s} - a_z + \frac{z}{R^3} = 0$$

(D-1)

Where a_x , a_y and a_z are the components of the thrust acceleration in the x, y, and z directions respectively, i.e., $a_x^2 + a_y^2 + a_z^2 = a^2$; q, r and s are the components of the vehicle velocity vector and $R^2 = x^2 + y^2 + z^2$. The units of time and length have been chosen such that the value of the Gaussian constant is unity. The optimization problem can now be restated as the minimization of the integral

$$I = \int_0^T F(\dot{X}, X, t) dt \quad (D-2)$$

where

$$F = \frac{a_x^2 + a_y^2 + a_z^2}{2P(X, t)} + \sum_{i=1}^6 \lambda_i g_i(\dot{X}, X) \quad (D-3)$$

and the λ_i are the Lagrange multipliers associated with the constraints g_i , and are, in general, functions of time, t. Setting the first variation of I equal to zero yields the Euler-Lagrange necessary conditions for the extremization of I:

$$\frac{d}{dt} \frac{\partial F}{\partial \dot{X}_i} - \frac{\partial F}{\partial X_i} = 0 \quad i = 1, 2, \dots, 9 \quad (D-4)$$

where the X_i are now specified as the set $[x, y, z, q, r, s, a_x, a_y, a_z]$.

Before proceeding further it is advantageous to specify the functional form of $P(X, t)$. We have chosen

$$P(X, t) = \frac{P_0 e^{-\gamma t}}{R^n} \quad (D-5)$$

where γ and n are time-independent parameters. The proportionality of exhaust power to $1/R^n$ is chosen to allow the solution latitude for taking into account the degrading of solar cell efficiency due to large thermal gradients encountered during close passage of the Sun. The exponential term may represent the time decay of a radioisotope power source or, perhaps more importantly, it may represent the reliability of the power source over the trip duration based upon a postulated powerplant component failure rate. The proper choice of n and γ can represent several different power modes as illustrated below.

Parameter Specification	Power Mode
$\gamma = 0; n = 0$	Constant Power, P_0 ; [1.0] reliability
γ given; $n = 0$	Constant Power, P_0 ; $[e^{-\gamma t}]$ reliability
$\gamma = \alpha + \beta; n = 0$	Radioisotope Power, $P_0 e^{-\alpha t}$; $[e^{-\beta t}]$ reliability
$\gamma = 0, n$ given	Solar Power, P_0/R^n ; [1.0] reliability
γ, n given	Solar Power, P_0/R^n ; $[e^{-\gamma t}]$ reliability

The differential equations governing the optimal trajectory can now be obtained from Eqs. (D-3), (D-4), and (D-5) by the appropriate operations.

$$\begin{aligned}\ddot{x} &= \frac{P_0 u}{e^{\gamma t} R^n} - \frac{x}{R^3}, & a_x &= \frac{P_0 u}{e^{\gamma t} R^n} \\ \ddot{y} &= \frac{P_0 v}{e^{\gamma t} R^n} - \frac{y}{R^3}, & a_y &= \frac{P_0 v}{e^{\gamma t} R^n} \\ \ddot{z} &= \frac{P_0 w}{e^{\gamma t} R^n} - \frac{z}{R^3}, & a_z &= \frac{P_0 w}{e^{\gamma t} R^n}\end{aligned}\tag{D-6}$$

$$\begin{aligned}\ddot{u} &= -\frac{m P_0 \lambda^2 x}{e^{\gamma t} R^{n+2}} + \frac{u(2x^2 - y^2 - z^2) + 3x(vy + wz)}{R^5} \\ \ddot{v} &= -\frac{m P_0 \lambda^2 y}{e^{\gamma t} R^{n+2}} + \frac{v(2y^2 - x^2 - z^2) + (ux + wz)}{R^5} \\ \ddot{w} &= -\frac{m P_0 \lambda^2 z}{e^{\gamma t} R^{n+2}} + \frac{w(2z^2 - x^2 - y^2) + 3z(ux + vy)}{R^5}\end{aligned}$$

where $2m = n$, $u \equiv \lambda_4$, $v \equiv \lambda_5$, $w \equiv \lambda_6$ and $\lambda^2 = u^2 + v^2 + w^2$

For the purpose of displaying the explicit form that these equations take on in the numerical solution as set forth in Appendix C, the problem is now considered for the specific trajectory mode of planetary rendezvous with hyperbolic excess velocities assigned to the vehicle at the boundaries. The transversality condition for this problem is

$$\sum_{k=1}^6 \frac{\partial F}{\partial \dot{x}_k} \delta x_k \Big|_0^T = 0 \quad (D-7)$$

where the departure date and trip time have been specified. Making use of Eqs. (D-1) and (D-3) and the fact that the positions at $t = 0, T$ are specified, Eq. (D-7) takes the form

$$u \delta \dot{x} + v \delta \dot{y} + w \delta \dot{z} \Big|_{t=0}^{t=T} = 0 \quad (D-8)$$

The case where an excess velocity of prescribed magnitude, V_0 , is added vectorially to the given planetary velocity at $t = 0$ has been treated elsewhere (Ref. D-1) and is briefly repeated here. Only the magnitude of the excess velocity, V_0 , has been fixed, hence it can be oriented in any direction and the locus of the tip of the resultant velocity vector is a circle as expressed by Eq. (D-9).

$$(\dot{x} - \dot{\xi}_0)^2 + (\dot{y} - \dot{\eta}_0)^2 + (\dot{z} - \dot{\zeta}_0)^2 - V_0^2 \Big|_{t=0} = 0 \quad (D-9)$$

where $\dot{\xi}_0$, $\dot{\eta}_0$ and $\dot{\zeta}_0$ are the velocity components of the departure planet. Taking the variation of (D-9) yields

$$(\dot{x} - \dot{\xi}_0) \delta \dot{x} + (\dot{y} - \dot{\eta}_0) \delta \dot{y} + (\dot{z} - \dot{\zeta}_0) \delta \dot{z} \Big|_{t=0} = 0 \quad (D-10)$$

Equations (D-8) and (D-10) yield the conditions

$$\frac{u}{v} = \frac{\dot{x} - \dot{\xi}_0}{\dot{y} - \dot{\eta}_0}, \quad \frac{v}{w} = \frac{\dot{y} - \dot{\eta}_0}{\dot{z} - \dot{\zeta}_0}, \quad \frac{w}{u} = \frac{\dot{z} - \dot{\zeta}_0}{\dot{x} - \dot{\xi}_0} \quad (D-11)$$

evaluated at $t = 0$. Considerations of Eqs. (D-6) and (D-11) show that V_0 is parallel and of the same sense as the low-thrust acceleration vector at $t = 0$. A similar result can be shown for an excess velocity, V_N , applied to the vehicle at the destination planet at $t = T$.

Now proceeding with knowledge of the positions and velocities of the departure and destination planets

$$\begin{aligned}
 x_1(0) &= \alpha_{10} & \dot{x}_1(0) &= \dot{\alpha}_{10} \\
 x_1(T) &= \alpha_{1(N+1)} & \dot{x}_1(T) &= \dot{\alpha}_{1(N+1)}
 \end{aligned}
 \tag{D-12}$$

the matrices associated with the algorithm of Appendix A can be explicitly set forth. The boundary value blocks take the form

$$B_0 = \left(\begin{array}{c|c} I & \beta_0 \\ \hline -I & \frac{P_0 h^2}{\Lambda_0^m} \cdot I \end{array} \right)$$

where

$$\beta_0 = \frac{2 h V_0}{\Lambda_0^{3/2}} \begin{pmatrix} (v_0^n)^2 + (w_0^n)^2 & -u_0^n v_0^n & -u_0^n w_0^n \\ -u_0^n v_0^n & (u_0^n)^2 + (w_0^n)^2 & -v_0^n w_0^n \\ -u_0^n w_0^n & -v_0^n w_0^n & (u_0^n)^2 + (v_0^n)^2 \end{pmatrix}$$

$$C_0 = \left(\begin{array}{c|c} -I & 0 \\ \hline -I & 0 \end{array} \right) = A_{N+1}$$

$$A_1 = \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & -I \end{array} \right) = C_N$$

and

$$\left(\begin{array}{c|c} \mathbf{I} & \beta_{N+1} \\ \hline -\mathbf{I} & \frac{Poh^2}{e \gamma^T A_{N+1}^m} \cdot \mathbf{I} \end{array} \right) = \beta_{N+1}$$

where

$$\beta_{N+1} = \frac{-2hV_N}{\Lambda_N^{3/2}} \begin{pmatrix} (v_N^n)^2 + (w_N^n)^2 & -u_N^n w_N^n & -u_N^n w_N^n \\ -u_N^n v_N^n & (u_N^n)^2 + (w_N^n)^2 & -v_N^n w_N^n \\ -u_N^n w_N^n & -v_N^n w_N^n & (u_N^n)^2 + (v_N^n)^2 \end{pmatrix}$$

The corresponding unknown vectors are

$$D_0 = \begin{pmatrix} x_{-1}^{n+1} - x_{-1}^n \\ y_{-1}^{n+1} - y_{-1}^n \\ z_{-1}^{n+1} - z_{-1}^n \\ u_0^{n+1} - u_0^n \\ v_0^{n+1} - v_0^n \\ w_0^{n+1} - w_0^n \end{pmatrix}$$

$$D_{N+1} = \begin{pmatrix} x_{N+1}^{n+1} - x_{N+1}^n \\ y_{N+1}^{n+1} - y_{N+1}^n \\ z_{N+1}^{n+1} - z_{N+1}^n \\ u_N^{n+1} - u_N^n \\ v_N^{n+1} - v_N^n \\ w_N^{n+1} - w_N^n \end{pmatrix}$$

The corresponding right-hand-side vectors are

$$P_0 = \begin{pmatrix} -x_{-1}^n + x_1^n - 2h \left(\frac{v_0 u_0^n}{\Lambda_0^{1/2}} + \dot{a}_{x_0} \right) \\ -y_{-1}^n + y_1^n - 2h \left(\frac{v_0 v_0^n}{\Lambda_0^{1/2}} + \dot{a}_{y_0} \right) \\ -z_{-1}^n + z_1^n - 2h \left(\frac{v_0 w_0^n}{\Lambda_0^{1/2}} + \dot{a}_{z_0} \right) \\ x_{-1}^n - 2a_{x_0} + x_1^n - h^2 \left(\frac{P_0 u_0^n}{A_0^m} - \frac{a_{x_0}}{A_0^{3/2}} \right) \\ y_{-1}^n - 2a_{y_0} + y_1^n - h^2 \left(\frac{P_0 v_0^n}{A_0^m} - \frac{a_{y_0}}{A_0^{3/2}} \right) \\ z_{-1}^n - 2a_{z_0} + z_1^n - h^2 \left(\frac{P_0 w_0^n}{A_0^m} - \frac{a_{z_0}}{A_0^{3/2}} \right) \end{pmatrix}$$

where

$$A_0 = a_{x_0}^2 + a_{y_0}^2 + a_{z_0}^2$$

$$\Lambda_0 = (u_0^n)^2 + (v_0^n)^2 + (w_0^n)^2$$

and

$$P_{N+1} = \begin{pmatrix} x_{N-1}^n - x_{N+1}^n - 2h \left(\frac{v_N u_N^n}{\Lambda_N^{1/2}} - \dot{a}_{x(N+1)} \right) \\ y_{N-1}^n - y_{N+1}^n - 2h \left(\frac{v_N v_N^n}{\Lambda_N^{1/2}} - \dot{a}_{y(N+1)} \right) \\ z_{N-1}^n - z_{N+1}^n - 2h \left(\frac{v_N w_N^n}{\Lambda_N^{1/2}} - \dot{a}_{z(N+1)} \right) \\ x_{N-1}^n - 2a_{x(N+1)} + x_{N+1}^n - h^2 \left(\frac{P_0 u_N^n}{e^{\gamma_T} A_{N+1}^m} - \frac{a_{x(N+1)}}{A_{N+1}^{3/2}} \right) \\ y_{N-1}^n - 2a_{y(N+1)} + y_{N+1}^n - h^2 \left(\frac{P_0 v_N^n}{e^{\gamma_T} A_{N+1}^m} - \frac{a_{y(N+1)}}{A_{N+1}^{3/2}} \right) \\ z_{N-1}^n - 2a_{z(N+1)} + z_{N+1}^n - h^2 \left(\frac{P_0 w_N^n}{e^{\gamma_T} A_{N+1}^m} - \frac{a_{z(N+1)}}{A_{N+1}^{3/2}} \right) \end{pmatrix}$$

where

$$A_{N+1} = a_{x(N+1)}^2 + a_{y(N+1)}^2 + a_{z(N+1)}^2$$

$$\Lambda_N = (u_N^n)^2 + (v_N^n)^2 + (w_N^n)^2$$

The general unknown vector is

$$D_j = \begin{pmatrix} x_j^{n+1} - x_j^n \\ y_j^{n+1} - y_j^n \\ z_j^{n+1} - z_j^n \\ u_j^{n+1} - u_j^n \\ v_j^{n+1} - v_j^n \\ w_j^{n+1} - w_j^n \end{pmatrix}$$

and the general right-hand-side vector is given by

$$P_j = \begin{pmatrix} x_{j-1} - 2x_j + x_{j+1} - h^2 \left(\frac{P_0 u_j}{e^{\gamma_j} A_j^m} - \frac{x_j}{A_j^{3/2}} \right) \\ y_{j-1} - 2y_j + y_{j+1} - h^2 \left(\frac{P_0 v_j}{e^{\gamma_j} A_j^m} - \frac{y_j}{A_j^{3/2}} \right) \\ z_{j-1} - 2z_j + z_{j+1} - h^2 \left(\frac{P_0 w_j}{e^{\gamma_j} A_j^m} - \frac{z_j}{A_j^{3/2}} \right) \\ u_{j-1} - 2u_j + u_{j+1} - h^2 \left(\frac{P_0 m x_j \Lambda_j}{A_j^{m+1}} + \frac{u_j (2x_j^2 - y_j^2 - z_j^2) + 3x_j (v_j y_j + w_j z_j)}{A_j^{5/2}} \right) \\ v_{j-1} - 2v_j + v_{j+1} - h^2 \left(\frac{P_0 m y_j \Lambda_j}{A_j^{m+1}} + \frac{v_j (2y_j^2 - x_j^2 - z_j^2) + 3y_j (u_j x_j + w_j z_j)}{A_j^{5/2}} \right) \\ w_{j-1} - 2w_j + w_{j+1} - h^2 \left(\frac{P_0 m z_j \Lambda_j}{A_j^{m+1}} + \frac{w_j (2z_j^2 - x_j^2 - y_j^2) + 3z_j (u_j x_j + v_j y_j)}{A_j^{5/2}} \right) \end{pmatrix}$$

Where

$$\Lambda_j = u_j^2 + v_j^2 + w_j^2$$

$$A_j = x_j^2 + y_j^2 + z_j^2$$

and each variable has superscript n. The general coefficient matrix B_j is given on the next page; each variable in the matrix has superscript n and subscript j.

Transversality Conditions for Variable Low-Thrust Planetary Flyby

The components of the vehicle velocity are unspecified at $t = T$, hence the transversality condition (D-7) takes the form

$$u\dot{\delta x} + v\dot{\delta y} + w\dot{\delta z} \Big|_{t=T} = 0 \quad (\text{D-13})$$

There is no preference regarding the magnitude nor direction of the final velocity, hence $\delta\dot{x}$, $\delta\dot{y}$, and $\delta\dot{z}$ are independent.

$$\therefore u(T) = v(T) = w(T) = 0 \quad (\text{D-14})$$

The following alterations occur in the matrices associated with the algorithm.

$$P_{N+1} = \begin{pmatrix} x_{N-1}^n - 2a_{x(N+1)} + x_{N+1}^n + h^2 \frac{a_{x(N+1)}}{A_{N+1}^{3/2}} \\ y_{N-1}^n - 2a_{y(N+1)} + y_{N+1}^n + h^2 \frac{a_{y(N+1)}}{A_{N+1}^{3/2}} \\ z_{N-1}^n - 2a_{z(N+1)} + z_{N+1}^n + h^2 \frac{a_{z(N+1)}}{A_{N+1}^{3/2}} \\ U_{N+1}^n + U_{N-1}^n \\ V_{N+1}^n + V_{N-1}^n \\ W_{N+1}^n + W_{N-1}^n \end{pmatrix} \quad D_{N+1} = \begin{pmatrix} x_{N+1}^{n+1} - x_{N+1}^n \\ y_{N+1}^{n+1} - y_{N+1}^n \\ z_{N+1}^{n+1} - z_{N+1}^n \\ U_{N+1}^{n+1} - U_{N+1}^n \\ V_{N+1}^{n+1} - V_{N+1}^n \\ W_{N+1}^{n+1} - W_{N+1}^n \end{pmatrix}$$

$$B_{N+1} = \left(\begin{array}{c|c} -I & 0 \\ \hline 0 & -I \end{array} \right) = A_{N+1} \quad C_N = (0)$$

Transversality Conditions for Variable Low-Thrust Solar Probe

The components of the vehicle position and velocity are unspecified at $t = T$, hence the transversality condition (D-7) takes the form

$$-u\dot{\delta x} - v\dot{\delta y} - w\dot{\delta z} + u\dot{\delta x} + v\dot{\delta y} + w\dot{\delta z} \Big|_{t=T} = 0 \quad (D-15)$$

The magnitude of the radius vector to the vehicle at $t = T$ is constrained to satisfy Eq. (D-16)

$$x^2 + y^2 + z^2 - R_T^2 \Big|_{t=T} = 0 \quad (D-16)$$

Taking the variation of Eq. (D-16) and solving the resulting expression for δx , Eq. (D-17) is obtained

$$\delta x \Big|_{t=T} = - (y/x \delta y + z/x \delta z) \Big|_{t=T} \quad (D-17)$$

Substituting Eq. (D-17) into (D-15)

$$(-\dot{v} + \dot{u} y/x) \delta y + (-\dot{w} + \dot{u} z/x) \delta z + u\dot{\delta x} + v\dot{\delta y} + w\dot{\delta z} \Big|_{t=T} = 0 \quad (D-18)$$

The five variations appearing in Eq. (D-18) are independent; it follows that the conditions to be imposed upon the differential equations at $t = T$ are

$$\dot{v} - \dot{u} y/x = 0$$

$$\dot{w} - \dot{u} z/x = 0$$

$$u = v = w = 0$$

$$x^2 + y^2 + z^2 - R_T^2 = 0$$

(D-19)

The following alterations occur in the matrices associated with the algorithm.

$$C_N = \left(\begin{array}{c|c} 0 & -I \\ \hline 0 & 0 \end{array} \right)$$

$$B_{N+1} = \left(\begin{array}{ccc|ccc} & & & & & \\ & I & & & 0 & \\ \hline U_N^n & -X_N^n & 0 & -V_{N+1}^n + V_{N-1}^n & U_{N+1}^n - U_{N-1}^n & 0 \\ Z_N^n & 0 & -X_N^n & -W_{N+1}^n + W_{N-1}^n & 0 & U_{N+1}^n - U_{N-1}^n \\ 0 & 0 & 0 & -2X_N^n & -2Y_N^n & -2Z_N^n \end{array} \right)$$

$$P_{N+1} = \left(\begin{array}{l} (V_{N+1}^n - V_{N-1}^n) X_N^n - (U_{N+1}^n - U_{N-1}^n) Y_N^n \\ (W_{N+1}^n - W_{N-1}^n) X_N^n - (U_{N+1}^n - U_{N-1}^n) Z_N^n \\ (X_N^n)^2 + (Y_N^n)^2 + (Z_N^n)^2 - R_T^2 \\ U_{N+1}^n + U_{N-1}^n \\ V_{N+1}^n + V_{N-1}^n \\ W_{N+1}^n + W_{N-1}^n \end{array} \right)$$

$$D_{N+1} = \begin{pmatrix} x_N^{n+1} - x_N^n \\ y_N^{n+1} - y_N^n \\ z_N^{n+1} - z_N^n \\ u_{N+1}^{n+1} - u_{N+1}^n \\ v_{N+1}^{n+1} - v_{N+1}^n \\ w_{N+1}^{n+1} - w_{N+1}^n \end{pmatrix} \quad A_{N+1} = \begin{pmatrix} 0 & -I \\ \hline 0 & -y_N^n & x_N^n & 0 \\ & -z_N^n & 0 & x_N^n \\ & 0 & 0 & 0 \end{pmatrix}$$

Constant Thrust with Coast

Optimal Control Analysis

In three dimensions, the classical methods of the calculus of variations are applied to the minimization of the integral

$$J = \int_0^T a^2 dt \quad (D-20)$$

Subject to the constraints

$$z_1 = \dot{q} - a_x + \frac{x}{R^3} = 0$$

$$z_2 = \dot{r} - a_y + \frac{y}{R^3} = 0$$

$$z_3 = \dot{s} - a_z + \frac{z}{R^3} = 0$$

$$z_4 = \dot{x} - q = 0$$

$$z_5 = \dot{y} - r = 0$$

$$z_6 = \dot{z} - s = 0$$

(D-21)

where x, y, z are the position coordinates of the vehicle, q, r, s are the components of velocity, a_x, a_y, a_z are the components of the thrust acceleration, i.e., $a_x^2 + a_y^2 + a_z^2 = a^2$, and $R^2 = x^2 + y^2 + z^2$. By virtue of the fact that constant I_{sp} and constant exhaust power characterize the powerplant, the magnitude of the thrust acceleration is given by

$$|a(t)| = \frac{\dot{m}c\alpha_p}{1-\dot{m}t} \quad (D-22)$$

where

$$\alpha_p = \begin{cases} 1, & \text{during thrust} \\ 0, & \text{during coast,} \end{cases}$$

c is the velocity of the exhaust jet relative to the vehicle and $\dot{m}$ is the rate at which the propellant leaves the vehicle, normalized with respect to the vehicle mass at time $t = 0$. Defining $l_i, i = 1, 2, 3$ as the direction cosines of the thrust acceleration vector with respect to the x, y, z directions, respectively, the components of the thrust acceleration are given by:

$$\begin{aligned} a_x &= \frac{\dot{m}c\alpha_p}{1-\dot{m}t} l_1 \\ a_y &= \frac{\dot{m}c\alpha_p}{1-\dot{m}t} l_2 \\ a_z &= \frac{\dot{m}c\alpha_p}{1-\dot{m}t} l_3 \end{aligned} \quad (D-23)$$

Defining the augmented function

$$F = a^2 + \sum_{i=1}^6 \lambda_i z_i(\dot{X}, X, t) \quad (D-24)$$

And proceeding as above, we can write the Euler-Lagrange Necessary conditions and the restated equations of motion:

$$\ddot{x} - f_1 = \ddot{x} - \frac{\dot{m}c\alpha_p}{1-\dot{m}t} \frac{u}{p} + \frac{x}{R^3} = 0$$

$$\ddot{y} - f_2 = \ddot{y} - \frac{\dot{m}c\alpha_p}{1-\dot{m}t} \frac{v}{p} + \frac{y}{R^3} = 0$$

$$\ddot{z} - f_3 = \ddot{z} - \frac{\dot{m}c\alpha_p}{1-\dot{m}t} \frac{w}{p} + \frac{z}{R^3} = 0$$

$$\ddot{u} - g_1 = \ddot{u} - \frac{u(2x^2 - y^2 - z^2) + 3x(vy + wz)}{R^5} = 0$$

$$\ddot{v} - g_2 = \ddot{v} - \frac{v(2y^2 - x^2 - z^2) + 3y(wz + ux)}{R^5} = 0 \quad (D-25)$$

$$\ddot{w} - g_3 = \ddot{w} - \frac{w(2z^2 - x^2 - y^2) + 3z(ux + vy)}{R^5} = 0$$

where $l_i = \lambda_i / (\lambda_1^2 + \lambda_2^2 + \lambda_3^2)^{1/2}$, $i = 1, 2, 3$ and $\lambda_1 \equiv u$, $\lambda_2 \equiv v$, $\lambda_3 \equiv w$ and $p = (\lambda_1^2 + \lambda_2^2 + \lambda_3^2)^{1/2}$

Since Eqs. (D-25) are homogeneous in the adjoint variables, u , v and w , they may be scaled by the expression

$$[p(0)]^2 = 1$$

or

$$[u(0)]^2 + [v(0)]^2 + [w(0)]^2 = 1 \quad (D-26)$$

to eliminate arbitrariness in the solution.

As set forth in Ref. D-2, satisfaction of the Weierstrass necessary condition leads to the definition of a switching function, $k(t)$, characterized by

- a) $k(t) \geq 0$ during thrusting
- b) $k(t) \leq 0$ during coasting
- c) $\dot{k}(t) = \frac{c}{\mu(t)} \dot{p}(t)$ throughout the interval $0 \leq t \leq T$,

where $\mu(t)$ is the vehicle mass at time, t , normalized with respect to vehicle mass at time $t = 0$.

Conditions (a) and (b) indicate that at the thruster switch-off and switch-on times, t_1 and t_2 , respectively, $k(t)$ vanishes, i.e., $k(t_1) = k(t_2) = 0$. Hence, integration of condition (c) between the times t_1 and t_2 leads to the expression

$$[p(t_1)]^2 = [p(t_2)]^2$$

or

$$[u(t_1)]^2 + [v(t_1)]^2 + [w(t_1)]^2 = [u(t_2)]^2 + [v(t_2)]^2 + [w(t_2)]^2 \quad (D-27)$$

since $\mu(t) = \text{constant}$ between these limits.

Boundary conditions are given by matching the state and its derivative (velocity) to some specified orbit; additionally, a given excess speed may be optimally applied to the orbital velocity.

The formal modification of the algorithm can now be set forth. Setting $x = x_1$, $y = x_2$, $z = x_3$, $u = \lambda_1$, $v = \lambda_2$, $w = \lambda_3$, the problem is now discretized by using a fixed number of equally spaced mesh points in each of the three regions (thrust, coast, thrust). Thus t_1 is always associated with mesh point n_1 , t_2 with mesh point n_2 , and the total time, $T \equiv t_3$, with the final mesh point n_3 . Defining the initial time, t_0 , at mesh point $n_0 = 1$, the mesh spacing, h , in each region becomes

$$h_k = \frac{t_k - t_{k-1}}{n_k - n_{k-1}}, \quad k = 1, 2, 3. \quad (D-28)$$

At the interior points in each region difference equations for Eqs. (D-25) are derived by using the standard three-point formula.

$$x_1(t-h) - 2x_1(t) + x_1(t+h) = h^2 f_1 \quad (D-29a)$$

$$\lambda_1(t-h) - 2\lambda_1(t) + \lambda_1(t+h) = h^2 g_1 \quad (D-29b)$$

At the exterior boundary points, n_0 and n_3 , equations for the λ_1 must be written. By using (D-29a) and the difference equations for the velocity at the boundary,

$$-x_1(t-h) + x_1(t+h) = 2h\left(x_1 + \frac{V\lambda_1}{p}\right) = 2hb_1, \quad (D-30)$$

where V is the given excess speed, the value of x_1 which occurs outside the interval $[t_0, t_3]$ can be eliminated. (The sign of V is $+$ at t_0 and $-$ at t_3 .)

$$-x_1(t_0) + x_1(t_0+h_1) = h_1b_1 + \frac{h_1^2}{2}f_1 \quad (D-31a)$$

$$x_1(t_3-h_3) - x_1(t_3) = -h_3b_1 + \frac{h_3^2}{2}f_1. \quad (D-31b)$$

Similarly, at the switching points, n_1 and n_2 , equations for x_1 and λ_1 are written. The subscripts $-$ and $+$ will be used to denote the variables which occur before and after the switching point. Since (D-25₁), (D-25₂) and (D-25₃) are discontinuous at these points, (D-29a) is written for both f_{1-} and f_{1+} and combined with the equation of continuity for the velocity,

$$\frac{-x_1(t-h_-) + x_1(t+h_-)}{2h_-} = \frac{-x_1(t-h_+) + x_1(t+h_+)}{2h_+} \quad (D-32)$$

in order to eliminate the extraneous variables $x_1(t+h_-)$ and $x_1(t-h_+)$. Since (D-25₄), (D-25₅) and (D-25₆) are continuous, the equation for λ_1 may be written using a standard divided difference formula. The resulting equations are

$$h'x_1(t-h_-) - h''x_1(t) + x_1(t+h_+) = \frac{h_+}{2}(h_-f_{1-} + h_+f_{1+}) \quad (D-33)$$

$$h'\lambda_1(t-h_-) - h''\lambda_1(t) + \lambda_1(t+h_+) = \frac{h_+(h_-+h_+)}{2}g_1 \quad (D-33b)$$

where $h' = h_+/h_-$ and $h'' = (h_- + h_+)/h_-$.

Equations (D-26), (D-27), (D-29), (D-31), and (D-33) now represent $6n_3^3 - 4$ nonlinear algebraic equations in the unknowns

$$x_{ij}, 1 \leq i \leq 3, 2 \leq j \leq n_3 - 1;$$

$$\lambda_{ij}, 1 \leq i \leq 3, 1 \leq j \leq n_3;$$

$$t_1, \text{ and } t_2.$$

After these equations are appropriately ordered, a numerical solution may be determined by successively solving the linear systems defined by the generalized Newton-Raphson iteration. The method essentially follows the details laid down in Ref. D-1.

Optimal Propulsion Parameter Analysis

This analysis is carried out under the assumption that $J = \int_0^T a^2 dt$ and $\bar{a}$, the arithmetic mean thrust acceleration, are nearly invariant under variations of the powerplant fraction, μ_w , in the neighborhood of its optimal value. The rationale upon which these assumptions are based is discussed in detail in Ref. D-3 and that analysis is summarized and modified here.

The values of J and $\bar{a}$ are obtained from trajectories utilizing constant thrust with coast periods that are optimal in the sense that J is minimized with respect to the control, a slight deviation from the analysis of the above reference. Thus we know the values for

$$\bar{a} = \frac{a_0 + a_1}{2} \quad \text{and} \quad J = \int_0^T a^2 dt \quad (\text{D-34})$$

from a previous trajectory optimization. The first of Eqs. (D-34) may be written

$$\bar{a} = \frac{a_0}{2} \left(1 + \frac{a_1}{a_0} \right) \quad (\text{D-35})$$

Also we have

$$a(t) = \frac{2\mu_w \eta}{\alpha c \mu} = \frac{a_0}{\mu} \quad (\text{D-36})$$

where μ is the normalized mass of the vehicle as a function of time. The rocket equation gives

$$\frac{1}{\mu_1} = 1 + \frac{\alpha J}{2\mu_w \eta_1} \quad (D-37)$$

where μ_1 is the final total mass of the vehicle normalized with respect to the initial mass. Combining Eqs. (D-35), (D-36) and (D-37), we obtain Eq. (D-38):

$$\bar{a}\alpha c = 2\mu_w \eta + \frac{\alpha J}{2} \quad (D-38)$$

As done in Ref. D-3, setting the first variation of the payload fraction equal to zero we obtain Eq. (D-39)

$$\mu_w = \frac{\mu_1(1 - \mu_1)}{1 - \left(\frac{2\mu_1}{1+\mu_1}\right)\left(\frac{\eta' c}{\eta}\right)} \quad (D-39)$$

where $\eta' \equiv d\eta/dc$ and $\eta \equiv \frac{1}{1 + d^2/c^2}$, d being a parameter dependent upon the constant value of c .

Thus, Eqs. (D-38) and (D-39) are two equations in the unknowns μ_w and c . They can be numerically solved for the optimal values μ_w^* and c^* when α and η are specified. New values for the thrust magnitude and mass flow rate can then be computed from

$$\dot{m}c = \frac{a\mu_w^* \eta}{ac^*}, \quad \dot{m} = \frac{2\mu_w^* \eta}{\alpha(c^*)^2} \quad (D-40)$$

These values are used in the numerical solution for optimal control (modified Newton-Raphson algorithm) which yields new values for J and $\bar{a}$. This completes one iteration in the propulsion parameter optimization.

APPENDIX D REFERENCES

- D-1. Van Dine, C. P.: "Application of a Finite-Difference Newton-Raphson Algorithm to a Problem of Low-Thrust Trajectory Optimization." Vol. 17, Progress in Astronautics and Aeronautics, Methods of Astrodynamics and Celestial Mechanics. Edited by R. L. DunCombe and V. G. Szebehely, Academic Press, May 1966.
- D-2. Lawden, D. F.: Optimal Trajectories for Space Navigation, Butterworths, 1963.
- D-3. Melbourne, W. G. and C. G. Sauer, Jr.: "Payload Optimization for Power-Limited Vehicles", Vol. 9, Progress in Astronautics and Aeronautics, Academic Press, 1963. Also in JPL Space Programs Summary No. 37-17, Vol. IV, October, 1962; also JPL TR 32-250.

APPENDIX E

OPERATIONS IN A PLANET-MOON SYSTEM

Missions which are made to planetary moons involve additional complications. During the trajectory optimization for these missions, characteristic speeds are determined with respect to the parent bodies only. The mass computational program (MCP) considers arrival or departure from a specific target body in one coordinate reference system. Therefore, for planetary moons, when the method of patching at the moon's activity sphere is employed, the MCP is not designed to handle the change from the moon's referenced coordinates to planetocentric coordinates.

A method to circumvent this complication has been derived. Since arrival at or departure from a planetary moon involves only one impulse (in the moon parking orbit), a conversion from planet-centered coordinates to moon-centered coordinates is made.

For a given circular orbit of a specified moon (within the moon's activity sphere), a tangential impulse is applied to the vehicle which places it on an escape trajectory. The corresponding hyperbolic excess speed is calculated. When the vehicle arrives at the moon's activity sphere, it is assumed that its planetocentric velocity is parallel to the moon's orbit. The vehicle's planetocentric velocity may be readily calculated. Once the vehicle's planetocentric velocity is determined, its planetocentric flight path angle and distance from the parent planet may be found. With the knowledge of the above three variables, the planetocentric trajectory of the vehicle may now be determined. If this trajectory is hyperbolic the vehicle's hyperbolic excess speed with respect to the parent planet is known. Now, the correspondence between the planetocentric hyperbolic excess speed (calculated from trajectory analysis) and the moon's hyperbolic excess speed may now be made (Figs. E-1 and E-2) for a specified moon orbit and tangential impulse.

Reversing the above procedure, for a specified planetocentric hyperbolic excess speed and moon circular parking orbit, the associated moon's characteristic speed is known and, hence, the corresponding hyperbolic excess speed. Therefore, in terms of the MCP, the necessity of the inclusion of the parent planet for lunar arrival or escape is uncoupled and eliminated from the analysis. The significance of the parent planet, i.e., the planetocentric hyperbolic excess speed of the vehicle, is employed to determine the equivalent moon's hyperbolic excess speed external to the MCP.

The only necessary variables for computation within the MCP are the circular speed of the parking orbit about the moon and the moon-centered hyperbolic excess speed. The analytical development of the problem is presented below.

Analysis

Small letters refer to the moon's (m) and planet's (p) coordinate systems.

Figure E-3 depicts the geometry of the lunar escape.

The following constants are assumed to be known:

- K_m Moon's gravitational parameter
- K_p Planet's gravitational parameter
- r Circular parking orbit radius about the moon
- r^* Moon's activity sphere radius ($r^* (m_m/m_p)^{2/5}$)
- R^1 Moon's radial distance from the parent planet

It is assumed that departure from circular orbit about the moon is tangential, hyperbolic with respect to the moon, and in the plane of the circular parking orbit about the moon.

Now,

$$V_{c_r} = (K_m/r)^{1/2} \quad (E-1)$$

Choose a ΔV_m ...

$$V_{\infty_m}^2 = V_{c_r}^2 [(\Delta V_m/V_{c_r} + 1)^2 - 2] \quad (E-2)$$

and

$$V_{\infty_m}^2 = K_m/a_m \quad (E-3)$$

Since the departure is tangential:

$$r = a_m(e_m - 1) \quad (E-4)$$

and

$$\cos \eta^* = \frac{1}{e_m} \left[\frac{a_m(e_m^2 - 1)}{r^*} - 1 \right] \quad (E-5)$$

where,

$$\cos \eta^* = \begin{array}{l} (+) \text{ first quadrant} \\ (-) \text{ second quadrant} \end{array} \quad (\text{E-6})$$

and η^* , the true anomaly at the activity sphere, is known.

Assume that V^* is parallel to V_{c_m} at r^* , where

$$V_{c_m} = (K_p/R')^{1/2} \quad (\text{E-7})$$

and

$$\theta^* = \eta^* + \psi - \pi/2 \quad (\text{E-8})$$

Now,

$$\cos \theta^* = \frac{[K_m a_m (e_m - 1)]^{1/2}}{r^* V^*} \quad (\text{E-9})$$

where

$$V^{*2} = \frac{2K_m}{r^*} + V_{\infty m}^2 \quad (\text{E-10})$$

therefore, θ^* is known.

But,

$$\psi = \theta^* - \eta + \pi/2$$

therefore ψ is known. Now in planet coordinates,

$$V^* = v^* + V_{c_m} \quad (\text{E-11})$$

From the law of cosines,

$$R^2 = r^{*2} + R'^2 - 2r^*R' \cos (3\pi/2 - \eta^* - \psi) \quad (\text{E-12})$$

now

$$V^{*2} - 2K_p/R = K_p/A \quad (\text{E-13})$$

If K_p/A is (+), $V_{\infty p} = K_p/A$ and A and $V_{\infty p}$ are known.

If K_p/A is (-), we must return and increase Δv_m .

Assuming K_p/A is (+):

$$\cos v = \frac{R^2 + R'^2 - r^{*2}}{2RR'} \quad (E-14)$$

and

$$\alpha = \psi + \eta^* - v - \pi/2 \quad (E-15)$$

From Fig. E-4

$$\theta_p = \theta^* - \alpha \quad (E-16)$$

and

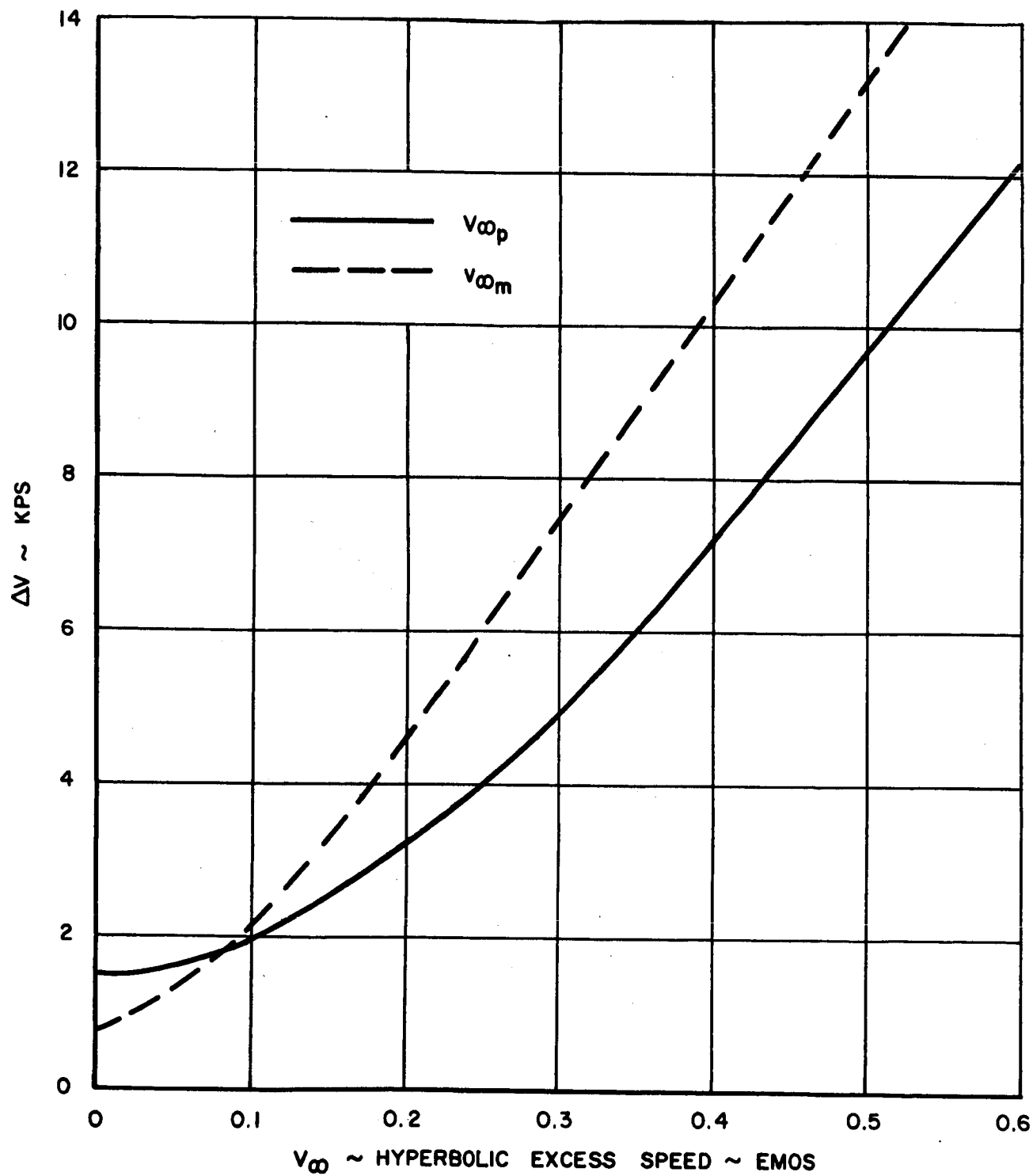
$$E_p = \left(1 + \frac{R^2 V^{*2} \cos^2 \theta_p}{K_p A_p} \right)^{1/2} \quad (E-17)$$

and

$$\cos \eta = \frac{1}{E_p} \left[\frac{A_p (E_p^2 - 1)}{R} - 1 \right] \quad (E-18)$$

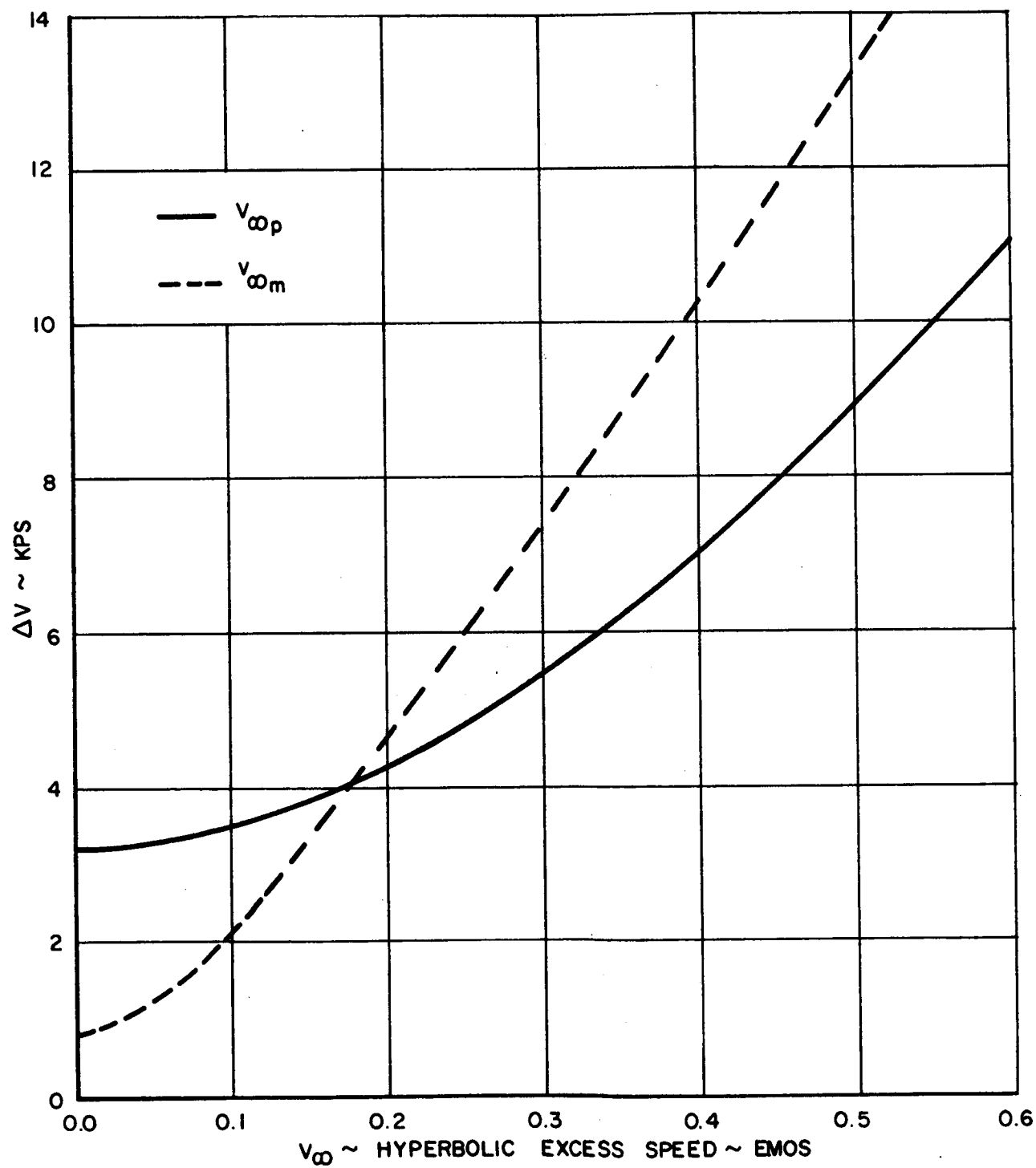
VELOCITY CONVERSIONS

SATURN-TITAN SYSTEM

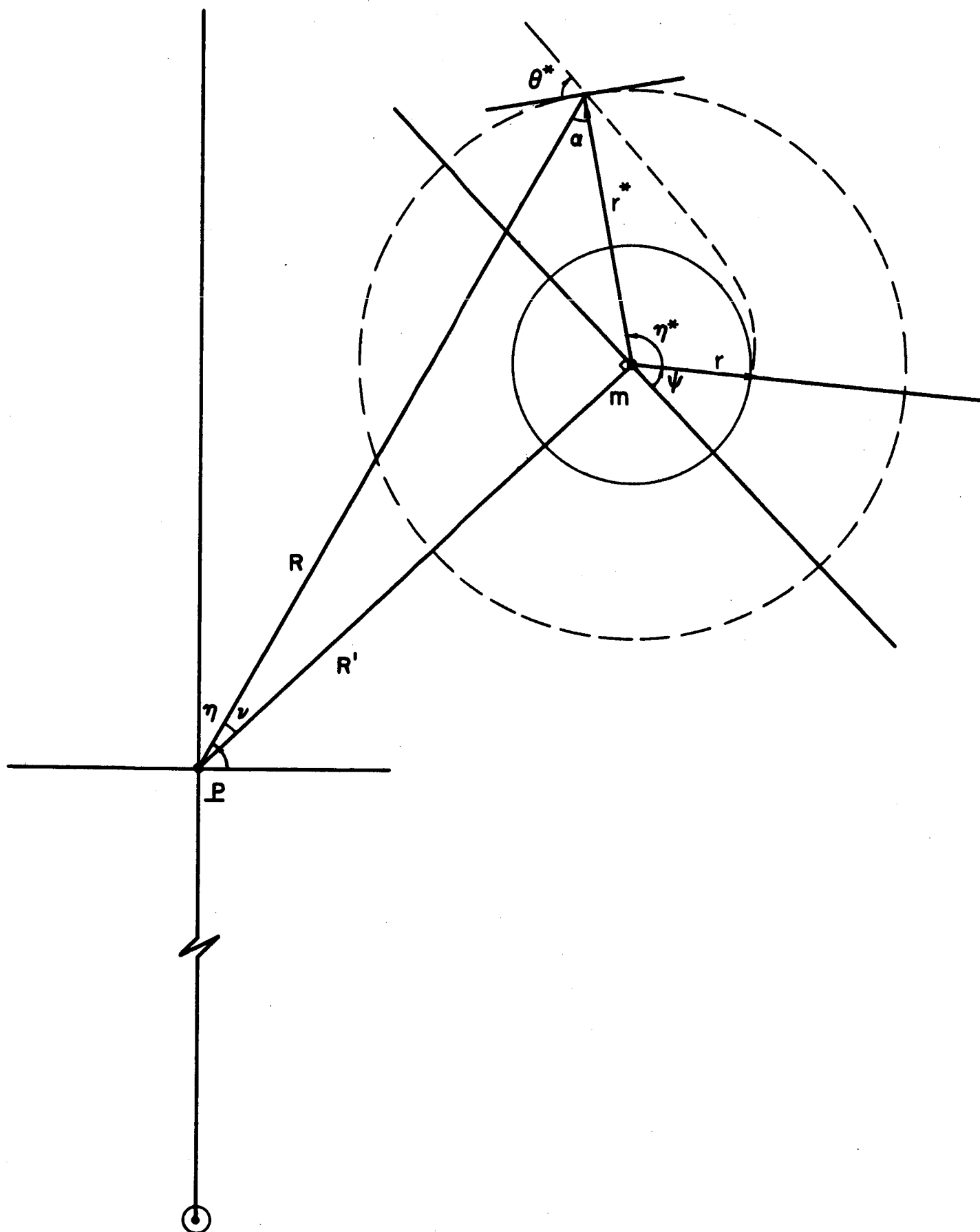


VELOCITY CONVERSIONS

JUPITER - GANYMEDE SYSTEM



GEOMETRY OF PLANET - MOON SYSTEM



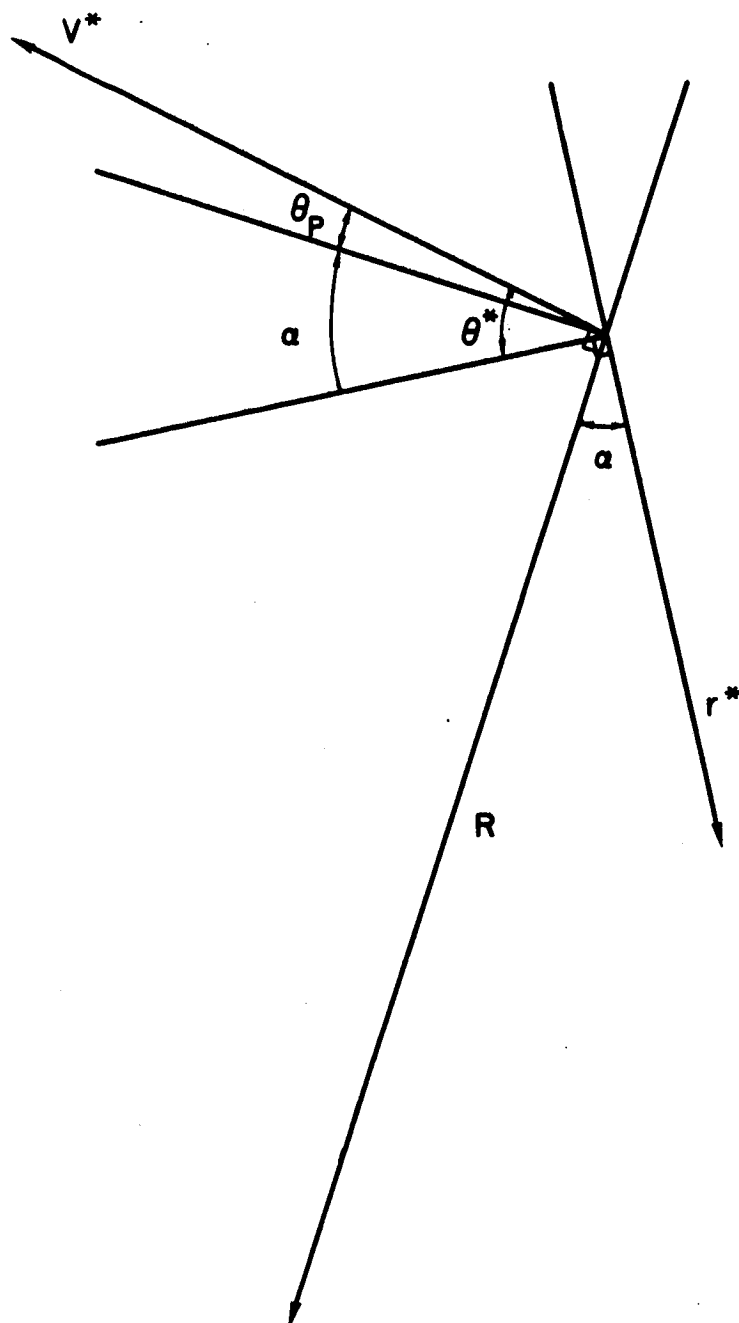
ANGULAR GEOMETRY AT THE SYSTEM
ACTIVITY SPHERE

TABLE E-I

Jupiter - Ganymede System

$$K_{\text{J}} = 1.0319 \times 10^4 \text{ km}^3/\text{sec}^2$$

$$K_{\text{p}} = 1.26498 \times 10^8 \text{ km}^3/\text{sec}^2$$

$$r^* = 2.47971 \times 10^4 \text{ km}$$

$$r = 2.6676 \times 10^3 \text{ km}$$

$$R' = 1.071 \times 10^6 \text{ km}$$

Saturn - Titan System

$$K_{\text{S}} = 9.390 \times 10^3 \text{ km}^3/\text{sec}^2$$

$$K_{\text{p}} = 3.78811 \times 10^7 \text{ km}^3/\text{sec}^2$$

$$r^* = 2.13952 \times 10^4 \text{ km}$$

$$r = 2.656 \times 10^3 \text{ km}$$

$$R' = 1.223 \times 10^6 \text{ km}$$

APPENDIX F

EXTREMIZATION OF PAYLOAD FRACTION FOR THE CASE OF CONSTANT THRUST,
CONSTANT POWER WITH OPTIMUM COAST AND SPECIFIED CONSTANT SPECIFIC IMPULSE

The payload fraction of a power-limited rocket operating at constant power is given by

$$\mu_{p1} = \mu_w \left[\frac{1}{\mu_w + \theta} - 1 \right] \quad (F-1)$$

where

μ_{p1} = payload fraction

μ_w = powerplant fraction

$$\theta = \frac{\alpha}{2} \int_0^T a^2 dt$$

with α = powerplant specific mass

$\vec{a}(t)$ = thrust acceleration

For the case of constant thrust and constant power, θ is considered to be a function of μ_w . Hence, maximizing μ_{p1} with respect to μ_w , $d\mu_{p1}/d\mu_w = 0$, we obtain

$$\frac{d\theta}{d\mu_w} + \theta \left(2 - \frac{1}{\mu_w} \right) + \frac{\theta^2}{\mu_w} = -\mu_w \quad (F-2)$$

This is Riccati's Equation and yields a solution of the form

$$\theta(\mu_w) = \frac{\mu_w [b(1-\mu_w)-1]}{1 + b\mu_w} \quad (F-3)$$

where b is the integration constant. Substituting Eq. (F-3) into (F-1) we obtain and evaluation of b ,

$$(\mu_{p1})_{\max} = \frac{1}{b} \quad (F-4)$$

Proceeding, $\theta(\mu_w)$ is minimized with respect to μ_w yielding

$$b = \frac{(1-2\mu_w) - \sqrt{1-4\mu_w}}{2\mu_w^2} \quad (F-5)$$

μ_w , as it appears in Eqs. (F-1), (F-2), (F-3), and (F-5), has still not attained its optimal value; these expressions hold for any value of μ_w . Consider these additional expressions for the payload and powerplant fractions, respectively:

$$\mu_{p1} = 1 - \dot{m}T_p - \mu_w \quad (F-6)$$

$$\mu_w = \frac{\dot{m}c^2}{2\eta} = v\dot{m} \quad (F-7)$$

where $T_p = T + t_1 - t_2$, the powered time (t_1 is engine cutoff time, t_2 is engine restart time); $\dot{m}$ and c are the mass flow rate and exhaust velocity, respectively, and η is the thruster efficiency.

Substituting Eqs. (F-4), (F-5), and (F-7) into (F-6) we obtain

$$\frac{2v^2\dot{m}^2}{1-2v\dot{m} - \sqrt{1-4v\dot{m}}} + \dot{m}(T_p + v) = 1 \quad (F-8)$$

Equation (F-8) provides the essential coupling of the equations of optimal motion and the propulsion parameters in the solution of this problem. As presented in Appendix D, the Euler-Lagrange necessary conditions and the equations of motion for the two-dimensional case are

$$\begin{aligned} \ddot{x} &= \frac{\dot{m}c}{1-\dot{m}t} \frac{u}{p} - \frac{x}{R^3}, & \ddot{u} &= \frac{(2x^2 - y^2)u + 3vxy}{R^5} \\ \ddot{y} &= \frac{\dot{m}c}{1-\dot{m}t} \frac{v}{p} - \frac{y}{R^3}, & \ddot{v} &= \frac{(2y^2 - x^2)v + 3uxy}{R^5} \end{aligned} \quad (F-9)$$

where $p = (u^2 + v^2)^{1/2}$.

These equations may be obtained also from an extremization of the integral

$$I = \int_0^T \left(\dot{m} + \frac{\mu_w}{T} \right) dt \quad (F-10)$$

where

$$\dot{m} = \begin{cases} A, & 0 \leq t \leq t_1, \quad t_2 \leq t \leq T \\ 0, & t_1 \leq t \leq t_2 \end{cases}$$

But, since $\dot{m}$, μ_w and T are constants this integral becomes

$$I = \int_0^{t_1} dt + \int_{t_2}^T dt = t_1 + T - t_2 = T_p \quad (F-11)$$

Hence, from a consideration of Eq. (F-6) we see that minimizing I maximizes the payload fraction with respect to the thrust control. Now Eqs. (F-9) are four expressions for determining the four functions $x(t)$, $y(t)$, $u(t)$ and $v(t)$; further, we have three unknown parameters $\dot{m}$, t_1 and t_2 . These can be determined by the three expressions

$$\begin{aligned} [u(0)]^2 + [v(0)]^2 &= 1 \\ [u(t_1)]^2 + [v(t_1)]^2 &= [u(t_2)]^2 + [v(t_2)]^2 \\ \frac{2v^2 \dot{m}^2}{1-2v\dot{m} - \sqrt{1-4v\dot{m}}} + \dot{m}(T + t_1 - t_2 + v) &= 1 \end{aligned} \quad (F-12)$$

It is noted that c and α/η are specified parameters during the solution.

The Newton-Raphson algorithm can solve the system of differential Eqs. (F-9) (in finite difference form) and the constraint Eqs. (F-12) for the optimal trajectory and mass flow rate.

The trajectory is optimal in this sense:

- a. the powered time is minimized
- b. the payload fraction is maximized with respect to powerplant fraction
- c. the mass of expellant in the exhaust jet is minimized with respect to powerplant fraction
- d. a. and c. infer that the mass flow rate, $\dot{m}$, hence the powerplant fraction $\mu_w = \alpha \dot{m} c^2 / 2\eta$, are simultaneously minimized.

It is noted that Eq. (F-3) may be obtained directly by solving Eq. (F-1) for θ :

$$\theta(\mu_w) = \frac{\mu_w}{\mu_{p1} + \mu_w} - \mu_w \quad (F-13)$$

If the analysis was then to proceed from Eq. (F-3) as it did above, the same governing equations [(F-9) and (F-12)] would be obtained. But the trajectory would no longer be formally optimal in the sense that the payload fraction is maximized with respect to the powerplant fraction. The implication is that minimizing the propellant expenditure with respect to the powerplant mass is tantamount to maximizing the payload fraction with respect to powerplant mass. In essence, what has been obtained is a definitive expression for the coupling between optimal control and optimal powerplant characteristics in the constant power, constant thrust with optimum coast trajectory mode.

The analysis must be flavored by those final remarks. The specific mass, α , the thruster efficiency, η , and the specific impulse are chosen at the outset of the analysis, hence the powerplant becomes a linear function of $\dot{m}$. It is felt that this approach to the problem of mission systems design is advantageous since the present state of powerplant technology is characterized by definite limits on these three parameters: α (lower limit), η (upper limit) and I_{sp} (upper limit).

Payload fraction has been maximized by minimizing the integral $I = \int_0^T a^2 dt$. This is not quite the same as the optimization process that occurred in the variable-thrust case. In the variable-thrust case I is minimized with respect to the thrust control as a function of time (calculus of variations); in the constant-thrust case I is similarly minimized with respect to the thrust control but is further minimized with respect to the powerplant fraction (theory of maxima and minima). This adjoining of the extremization of functions and functionals into a single operation, as the Newton-Raphson algorithm is capable of doing, is the means by which the simultaneous solution of optimal control and optimal powerplant parameters is obtained.

APPENDIX G

NUMERICAL PROCEDURE FOR HYBRID-THRUST OPTIMIZATION

The basic propulsion mixes and resulting payload ratio equations are given in Section V, Flight Profile Studies. As mentioned in Section V, the over-all plan of the hybrid-thrust optimization procedure is to determine by analytical means the idealized hyperbolic excess speeds which result in maximum payload ratios for a given propulsion system mix. These hyperbolic speeds and the corresponding value of the intervening low-thrust trajectory requirement (J), are then used in a mass computation procedure which accounts for velocity losses and the variation of high-thrust step inert mass fraction (structural factor).

The purpose of this appendix is to describe in further detail the actual numerical search procedure employed to solve for maximum payload ratio. Actually the method is applicable to other than the solution of maximization (or minimization) problems. The general technique was slightly revised for the purposes of the problem at hand.

Much of the basic theory and the development of the systematic search technique are contained in Ref. V-2. According to Ref. V-2, the direct search method has been found to be attractive for the following reasons:

1. No techniques of classical analysis are necessarily involved
2. Repeated arithmetic operations are used with simple logic
3. An approximate solution, improving continuously, is provided at all phases of the computation
4. Other classes of problems are readily attacked

Systematic Search Technique

The basic theory of the method is briefly summarized here for the sake of completeness. For an exhaustive treatment of the subject as well as a formalized definition of direct search the reader is referred to Ref. V-2.

The problem is to minimize a function $f(x_1, x_2, \dots, x_n)$. A solution vector or "point" P_i consists of n components $(x_{1i}, x_{2i}, \dots, x_{ni})$ which when compared to some other solution P_j is better if and only if

$$f(x_{1i}, x_{2i}, \dots, x_{ni}) < f(x_{1j}, x_{2j}, \dots, x_{nj})$$

A base point B_0 is determined from initial guesses of the values for the n components or coordinates. Using the strategy discussed below, an adjacent point P_1 is generated and compared to the base point B_0 . If P_1 is an improved solution compared to B_0 , then P_1 becomes the new base point B_1 , and the "move" which resulted in P_1 is termed a success. If P_1 is not better than B_0 , then the move was a failure. A success or failure in a move or step is judged solely by the above inequality.

The next trial point P_r is determined relative to B_r by the present state S_r . The states make up part of the logic, since they determine directions for moves in the solution space. They provide new directions if recent moves fail, and they decide when no further progress can be made.

The search procedure employs two types of moves - exploratory and pattern moves. Explorations in the n -coordinates are made to determine how the function $f(x_1, \dots, x_n)$ behaves in the neighborhood of the base point. The pattern move utilizes the behavioral information to provide a substantial reduction of the function.

The exploratory moves are made one coordinate at a time. Thus x_1 is varied by an increment $+\delta$ while $x_2, \dots, x_n$ remain fixed. This new vector $(x_1 + \delta, x_2, \dots, x_n)$ is tested against the base point $(x_1, \dots, x_n)$. If it is better, the new coordinate value is retained. If it is not, x_1 is varied by $-\delta$ while $x_2, \dots, x_n$ remains fixed. If this vector yields a smaller f , $x_1 - \delta$ is retained. If both $+$ and $-$ variations do not reduce f , then the original value, x_1 , is retained.

The entire procedure is repeated for the remaining coordinates x_2 through x_n . At the completion of the procedure, each coordinate will have associated with it a direction and a slightly reduced value for f if at least one variation succeeded. The set of directions is referred to as a pattern. Hence the pattern move consists of changing all the coordinates simultaneously in the indicated directions or patterns as obtained from the exploratory moves.

The new values of the coordinates after the pattern move form the new base point from which exploratory moves may be made as discussed above. Alternatively, the same pattern may be used repeatedly with a test for improvement in the value of the function made after each move. Each success updates the base point. In this approach, if a pattern move fails, exploratory moves are then made from the current base point. The present version of the computer program uses this approach. The justification for this approach is based on the fact that, for problems so far encountered, shorter machine times are realized.

If a combination pattern and exploratory move fails and if exploratory moves from the last base point fail, a decrease in the variation, or step size, δ , is required. The criterion for a final solution is when δ is reduced below some input tolerance, ϵ . Ideally, this final solution occurs

when the function is at a minimum or near-minimum solution. However, the fact that no further progress can be made beyond the tolerance ϵ does not always indicate that a solution has been found. As is characteristic of direct search methods, no sufficiency conditions are available for the success of the method. Thus, Hooke and Jeeves recommend the search technique for the following types of problems:

1. problems for which the answers may be tested,
2. problems consisting of many separate cases, a few of which can be checked by alternative means.

If (1) and (2) are not feasible, partial checks may be obtained by using the method several times, with different starting solutions.

An over-all view of the systematic search technique may be obtained from Figs. G-1, -2, and -3 which present the basic logic in flow chart form.

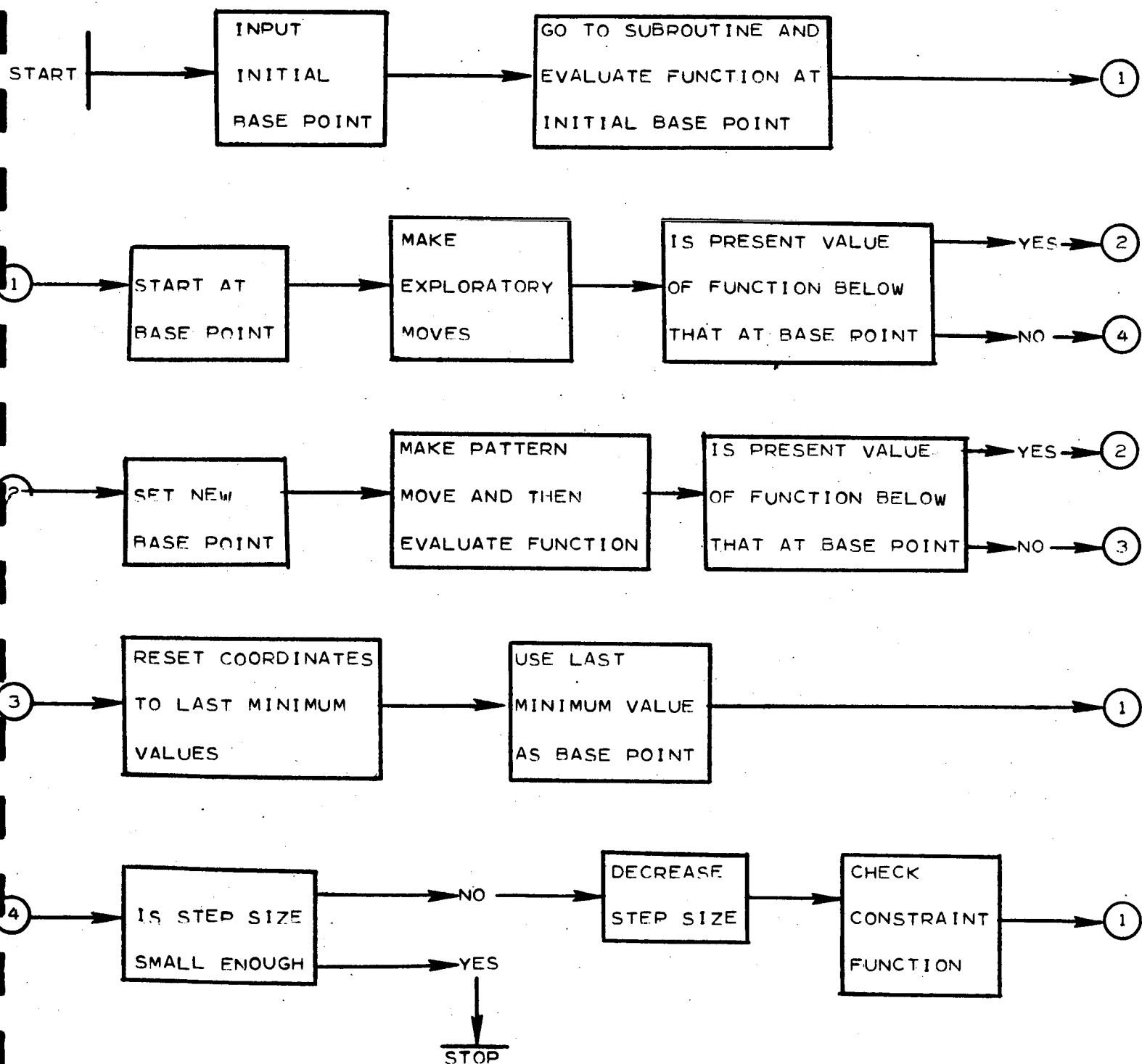
Application to Hybrid-Thrust Optimization

As noted above the procedure is based primarily on the minimization of a function. Since a maximization of the payload ratio function is required, the problem is reformulated whereby a minimum to the inverse function (i.e., $1/\mu$) is sought. The important function for the numerical procedure is $\Gamma(v_A, v_B)$ which in general is known only through the trajectory optimization program. Because certain fixed dates of departure and arrival were used which infer minimum vehicle mass from a leg-time distribution viewpoint, it was expedient to utilize a two-dimensional table whose entries are the Γ 's which correspond to the pair of normalized speeds (v_A, v_B) . Such a table was generated for each given set of dates and was used repeatedly for different propulsion parameters.

The method requires a starting guess for the variables (v_A, v_B) which can be easily given since these normalized variables range from 0 to 1. In practically all cases convergence is quite rapid and starting guesses far from the solution pose no problem. The major exception is the case of atmospheric Earth entry for the return leg. In those instances where the entry system mass changes slowly with entry speed the optimization procedure attempts to assign a value of 1.0 or greater to v_B . An automatic stop is written into the program such that if this does occur v_B is set equal to unity. The maximum allowable entry speed (20 km/sec) could be used as the normalizing parameter in v_B thereby indirectly imposing this restriction on the solution.

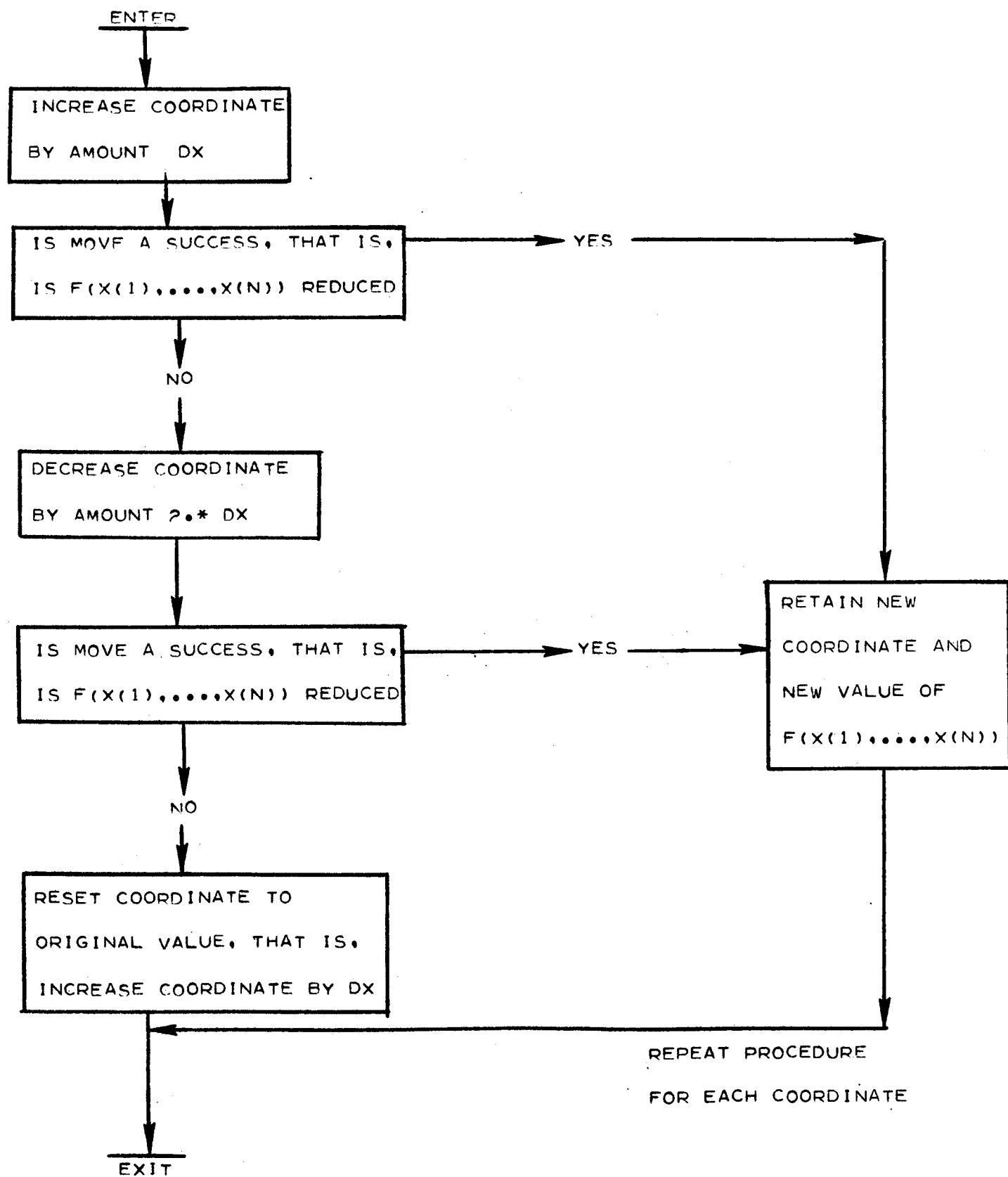
The main computer program which implements the basic logic given by the general flow charts is quite simple to write. The information presented in the flow charts is sufficient to code a program for a given problem or general use. Thus it was not deemed necessary to include a description of the computer program.

DECK F365 SEARCH PROCEDURE



FLOW CHARTS DESCRIBING
EXPLORATORY AND PATTERN
MOVES ARE DESCRIBED
ON THE FOLLOWING PAGES

EXPLORATORY MOVES



PATTERN MOVE

